Well-conditioned problems

```
t = [3.4935 4.2853 5.1374 5.8181 6.8632 8.1841]';

x = [6 10.1333 14.2667 18.4000 22.5333 26.6667]';
```

No.1

a). Find the least squares solutions for the model parameters t_o and s2

```
exp_val = 0; % expected value
sig = 0.1; % standard deviation

% model
m = length(t);

G = [ones(m,1) x]; % matrix G
M_L2 = inv(G'*G)*G'*t; % least square solution

t_o = M_L2(1) % to
```

```
t_o = 2.0323
```

```
s_2 = M_L2(2) % s2
```

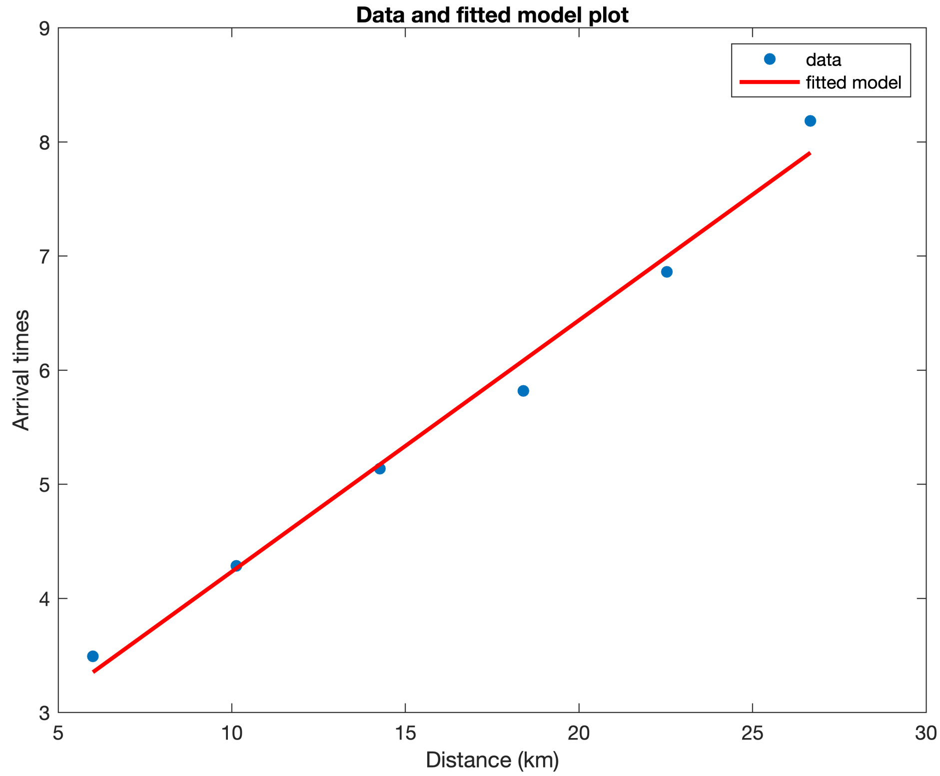
```
s_2 = 0.2203
```

Plot the data and the fitted model on the same graph, and the residuals on the second graph.

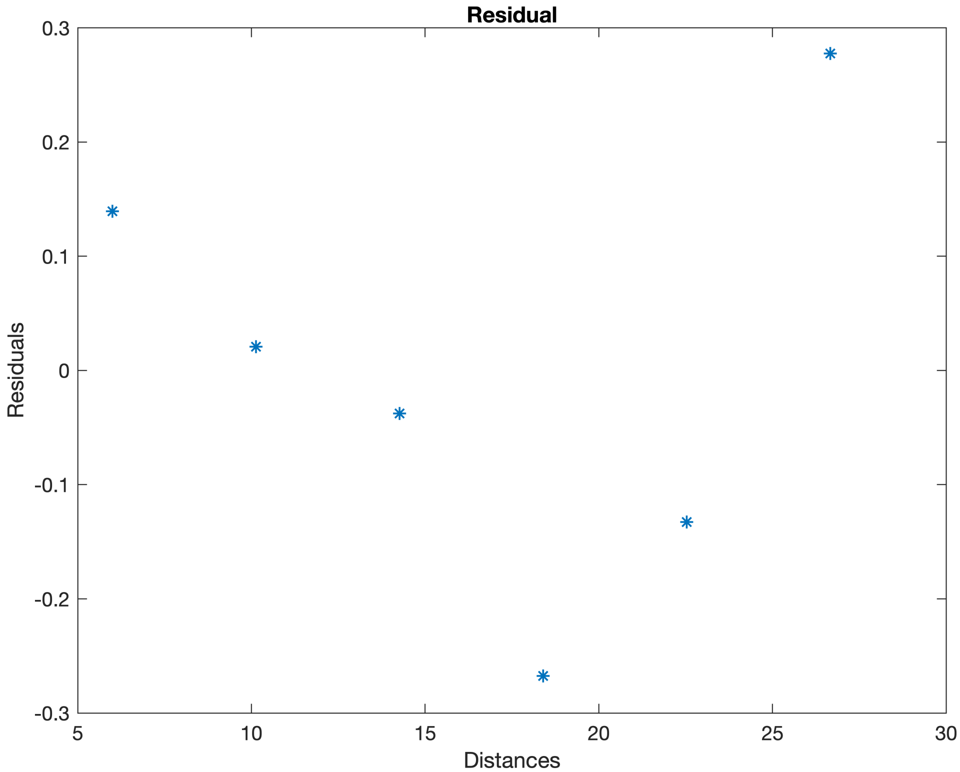
```
% fitted model
T = zeros(m,1);
for i =1:m
    T(i) = t_o + s_2*x(i);
end

% residuls
GM = G*M_L2;
r = t - GM;

%plot
figure(1)
plot(x,t,'.','MarkerSize',20)
hold on
plot(x,T,'r',LineWidth=2)
legend('data','fitted model')
ylabel('Arrival times')
xlabel('Distance (km)')
title('Data and fitted model plot')
```




```
figure(2)
plot(x,r,'*',LineWidth=2)
ylabel('Residuals')
xlabel('Distances')
title('Residual')
```



