MATH 365

Introduction to Inverse Methods, individual activity

Consider using the following quadratic polynomial to model a ballistic trajectory with linear regression as described in Example 1.1 of the textbook:

$$y(t) = m_1 + m_2 t - \frac{1}{2} m_3 t^2.$$

The inverse problem is to estimate m_1, m_2 and m_3 given observed altitudes $y(t_j)$, $j = 1, \ldots, m$.

1. Write out the system of equations in matrix-vector form that define the inverse problem $\mathbf{Gm} = \mathbf{d}$ when
   (a) the altitude is observed at times t_1, t_2 and t_3, i.e. when $m = 3$.
   (b) the altitude is observed at times t_1 and t_2, i.e. when $m = 2$.

2. Assume that

$$\mathbf{m}_{true} = \begin{bmatrix} 0.18 \\ 16.21 \\ 9.81 \end{bmatrix},$$

   and we have observations at 20 times with $t_j = 0, \ldots, 3$. Use MATLAB or other software to
   (a) form (i) $\mathbf{G}$, (ii) $\mathbf{d}_{true}$ and (iii) noisy data $\mathbf{d}$ where noise is normally distributed with mean $\mathbf{0}$ and standard deviation 2. The noise can be generated in MATLAB using the command `2*randn(m,1)`. Plot $\mathbf{d}_{true}$ and the noisy data as points on the same graph with appropriate labels. Discuss the difference between the data.
   (b) solve $\mathbf{Gm} = \mathbf{d}$ for $\mathbf{m}$ using the noisy data in 2a. Plot $y(t)$ using $\mathbf{m}$ and $\mathbf{m}_{true}$ on the same graph with appropriate labels. Discuss the difference between the trajectories.