Superposition and Dipole $\vec{E}$ field

PHYS 272 - David Blasing

Tuesday June 11th

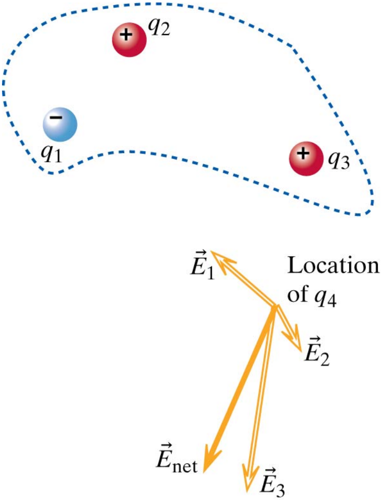
Principle of Superposition

Definition: Superposition

The net electric field at a position in space is the vector sum of every electric field made at that location by all the *other* charged particles around.

The electric field created by a charged particle is *not* affected by the presence of other charged particles or electric fields nearby.

Superposition Example

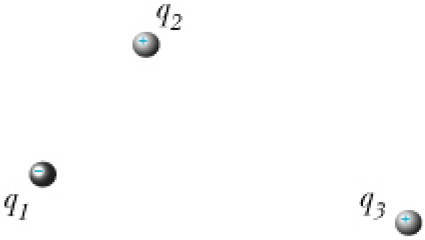


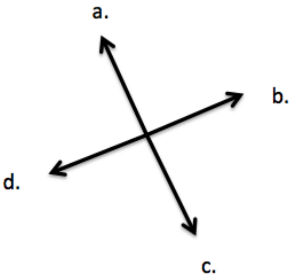
$\vec{E}_{net}$ is related to $\vec{F}_{net}$

- Once you have calculated the $\vec{E}_{net}$ due to all the other charges, then you can quickly calculate the force $\vec{F}_{net} = Q * \vec{E}_{net}$ on **any** amount of charge Q placed at that location

Clicker Question 1

What is the direction of $\vec{E}_{net}$ at the location of q_3 if $|q_2| \approx 2|q_1|$?





e. There is no electric field

Clicker Question 1

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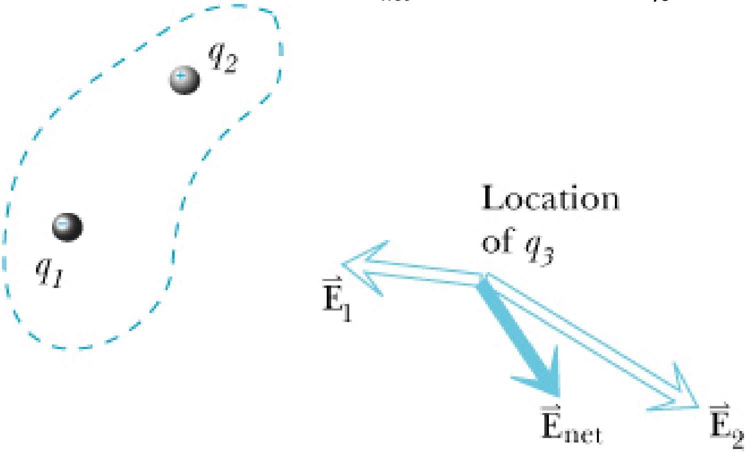
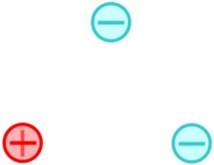


Clicker Question 2

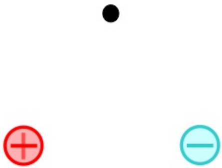
Tom places a negative charge at the top corner of an isosceles triangle to test the electric field produced by the +Q and –Q charges at the bottom of the triangle. What is the direction of the ***net force*** on the ***top*** negative charge?



A. Left.
B. Down.
C. Right.
D. Up.
E. The net force is zero

Clicker Question 3

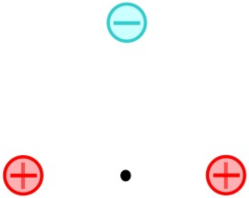
Now, Tom removes the test charge. What is the direction of the ***electric field*** at the previous point (top of triangle)?



