```
clear all
close all


%Rank-deficient problems and regularization
%Consider a 2pi periodic signal x(t) sampled at the N equally spaced points
%tj = hj

N = 256;
delta = 0.1;

j = [0:N-1]';
h = 2*pi/N;
tj = h*j;

%periodic version of the Gaussian function
k = @(t,delta) (1/(delta*sqrt(2*pi)))*exp(-(2-(2*cos(t)))/(2*delta^2));

aj = h*k(tj,delta);

fprintf('2a). Construct A and compute its condition number. \n\n');
```

```
2a). Construct A and compute its condition number.
```

```
%matrix A
A = circulant([aj]);

%condition number
kapa = cond(A);
fprintf('The condition number of A is: %e \n\n',kapa);
```

```
The condition number of A is: 1.144935e+16
```

```

fprintf('2b). Construct vector x. \n\n');
```

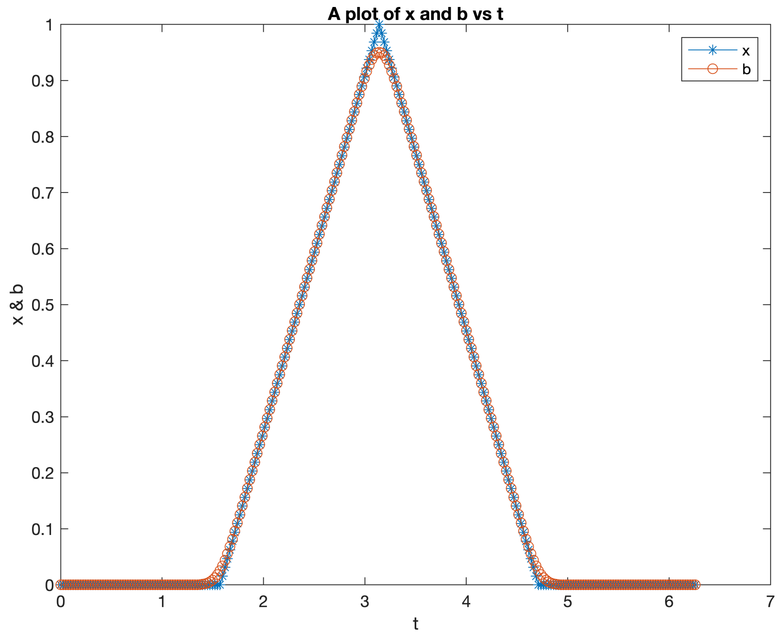
```
2b). Construct vector x.
```

```
%vector x
X = zeros(N,1);
for j = 1:N
    X(j) = signal(tj(j));
end

%Compute the blurred signal b
b = A*X;

%Make a plot of x vs t and b vs t
figure(1)
plot(tj,X,'-*')
hold on
plot(tj,b,'-o')
legend('x','b');
xlabel('t'); ylabel('x & b');
```

```
title('A plot of x and b vs t');
```



