Herbeeb and Brian

Home Work 1

4.(a) Note the rank of your G matrix that relates the data and model.

```
p = 74
```

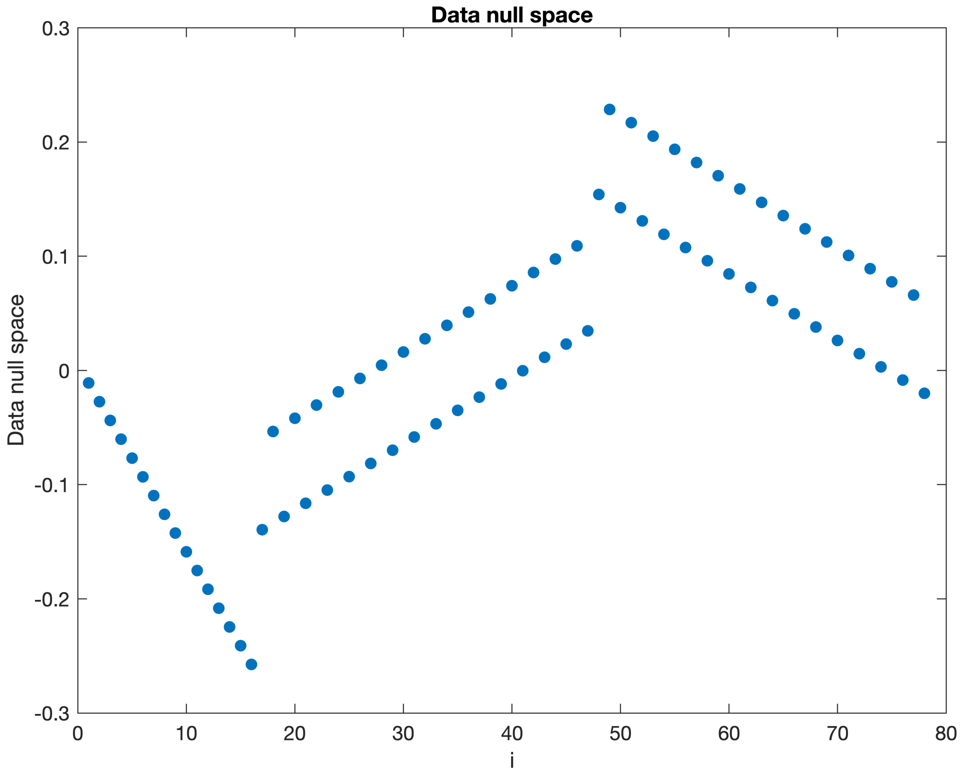
4.b) i. State and discuss significance of the elements and dimensions of the data and model null spaces.

