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Homework #3

2. Weighted least squares.

a) Derive the normal Equations for Computing the solution x that minimizes $\|Ax-b\|_w$

$$\|y\|_w = \sqrt{y^T w y}$$

So,

$$\|Ax-b\|_w = \sqrt{(Ax-b)^T w (Ax-b)}$$

$$\|Ax-b\|_w^2 = (Ax-b)^T w (Ax-b)$$

$$\|Ax-b\|_w^2 = (x^TA^TwAx - x^TA^Twb - b^TwAx + b^Twb)$$

at minimum $\frac{d}{dx}\left(\|Ax-b\|_w^2\right) = 0$

$$\frac{d}{dx}\left(x^TA^TwAx - x^TA^Twb - b^TwAx + b^Twb\right) = 0$$

$$\frac{d}{dx}\left((A^TwAx)^Tx\right) + x^TA^T\frac{d}{dx}(wAx) - \frac{d}{dx}\left((A^Twb)^Tx\right)$$

$$-b^TwA = 0$$

$$x^TA^TwA + x^TA^TwA - b^Tw^TA - b^TwA = 0$$

$$2x^TA^TwA = 2b^Tw^TA \quad , \text{ since } w^T = w$$

Taking Transpose both sides.

$$(x^TA^TwA)^T = (b^TwA)^T$$

$$A^T W A X = A^T W b$$

3) Unstable algorithm.

a) Compute the Conditional number of this function for $x = 0$ and some values close to zero.

$$K = \frac{\|J(x)\|}{\|f\| / \|X\|}$$

$$f(x) = \log(x+1)/x \Rightarrow J(x) = \frac{df}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$J(x) = \frac{d}{dx}\left(\frac{\log(x+1)}{x}\right)$$

$$J(x) = \frac{x\left(\frac{1}{x+1}\right) - \log(x+1)}{x^2}$$

$$K(x) = \frac{\|x\| \cdot \left\| \frac{x - (x+1)\log(x+1)}{x^2(x+1)} \right\|}{\| \log(x+1)/x \|}$$

2.

$$K(x) = \left\| \frac{x - (x+1)\log(x+1)}{(x+1)\log(x+1)} \right\|$$

At, ~~as~~ $x=0$, the Condition number $K(x=0)$ is undefined and function $f(x=0)$ is also undefined.