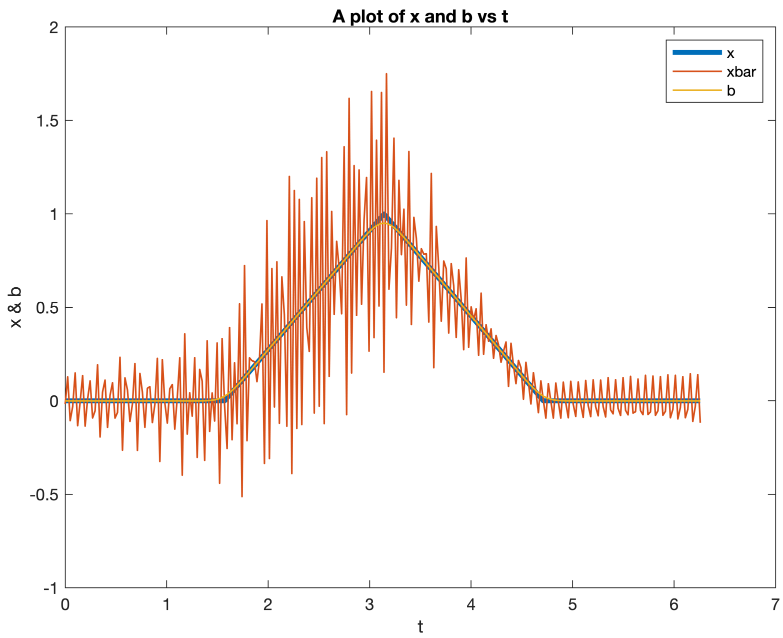

```
fprintf('2c). Solve Axbar = b  \n\n');
```

```
2c). Solve Axbar = b
```

```
xbar = A\b;
```

```
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
 6.559665e-17.
```

```
figure(2)
plot(tj,X,'linewidth',3)
hold on
plot(tj,xbar,'linewidth',1)
hold on
plot(tj,b,'linewidth',1)
legend('x','xbar','b');
xlabel('t'); ylabel('x & b');
title('A plot of x and b vs t');
```




```
fprintf('xbar doesnot look anything close to x, since A is ill conditioned, xbar has al
```

```
xbar doesnot look anything close to x, since A is ill conditioned, xbar has alot of noise.
```

```
fprintf('2d). Compute a reduced rank least squares solution  \n\n');
```

```
2d). Compute a reduced rank least squares solution
```

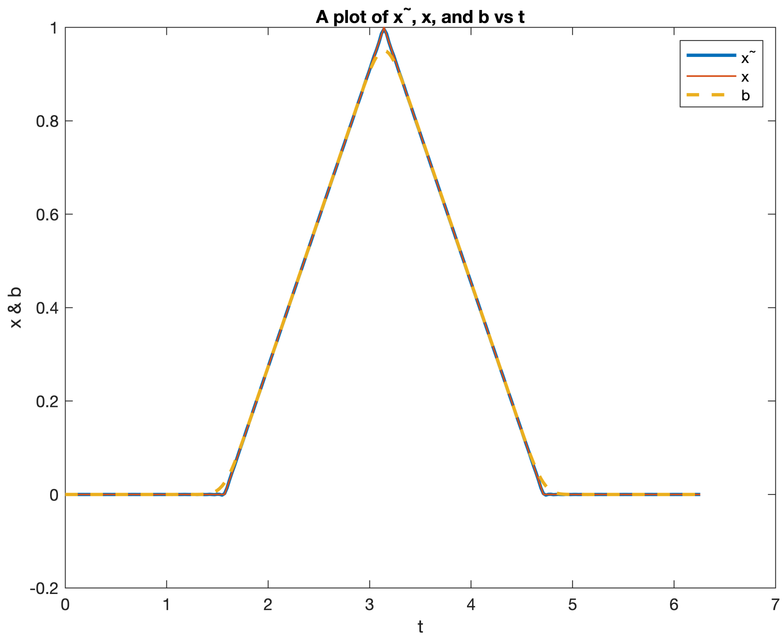
```
[u,s,v] = svd(A);

%sigular values
sigma = [];
U = [];
V = [];
for i = 1:N
   sig = s(i,i);
   if sig >= 1e-12
      sigma = [sigma,sig];
      U = [U,u(:,i)];
      V = [V,v(:,i)];
      continue
   end
end

%compute xtilda
xtilda = rls(sigma,U,V,b);

%plot
figure(3)
```

```
plot(tj,xtilda,'linewidth',2)
hold on
plot(tj,X,'linewidth',1)
hold on
plot(tj,b,'--','linewidth',2)
legend('x^{~}','x','b');
xlabel('t'); ylabel('x & b');
title('A plot of x^{~}, x, and b vs t');
```




```
fprintf('The reduced rank least squares solution ,xhat, fits the data much better than
```

