MATH 568

Ch2: Confidence regions, individual activity

Continue with Exercise 1. in the textbook.

1. Assuming we are solving $\mathbf{Gm} = \mathbf{d}$, use the estimated noise in the first arrival time measurements to define $\mathrm{Cov}(\mathbf{d}) = \sigma^2\mathbf{I}$. Simplify the expression for the covariance matrix $\mathbf{C}$ for the parameter estimates, given by (2.25), so that it is only a function of $\mathbf{G}$ and σ (make sure to show your work). Use software to compute $\mathbf{C}$ and discuss the meaning of the elements of the matrix.

2. Use the covariance matrix for the parameter estimates to compute the individual 95% confidence interval for each parameter. Discuss why this may not be a good estimate of the parameter uncertainty.

3. Answer 1b. in the textbook.

4. Define the inequality for the ellipsoid that defines the confidence region and explain its meaning.

5. Use software to diagonalize $\mathbf{C}^{-1}$, i.e. find its eigenvectors and eigenvalues and specify $\mathbf{Q}$ and $\mathbf{\Lambda}$ so that $\mathbf{C}^{-1} = \mathbf{Q}\mathbf{\Lambda}\mathbf{Q}^T$. Discuss the benefits of using this decomposition to define a confidence region for the parameter estimates.

6. Answer 1c. in the textbook. Note that you already computed the 95% confidence intervals in 1. Compare the ellipsoids as uncertainty estimates to the confidence intervals as uncertainty estimates.