**INVERSE METHODS FOR SOLVING SHALLOW WATER EQUATIONS**

**Project Pre-proposal**

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# Problem description

Many problems that arise from physical applications give us a natural feel to the inverse methods. These methods have been applied directly and inversely to nonlinear problems[4], and many researchers have used them to perform different studies on a variety of problems. However here we are going to concentrate on;

- Examine why the method works [1].
- Discretization of the governing equations for the forward and inverse models.
- Using the method to solve forward and Inverse models.
- Applying the models on a dam break problem.
- Experiment with different test cases i.e, a one-dimensional steady sub-critical flow over a frictionless channel [2].
- Validation and estimation of the inverse model using results from the forward model.

# Method used

The Methods that will be used in this study are the forward and inverse methods for solving a Riemann problem. The idea behind the forward approach is to find parameters: height field ($h$ $(m)$) and the momentum field ($hu$ $m^2s^{-1}$) for a Riemann problem at every state. In contrast with the forward method, the inverse method uses known height and momentum parameters to determine the unknown height and momentum fields. The only major difference is using the explicit backward difference scheme to discretize the inverse problem rather than the forward difference scheme. This is because we have no data exchange from down to up direction, hence making the update expression also different. To confirm whether the method is working, the results from the forward model are used to validate and assess the inverse method [2].

## Case Study

The above solvers are used to solve a one-dimensional shallow water equation (1), with its associated Riemann problem (2).

$$\begin{aligned} h_t + (hu)_x &= 0, \\ (hu)_t + \left(hu^2 + \frac{1}{2}gh^2\right)_x &= 0, \end{aligned} \tag{1}$$

where $g$ $(m/s^2)$ is acceleration due to gravity, $hu$ measures the flow rate of water past a point and $u(x,t)$ $(m/s)$ is the horizontal velocity [3, 5].

$$q(x,t) \quad = \quad \begin{cases} q_l, & \text{if} \quad x \leq 0, \\ q_r, & \text{if} \quad x > 0, \end{cases} \tag{2}$$

where $q_l$ and $q_r$ are two piece-wise constant states separated by a discontinuity. At $x = 0$ and $t = 0$, the discontinuity is located between the left and right state, so the solution at the left $(q_l)$ and right $(q_r)$ states are given by:

$$q_l = \begin{bmatrix} h_l \\ (hu)_l \end{bmatrix} \quad \text{and} \quad q_r = \begin{bmatrix} h_r \\ (hu)_r \end{bmatrix}, \tag{3}$$

where $h_l$ is the left height field, $h_r$ is the right height fields, $(hu)_l$ and $(hu)_r$ are the left and right momentum fields respectively. For this study, several values for the height and momentum field will be experimented with, depending on the problem in question.

# References


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[4] Jerome Monnier, Frédéric Couderc, Denis Dartus, Kévin Larnier, Ronan Madec, and J-P Vila. Inverse algorithms for 2d shallow water equations in presence of wet dry fronts: Application to flood plain dynamics. *Advances in Water Resources*, 97:11–24, 2016.

[5] Eleuterio F Toro. *Shock-capturing methods for free-surface shallow flows.* Wiley-Blackwell, 2001.