Math 566 1. (Another way to solve the laast Squares Let A be a read m-by-n matrix with m>, n and rank (A)=n, and b € Pm Of Show that the X that minimizes

11 AX-6112 is given by the Solution to The
Square Innear System. I A r b x b Guin A = I A D] The 2-by-2 block Granssten Eliminostran A A viril be RITAIBATE TAIB thin I A TY = b

O ATA X = ATb] Tr + Ax = b $A^TAX = A^Tb$ X = (ATA) AT b is the Solution that minimizes 1/ +x- blg.

Brian KYXNJO

Home work 4



thm Ir + Ax = b

rt Ax=b => r= b-Ax, which is the

b) Determine the 2-norm Conduter number of At in terms of the Singular Values of A Let A = UZ V be full SVD of A,

Consider $A = \begin{bmatrix} I & J \sum V^T J \\ V \sum^T U^T & D \end{bmatrix}$

At $\beta = \begin{bmatrix} 1 & 0 \\ 0 & V \end{bmatrix}$, β is orthogonal

Computing Eregen Values of A and relate there to emopular values of A

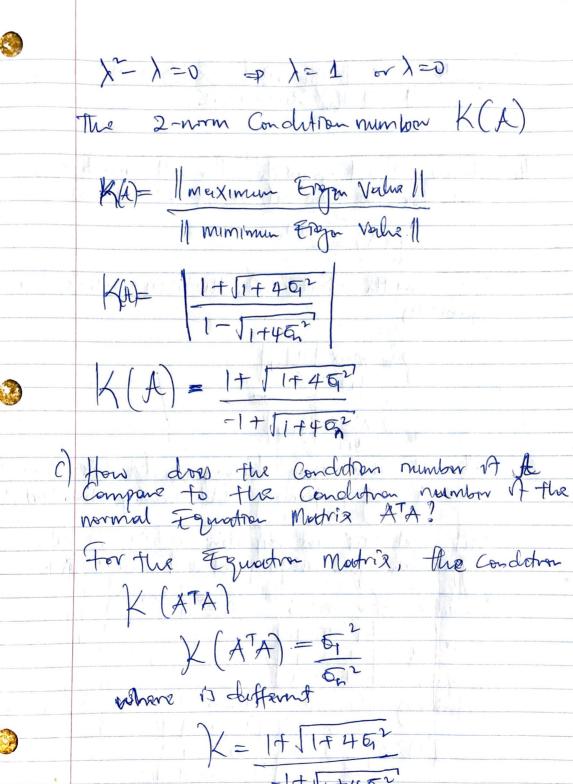
A- >I = 0

$$|I-\lambda I| \geq 0$$
 $|\Sigma^{T}-\lambda I| = 0$



Girm q 2x2 block matrix $M = \begin{pmatrix} A & B \\ C & B \end{pmatrix}$ where $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, $C \in \mathbb{R}^{m \times n}$ and, $A \in \mathbb{R}^{m \times m}$ If Dis montible then de det (M) = det (A-BD-C) det (D) Sunce (1-X)I E Rnxn ∑ e p^{nxm}, ∑ T∈ pp^{mxn} -XI E RMXM ad mundible then $|I-\lambda I| \sum_{n=1}^{\infty} dx + ((1-\lambda)I - \sum_{n=1}^{\infty} dx + (\lambda I) \sum_$ Since 5= IT $det ((1-x)I + \lambda^{-1} \Sigma^{2}) det (-\lambda I) = 0$ do+ (m) (λ(1-λ) I + Σ²)) de+ (-λI)=0 det (-XI) = (-X) $(-\lambda)^{m}(\lambda)^{-m}$ det $(\lambda(1-\lambda)I+Z^{2})=0$ (-1) det () (1-x) I + Z2) = 0 det $(\lambda(\lambda-1)I-\Sigma^2)=0$

$$\sum_{i=1}^{2} \frac{1}{2} = \frac{$$



A) Compose Atre Atre Money work work ACO Show thout growth for the mostrix A (but for the gleneral n-by-n case) is excutly $g(A) = 2^{n-1}$ The growth factor of the nxn matrix A is $p = \max_{\hat{c}_i, \hat{s}} | U_i \hat{s} |$ Max ais from A by parforming LU decomposition of A Ulis and Chis are respective Entries of the upper transpolar Matrix U and Motrix . Consider for n=3 A E P 3x3 $A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ -1 & -1 & 1 \end{bmatrix}$

The maximum Every in U is 4, and in A is 1 So the growth factor.

$$P = 4 = 2^{3-1} = 2^{m-1}$$

for $n = 4$, $A \in \mathbb{R}^{9 \times 4}$

max Ui =8, mox Ou =4 9 = 2 = 2ⁿ⁻¹

max (Us) = 22-2 Max ais =1

 $\beta = \frac{2^{k-2}}{1} = \frac{2^{k-2}}{1} = \frac{2^{k-1}-1}{1}$ If it free for n=k-1, then the for n=k, then the maximum Entry, Max 7llis The maximum (Ui) = 2", and Max ais =1 P = Max Mil = 2*-1 Max (asi) There fore the growth factor for non Moutris House proved by Induction.

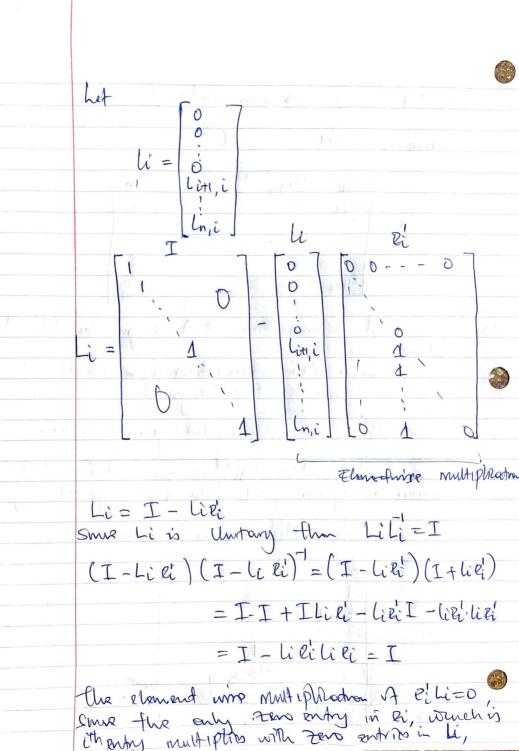
3 (Unit Lower trangela Madria) Girm a matrix $A \in \mathbb{R}^{m \times n}$

A = 912 92 = 920

The LU decomposition of A is given by LV=A L 11 a wint Land fribugular Matrix with

U is the Upper trangular Madria. - The Multiplication, Lie, accumulated into the Lower transgular part with a Change of Sign

Lil = gii , J= itt, ..., n



6.
Since non teno entries in Li, Stout at Ett, fort
This implies that for any i,
$\begin{array}{c} -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 $
Throfore mothemostically the mouth is frue