MATH 568

Ch2: Well-conditioned problems, individual activity

Consider Exercise 1. in the textbook. The data are in the MATLAB file **profile.mat** and contain the first arrival times

$$\mathbf{t} = \begin{bmatrix} 3.4935 \\ 4.2853 \\ 5.1374 \\ 5.8181 \\ 6.8632 \\ 8.1841 \end{bmatrix}$$

of seismic energy from a mid-crustal refractor, measured in seconds after the source origin time. The arrival times are observed at distances (in kilometers) of

$$\mathbf{x} = \begin{bmatrix} 6 \\ 10.1333 \\ 14.2667 \\ 18.4000 \\ 22.5333 \\ 26.6667 \end{bmatrix}$$

from the source.

1. Answer 1a. in the textbook and estimate the parameters by solving the normal equations for the least squares estimate. Plot the data and the fitted model on the same graph, and the residuals on a second graph.

2. Use the standard deviation to form a diagonal weighting matrix $\mathbf{W} = \sigma^{-1}\mathbf{I}$. Find the maximum likelihood estimate of the parameters by solving the normal equations, and use it to estimate the travel times. Plot the travel time estimate estimate along with the data on the same graph, and the residuals on a second graph. Discuss and explain the difference between the weighted parameter estimates, graphs of their corresponding travel times and residuals, to those you found in 1.

3. Calculate the χ^2 statistic for the data, i.e. (2.20), using vector multiplation rather than the summation. Find the χ^2 statistic with the least squares parameter estimate you found in 1 and the statistic the the maximum likelihood parameter estimate you found in 2. In each case, and simply by inspection, discuss if the value of the χ^2 statistic is near the expected value of the χ^2 random variable with appropriate degrees of freedom.

4. Answer 1d. in the textbook, and interpret the p-value. Discuss if this conclusion matches the conclusion you made by inspection in 3.