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Homework 2

1. Question 5-3

Consider the matrix

$$A = \begin{bmatrix} -2 & 11 \\ -10 & 5 \end{bmatrix}$$

a) Determine, on a paper, a real SVD of A in the form $A = U\Sigma V^*$.

The non zero singular values of A = $\sqrt{\text{eigen values of } A^*A}$

$$A^*A = \begin{bmatrix} -2 & -10 \\ 11 & 5 \end{bmatrix} \begin{bmatrix} -2 & 11 \\ -10 & 5 \end{bmatrix} = \begin{bmatrix} 104 & -72 \\ -72 & 146 \end{bmatrix}$$

let $A^*A = B$

Eigen values of B

$$|B - \lambda I| = 0 \quad \Rightarrow \lambda_1 = 200, \ \lambda_2 = 50$$

the singular values of A = $\sqrt{\text{eigen value of } B}$.

$$\sigma_1 = \sqrt{200} = 10\sqrt{2}$$

$$\sigma_2 = \sqrt{50} = 5\sqrt{2}$$

U and V will be the eigen vectors of AA^* and A^*A respectively.

$Bv = \lambda v$, let $v = \begin{pmatrix} x \\ y \end{pmatrix}$

$$\begin{bmatrix} 104 & -72 \\ -72 & 146 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \lambda \begin{bmatrix} x \\ y \end{bmatrix}$$

$104x - 72y = \lambda x$

for $\lambda = 200 \Rightarrow x = -3,\ y = 4$.

$$V_1 = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$$

for $\lambda = 50 \Rightarrow x = 4,\ y = 3$

$$V_2 = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

Therefore the Right Singular vectors.

$$V_1 = \frac{1}{\sqrt{3^2+4^2}} \begin{bmatrix} -3 \\ 4 \end{bmatrix} = \frac{1}{5} \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$

$$V_2 = \frac{1}{\sqrt{4^2+3^2}} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

$$\Rightarrow V = \begin{bmatrix} -3/5 & 4/5 \\ 4/5 & 3/5 \end{bmatrix}$$

let $C = XX^T = \begin{bmatrix} -2 & 11 \\ -10 & 5 \end{bmatrix} \begin{bmatrix} -2 & -10 \\ 11 & 5 \end{bmatrix} = \begin{bmatrix} 125 & 75 \\ 75 & 125 \end{bmatrix}$

Eigen values of C

$|C - \lambda I| = 0 \Rightarrow \lambda_1 = 200,\ \lambda_2 = 50.$

Eigen vector of C

$CU = \lambda U$, let $U = \begin{pmatrix} x \\ y \end{pmatrix}$

$$\begin{bmatrix} 125 & 75 \\ 75 & 125 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \lambda \begin{bmatrix} x \\ y \end{bmatrix}$$

$125x + 75y = \lambda x$

for $\lambda_1 = 200$, $x = 1$, $y = 1$ $\Rightarrow U_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

for $\lambda_2 = 50$, $x = -1$, $y = 1$ $\Rightarrow U_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

left singular vectors

$$U_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad U_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$U = \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}$$

therefor the SVD is

$$A = U \Sigma V^T = \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix} \begin{bmatrix} 10\sqrt{2} & 0 \\ 0 & 5\sqrt{2} \end{bmatrix} \begin{bmatrix} -3/5 & 4/5 \\ 4/5 & 3/5 \end{bmatrix}$$

b) List the singular values,

$$\sigma_1 = 10\sqrt{2}, \quad \sigma_2 = 5\sqrt{2}$$

left singular vectors

$$u_1 = \begin{bmatrix} \sqrt{2}/2 \\ \sqrt{2}/2 \end{bmatrix}, \quad u_2 = \begin{bmatrix} -\sqrt{2}/2 \\ \sqrt{2}/2 \end{bmatrix}$$

Right singular vectors

$$v_1 = \begin{bmatrix} -3/5 \\ 4/5 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 4/5 \\ 3/5 \end{bmatrix}$$

c) What are the 1-, 2-, ∞-, and frobenius norm of A?

$$\|A\|_1 = \max\{12, 16\} = 16$$

$$\|A\|_2 = \sqrt{\rho(A^*A)} = 10\sqrt{2}$$

$$\|A\|_\infty = \max\{13, 15\} = 15$$

$$\|A\|_F = \sqrt{2^2 + 11^2 + 10^2 + 5^2} = \sqrt{250} = \underline{\underline{5\sqrt{10}}}$$

d) Find A^{-1} not directly, but via the SVD.