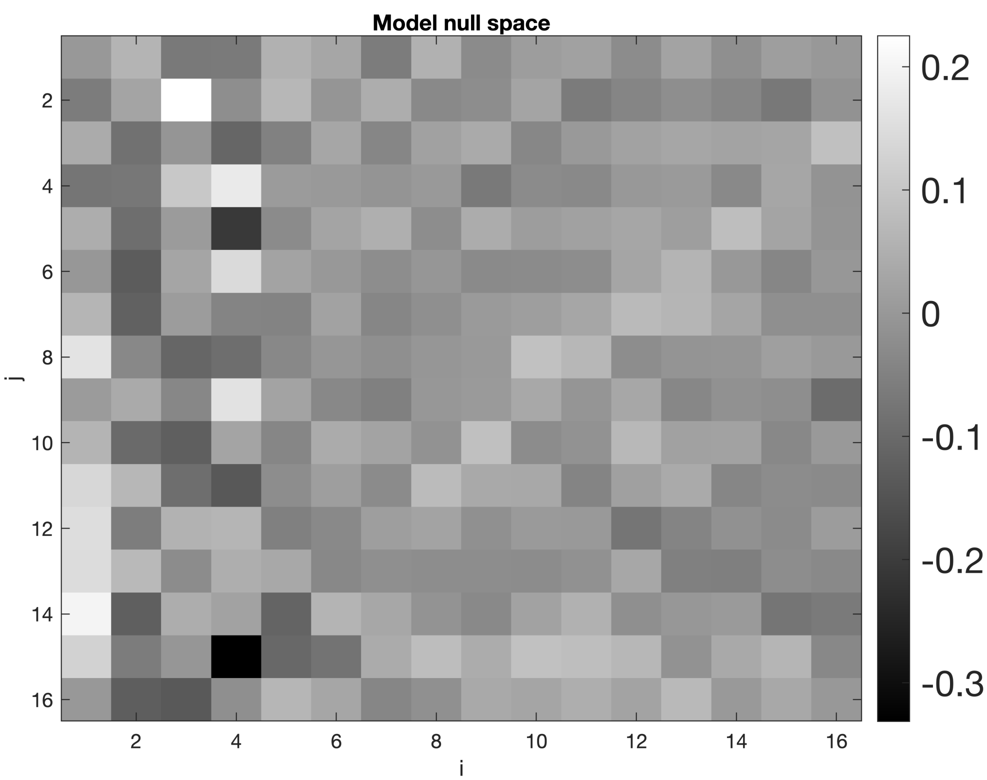
```
data_null_space = 78×4
   -0.0109   -0.3172         0         0
   -0.0274   -0.2920   -0.0033    0.0112
   -0.0438   -0.2668   -0.0066    0.0224
   -0.0602   -0.2416   -0.0099    0.0337
   -0.0767   -0.2164   -0.0132    0.0449
   -0.0931   -0.1912   -0.0165    0.0561
   -0.1095   -0.1660   -0.0198    0.0673
   -0.1260   -0.1408   -0.0231    0.0785
   -0.1424   -0.1156   -0.0264    0.0897
   -0.1588   -0.0904   -0.0296    0.1010
      ⋮

dim_data_null_space = 4

model_null_space = 256×182
   -0.0000    0.0000    0.0000   -0.0000    0.0000    0.0000   -0.0000   -0.0000 ⋯
   -0.0603    0.1240   -0.0442   -0.0446    0.0517    0.0294    0.1198   -0.1210
    0.0405    0.3129   -0.0141    0.0560    0.0329   -0.0212    0.0444    0.0963
   -0.0752    0.2866   -0.1164   -0.0217   -0.0985    0.0471    0.0490    0.0548
    0.0459    0.2216   -0.0506   -0.0106    0.1276   -0.0096    0.0003    0.0768
   -0.0029    0.1732    0.1082   -0.0355    0.0446    0.0764   -0.2475    0.0134
    0.0620    0.1162    0.0407   -0.0604    0.0163    0.0876   -0.1000   -0.1393
    0.1625    0.0933    0.0781   -0.0810    0.0840    0.0052    0.0315   -0.0498
    0.0063    0.0820    0.0274   -0.0845    0.0508    0.0110    0.0220   -0.0001
    0.0619    0.1185    0.0172   -0.2029    0.0401    0.0056   -0.0105    0.0159
      ⋮

dim_model_null_space = 182
```

Discuss the significance of the elements in each null space

Since the model and data null spaces contain non-zero vectors, then the problem under consideration is rank deficient. So, we will have infinitely many models that can fit the data exactly due to the existence of non-trivial model model null space. Also, we have infinitely many data lying in the data null space, this means there are many datasets that can produce the same model parameters.

