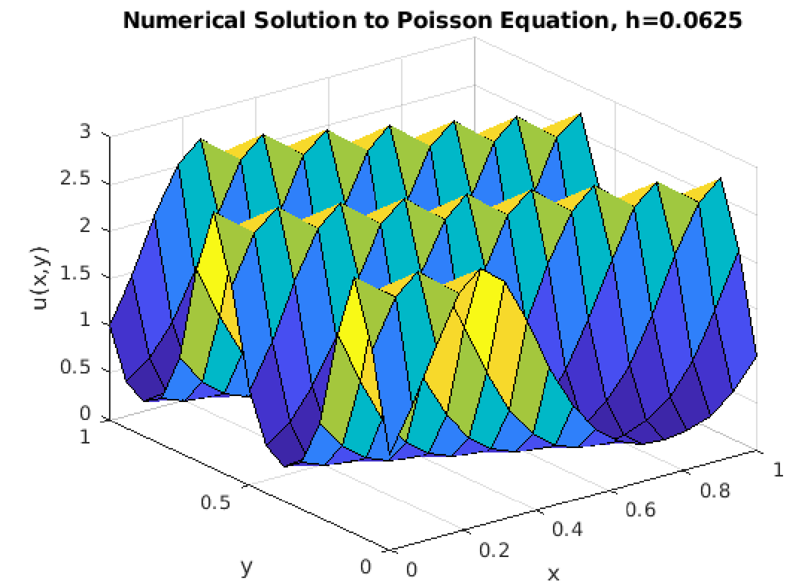
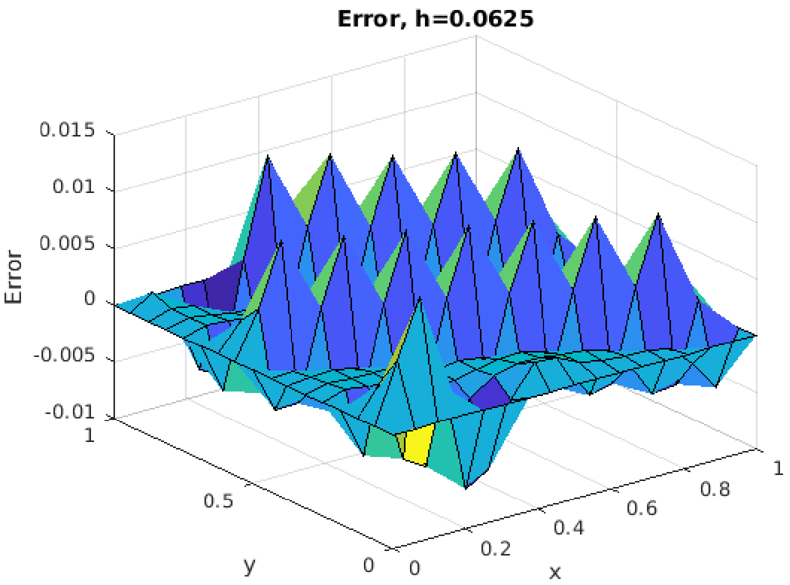
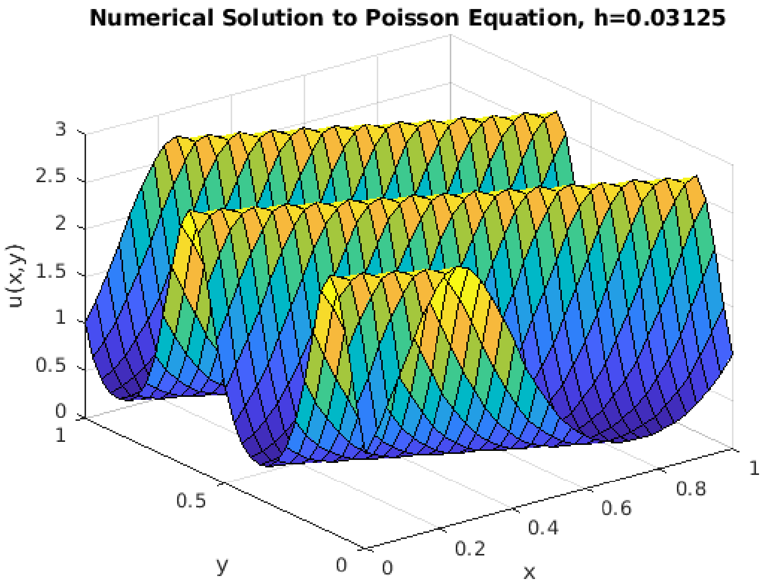
```
% Uses SOR function to to solve the poisson eqaution from problem 2 for
% various values of m and produce plots and tables that clearly show the
% forth order accuracy of the method.

a=0; b=1;

% Laplacian(u) = f
f = @(x,y) 10*pi^2*(1+cos(4*pi*(x+2*y))-2*sin(2*pi*(x+2*y))).*exp(sin(2*pi*(x+2*y)));
% u = g on Boundary
g = @(x,y) exp(sin(2*pi*(x+2*y)));

%Table showing the forth order acuracy of the method.
k1 = zeros(4,1);
h1=zeros(4,1);
L2=zeros(4,1);
m1=zeros(4,1);

for k = 4:7
    k1(k-3) = k;
    m1(k-3) = (2^k) - 1;
    m = (2^k) - 1;
    h1(k-3) = (b-a)/(m+1);
    h = (b-a)/(m+1);

    w = 2/(1+sin(pi*h)); %optimal relaxation parameter

    [x,y] = meshgrid(a:h:b);

    %Numerical solution
    [u,x,y] = SOR(f,g,a,b,m,w);

    % Exact solution is g.
    uexact = @(x,y) g(x,y);

    %Error
    error = u -uexact(x,y);

    %Relative 2-norm
    L2(k-3) = R2Norm(error,uexact(x,y));

    % Plot solution
    figure, set(gcf,'DefaultAxesFontSize',10,'PaperPosition', [0 0 3.5 3.5]),
    surf(x,y,u), xlabel('x'), ylabel('y'), zlabel('u(x,y)'),
    title(strcat('Numerical Solution to Poisson Equation, h=',num2str(h)));

    % Plot error
    figure, set(gcf,'DefaultAxesFontSize',10,'PaperPosition', [0 0 3.5 3.5]),
    surf(x,y,u-uexact(x,y)),xlabel('x'),ylabel('y'), zlabel('Error'),
    title(strcat('Error, h=',num2str(h)));

end

%table
T = table(k1(:),m1(:),h1(:),L2(:), 'VariableNames',{'k','m','h','R2-norm'})

%polyfit
p=polyfit(log(circshift(h1,size(h1))),log(L2),1);
p
fprintf('Since the order of convergence,p, is 4.1172, which is approximately 4, \n hence the method is fourth order accurate.\n')

plot(h1,L2);
xlabel('h');
ylabel('R 2-norm');
title('A graph of h against R 2-norm');

function L2 = R2Norm(error, uexact)
    R = error .^2;
    u_ex = uexact.^2;
    L2 = sqrt(sum(R,'all')/sum(u_ex,'all'));
end
```