A. Left.
B. Down.
C. Right.
D. Up.
E. The electric field is zero

Clicker Question 4

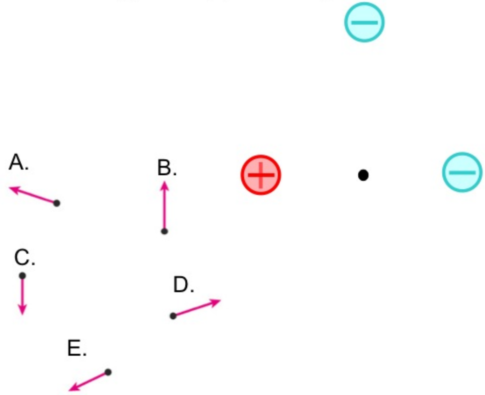
Tom never quits. He now wishes to find the direction of the electric field at the origin, as shown by the black dot. The ***electric field*** at the origin points



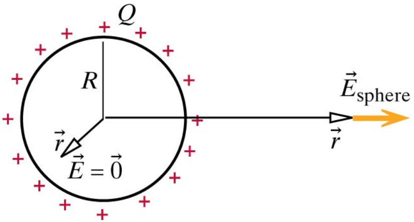
A. Left.
B. Down.
C. Right.
D. Up.
E. The net field is zero

Clicker Question 5

Now, Tom changes one of the positive charges on the bottom to negative, as shown below. At the position of the dot, the ***electric field*** points approximately



- Electric field of a uniformly charged sphere



- for $r > R$

$$\vec{E}_{sphere} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r}$$

- for $r < R$

$$\vec{E}_{sphere} = 0$$

$\vec{E}$ from a Uniformly Charged Sphere

1. Note: for $r > R$, $\vec{E}_{sphere} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r}$. Does this formula look familiar?

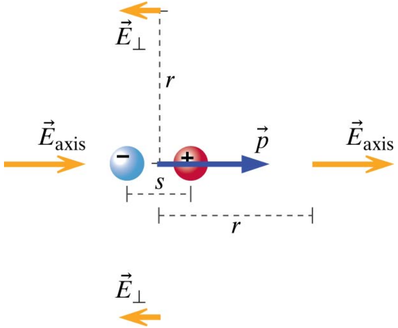
$\vec{E}$ from a Uniformly Charged Sphere

1. Note: for $r > R$, $\vec{E}_{sphere} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r}$. Does this formula look familiar?

2. A uniformly charged sphere at locations outside the sphere produces the same $\vec{E}$ field of a point charge with charge equal to the total charge on the sphere.

   The charged sphere responds to applied electric fields in the same way as a point charge at its center would

$\vec{E}$ field of a dipole and the electric dipole moment vector



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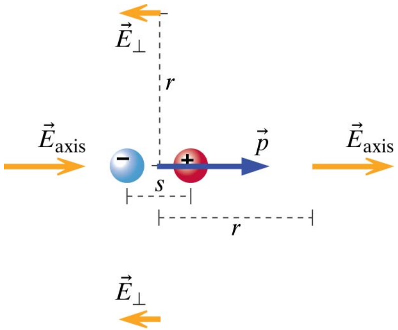
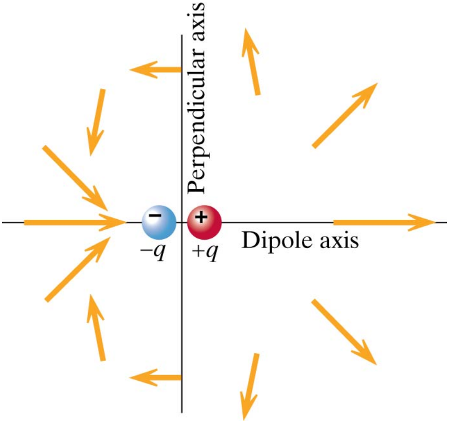


Definition: Dipole Moment Vector $\vec{p}$

$\vec{p} = q\vec{s}$ where q is the *magnitude* of both charges that make up a dipole, $\vec{s}$ is the position of the positive charge relative to the negative charge.

$\vec{E}$ field of a dipole at other locations



Part of one of your labs will recreate this plot.

