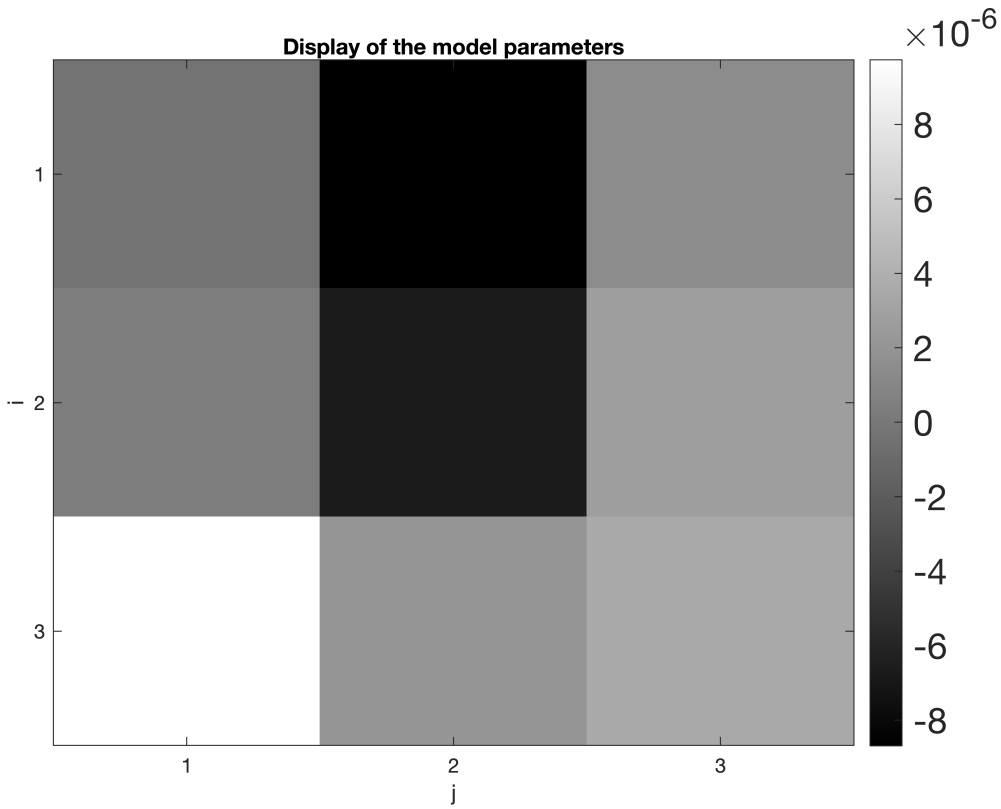
3.a) i) Use the generalized inverse of G, with the compact SVD decomposition.

```
model_parameters_1 = 9×1
10^-5 ×
   -0.0369
   -0.8697
    0.1399
    0.0303
   -0.6702
    0.2732
    0.9732
    0.2066
    0.3536
```

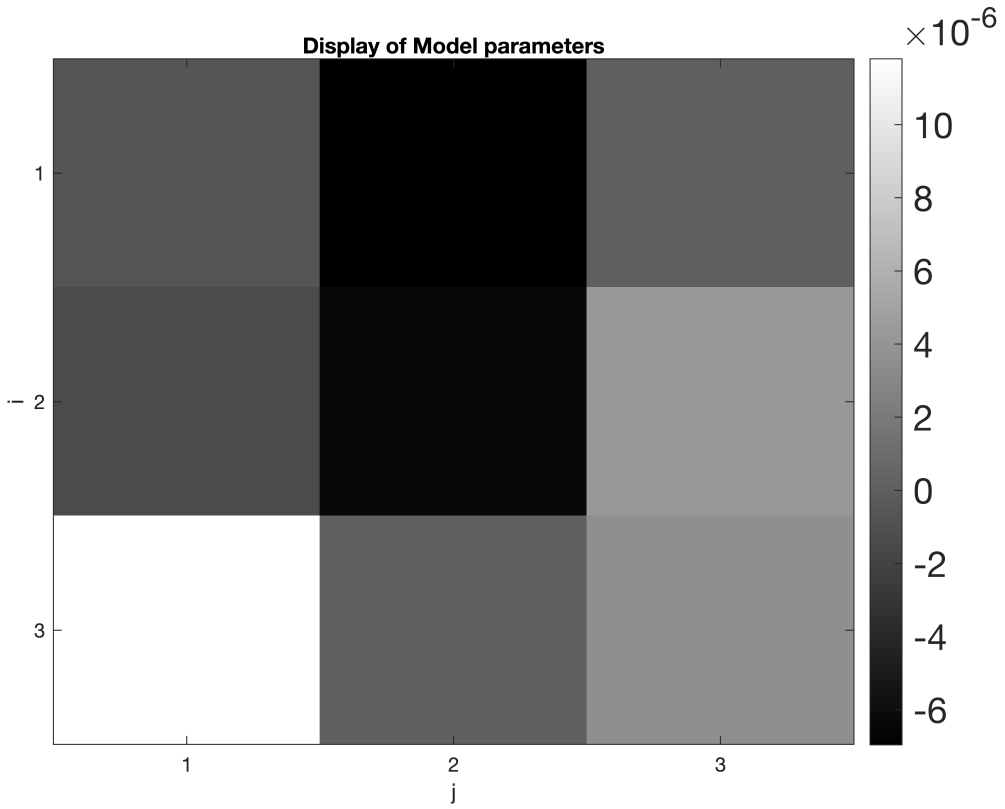
ii) Use available software, e.g. the backslash operator in MATLAB.

```
Warning: Rank deficient, rank = 7, tol =  4.895064e-15.
model_parameters_2 = 9×1
10^-4 ×
   -0.0070
   -0.0696
         0
   -0.0143
   -0.0637
    0.0413
    0.1180
         0
    0.0354
```