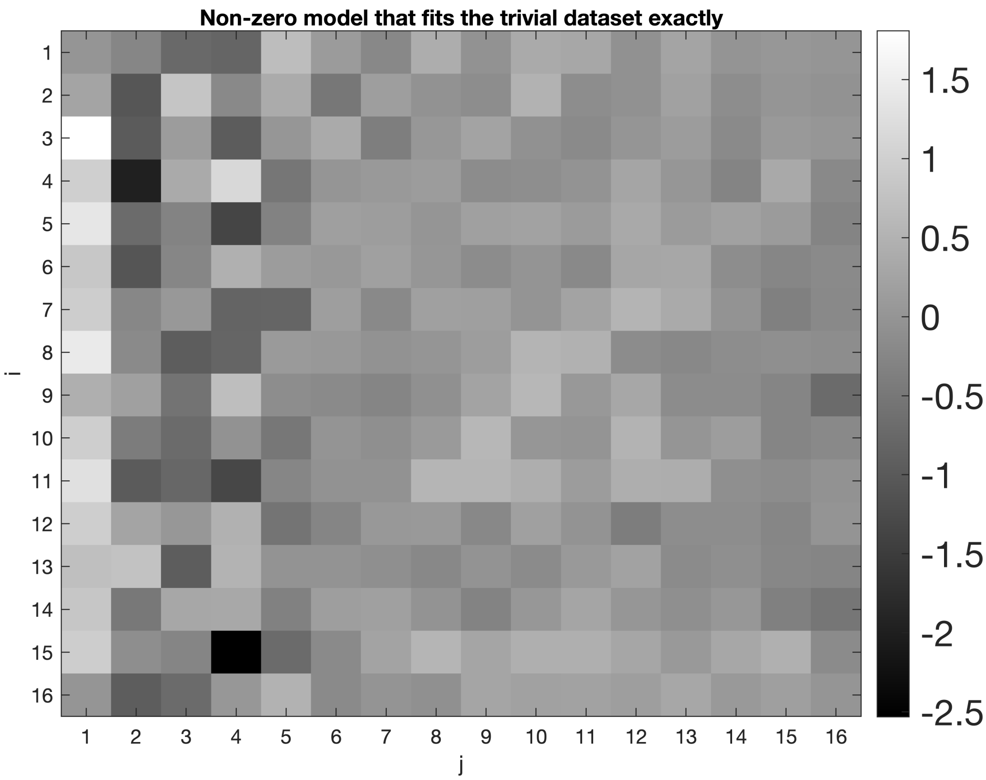
Plot and interpret at least one element of each space,



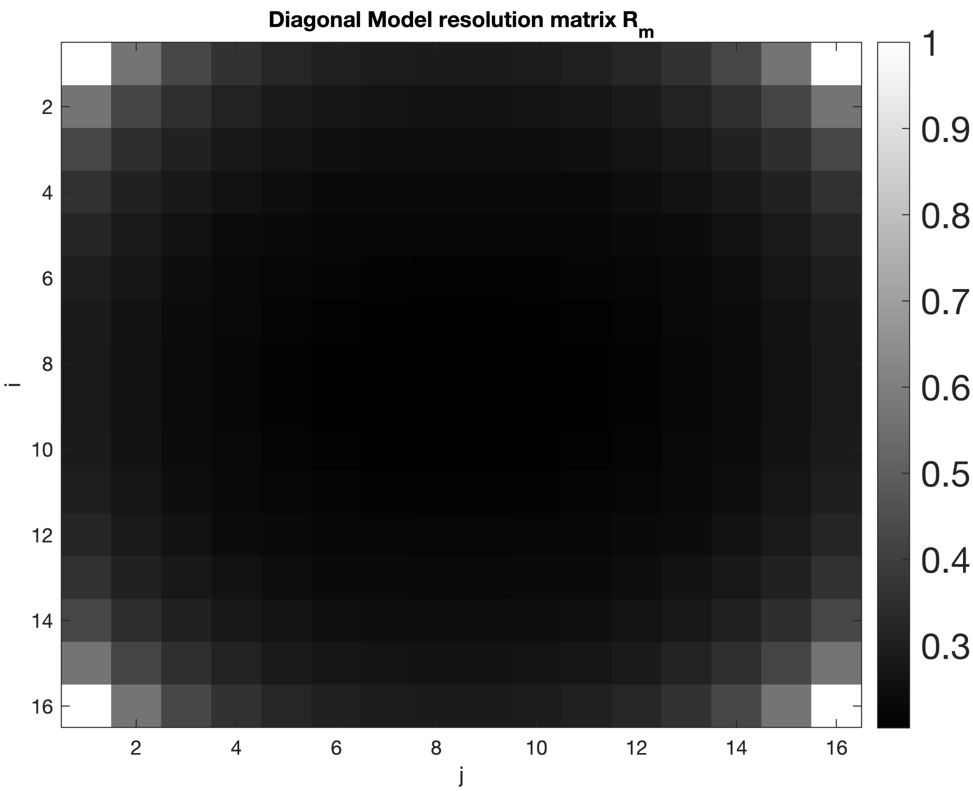


ii. Contour or otherwise display a nonzero model that fits the trivial data set exactly

Any vector in the V_i in the model null space satisfies the equation $GV_i = 0$ hence we can form a model m_o where $m_o = \sum_{i=p+1}^{n} \alpha_i V_{:,i}$. We have some many vectors in the model null space so we can pick the first two and set $\alpha = 0$ for the rest.