No.2

Use the standard deviation to form a diagonal weighting matrix

```
W = (1/sig)*eye(m)
```

```
W = 6×6
    10     0     0     0     0     0
     0    10     0     0     0     0
     0     0    10     0     0     0
     0     0     0    10     0     0
     0     0     0     0    10     0
     0     0     0     0     0    10
```

Find the maximum likelihood estimate of the parameters

```
M_mle = inv(G'*(W^2)*G)*G'*(W^2)*t
```

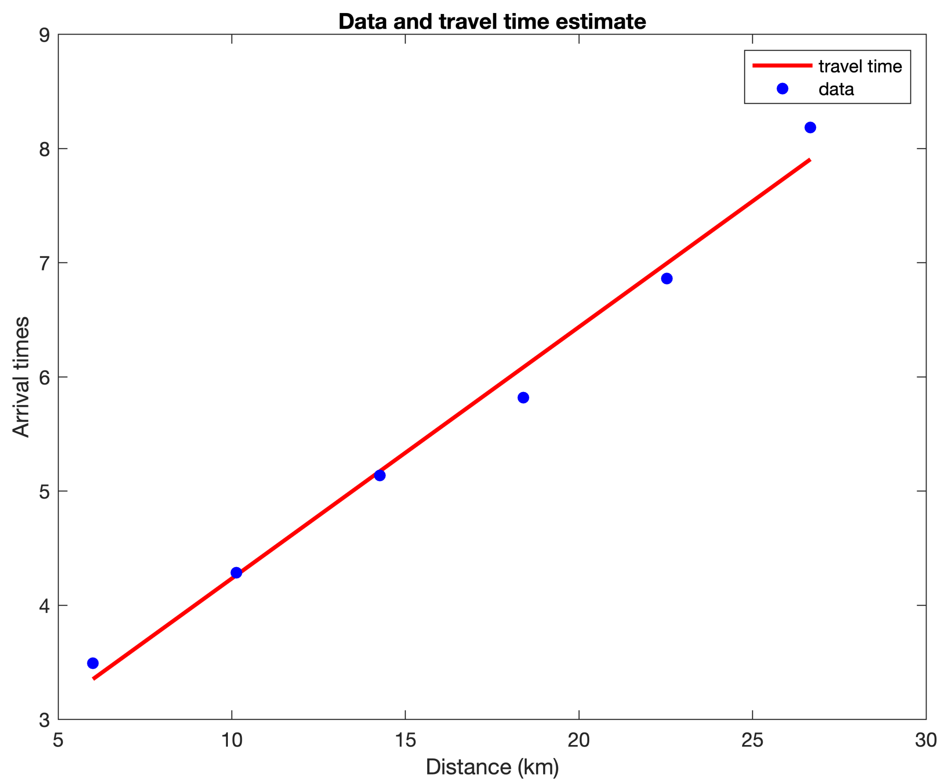
```
M_mle = 2×1
    2.0323
    0.2203
```

```
% Estimate travel times
t_times = zeros(m,1);
for i =1:m
    t_times(i) = M_mle(1) + M_mle(2)*x(i);
end
t_times
```

```
t_times = 6×1
```