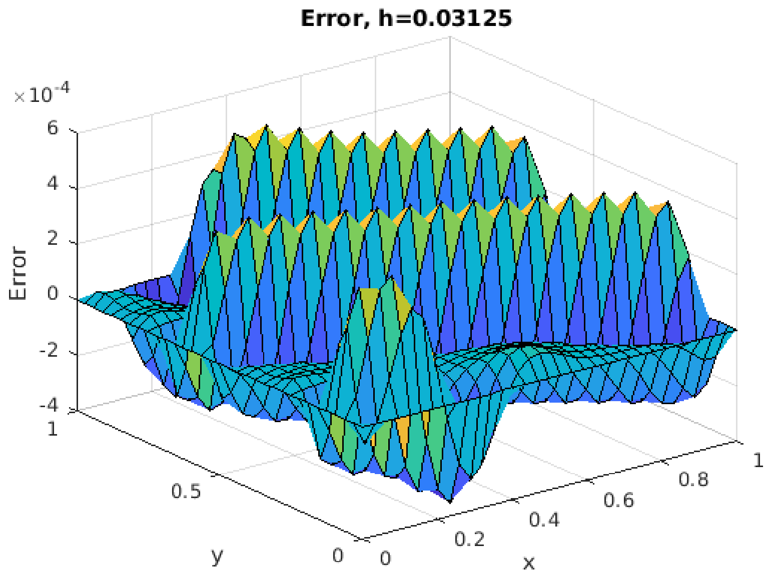
```
T =

  4×4 table

    k     m         h          R2-norm
    _    ___    _________    __________

    4     15       0.0625     0.0021715
    5     31      0.03125     0.0001109
    6     63     0.015625    6.6201e-06
    7    127    0.0078125    4.1065e-07
```

