## **Appendix**

```
clear
clc
load ifk.mat
[m,n] = size(d);
a = 0; b = 1;
n = m;
sig = 5e-5;
dx = (b-a)/m;
g = @(x,y) x*exp(-x*y);
x = [];
for j = 1:n
  x = [x \ a + (dx/2) + (j-1)*dx]; \% form x
end
y = x;
G = zeros(m,n);
for i = 1:m
  for j = 1:n
     G(i,j) = g(x(j),y(i)).*dx; %form G
  end
end
%form L1 and L2 matrices
L1 = zeros(n-2,n-1);
for i=2:n-1
  for j=1:n
     L1(1,1) = -1;
     L1(1,2) = 1;
     if (i==j)
       L1(i,j) = -1;
     elseif (j==i+1)
        L1(i,j) = 1;
     end
  end
end
L2 = zeros(n-3,n-1);
for i=2:n-2
  for j=1:n
     L2(1,1) = 1;
     L2(1,2) = -2;
     L2(1,3) = 1;
```

```
if (i==j)
        L2(i,j) = 1;
     elseif (i==i+1)
        L2(i,j) = -2;
     elseif (i==i+2)
        L2(i,j) = 1;
     end
  end
end
No.2 Plot the predictive errors.
alpha max = 0.0148;
alpha_min = 2.5058e-5;
nbar = 100;
alp_i = [];
g0 = []; g1 = []; g2 = [];
for i = 1:nbar
  apha = alpha min*(alpha max/alpha min)^((nbar-i)/(nbar-1));
  alp_i = [alp_i apha];
  g0 = [g0 galpha(m,G,eye(m),d,apha)];
  g1 = [g1 galpha(m,G,L1,d,apha)];
  g2 = [g2 galpha(m,G,L2,d,apha)];
end
[op0 i0] = min(g0);
[op1 i1] = min(g1);
[op2 i2] = min(g2);
figure(1)
subplot(1,3,1)
loglog(alp_i,g0); hold on
loglog(alp i(i0),op0,'or'); grid on
title('zeroth order');
xlabel('\alpha'); %ylabel('g(\alpha)')
subplot(1,3,2)
loglog(alp_i,g1); hold on
loglog(alp_i(i1),op1,'or'); grid on
title('first order');
xlabel('\alpha'); %ylabel('g(\alpha)')
subplot(1,3,3)
loglog(alp_i,g2); hold on
loglog(alp_i(i2),op2,'or'); grid on
title('second order');
xlabel('\alpha'); %ylabel('g(\alpha)')
function g = galpha(m,G,L,d,alpha)
  Gpound = (G'*G + (alpha^2).*L'*L)\G';
  m_alpha = Gpound*d;
```

 $g = m^*(norm(G^*m\_alpha - d)^2)/trace(eye(m) - G^*Gpound)^2;$ 

end