Consider

$$A = (U\Sigma V^*)$$

$$A^{-1} = (U\Sigma V^*)^{-1} = (V^*)^{-1}\Sigma^{-1}U^{-1}$$

but V and U are Unitary, $V^* = V^{-1}$, $U^* = U^{-1}$

$$A^{-1} = V\Sigma^{-1}U^* = \begin{bmatrix} -3/5 & 4/5 \\ 4/5 & 3/5 \end{bmatrix} \begin{bmatrix} 10\sqrt{2} & 0 \\ 0 & 5\sqrt{2} \end{bmatrix}^{-1} \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 0.05 & -0.11 \\ 0.1 & -0.02 \end{bmatrix}$$

---

e) Find the eigen values λ_1, λ_2 of A.

$$\lambda^2 - \text{trace}(A)\lambda + \det(A) = 0$$

$$\det(A) = 100, \quad tr(A) = 3$$

$$\lambda^2 + 3\lambda + 100 = 0$$

$$\lambda_1 = \frac{3 + \sqrt{391}\,i}{2}, \quad \lambda_2 = \frac{3 - \sqrt{391}\,i}{2}$$

---

f) Verify that $\det A = \lambda_1\lambda_2$ and $|\det(A)| = \sigma_1\sigma_2$

$$\lambda_1 \cdot \lambda_2 = \left(\frac{3+\sqrt{391}\,i}{2}\right)\left(\frac{3-\sqrt{391}\,i}{2}\right)$$

$$\lambda_1 \cdot \lambda_2 = \frac{1}{4}\left(3^2 - (\sqrt{391}\,i)^2\right) = 100.$$

have $\det(A) = \lambda_1 \lambda_2$

$|\det A| = \sigma_1 \sigma_2$

Since U is Unitary.

$$\det(UU^*) = \det(U) \cdot \det(U^*) = 1$$

but $\det(U) = \det(U^*) = \pm 1$

$\det(A) = \det(U \Sigma V^*) \Rightarrow (\det(A))^2 = \det(A^* A)$

$\cancel{(\det(A))^2 = (\det(U\Sigma V^*))^2 = \det(U\Sigma V^* U \Sigma V^*)}$

$$A^* = V \Sigma^* U^*$$

$(\det(A))^2 = \det(V \Sigma^* U^* U \Sigma V^*)$

$(\det(A))^2 = \det(V \Sigma^* \Sigma V^*)$

$(\det(A))^2 = \det(V) \cdot \det(\Sigma^* \Sigma) \cdot \det(V^*)$

$$\text{but } \det(V) = \det(V^*) = -1$$

$(\det(A))^2 = \det(\Sigma^* \Sigma)$

$|\det(A)| = \sqrt{\det(\Sigma^* \Sigma)}$

$|\det(A)| = \sqrt{\sigma_1^2 \sigma_2^2}$

$|\det(A)| = \sigma_1 \sigma_2$

(g) What is the area of the ellipsoid onto which A maps the unit ball of $\mathbb{R}^2$?

$$A = \pi r_1 r_2$$

$$A = \pi(\sigma_1 \sigma_2) = \pi(10\sqrt{2})(5\sqrt{2})$$

$$\underline{A = 100\pi}$$

2. Question 5.4

Consider $A = U\Sigma V^* \Rightarrow A^* = V\Sigma^* U^*$

Let $B = \begin{bmatrix} 0 & A^* \\ A & 0 \end{bmatrix}$

$$B = \begin{bmatrix} 0 & V\Sigma^* U^* \\ U\Sigma V^* & 0 \end{bmatrix}$$

B can be flipped, let I be an $m \times m$ Identity Matrix

$$B = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} \begin{bmatrix} U\Sigma V^* & 0 \\ 0 & V\Sigma^* U^* \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} \begin{bmatrix} U & 0 \\ 0 & V \end{bmatrix} \begin{bmatrix} \Sigma & 0 \\ 0 & \Sigma^* \end{bmatrix} \begin{bmatrix} V^* & 0 \\ 0 & U^* \end{bmatrix} \quad \text{--- (1)}$$