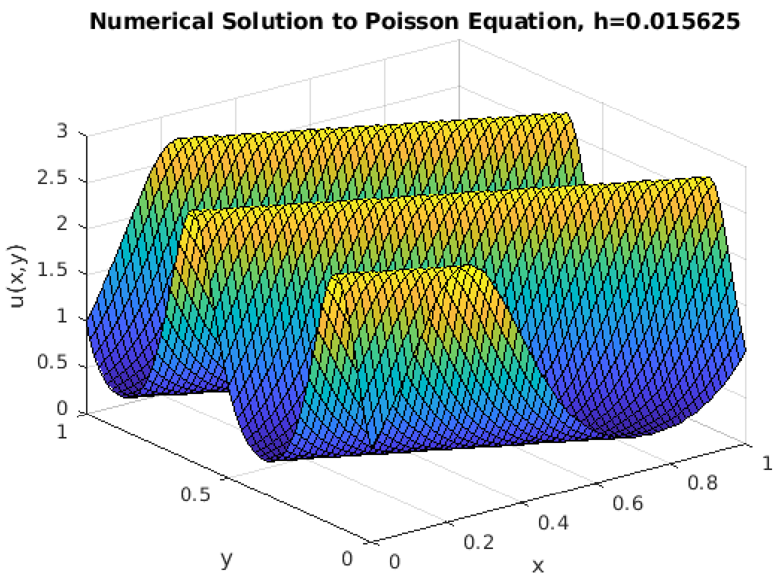
```
p =

    4.1172    5.2284

Since the order of convergence,p, is 4.1172, which is approximately 4,
 hence the method is fourth order accurate.
```