Plot the model parameters from 3(a)i



Plot the model parameters from 3(a)ii



Discuss the difference between estimates in 3(a)i and 3(a)ii

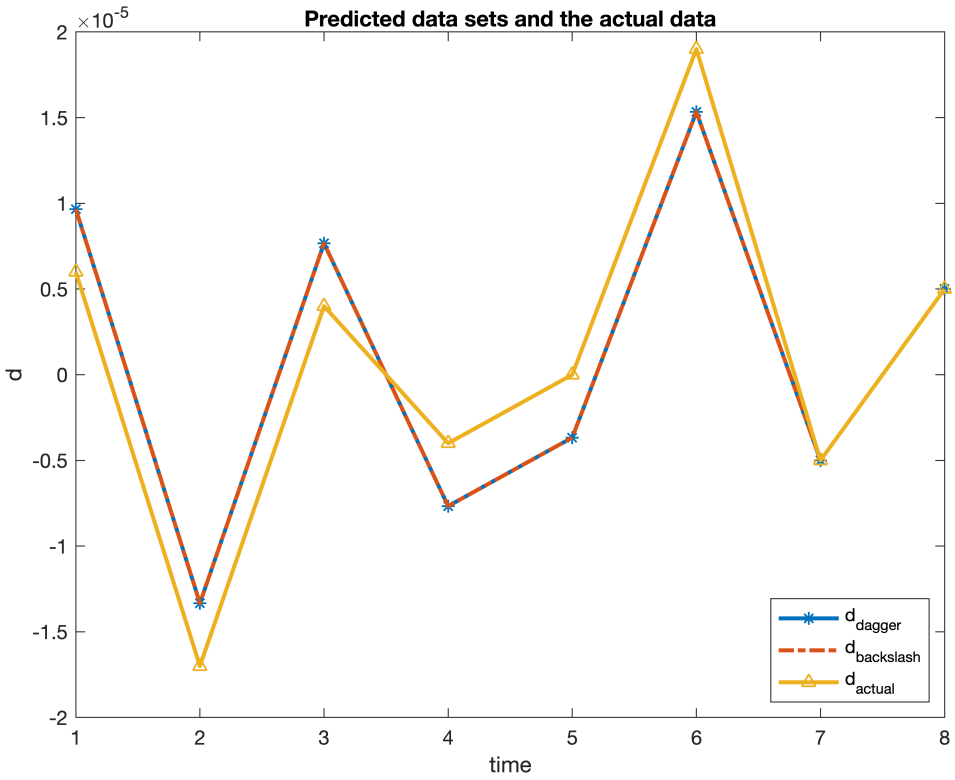
The estimates in 3(a) i, are to the order of 10^{-5} while those in 3(a)ii, are to the order of 10^{-4}.

Use each set of model parameter estimates to predict data.

```
d_dagger = 8×1
10^-4 ×
    0.0967
   -0.1333
    0.0767
   -0.0767
   -0.0367
    0.1533
   -0.0500
    0.0500

d_back_slash = 8×1
10^-4 ×
    0.0967
   -0.1333
    0.0767
   -0.0767
   -0.0367
    0.1533
   -0.0500
    0.0500
```

Compare both sets of predicted data to each other, and to the actual data.