Since Σ is diagonal matrix then $\Sigma^* = \Sigma$

from (1), let $\begin{bmatrix} U & 0 \\ 0 & V \end{bmatrix}$ be C, $C^{-1} = \begin{bmatrix} V^* & 0 \\ 0 & U^* \end{bmatrix}$,

Since

$$\begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}^{-1} = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} \text{ then}$$

$$\begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}\begin{bmatrix} U & 0 \\ 0 & V \end{bmatrix} = \begin{bmatrix} V^* & 0 \\ 0 & U^* \end{bmatrix}\begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}$$

therefore,

$$B = \left(\begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}\begin{bmatrix} U & 0 \\ 0 & V \end{bmatrix}\right)\left(\begin{bmatrix} \Sigma & 0 \\ 0 & \Sigma \end{bmatrix}\begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}\right)\left(\begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}\begin{bmatrix} U & 0 \\ 0 & V \end{bmatrix}\right)^{-1}$$

$$B = \left(\begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}\begin{bmatrix} U & 0 \\ 0 & V \end{bmatrix}\right)\left(\begin{bmatrix} 0 & \Sigma \\ \Sigma & 0 \end{bmatrix}\right)\left(\begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}\begin{bmatrix} U & 0 \\ 0 & V \end{bmatrix}\right)^{-1} \quad \ldots (2)$$

Since Σ has non negative ~~entri~~ elements then

$$\begin{bmatrix} 0 & \Sigma \\ \Sigma & 0 \end{bmatrix}\begin{bmatrix} I & I \\ I & -I \end{bmatrix} = \begin{bmatrix} I & I \\ I & -I \end{bmatrix}\begin{bmatrix} \Sigma & 0 \\ 0 & -\Sigma \end{bmatrix} \quad \text{— (3)}$$

but $\begin{bmatrix} I & I \\ I & -I \end{bmatrix}\begin{bmatrix} I & I \\ I & -I \end{bmatrix} = 2I_2$, where I_2 is a $2n \times 2n$ identity matrix.

So equation (3) becomes.

$$\begin{bmatrix} 0 & \Sigma \\ \Sigma & 0 \end{bmatrix} = \begin{bmatrix} I & I \\ I & -I \end{bmatrix}\begin{bmatrix} \Sigma & 0 \\ 0 & -\Sigma \end{bmatrix}\begin{bmatrix} I & I \\ I & -I \end{bmatrix}^{-1}$$

$$\begin{bmatrix} 0 & \Sigma \\ \Sigma & 0 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} I & I \\ I & -I \end{bmatrix}\begin{bmatrix} \Sigma & 0 \\ 0 & -\Sigma \end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix} I & I \\ I & -I \end{bmatrix}$$

Subsituting $\begin{bmatrix} 0 & \Sigma \\ \Sigma & 0 \end{bmatrix}$ back into Equation ②

$$B = \left(\begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}\begin{bmatrix} U & 0 \\ 0 & V \end{bmatrix}\right)\left(\frac{1}{\sqrt{2}}\begin{bmatrix} I & I \\ I & -I \end{bmatrix}\begin{bmatrix} \Sigma & 0 \\ 0 & -\Sigma \end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix} I & I \\ I & -I \end{bmatrix}\right)\left(\begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}\begin{bmatrix} U & 0 \\ 0 & V \end{bmatrix}\right)^{-1} \quad \text{--- ④}$$

If we let

$$Q = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}\begin{bmatrix} U & 0 \\ 0 & V \end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix} I & I \\ I & -I \end{bmatrix}$$

Then Equation ④ becomes

$$B = Q\begin{bmatrix} \Sigma & 0 \\ 0 & -\Sigma \end{bmatrix}Q^{-1}$$

3. Question 6.2

Given $Ex = \dfrac{(x + fx)}{2}$

To know the nature of E we need to know the nature of F.