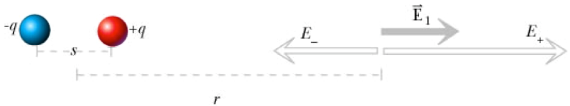
Parallel to the Axis

1. Now we are going to apply the principal of superposition to get the electric field of a dipole.
2. As a general hint, symmetry is your friend. Doing math is usually simpler when you somehow preserve or take advantage of a physical symmetry.

Parallel to the Axis

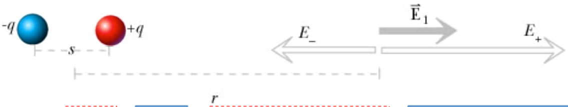
1. Now we are going to apply the principal of superposition to get the electric field of a dipole.
2. As a general hint, symmetry is your friend. Doing math is usually simpler when you somehow preserve or take advantage of a physical symmetry.
3. There are two lines of symmetry for a dipole, and we are going to derive simpler formulas along each of those lines.

Parallel to the Axis



$$E_{1,x} = \boxed{E_{+,x}} + \boxed{E_{-,x}} = \boxed{\frac{1}{4\pi\varepsilon_0}\frac{q}{(r-s/2)^2}} + \boxed{\frac{1}{4\pi\varepsilon_0}\frac{-q}{(r+s/2)^2}}$$

Parallel to the Axis



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$$E_{1,x} = \frac{1}{4\pi\varepsilon_0}\frac{qr^2 + qrs + qs^2/4 - qr^2 + qrs - qs^2/4}{(r-s/2)^2(r+s/2)^2}$$

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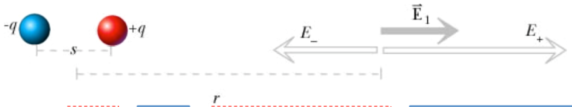

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Parallel to the Axis

$$E_{1,x} = \frac{1}{4\pi\varepsilon_0}\frac{2srq}{\left(r - \frac{s}{2}\right)^2\left(r + \frac{s}{2}\right)^2}$$

Parallel to the Axis

$$E_{1,x} = \frac{1}{4\pi\varepsilon_0}\frac{2srq}{\left(r-\frac{s}{2}\right)^2\left(r+\frac{s}{2}\right)^2}$$

if r>>s, then $\left(r-\frac{s}{2}\right)^2 \approx \left(r+\frac{s}{2}\right)^2 \approx r^2$

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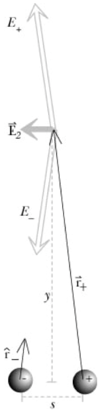
$$\text{if r>>s, then}\quad \left(r-\frac{s}{2}\right)^2 \approx \left(r+\frac{s}{2}\right)^2 \approx r^2$$

$$E_{1,x} = \frac{1}{4\pi\varepsilon_0}\frac{2sq}{r^3} \qquad \vec{E}_1 = \left\langle \frac{1}{4\pi\varepsilon_0}\frac{2sq}{r^3},0,0 \right\rangle$$

While the electric field of a point charge is proportional to $\frac{1}{r^2}$, electric fields created by several charges may have a different distance dependence.

Perpendicular to the Axis



$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

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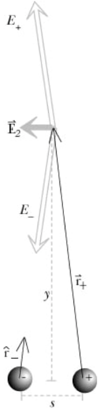

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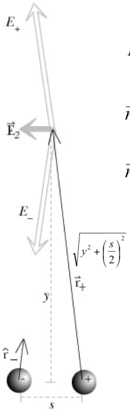
$$\vec{r}_+ = \langle 0, y, 0\rangle - \left\langle \frac{s}{2}, 0, 0\right\rangle = \left\langle -\frac{s}{2}, y, 0\right\rangle$$

$$\vec{r}_- = \langle 0, y, 0\rangle - \left\langle -\frac{s}{2}, 0, 0\right\rangle = \left\langle \frac{s}{2}, y, 0\right\rangle$$

Perpendicular to the Axis



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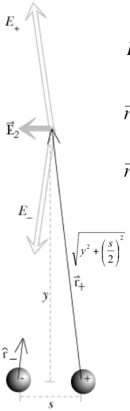

