MATH 568

Ch9: Nonlinear regression part 2, individual activity

Consider the problem of fitting the exponential function $y(t) = m_1 e^{m_2 t}$ to the $\{(t, y)\}$ data $(1, 3.2939), (2, 4.2699), (4, 7.1749), (5, 9.3008), (8, 20.259)$, and assume the standard deviation in the data errors is 0.15.

1. Use your own code for the Gauss-Newton method to fit the data, with initial parameter estimates $m_1 = 2, m_2 = 0$. Stop the iterations when the change in the parameter estimate is less than 10^{-6}.

   (a) Approximate the Jacobian with finite differences, as given in (9.49), with a step size $h = 10^{-2}$. Report the resulting parameter estimates, the number of iterations it took to converge, and calculate the χ^2_{obs} and p-values. Discuss how good you think your parameter estimates are.

   (b) Report uncertainty estimates: Covariance matrix, confidence intervals, correlation matrix and linearized confidence ellipsoid.

2. Write your own code that use Levenberg-Marquardt method to fit the data.

   (a) Choose initial parameter estimates $m_1 = 1.7, m_2 = 0$, and through trial and error find a value of $0 < \lambda < 1$ for which the solution converges. Stop the iterations when the change in the parameter estimate is less than 10^{-6}. In addition to reporting the resulting parameter estimates, calculate the resulting χ^2_{obs} and p-values. Discuss how good you think your parameter estimates are.

   (b) Choose initial parameter estimates $m_1 = 1.6, m_2 = 0$, and through trial and error find two values of λ, of different orders of magnitude, for which the solution converges. Stop the iterations when the change in the parameter estimate is less than 10^{-6}. Report the number of iterations it took LM to converge for each value of λ.