Appendix

```
d =  [6e-06 -1.7e-05 4e-06 -4e-06 0 1.9e-05 -5e-06 5e-06]';

% Construct system matrix for the ray path models
s2=sqrt(2);
G = [1,0,0,1,0,0,1,0,0;
    0,1,0,0,1,0,0,1,0;
    0,0,1,0,0,1,0,0,1;
    1,1,1,0,0,0,0,0,0;
    0,0,0,1,1,1,0,0,0;
    0,0,0,0,0,0,1,1,1;
    s2,0,0,0,s2,0,0,0,s2;
    0,0,0,0,0,0,0,0,s2];

% Get the singular values for the system matrix
[U,S,V] = svd(G);
```

3.a) i) Use the generalized inverse of G, with the compact SVD decomposition.

```
% Find dimensions of G
[m,n]=size(G);

%disp('System rank:')
p=rank(G);

Gdagger = V(:,1:p)*inv(S(1:p,1:p))*U(:,1:p)';

%Estimates
model_parameters_1 = Gdagger*d
model_parameters_11 = pinv(G, 4.8e-15)*d;
```

ii) Use available software, e.g. the backslash operator in MATLAB.

```
model_parameters_2 = G\d
```

Plot the model parameters from 3(a)i

```
figure(7)
clf
colormap('gray')
imagesc(reshape(model_parameters_1,3,3)');
```

```
%caxis([-0.9e-5 1e-5])
set(colorbar,'Fontsize',18);
set(gca,'xtick',[1,2,3]);
set(gca,'ytick',[1,2,3]);
xlabel('j')
ylabel('i')
title('Display of the model parameters')
```

Plot the model parameters from 3(a)ii

```
figure(8)
clf
colormap('gray')
imagesc(reshape(model_parameters_2,3,3)');
%caxis([-0.1e-4 0.2e-4])
set(colorbar,'Fontsize',18);
set(gca,'xtick',[1,2,3]);
set(gca,'ytick',[1,2,3]);
xlabel('j')
ylabel('i')
title('Display of Model parameters')
```

Use each set of model parameter estimates to predict data.

```
d_dagger = G*model_parameters_1 %using model parameter_1
d_back_slash = G*model_parameters_2 %using model parameter_2
```

Compare both sets of predicted data to each other, and to the actual data.

```
figure(10)
plot(d_dagger,'-*','LineWidth', 2)
hold on
plot(d_back_slash,'-.','LineWidth', 2)
hold on
plot(d,'-^','LineWidth', 2)
legend('d_{dagger}','d_{backslash}','d_{actual}','Location','southeast')
title('Predicted data sets and the actual data')
xlabel('time'); ylabel('d');
```

b) Determine the dimension of the data null space.

dim_data_Null_space = m-p

Plot the vectors in the data null space.

```
figure(9)
plot(U(:,8),'.','MarkerSize',20)
title('Data null space')
xlabel('j')
ylabel('i')
```

c) Determine the dimension of the model null space.
dim_model_Null_space = 2
Plot the vectors in the model null space on 3×3 grids, as they are illustrated in Figure 3.2.

```
m01=reshape(V(:,p+1),3,3)';
m02=reshape(V(:,p+2),3,3)';

figure(1)
clf
colormap('gray')
imagesc(m01)
%caxis([-0.6 0.6]);
set(colorbar,'Fontsize',18);
set(gca,'xtick',[1,2,3]);
set(gca,'ytick',[1,2,3]);
xlabel('j')
ylabel('i')
title('Display of the first vectors in the model null space');

figure(2)
clf
colormap('gray')
imagesc(m02)
caxis([-0.6 0.6]);
set(colorbar,'Fontsize',18);
set(gca,'xtick',[1,2,3]);
set(gca,'ytick',[1,2,3]);
xlabel('j')
ylabel('i')
title('Display of the second vectors in the model null space');
```

(d) Is it possible to have two sets of parameters that produce the same data?

```
M1 = model_parameters_1 + 6*V(:,8) + 5*V(:,9);
d1 = G*M1

M2 = model_parameters_2 + 0.6*V(:,8) + 0.9*V(:,9);
d2 = G*M2
```

(e) Is it possible to have two sets of data that produce the same model parameters? Explain why or why not, and give an example if possible.

```
d1 = d_dagger + 6*U(:,8);
M1 = G\d1

d2 = d_back_slash + 0.9*U(:,8);
M2 = G\d2
```