In [1]:

```
%matplotlib notebook
%pylab
from scipy.integrate import quad
from scipy.special import erf
```

```
Using matplotlib backend: nbAgg
Populating the interactive namespace from numpy and matplotlib
```

1.e) Verify that your formulation holds for the function

$$u(x) = e^x,$$

subject to boundary conditions u(0) = 1, u(1) = e.

The function to be plotted is

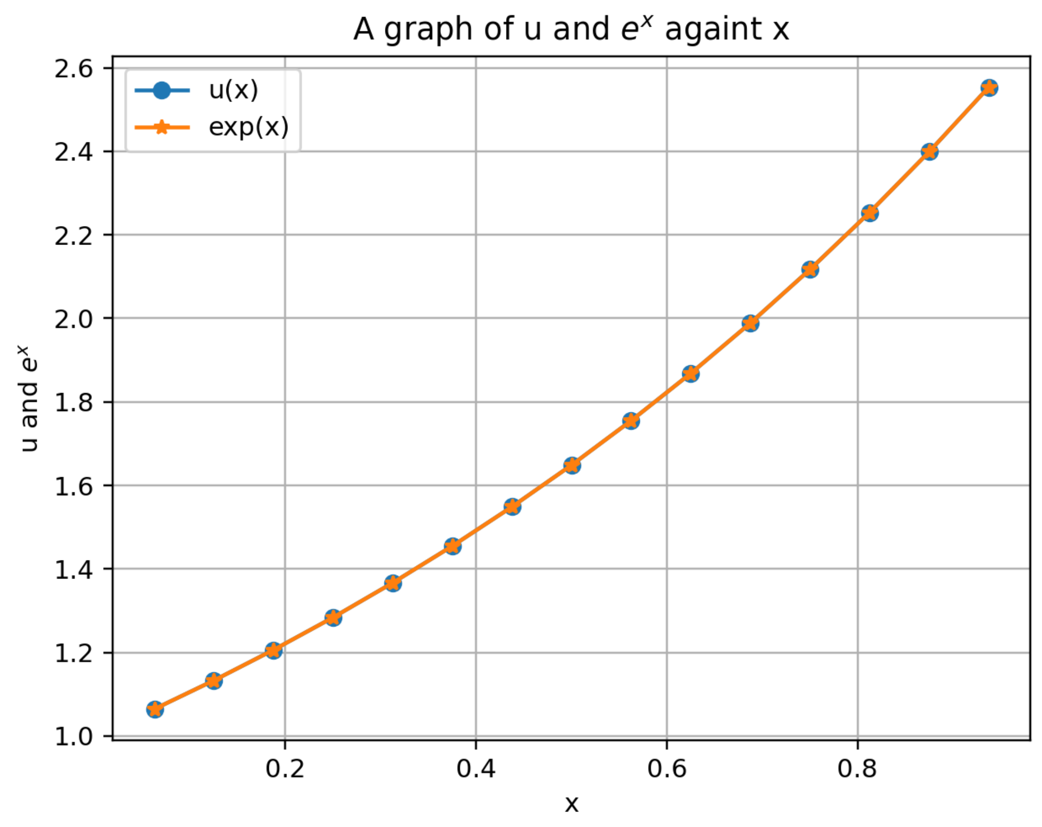
$$u(x) = \frac{1}{2}\int_0^1 (|x-y| - x + 2xy - y)e^y dy + 1 - x + xe$$

In [12]:

```
N = 16
a = 0; b = 1
h = 1/N

x = zeros(N+1)
I = zeros(N+1)

#rhs
def f(x):
    return exp(x)

#Green's function
def G(x,y):
    return 0.5*(abs(x-y)-x+2*x*y-y)

#Qudrature Rule
for j in range(N+1):
    x[j] = j*h
    xj = x[j]
    Gf = lambda y: G(xj,y)*f(y)
    I[j] =  quad(Gf,a,b)[0]
I = I + 1 - x +x*exp(1)
```

In [13]:

```
#ploting
figure(1)
clf()
plot(x[1:-1],I[1:-1],'-o',label='u(x)')
plot(x[1:-1],exp(x[1:-1]),'-*',label='exp(x)')
xlabel('x'); ylabel('u and $e^{x}$')
title('A graph of u and $e^{x}$ againt x')
grid()
legend()
show()
```