F flips $(x_1, x_2 \ldots, x_m)^*$ to $(x_m, \ldots x_2, x_1)^*$

$$F\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix} = \begin{bmatrix} 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 1 & 0 \\ \vdots & & \ddots & & \vdots \\ 1 & 0 & \cdots & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix} = \begin{bmatrix} x_m \\ x_{m-1} \\ \vdots \\ x_1 \end{bmatrix}$$

Since F just flips x, then it has the form above however its Unitary and Symmetrically, therefore $F^2 = I$.

So there for we can take advantage of the Square

$$Ex = \frac{(x + Fx)}{2} = \frac{(I+F)}{2} x$$

$$E = \frac{I+F}{2} \Rightarrow E^2 = \frac{1}{4}(I+F)^2$$

$$E^2 = \frac{1}{4}(I^2 + F^2 + 2IF) \quad ,$$

Since $F^2 = I$

$$E^2 = \frac{1}{4}(I^2 + I^2 + 2IF) = \frac{1}{4}(2I^2 + 2IF)$$

$$E^2 = \frac{1}{4} 2(F+I) = \frac{(F+I)}{2}$$

$$E^2 = \frac{(F+I)}{2}$$

$$\underline{\underline{E^2 = E}}$$

Since both E and F are Unitary and Symmetrical

$E = E^*$, $F = F^*$, thi means E is orthogonal projector.

What are the entries of E?

$$E = \frac{1}{2}(F + I)$$

$$E = \frac{1}{2}\left(\begin{bmatrix} 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 1 & 0 \\ \vdots & & ⋰ & & \vdots \\ 1 & 0 & \cdots & 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & & \ddots & & 0 \\ 0 & & & & 1 \end{bmatrix} \right)$$

$$E = \frac{1}{2} \begin{bmatrix} 1 & 0 & \cdots & 0 & 1 \\ 0 & 1 & \cdots & 1 & 0 \\ & & \ddots & & \\ & & 2 & & \\ & & & \ddots & \\ 0 & 1 & \cdots & 1 & 0 \\ 1 & 0 & \cdots & 0 & 1 \end{bmatrix}$$

5. (Householder reflections)

a) verify directly that if we let $v = \frac{y-x}{\|y-x\|}$ and $H = I - 2vv^T$ then $Hx = (I-2vv^T)x = y$

from $H = I - 2vv^T$

$$Hx = (I - 2vv^T)x$$
$$Hx = x - 2vv^Tx$$

$$Hx = x - 2\left(\frac{y-x}{\|y-x\|}\right)\left(\frac{y-x}{\|y-x\|}\right)^T x$$

$$Hx = x - \frac{2(y-x)(y-x)^T}{\|y-x\|^2}x$$

Introducing a useful zero $-y+y$

$$Hx = x - y + y - \frac{2(y-x)(y-x)^T}{\|y-x\|^2}x + y$$

$$Hx = \frac{-(y-x)}{\|y-x\|^2}\left[\|y-x\|^2 + 2(y-x)^Tx\right] + y$$

Since $x^Tx = \|x\|^2 \Rightarrow \|y-x\|^2 = (y-x)^T(y-x)$

$$Hx = \frac{-(y-x)}{\|y-x\|^2}\left[(y-x)^T(y-x) + 2(y-x)^Tx\right] + y$$

$$Hx = \frac{-(y-x)}{\|y-x\|^2}\left[(y^T-x^T)(y-x) + 2(y^T-x^T)x\right] + y$$

$$Hx = \frac{-(y-x)}{\|y-x\|^2}\left[y^Ty - y^Tx - x^Ty + x^Tx + 2y^Tx - 2x^Tx\right] + y$$

Since $y^Ty = x^Tx$, $y^Tx = x^Ty$

$$Hx = \frac{-(y-x)}{\|y-x\|^2}\left[-2y^Tx + 2y^Tx\right] + y = y$$

$$\underline{\underline{Hx = y}}$$

b) H be a Householder matrix of size m., $x^Tx = y^Ty$
H is both symmetrical and orthogonal, eigen values
$(H) = I1$

(i) $Tr(H)$
Consider $H = I - 2vv^T$

$$Tr(H) = Tr(I - 2vv^T) = Tr(I) - Tr(2vv^T)$$

$$= Tr(I) - 2Tr(vv^T)$$

but $v = \frac{y-x}{\|y-x\|}$, $\Rightarrow vv^T = \frac{(y-x)(y-x)^T}{\|(y-x)\|^2} = 1$

$$Tr(H) = Tr(I) - 2Tr(1)$$
$$Tr(H) = Tr(I) - 2$$

Since I is $m \times m$ then $Tr(I) = m$

$$\underline{\underline{Tr(H) = m-2}}$$

Eigen values of H,

Let the eigen values of H be λ.