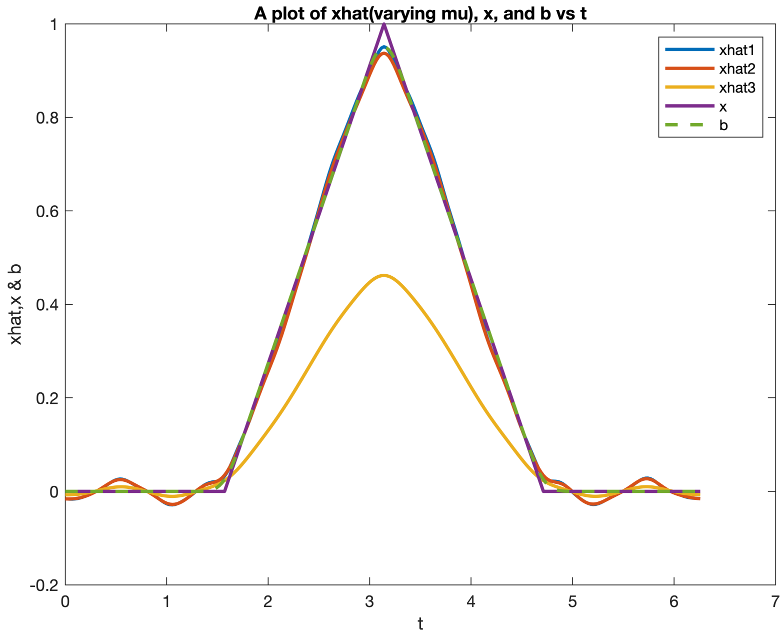
```
The reduced rank least squares solution ,xhat, fits the data much better than xbar.
```

```
fprintf('2e).Use ridge regression \n\n');
```

```
2e).Use ridge regression
```

```
%compute xhat
mu1 = 1e-4; mu2 = 1e-2; mu3 = 1;
xhat1 = ridge(mu1,s,u,v,b,N);
xhat2 = ridge(mu2,s,u,v,b,N);
xhat3 = ridge(mu3,s,u,v,b,N);
%plot
figure(4)
plot(tj,xhat1,'linewidth',2)
hold on
plot(tj,xhat2,'linewidth',2)
hold on
plot(tj,xhat3,'linewidth',2)
hold on
plot(tj,X,'linewidth',2)
```

```
hold on
plot(tj,b,'--','linewidth',2)
legend('xhat1','xhat2','xhat3','x','b');
xlabel('t'); ylabel('xhat,x & b');
title('A plot of xhat(varying mu), x, and b vs t');
```




```
fprintf('According to the plot above, the small the value of the regularization paramet
```

```
According to the plot above, the small the value of the regularization parameter the better the
 approximation, as its seen for mu = 1, the solution is completely off.
```

```
fprintf('Xhat perfomance interns of approximation depends on parameter, mu, so if we se
```

```
Xhat perfomance interns of approximation depends on parameter, mu, so if we select a good parameter,
 then it approximates better than the rest
```

```
fprintf('2f). Repeat (c) - (e) by perturbing each entry of b \n\n');
```

```
2f). Repeat (c) - (e) by perturbing each entry of b
```

```
b = b + 1e-5*randn(N,1);
fprintf('Repeated 2c). \n\n');
```