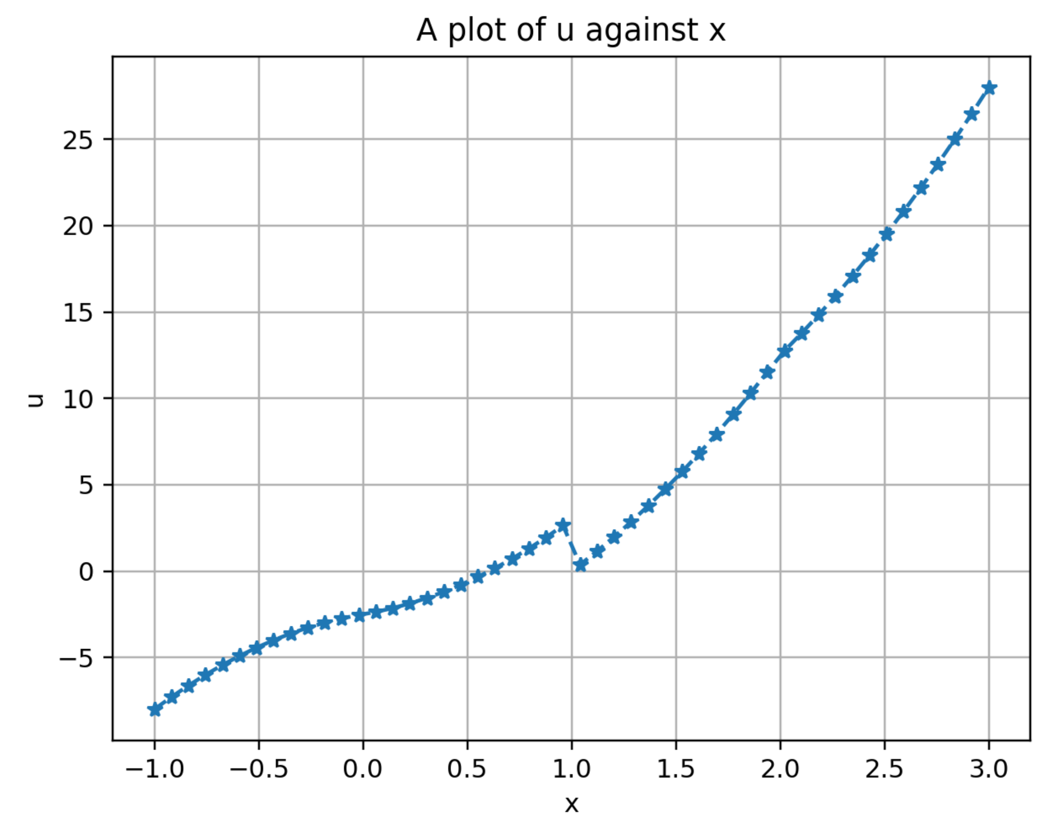


Hence the formulation holds, since the solution is well approximated

2.d) Sketch a plot of the resulting solution over the interval $[-1, 3]$

In [14]:

```
a = -1; b = 3
x = linspace(a,b)

b = array([2,6,0])
A = array([[1,1,2],[1,1,1],[1,-1,-1]])
y = linalg.solve(A,b)

def u(x):
    A = y[0]*(1-x)/(2*abs(x-1))
    B = y[1]*(x/2*abs(x))
    C = y[2]*(0.5*abs(x-2))
    return A + B + C

figure(2)
plot(x,u(x),'--*')
xlabel("x")
ylabel("u")
title('A plot of u against x')
grid()
show()
```



There is a jump, before and after $x = 1$

2.e) Solve the problem

$$u^{''}(x) = e^x$$

$\forall x \in [0, 1]$ subject to u(0) = 1 and u(1) = e, using a volume integral plus a distribution of sources and dipoles.