$\det(H - \lambda I) = 0$, $H = I - 2vv^T$

$\det(I - 2vv^T - \lambda I) = 0$

$\det((1-\lambda)I - 2vv^T) = 0$

$(-1)^m \det((\lambda - 1)I + 2vv^T) = 0$

$\det((\lambda-1)I + 2vv^T) = 0$

$\det((\lambda-1)I(I + 2((\lambda-1)I)^{-1} vv^T) = 0$

$(\lambda-1)^m \det(I + 2((\lambda-1)I)^{-1} vv^T) = 0$

Since $vv^T = 1$

$(\lambda-1)^m \det[I + 2((\lambda-1)I)^{-1}] = 0$

$(\lambda-1)^m (1 + 2(\lambda-1)^{-1}) \det(I) = 0$

$(\lambda-1)^m (1 + 2(\lambda-1)^{-1}) = 0$

Either

$(\lambda-1)^m = 0$ or $\lambda = -1$

(ii) Show that $Hv = -v$ and that $Hu = u$ for any $u \in \mathbb{R}^m$ that is orthogonal to v

$H = I - 2vv^T$

$$Hv = (I - 2vv^T)v$$

$$Hv = v - 2(vv^T)v \quad , \text{ but } v^Tv = 1$$

$$Hv = v - 2v = -v$$

$$\underline{Hv = -v} \quad \text{———— (1)}$$

$$Hu = (I - 2vv^T)u$$

$$Hu = (u - 2(vv^T)u)$$

$$Hu = u - 2(v^Tu)v \quad ; \text{ but } v^Tu = 0$$

$$\underline{Hu = u} \quad \text{———— (2)}$$

Using the definition $Hv = \lambda v$, where λ are the eigen values and v are the eigen vectors

