MATH 568

Probability and Statistics review, individual activity

1. Let $\mathbf{X} = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$, $E(\mathbf{X}) = \boldsymbol{\mu} = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}$, and consider a 2×2 matrix of scalars

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}.$$

   (a) Use the fact that $E(aX) = aE(X)$ for scalar a and random Variable X to show that $E(\mathbf{AX}) = \mathbf{A}E(\mathbf{X})$.
   Hint: form a vector $\mathbf{Y} = \mathbf{AX}$ *and take the mean of each element of* $\mathbf{Y}$.

   (b) If $\mathbf{Y} = \mathbf{AX}$, find $\mathrm{Var}(Y_1)$, $\mathrm{Var}(Y_2)$ and $\mathrm{COV}(Y_1, Y_2)$ in terms of $\mathrm{Var}(X_1)$, $\mathrm{Var}(X_2)$ and $\mathrm{COV}(X_1, X_2)$.
   Hint: Use Theorem B.1 and Exercise 3. in the textbook, along with $COV(aX, bY) = abCOV(X, Y)$ *and* $COV(X, X) = Var(X)$.

2. Do Exercise 8 in Appendix B.

3. Generate 5 exponentially distributed random numbers with expected value 10 (you can do this in MATLAB with the function *exprnd(10,5,1)*) and calculate the mean of the 5 numbers. Do this 1000 times and determine if the averages are normally distributed in two ways:

   (a) Plot a histogram of the 1000 averages (*hist* in MATLAB).

   (b) Make a Q-Q plot of the 1000 averages (*qqplot* in MATLAB).

   Explain why or why not the averages are approximately normally distributed in each case.

4. Repeat the experiment in 3. but instead generate 50 exponentially distributed random numbers with expected value 10 and calculate the mean of the 50 numbers. Explain why or why not the averages are approximately normally distributed by studying the histogram and Q-Q plot.