In [15]:

```
def u(x):
    b = array([2,2*exp(1),0])
    A = array([[1,1,2],[1,1,1],[1,-1,-1]])
    y = linalg.solve(A,b)
    A = y[0]*((1-x)/(2*abs(x-1)))
    B = y[1]*(x/(2*abs(x)))
    C = y[2]*((0.5*abs(x-2)))
    return A + B + C
```

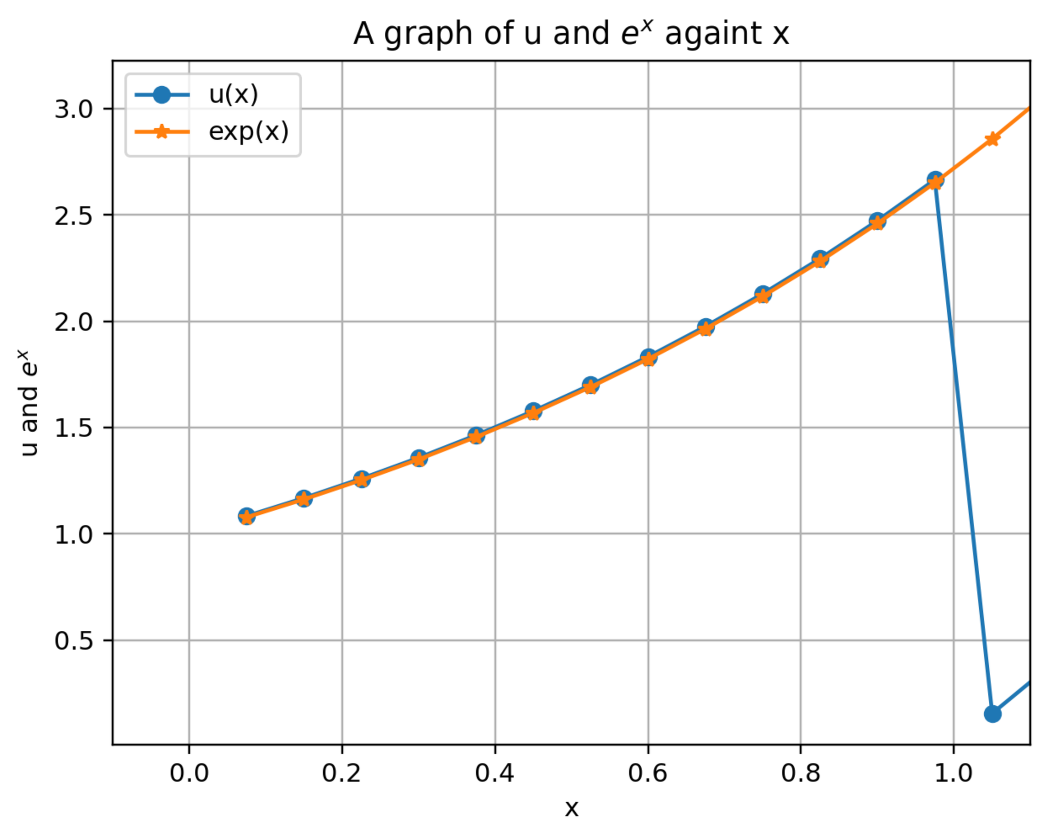
In [16]:

```
N = 16
a = -.1; b = 1.1
h = ((b-a)/N)

x = zeros(N+1)
I = zeros(N+1)

#rhs
def f(x):
    return exp(x)

#Green's function
def G(x,y):
    return 0.5*(abs(x-y)-x+2*x*y-y)


#Qudrature Rule
for j in range(N+1):
    x[j] = j*h
    xj = x[j]
    Gf = lambda y: G(xj,y)*f(y)
    I[j] =  quad(Gf,a,b)[0]
I = I + u(x)
```

```
<ipython-input-15-8ea98c7eb7bb>:6: RuntimeWarning: invalid value encountered in true_divide
  B = y[1]*(x/(2*abs(x)))
```

In [17]:

```
#plotting
figure(3)
clf()
plot(x[1:-1],I[1:-1],'-o',label='u(x)')
plot(x[1:-1],exp(x[1:-1]),'-*',label='exp(x)')
xlabel('x'); ylabel('u and $e^{x}$')
title('A graph of u and $e^{x}$ againt x')
xlim(-0.1,1.1)
grid()
legend()
show()
```



Hence the solution has the correct jump behavior at x = 0 and x = 1.

In [ ]: