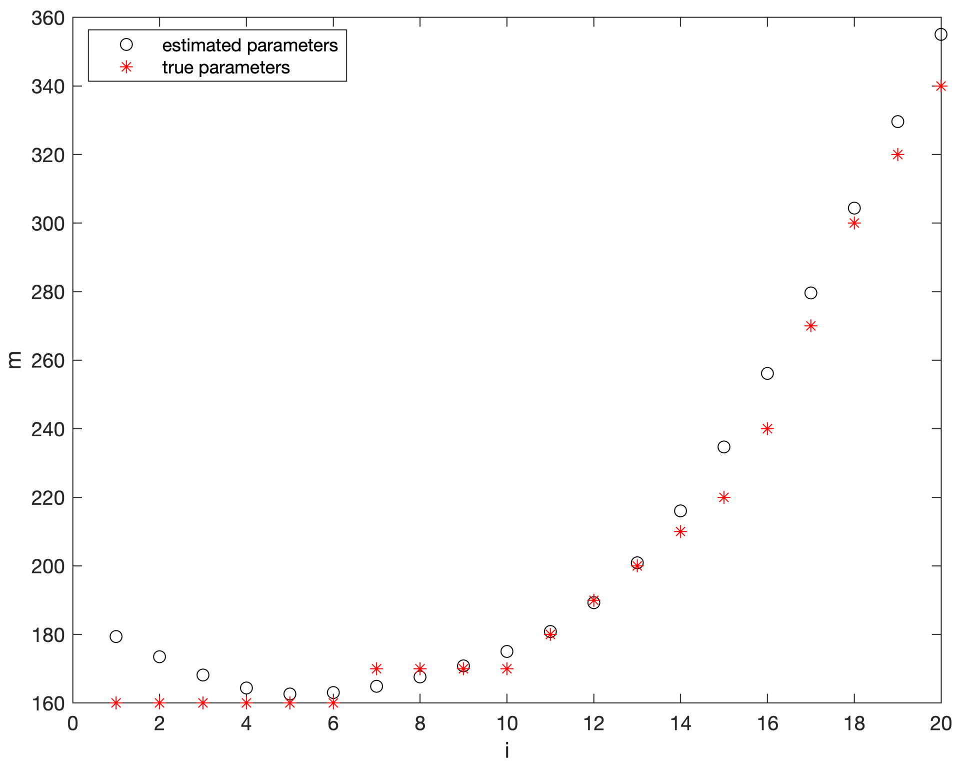
Habeeb and Brian

Home work 3

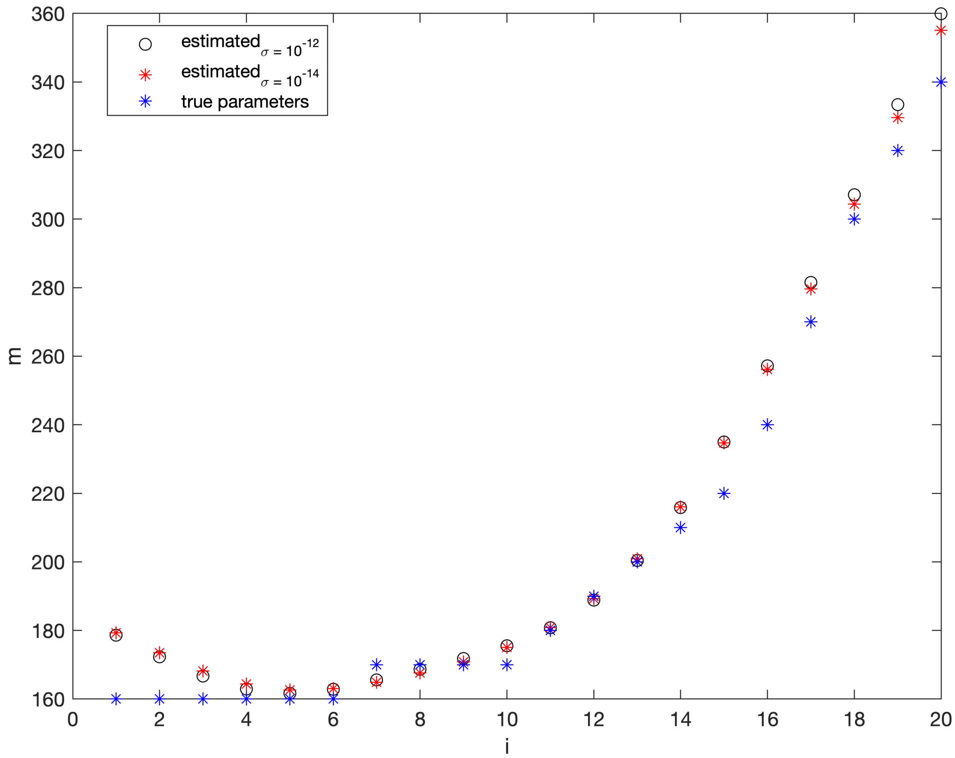
No.1

The Jacobian was coded based on the centered difference approach as follows: 1). Created an empty container, J, of dimension (20 by 20) to store a (20 by 1) vector: $\frac{(G(m+he_j)-G(m-he_j))}{(2*h)}$ at every iteration $j \ \forall \ j \in [1,n]$, forming a (20 by 20) Jacobian matrix. This is posible since m and e_j are (20 by 1) vectors, with $G(m \pm he_j)$, a function returning a (20 by 1) vector as well. The vector e_j is a standard basis vector whose single non zero element one is update at every j iteration.



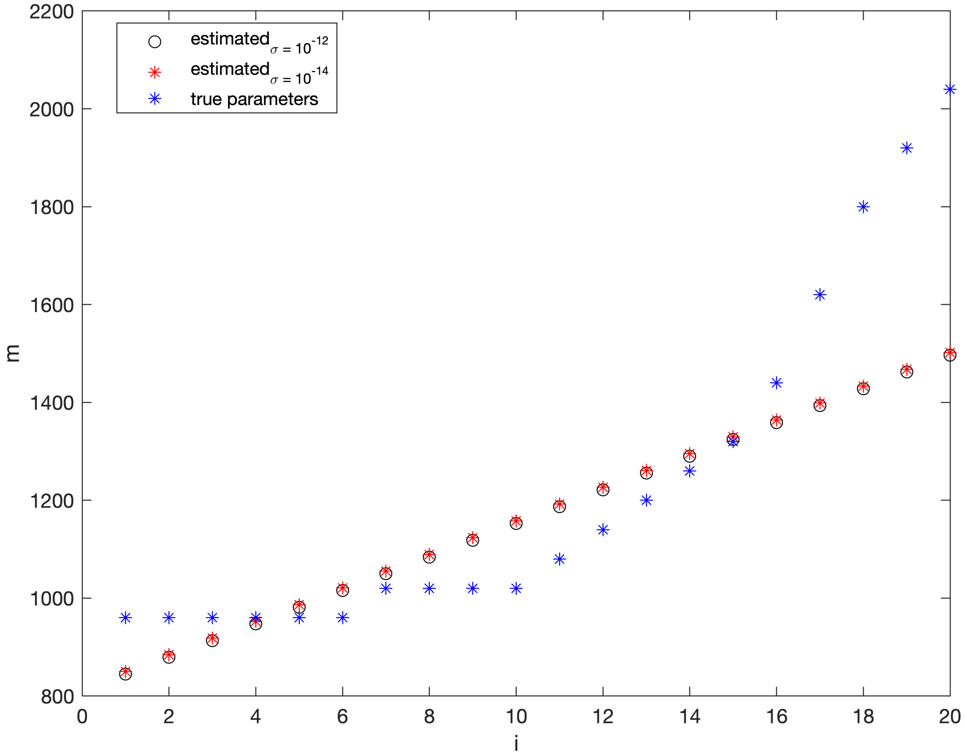
We had to downsize the value of the initial estimates from 400 to 200 for all the parameters inorder to achieve a reasonable solution, this was due to the nature of the intergration rule used as its not good enough to yeild the right approximates. On doing that we where able to obtain a close fit of the true parameter estimates as shown in the figure above.

No.1 d)