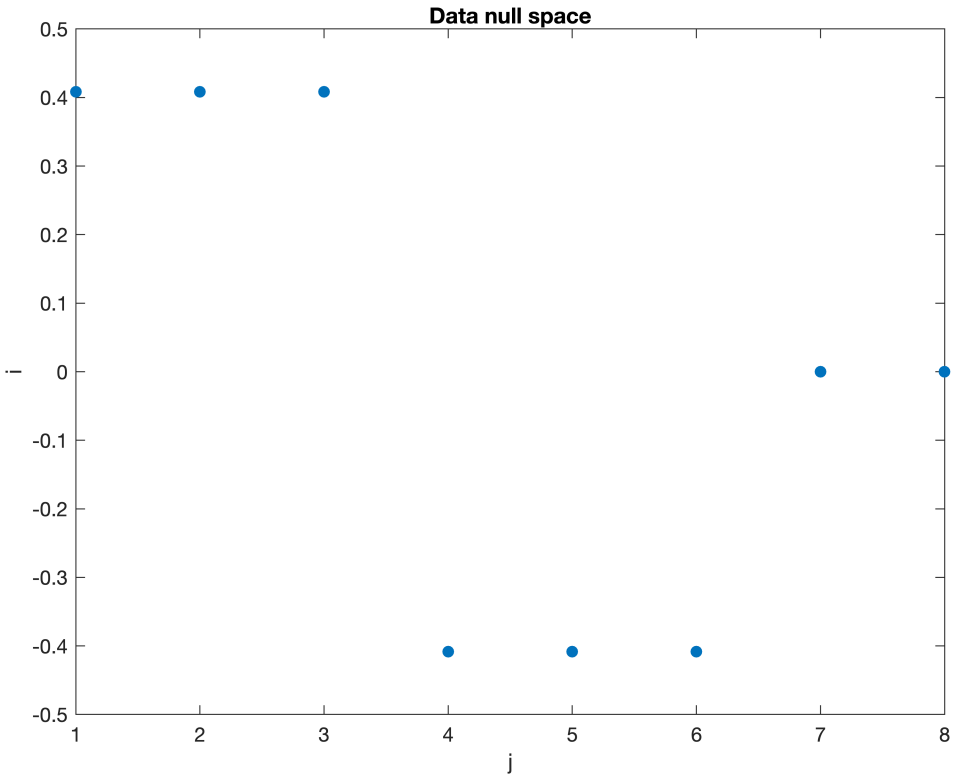


Both sets of predicted data are the same basing to the graph compared tot the actual data.

b) Determine the dimension of the data null space.

```
dim_data_Null_space = 1
```

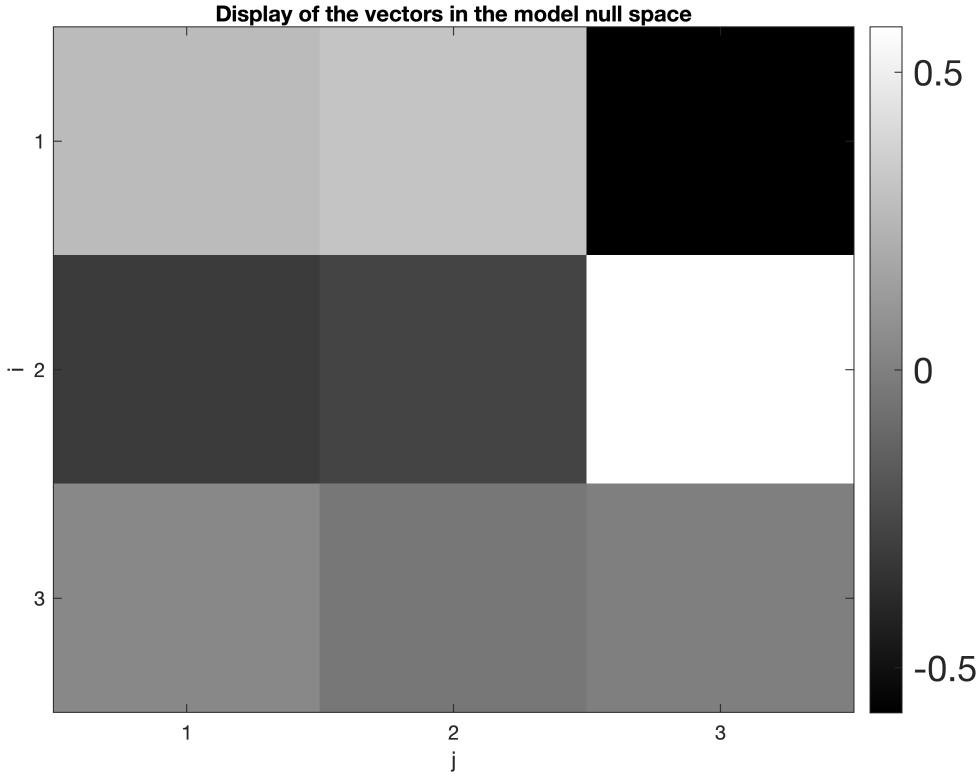
Plot the vectors in the data null space.

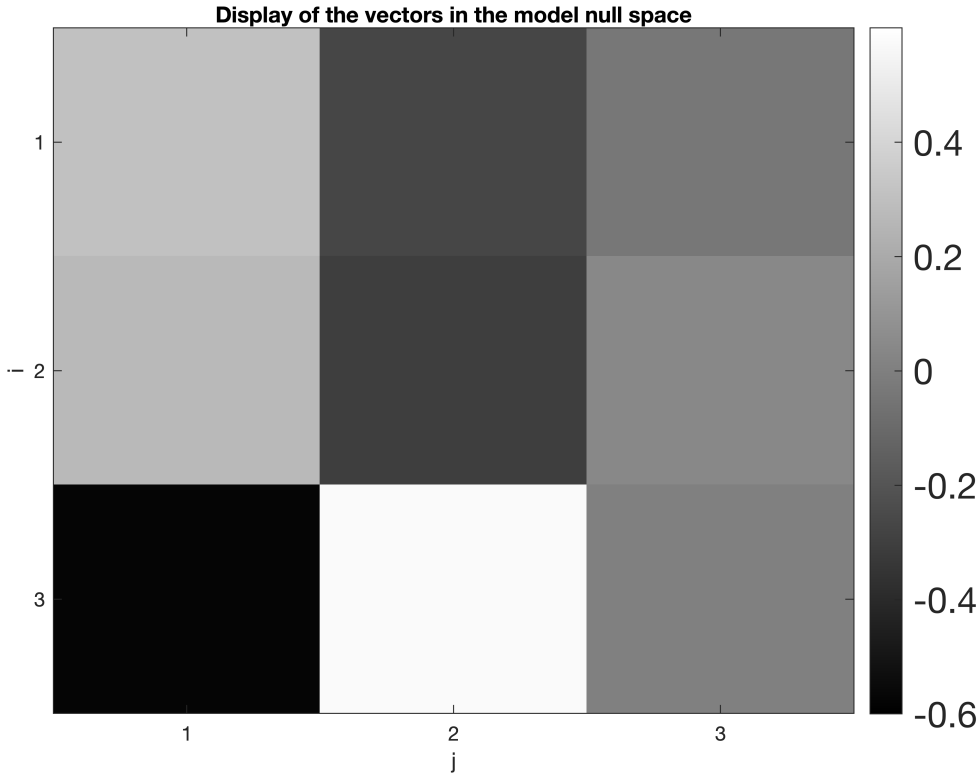


c) Determine the dimension of the model null space.

```
dim_model_Null_space = 2
```

Plot the vectors in the model null space on 3×3 grids, as they are illustrated in Figure 3.2.





(d) Is it possible to have two sets of parameters that produce the same data? Explain why or why not, and give an example if possible.

Yes, since all the model parameters are in the model null space, hence any model parameter obtained as aresult of the linear combination of the elements in the model null space set plus model parameter will always yield the same fitted data. see for instance, example 3.a) i and 3.a) ii. and also in the calculation below, the value of the fitted data is the same for different linear combinations of model null space vectors.

```
d1 = 8×1
10^-4 ×
    0.0967
   -0.1333
    0.0767
   -0.0767
   -0.0367
    0.1533
   -0.0500
    0.0500
```

```
d2 = 8×1
10^-4 ×
    0.0967
   -0.1333
    0.0767
   -0.0767
   -0.0367
    0.1533
   -0.0500
    0.0500
```

(e) Is it possible to have two sets of data that produce the same model parameters? Explain why or why not, and give an example if possible.

Yes, Since our data exists in the the data null space, hence any two sets of data produced as aresult of a linear combination of elements in the data null space plus any data will yeild the same model parameters. see for instance, the example below in the calculation gives the same model parameters for two different linear combinations.

```
Warning: Rank deficient, rank = 7, tol =  4.895064e-15.
M1 = 9×1
10^-4 ×
   -0.0070
   -0.0696
         0
   -0.0143
   -0.0637
    0.0413
    0.1180
         0
    0.0354
```

```
Warning: Rank deficient, rank = 7, tol =  4.895064e-15.
M2 = 9×1
10^-4 ×
   -0.0070
   -0.0696
         0
   -0.0143
   -0.0637
    0.0413
    0.1180
         0
    0.0354
```