Inverse methods for solving shallow water equations

Progress report

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Topic description

We are finding parameter estimates for the shallow water problem using inverse methods techniques. In this work, we chose various sets of initial estimates, m = [(hl,ul,hl*ul),(hm,um,hm*um)),(hr,ur,hr*ur)], and use the exact solver from the Forestclaw code (atmospheric code) to generate the data G(m). Obtaining G(m) enables us to obtain the Jacobian matrix J, which enables us to use the inverse methods techniques to recover the true parameter estimates: mtrue for the shallow water problem.

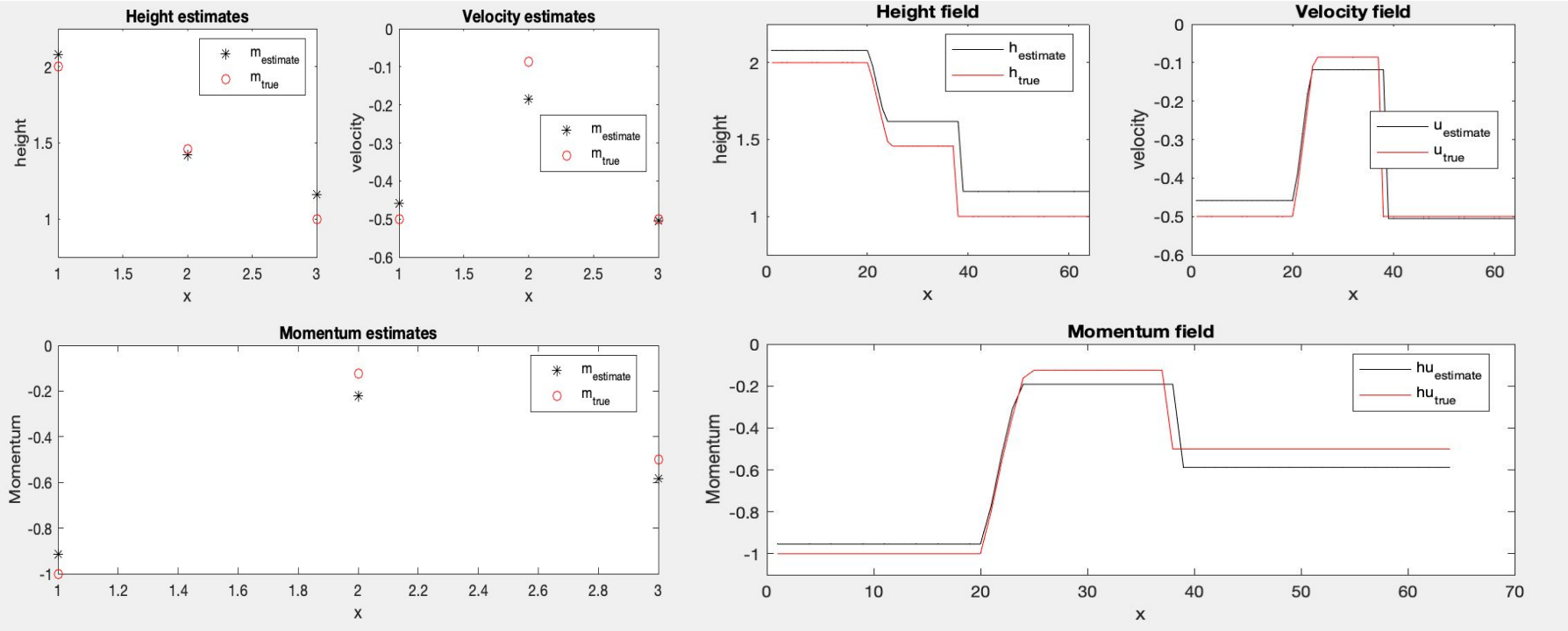
Changes

- Changed the dimension of the initial estimate from 2X2 to 3X3 to include in momentum.
- Changed from a forward difference Jacobian formulation to a centered difference.
- Used the Occam model instead of the Gauss Newton method due to the difficulties in obtaining the right alpha at each iteration.
- Changed from using the zeroth-order Tikhonov regularization roughening matrix to second-order, as this yielded better results.
- Shifted from Python to Matlab, to be able to use codes from the book: Occam’s model code.

Accomplishments

- Choosen my data to be m = [(hl,ul,hl*ul),(hm,um,hm*um),(hr,ur,hr*ur)]
- Extracted and implemented the forestclaw code to obtain G(m,x/t)
- Obtained the observed data d = G(m,x/t) + noise
- Computed the Jacobian J
- Used the Occam model to recover mtrue
- Obtained the Chi-square: [0.0069178 0.00064838   0.0055623]
- Obtained the p-value: [1 1 1]

Results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Findings

I discovered that recovering mtrue, depends on:

- The initial estimate ,mo, used.
- The value of the standard deviation used to generate the noise.
- The step size ,h,
- The inversion model used which high depends on the range of values of alpha used.
- The order of the roughening matrix used.

Remaining Tasks

- Validating the code on different sets of problems.
- Documenting the code.
- Preparing the final presentation.
- Starting and finishing the write up.

Thinking about:

- Covariance Matrix
- Confidence intervals
- Correlation Matrix
- Linearized confidence ellipsoid (between h and u)

Time frame

- 22nd - 25th, April : implementing the details from the progress report meeting, finalizing the remaining tasks, and drafting the final presentation for 26th, April
- 27th, April - 2nd, May : start and accomplishment of the first report draft.
- 3rd - 5th, May : implementation of final remarks and submission of the final work.