$$Hv = -v \implies \lambda = -1$$
$$Hu = u, \implies \lambda = 1$$

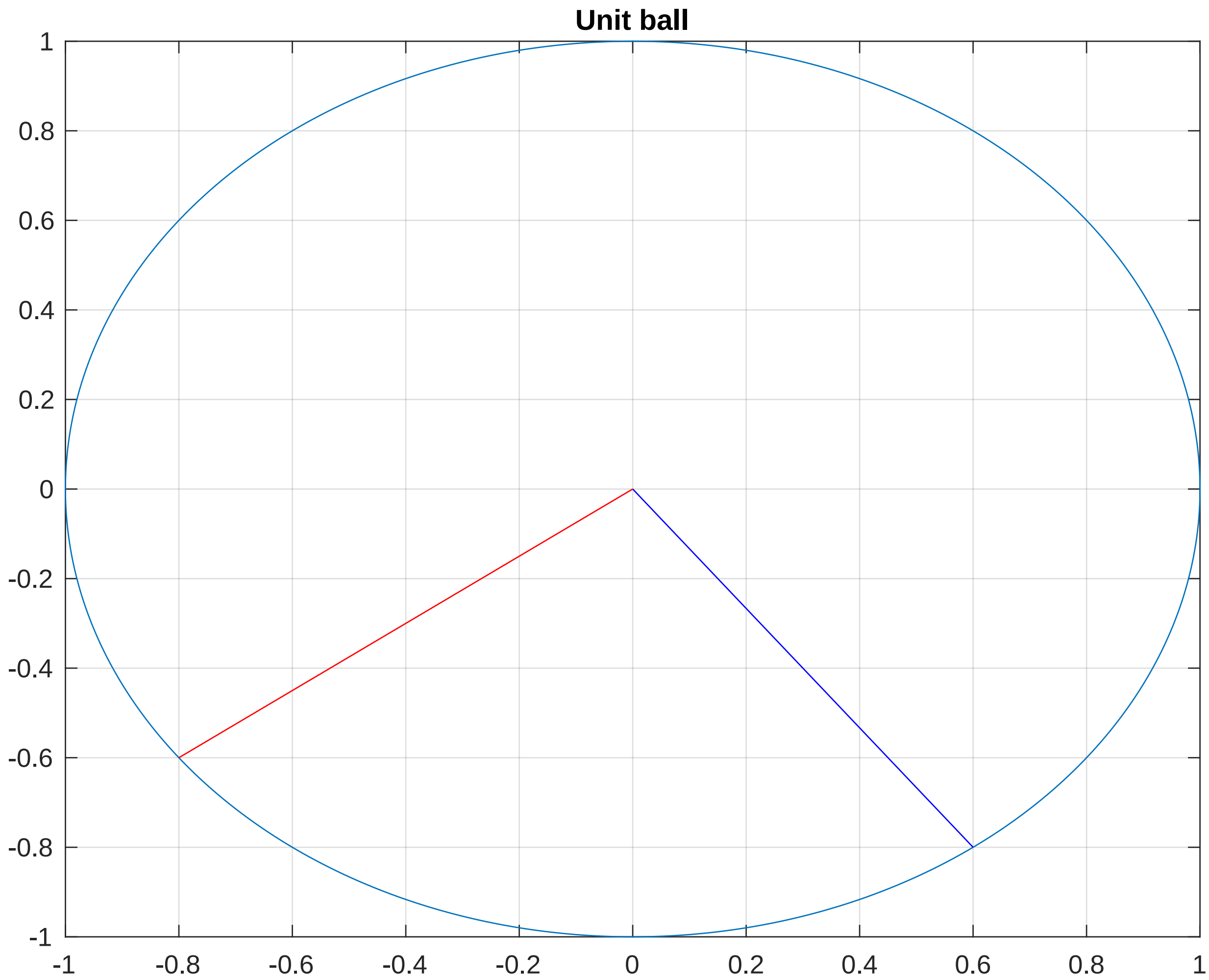
Therefore the Eigen values of H are $\lambda = \pm 1$, and this is true since H is both symmetric and orthogonal matrix.

(iii) Using the ~~Prog~~ properties of eigen values, determine det (H)

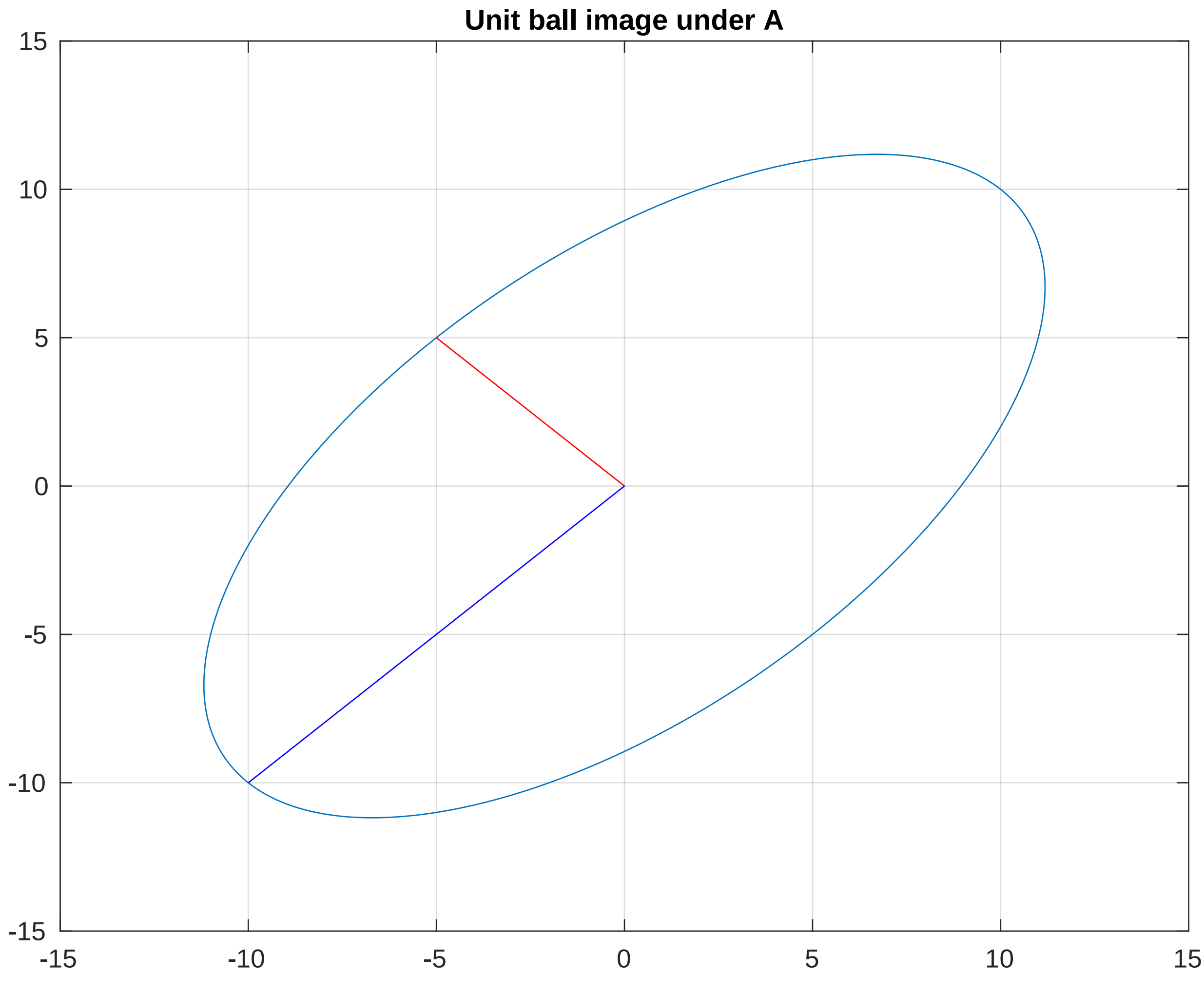
det(H) = Product of all eigen value of H