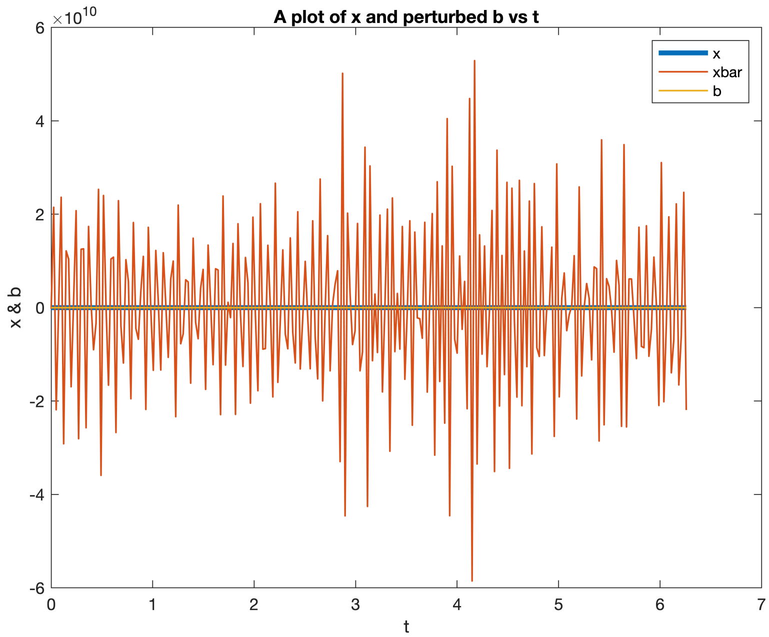
```
Repeated 2c).
```

```
xbar = A\b;
```

```
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
 6.559665e-17.
```

```
figure(5)
```

```
plot(tj,X,'linewidth',3)
hold on
plot(tj,xbar,'linewidth',1)
hold on
plot(tj,b,'linewidth',1)
legend('x','xbar','b');
xlabel('t'); ylabel('x & b');
title('A plot of x and perturbed b vs t');
```




```
fprintf('xbar depicts alot of noise compared to x and b, this is due to the pertubation
```

```
xbar depicts alot of noise compared to x and b, this is due to the pertubation caused at b
```

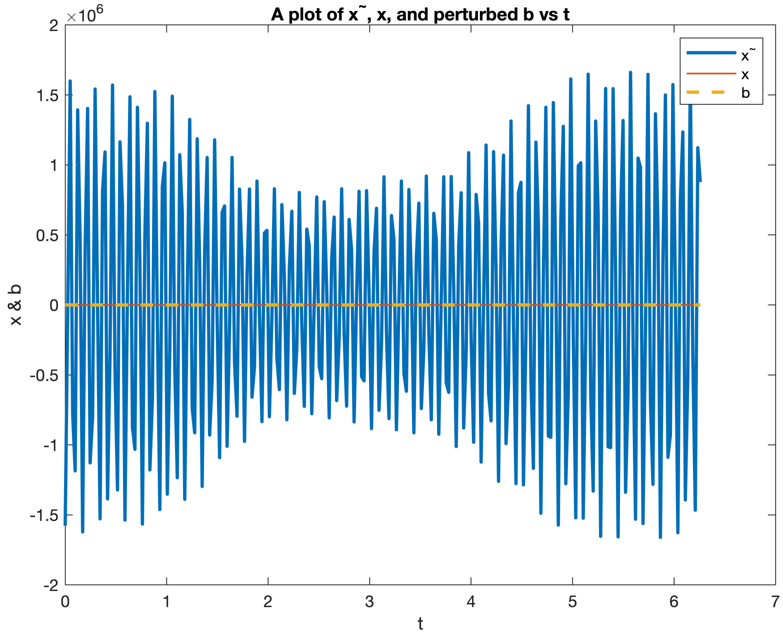
```
fprintf('Repeated 2d). Compute a reduced rank least squares solution  \n\n');
```

```
Repeated 2d). Compute a reduced rank least squares solution
```

```
b = A*X;
b = b + 1e-5*randn(N,1);
%compute xtilda
xtilda = rls(sigma,U,V,b);

%plot
figure(6)
plot(tj,xtilda,'linewidth',2)
hold on
plot(tj,X,'linewidth',1)
hold on
plot(tj,b,'--','linewidth',2)
legend('x^{~}','x','b');
xlabel('t'); ylabel('x & b');
```

```
title('A plot of x^{~}, x, and perturbed b vs t');
```




```
fprintf('The reduced rank least squares solution ,xhat, also exibits noise as xbar, but
```

```
The reduced rank least squares solution ,xhat, also exibits noise as xbar, but in large amplitudes.
```