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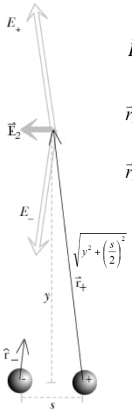

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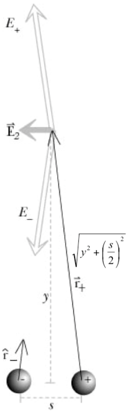

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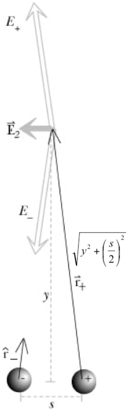

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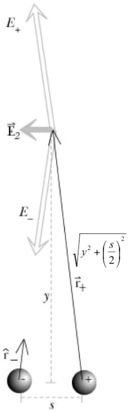
$$\vec{E}_2 = \vec{E}_+ + \vec{E}_- = \frac{1}{4\pi\varepsilon_0}\frac{q}{\left[y^2+\left(\frac{s}{2}\right)^2\right]^{\frac{3}{2}}}\langle -s,0,0\rangle$$

Perpendicular to the Axis



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$$\vec{E}_2 = \vec{E}_+ + \vec{E}_- = \frac{1}{4\pi\varepsilon_0}\frac{qs}{\left[y^2+\left(\frac{s}{2}\right)^2\right]^{\frac{3}{2}}}\langle -1,0,0\rangle$$

if y>>s, then $\vec{E}_2 \approx \left\langle \frac{-1}{4\pi\varepsilon_0}\frac{qs}{y^3}, 0, 0\right\rangle$ at <0,y,0>

Summary Dipole $\vec{E}$ Field

$\vec{E}$ Far From the Dipole

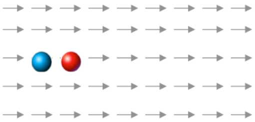
$$|\vec{E}_{perp}| = \frac{1}{4\pi\epsilon_0}\frac{p}{y^3} \qquad |\vec{E}_{onaxis}| = \frac{1}{4\pi\epsilon_0}\frac{2p}{r^3}$$

For the same distance r, $|\vec{E}_{onaxis}| = 2 * |\vec{E}_{perp}|$. At other locations you can still use superposition, it is just that the formula doesn't simplify as nicely.

Group Question 1

Dipole in a uniform $\vec{E}$ field, What is the net force?

$$\vec{F} = q\vec{E}$$



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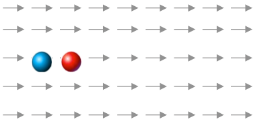

The force on the positive is equal in magnitude but opposite in direction of the force on the negative, so $\vec{F}_{net} = \vec{F}_{+} + \vec{F}_{-} = \vec{0}$

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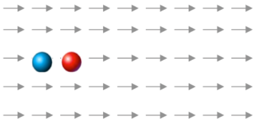

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So what could a dipole be used to measure?

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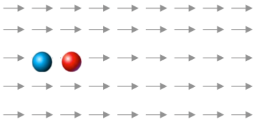
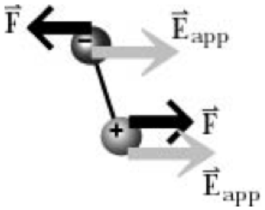


The force on the positive is equal in magnitude but opposite in direction of the force on the negative, so $\vec{F}_{net} = \vec{F}_{+} + \vec{F}_{-} = \vec{0}$

So what could a dipole be used to measure? A dipole would experience a force in a *non-uniform* $\vec{E}$ field. So they can measure the uniformity of the $\vec{E}$ field

Group Question 2

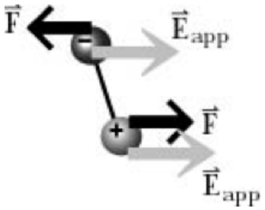
Dipole in a uniform $\vec{E}$ field



The $\vec{E}$ field results in a torque on the dipole, aligning $\vec{p}$ to $\vec{E}$. Given this, what additional piece of information might a dipole measure?

Group Question 2

Dipole in a uniform $\vec{E}$ field



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The direction of $\vec{E}$ (i.e. $\hat{E}$)