$$\det(H) = \prod_{i=1}^{m} \lambda_i = 1^{m-1} \cdot (-1) = -1$$

$\det(H) = -1$



Unit ball
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1


Unit ball image under A
15
10
5
0
-5
-10
-15
-15
-10
-5
0
5
10
15

```
%=================================================================
% Program that implements the classical Gram-Schmidt algorithm for
% computing the QR factorization of a m by n matrix A, where m>=n.
% m equally spaced points over [0,1]
% input : real m by n matrix A
% output : real m by n matrix q
%          real n by n matrix r
%==================================================================

%Classical Gram-Schmidt
function [q,r] = CGS(A)

[m,n] = size(A);

q = zeros(m,n);
r = zeros(n,n);
v = zeros(m,n);

for j=1:n

   v(:,j) = A(:,j);

   for i = 1:j-1

      r(i,j) = q(:,i)'*A(:,j);
      v(:,j) = v(:,j) - r(i,j)*q(:,i);

   end

   r(j,j) = norm(v(:,j),2);
   q(:,j) = v(:,j)/r(j,j);

end
end
```

```
Not enough input arguments.

Error in CGS (line 13)
[m,n] = size(A);
```

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```
%=====================================================================
% Program that implements the modified Gram-Schmidt algorithm for
% computing the QR factorization of a m by n matrix A, where m>=n.
% input : real m by n matrix A
% output : real m by n matrix q
%          real n by n matrix r
%=====================================================================

function [q,r] = MGS(A)

    [m,n] = size(A);

    v = zeros(m,n);
    r = zeros(n,n);
    q = zeros(m,n);

    for i = 1:n
        v(:,i) = A(:,i);
    end

    for i = 1:n

       r(i,i) = norm(v(:,i),2);
       q(:,i) = v(:,i)/r(i,i);

       for j = i+1:n

          r(i,j) = q(:,i)'*v(:,j);
          v(:,j) = v(:,j) - r(i,j)*q(:,i);

       end

    end

end
```

```
Not enough input arguments.