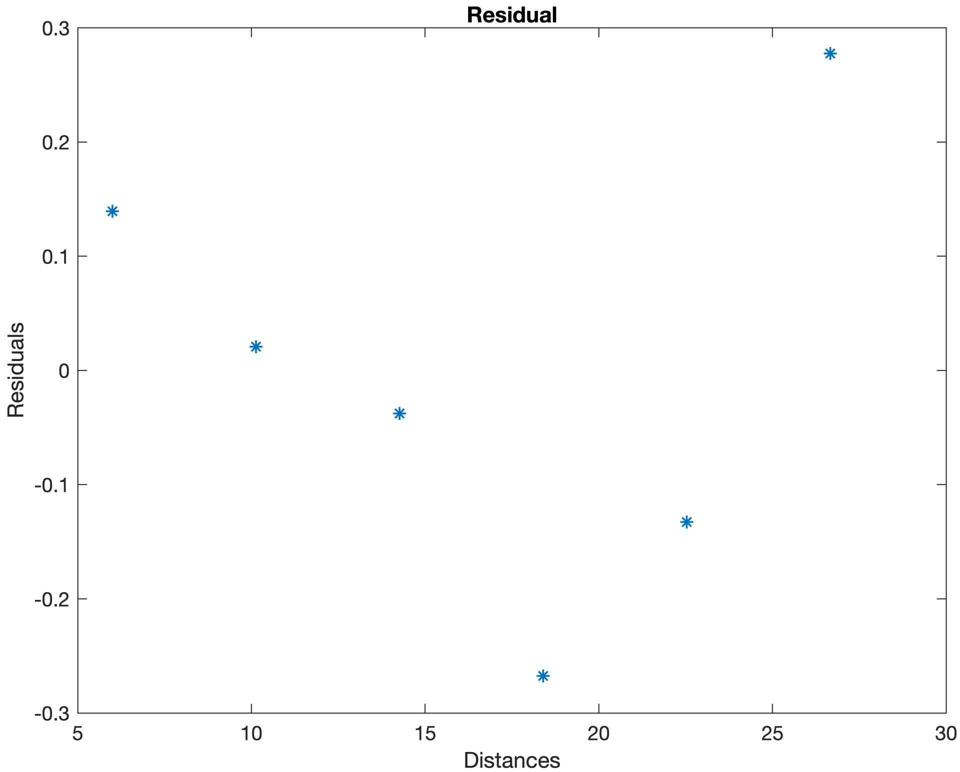
```
    3.3540
    4.2645
    5.1750
    6.0855
    6.9960
    7.9065
```

```
figure(3)
plot(x,t_times,'r',LineWidth=2)
hold on
plot(x,t,'b.','MarkerSize',20)
legend('travel time','data')
ylabel('Arrival times')
xlabel('Distance (km)')
title('Data and travel time estimate')
```




```
% residuls
Gm = G*M_mle;
r1 = t - Gm;

figure(4)
plot(x,r1,'*',LineWidth=2)
ylabel('Residuals')
xlabel('Distances')
title('Residual')
```



Discuss and explain the difference between the weighted parameter estimates, graphs of their corresponding travel times and residuals, to those you found in 1.

Since the standard deviation (σ = 0.1) is a constant value, this makes the weighted parameter estimates to be equal to the least squares estimates, i.e., $M_{mle} = (G^TW^2G)^{-1}G^TW^2t = (G^T(\sigma^{-1}I)^2G)^{-1}G^T(\sigma^{-1}I)^2t = \sigma^2(G^TG)^{-1}\sigma^{-2}(G^TIG) = (G^TG)^{-1}(G^TG) = M_{L2}$. So getting the equal parameters makes their corresponding travel times and residuals in the two cases to be the same.

No.3

Calculate the chi-square statistic for the data

```
% using the least squares parameter estimate
chi_lsq = (t-G*M_L2)'*(W^2)*(t - G*M_L2)
```

```
chi_lsq = 18.7502
```

```
 % using the maximum likelihood parameter
chi_ml = (t-G*M_mle)'*(W^2)*(t - G*M_mle)
```

```
chi_ml = 18.7502
```

In each case, and simply by inspection, discuss if the value of the χ^2 statistic is near the expected value of the χ^2 random variable with appropriate degrees of freedom.

Since $M_{mle} = M_{L2}$, this means that in both cases the χ^2 statistic is the same which is $\chi^2 = 18.7502$. This values is much greater than the expexted value ,$\nu = 4$, of the χ^2 random variable with appropriate degrees of freedom, hence the χ^2 statistic is not near χ^2 random variable and doesn't follow a chi-square distribution.

No.4

Evaluate the p-value for this model.

```
m = 6;
n = 2;
p = 1 - chi2cdf(chi_lsq,m-n)
```

```
p = 8.7992e-04
```

Interpret the p-value. Discuss if this conclusion matches the conclusion you made by inspection in 3.

The p-value is very close to 0, therefore we reject the null hypothesis. This conclusion matches the conclusion made in 3 by inspection, since the p-value being very small and near to zero, means that it exists in the skewed region, which is very far away from the expected value.