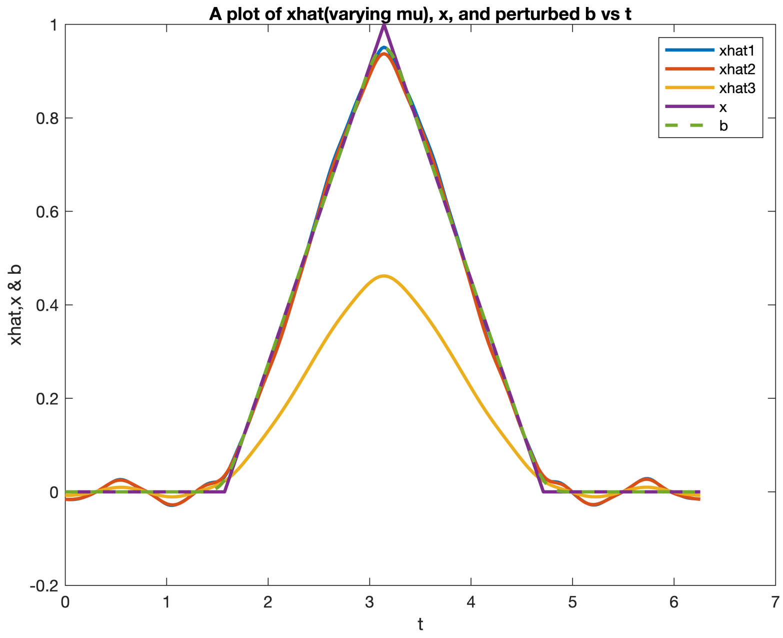
```
fprintf('Repeated 2e).Use ridge regression \n\n');
```

```
Repeated 2e).Use ridge regression
```

```
b = A*X;
b = b + 1e-5*randn(N,1);

%compute xhat
mu1 = 1e-4; mu2 = 1e-2; mu3 = 1;
xhat1 = ridge(mu1,s,u,v,b,N);
xhat2 = ridge(mu2,s,u,v,b,N);
xhat3 = ridge(mu3,s,u,v,b,N);
%plot
figure(7)
plot(tj,xhat1,'linewidth',2)
hold on
plot(tj,xhat2,'linewidth',2)
hold on
plot(tj,xhat3,'linewidth',2)
hold on
plot(tj,X,'linewidth',2)
hold on
plot(tj,b,'--','linewidth',2)
legend('xhat1','xhat2','xhat3','x','b');
xlabel('t'); ylabel('xhat,x & b');
```

```
title('A plot of xhat(varying mu), x, and perturbed b vs t');
```




```
fprintf('Pertubing b, does not affect xhat, due to obtaining the same plots, before and
```

```
Pertubing b, does not affect xhat, due to obtaining the same plots, before and after perturbing b
```

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fprintf('Xhat perfomance interns of approximation depends on parameter, mu, so if we se
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```
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 then it approximates better than the rest
```

```
%ridge regression solution
```

```
function xhat = ridge(mu,s,u,v,b,N)

    xhat = 0;
    for j = 1:N
        s = diag(s);
        xhat = xhat + ((s(j)./((s(j).^2)+mu)).*(u(:,j)'*b)).*v(:,j);
```

```
    end

end

%Reduced rank least squares solution
function xtilda = rls(sigma,U,V,b)
    r = length(sigma);
    xtilda = 0;
    for j = 1:r
        xtilda = xtilda + ((U(:,j)'*b)./sigma(j)).*V(:,j);
    end
end


%periodic signal
function [x] = signal(t)

    if abs(t-pi) < pi/2
        x = 1 - (2/pi)*abs(t-pi);
    else
        x = 0;
    end

end
```