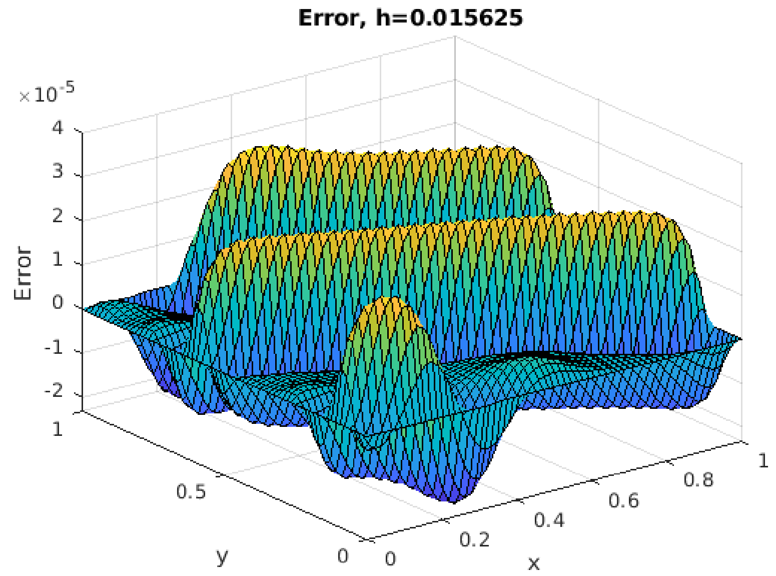


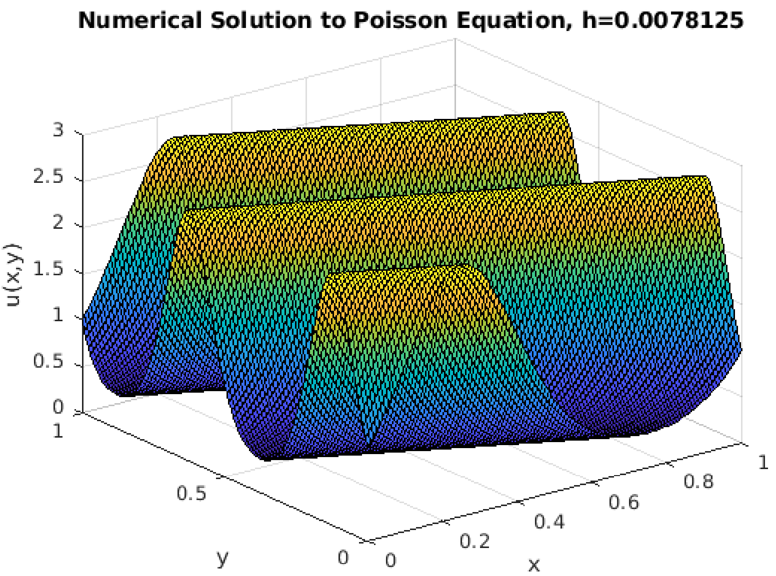


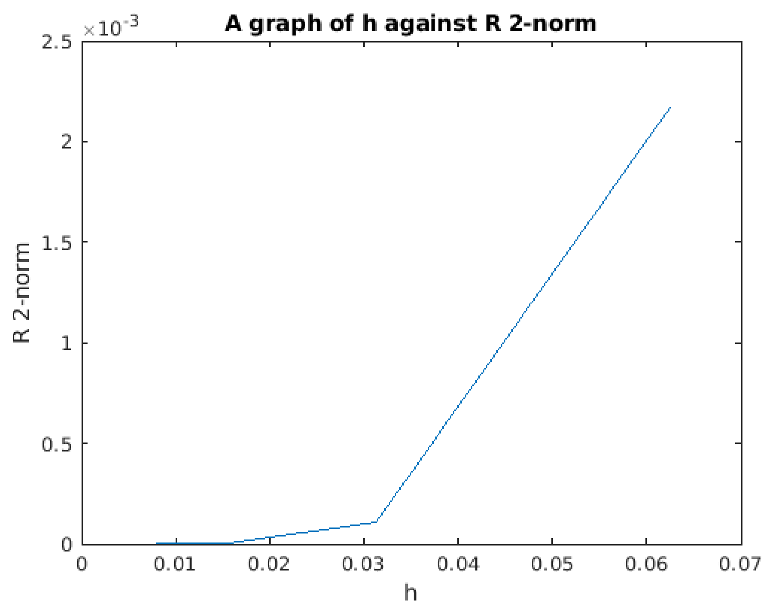

Numerical Solution to Poisson Equation, h=0.03125
u(x,y)
3
2.5
2
1.5
1
0.5
0
1
0.5
y
0
0
0.2
0.4
0.6
0.8
1
x


Error, h=0.03125
×10-4
6
4
2
0
-2
-4
Error
1
0.5
y
0
0
0.2
0.4
0.6
0.8
1
x


Numerical Solution to Poisson Equation, h=0.015625
u(x,y)
3
2.5
2
1.5
1
0.5
0
1
0.5
y
0
0
0.2
0.4
0.6
0.8
1
x







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