Appendix

load ifk.mat

N01. Discretize the integral equation using simple collocation to create a square G matrix

```
[m,n] = size(d);
a = 0; b = 1;

n = m;

dx = (b-a)/m
g = @(x,y) x*exp(-x*y)
x = [];
for j = 1:n
   x = [x  a + (dx/2) + (j-1)*dx]; %form x
end
y = x;

G = zeros(m,n);
for i = 1:m
   for j = 1:n
      G(i,j) = g(x(j),y(i)).*dx; %form G
   end
end
G
```

Plot the singular values, and for well chosen values of α on the same graph.

```
% Get the singular values for the system matrix
[U,S,V] = svd(G);

j = linspace(1,m,m);
c = S(1,1);
alpha1 = 5;
alpha2 = 1;
alpha_g1 =  c./(j.^alpha1); %alpha greater than 1
alpha_l1 = c./(j.^alpha2);  %alpha less or equal 1

figure(1)
plot(diag(S),'-*'); hold on
plot(alpha_g1,'-o'); hold on
plot(alpha_l1,'-^')
legend('singular values','\alpha > 1','\alpha <= 1')
ylabel('singular values, \alpha > 1 & \alpha <= 1'); xlabel('j')
```

N0.2 Plot the log of the singular values and identify a value for i above which the singular values can reasonably be considered to be nonzero, and below which the singular values can be considered to be zero.

```
i = 9
figure(2)
semilogy(diag(S),'*')
title('Singular values')
```

```
xlabel("i"); ylabel('Singular values')
```

N0.3 Plot the log of the magnitude of the Picard ratios UT:,id/si.

```
% Picard's ratios
Si = diag(S);
Picard = [];
for i = 1:m
   Picard = [Picard abs((U(:,i)'*d)/Si(i))];
end
figure(3)
semilogy(Picard,'*')
ylabel('Magnitude of Picard ratios'); xlabel('i');
title('Magnitude of Picard ratios');
```

Identify a value for i where these coefficients abruptly increase.
i = 5

N04. Use equation (3.80) to produce truncated SVD (TSVD) model parameter estimates

```
i = 4;
mdagger = 0;
for j = 1:i
   mdagger = mdagger + (U(:,j)'*d/Si(j))*V(:,j);
end
mdagger
```

Use trial and error to choose the largest possible i so that you get a reasonable looking graph.
i = 4

Plot the best looking estimates.

```
figure(4)
plot(mdagger,'*')
ylabel('mdagger'); xlabel('i')
```

N0.5 Plot the true parameters along with your best estimate from 4. On the same graph, also plot a TSVD estimate created by truncating at i −1 singular values, where i gave you the best TSVD estimate.

```
mtrue = @(x) exp(-10*(x-0.2).^2) + 0.4*exp(-10*(x-0.9).^2)
%TSVD estimate
i = 4;
Si = Si(1:i-1);
mest = 0;
for j = 1:i-1
   mest = mest + (U(:,j)'*d/Si(j))*V(:,j);
end
mest;

figure(5)
plot(mtrue(x),'-o'); hold on
plot(mdagger,'*'); hold on
plot(mest,'^')
xlabel('i'); ylabel('estimates')
```

```
legend('mtrue','m_{i}','m_{i-1}')
```