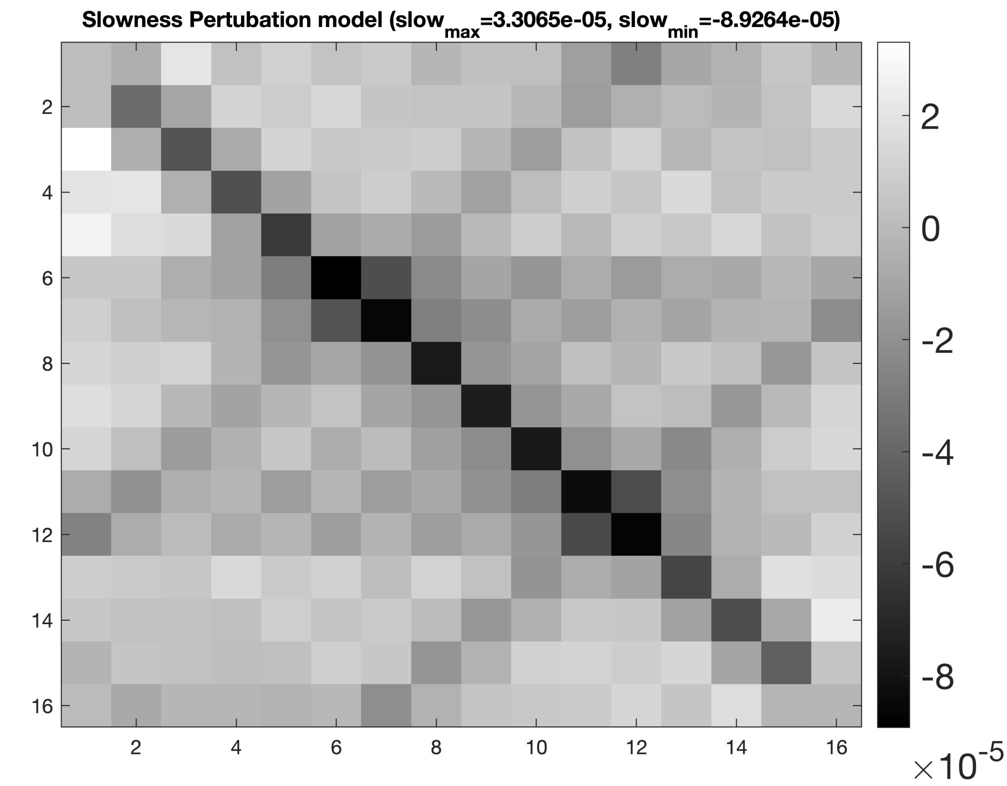
iii. Show the model resolution by contouring or otherwise displaying the 256 diagonal elements of the model resolution matrix, reshaped into an appropriate 16 by 16 grid. Note if there are any model parameters that have perfect resolution.