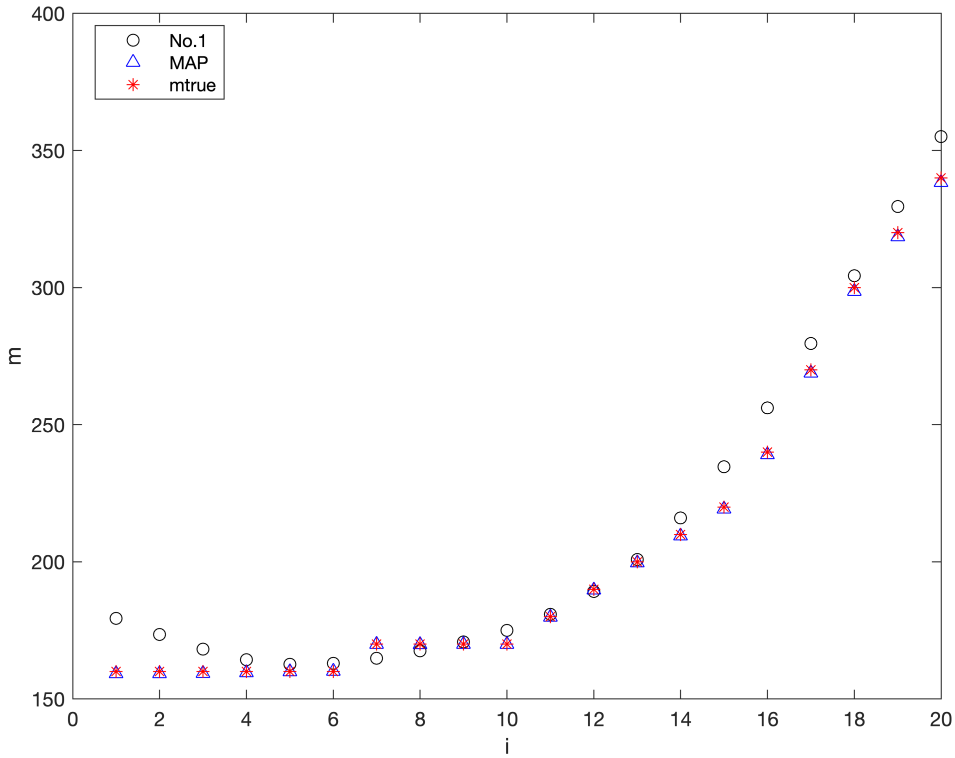
According to the plot above we somehow achieved good resolution of the true parameter estimates at the points where the inverse solution coincided with the true solution, however, the spiked model parameters are greater than the first four elements of the true parameter estimates, hence yeilding poor resolution at those points. And outside the first four elements/features, other features appeared to be real and well captured. The magnitude of σ has very less impact to the noise subjected to the data, and the spike model, since the solution coincide at all points. Although other varaibles like the step size used in the Jacobian matrix, α used in the occam model and the initial estimate high influence the inverse solution.

No.1 e)



When the density pertubation was changed to $1000\ m$ depth, this inturn increased the density pertubation and the lower and upper boundaries of the spike model test parameters. W had to adjust the m_{true} to the order of 1000 inorder to be able to use the occam model to retrive it, however due to the deep depth, the resolution is way so poor as seen from the figure above. This may be due to obstacles influencing the signal under the surface since it has to travel a penetrate along depth and bounce back in the same depth with result which is high drainged by the obstacles along the way. The solution isn't so much affected by the noise (value of σ) subjected to the data, as both solutions almost coincide with each other. Although other varaibles like the step size used in the Jacobian matrix, α used in the occam model and the initial estimate high influence the inverse solution.

No.2



The MAP solution approximates the true solution much better than the occam model in part 1, as seen in the graph above. This might be caused due to the value of the initial estimate and α used in the occam model which might not have been chosed properly. The MAP solution depended on the chosen values of the mean and covariance, and these where choosen basing on how welll the MAP model was fitting the true parameters.

The non-linear formulation was written as a regular least squares problem as follows:

$$\min \left(G(m) - d \right)^T C_D^{-1} \left(G(m) - d \right) + (m - m_{prior})^T C_M^{-1} (m - m_{prior})$$

$$= \min \left(G(m) - d \right)^T C_D^{-1/2} C_D^{-1/2} \left(G(m) - d \right) + (m - m_{prior})^T C_M^{-1/2} C_M^{-1/2} (m - m_{prior})$$

since $C_D^{-1/2} = (C_D^{-1/2})^T$ and $C_M^{-1/2} = (C_M^{-1/2})^T$

$$= \min \left((G(m) - d)^T (C_D^{-1/2})^T C_D^{-1/2} (G(m) - d) + (m - m_{prior})^T (C_M^{-1/2})^T C_M^{-1/2} (m - m_{prior}) \right)$$

$$= \min \left(\left(C_D^{-1/2} (G(m) - d) \right)^T C_D^{-1/2} (G(m) - d) + \left(C_M^{-1/2} (m - m_{prior}) \right)^T C_M^{-1/2} (m - m_{prior}) \right)$$

$$= \min \left\| \begin{bmatrix} C_D^{-1/2} G(m) \\ C_M^{-1/2} m \end{bmatrix} - \begin{bmatrix} C_D^{-1/2} d \\ C_M^{-1/2} m_{prior} \end{bmatrix} \right\|_2^2$$