Error in MGS (line 11)
    [m,n] = size(A);
```

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```
clear all;
%m equally spaced points over [0,1]
m = 100; n=15;

% Vandermonde matrix B
t = zeros(m,n);
for i = 1:n
    for j = 1:m
        t(j,i) = ((j-1)/(m-1))^(n-i);
    end
end

%fliping the vandermonde matrix t to form A
A = fliplr(t);

%calling the function for CGS
[q_c,r_c] = CGS(A);

%infinity norm of A - QR for CGS
Nc = norm((A - q_c*r_c),inf)
Nc1 = norm((eye(n) - q_c'*q_c),inf)

%calling the function for MGS
[q_m,r_m] = MGS(A);

%infinity norm of A - QR for MGS
Nm = norm((A - q_m*r_m),inf)
Nm2 = norm((eye(n) - q_m'*q_m),inf)

fprintf("The infinity norms (A - QR) for the two methods are very small to almost zero, \n while for (I - Q'Q), for the modified algorithm its near to
```

```
Nc =

   1.1102e-15


Nc1 =

    4.6752


Nm =

   1.2212e-15


Nm2 =

   3.5413e-07

The infinity norms (A - QR) for the two methods are very small to almost zero,
 while for (I - Q'Q), for the modified algorithm its near to zero, but its
 strange for the classical one, because its 4.6752 and this is a very big
 value sice we are almost subtracting the same things.
```

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```
%==========================================================================
% function computes the implicit representation of a full QR decomposition
% of a real m by n matrix A via Householder reflections.
% input : real m by n matrix A
% output : real m by n matrix V
%          real n by n matrix R
%==========================================================================

function [V,R] = house(A)

[m,n] = size(A);

R = A;

V = zeros(m,n);

for k=1:n

   x = R(k:m,k);
   V(k:m,k) = sign(x(1))*(norm(x,2))*eye(m-k+1,1) + x;
   V(k:m,k) = V(k:m,k)/(norm(V(k:m,k),2));
   R(k:m,k:n) = R(k:m,k:n) - 2*V(k:m,k)*(V(k:m,k)'*R(k:m,k:n));

end

end
```

```
Not enough input arguments.

Error in house (line 11)
[m,n] = size(A);
```

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```
%==========================================================================
% function computes tcomputes the corresponding m-by-m orthogonal matrix Q.
% input : real m by n matrix V
% output : real m by m matrix Q
%==============================================

function Q = house2q(V)

[m,n] = size(V);

x = eye(m);

for k = n:-1:1
    x(k:m,:) = x(k:m,:) - 2*V(k:m,k)*(V(k:m,k)'*x(k:m,:));
end
Q = x;
end
```

```
Not enough input arguments.

Error in house2q (line 9)
[m,n] = size(V);
```

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```
%matrix A
A = [1 2 3 ; 4 5 6 ; 7 8 7 ; 4 2 3 ; 4 2 2];

%calling function house
[V,R] = house(A)

%calling function house2q
Q = house2q(V)

%function qr from matlab
[q ,r] = qr(A)

fprintf('The Q and R, produced by the code satisy A = QR \n');
```

```
V =

    0.7420         0         0
    0.2723    0.7866         0
    0.4765    0.1192   -0.9800
    0.2723   -0.4284    0.1842
    0.2723   -0.4284   -0.0748


R =

   -9.8995   -9.4954   -9.6975
         0   -3.2919   -3.0129
         0         0    1.9701
         0         0         0
         0         0    0.0000


Q =

   -0.1010   -0.3162    0.5420   -0.6842   -0.3577
   -0.4041   -0.3534    0.5162    0.3280    0.5812
   -0.7071   -0.3906   -0.5248    0.0094   -0.2683
   -0.4041    0.5580    0.3871    0.3656   -0.4918
   -0.4041    0.5580   -0.1204   -0.5390    0.4695


q =

   -0.1010   -0.3162    0.5420   -0.6842   -0.3577
   -0.4041   -0.3534    0.5162    0.3280    0.5812
   -0.7071   -0.3906   -0.5248    0.0094   -0.2683
   -0.4041    0.5580    0.3871    0.3656   -0.4918
   -0.4041    0.5580   -0.1204   -0.5390    0.4695


r =

   -9.8995   -9.4954   -9.6975
         0   -3.2919   -3.0129
         0         0    1.9701
         0         0         0
         0         0         0
```

```
The Q and R, produced by the code satisy A = QR
```

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