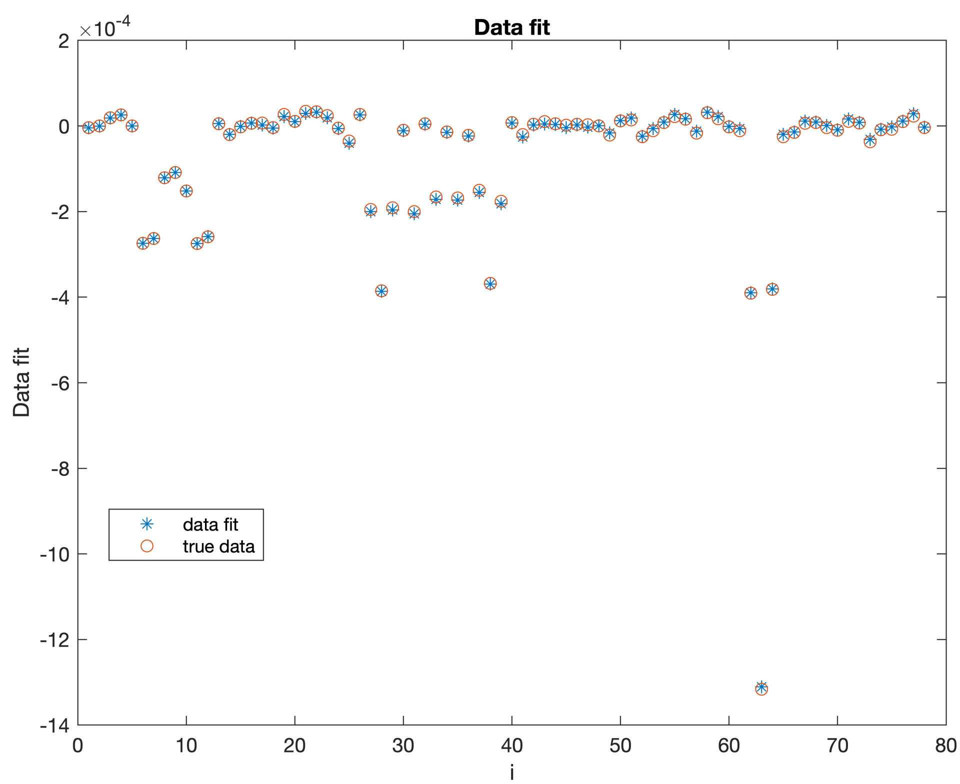
The model parameters corresponding to the four edges (m_1, m_16, m_241, and m_256) have perfect resolutions.

(c) Produce a 16 by 16 element contour or other plot of your slowness perturbation model, displaying the maximum and minimum slowness perturbations in the title of the plot. Interpret any internal structures geometrically and in terms of seismic velocity (in m/s).

The row and diagonal scan solution in the figure below, depicts that there is low slowness(high velocity) along the diagonal, this anomaly is around $-8.9x10^{-5}$ s/m, inthat incoporating the background velocity of 3000 m/s, yeilds a velocity approximately $\left(\frac{1}{3000} - 8.9x10^{-5}\right) \approx 2.44x10^{-4}$ = 4092m/s. However the rowscans aren't well depicted due to limited resolution.



In addition, produce a plot of the data fit and discuss it.



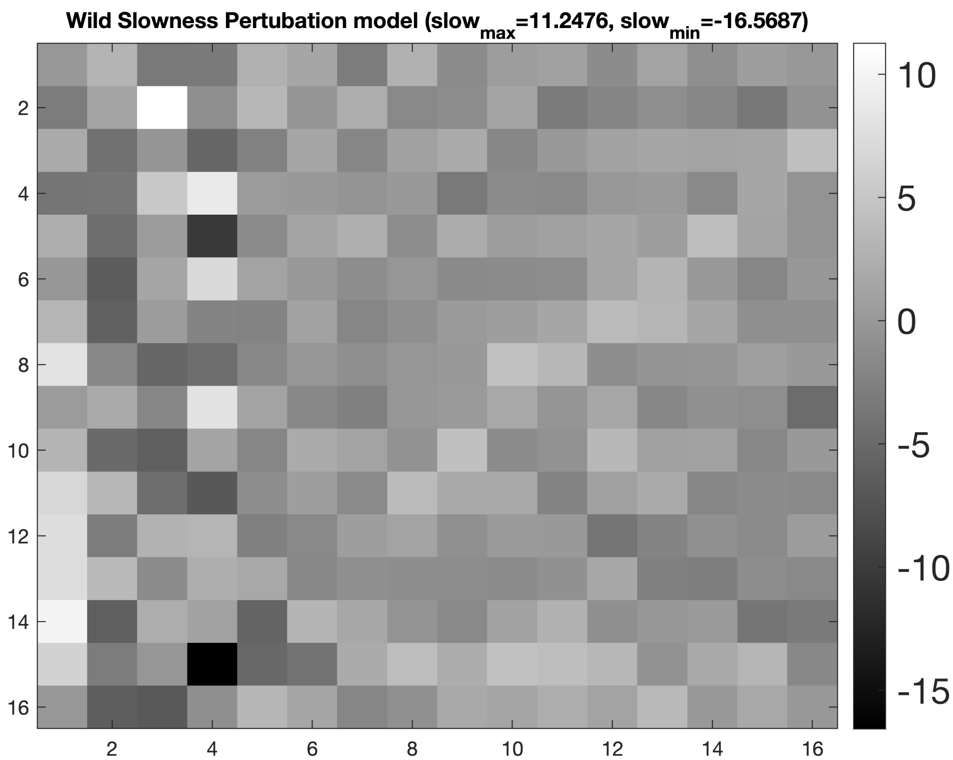
Comment

According to the plot, the estimated data almost fits the true data.

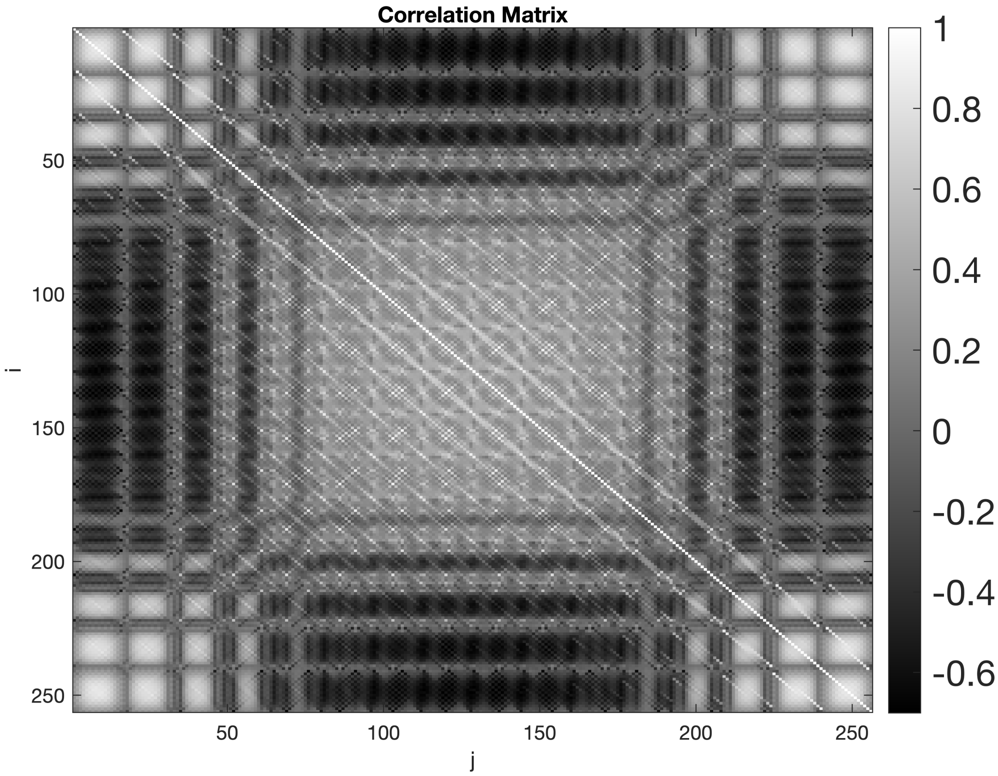
(d) Describe how one could use solutions to Gm = d = 0 to demonstrate that very rough models exist that will fit any data set just as well as a generalized inverse model.

A wild model can be generated by superimpossing scaled basis vectors from the model null space onto the generalized inverse solution. We used 50 times the first basis vector from the model nullspace to add to $m_{\dagger}$ to obtain a model in the figure below.

Show one such wild model.



(e) Plot and interpret the correlation matrix.



The plot shows how the parameter estimates correlate with each other. The diagonal elements shows how an estimate correlate with itself and as expected, we have perfect correlation along the diagonal. Also, the 4 edges show that the estimates in the edg have high correlation with each other

(f) Quantify and discuss stability of slowness perturbation estimates.

```
CondGd = 6.7064
```

Since the condition number (6.7064) is small, then the estimates are stable and a small change in the data, will not result in a large change in the estimate.

```
CondG = 9.5704e+15
```

The condition value of G is almost infinite. This means that small changes in the data may result in large changes in our parameter estimates.