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Class: GEOPH 605

Consider the following quadratic polynomial

$$y(t) = m_1 + m_2 t - \frac{1}{2} m_3 t^2$$

estimate: m_1, m_2, m_3

observed altitudes: $y(t_j)$, $j = 1, \ldots, m$

1. Matrix vector form

(a) when $m = 3$, altitude observed at t_1, t_2 and t_3

$$\begin{bmatrix} 1 & t_1 & -\frac{1}{2} t_1^2 \\ 1 & t_2 & -\frac{1}{2} t_2^2 \\ 1 & t_3 & -\frac{1}{2} t_3^2 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

(b) when $m = 2$, altitude observed at t_1 and t_2

$$\begin{bmatrix} 1 & t_1 & -\frac{1}{2} t_1^2 \\ 1 & t_2 & -\frac{1}{2} t_2^2 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

2. Assume that

$$m_{true} = \begin{bmatrix} 0.18 \\ 16.21 \\ 9.81 \end{bmatrix}$$

```
mtrue = [0.18 16.21 9.81]' %true m
```

```
mtrue = 3×1
    0.1800
   16.2100
    9.8100
```

```
m = 20; %rows
n = 3;  %columns

tj = linspace(0,3,m) %temporal domain
```

```
tj = 1×20
         0    0.1579    0.3158    0.4737    0.6316    0.7895    0.9474    1.1053 ···
```

N0.2 a)

(i). Form G

```
%forming G matrix
G = zeros(m,n); %m by n container

for i = 1:n
    for j = 1:m
       G(j,1) = 1;
       G(j,2) = tj(j);
       G(j,3) = -0.5*tj(j)^2;
    end
end
disp(G)
```

```
    1.0000         0         0
    1.0000    0.1579   -0.0125
    1.0000    0.3158   -0.0499
    1.0000    0.4737   -0.1122
    1.0000    0.6316   -0.1994
    1.0000    0.7895   -0.3116
    1.0000    0.9474   -0.4488
    1.0000    1.1053   -0.6108
    1.0000    1.2632   -0.7978
    1.0000    1.4211   -1.0097
    1.0000    1.5789   -1.2465
    1.0000    1.7368   -1.5083
    1.0000    1.8947   -1.7950
    1.0000    2.0526   -2.1066
    1.0000    2.2105   -2.4432
    1.0000    2.3684   -2.8047
    1.0000    2.5263   -3.1911
    1.0000    2.6842   -3.6025
    1.0000    2.8421   -4.0388
    1.0000    3.0000   -4.5000
```

(ii). Form dtrue

```
dtrue =  G*mtrue;
disp(dtrue)
```

```
    0.1800
    2.6172
    4.8098
```

```
    6.7579
    8.4613
    9.9202
   11.1346
   12.1043
   12.8295
   13.3102
   13.5462
   13.5377
   13.2846
   12.7869
   12.0447
   11.0579
    9.8265
    8.3506
    6.6301
    4.6650
```

(iii). Form noisy data d

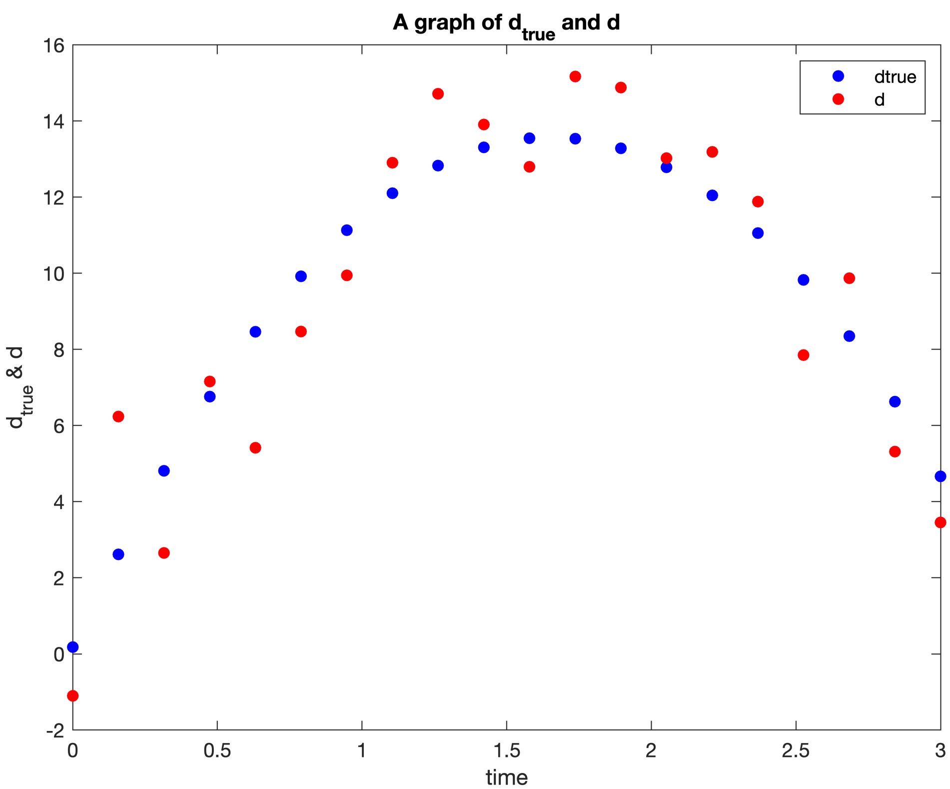
```
mean  = 0;
std = 2;

%noise
noise = 2*randn(m,1);

%noise data
d = dtrue + noise;
disp(d)
```

```
   -1.1014
    6.2349
    2.6501
    7.1562
    5.4193
    8.4730
    9.9481
   12.9070
   14.7138
   13.9111
   12.8001
   15.1687
   14.8824
   13.0273
   13.1872
   11.8835
    7.8526
    9.8697
    5.3157
    3.4572
```

Plot dtrue and d as points on the same graph

```
figure(1)
plot(tj,dtrue,'b.','MarkerSize',20)
hold on
plot(tj,d,'r.','MarkerSize',20)
title('A graph of d_{true} and d')
legend('dtrue','d')
xlabel('time');
```

```
ylabel('d_{true} & d')
```



Discuss the difference between the data

Comparing the trud data, dtrue, (blue) and the noisy data, d, (red), its clearly shown d has alot of dispersed data points due to the effect of noise while dtrue is a smooth curve without any disturbances. Even though the noisy data is distorted it will still depicts the trend of the true data.

N0.2 b)

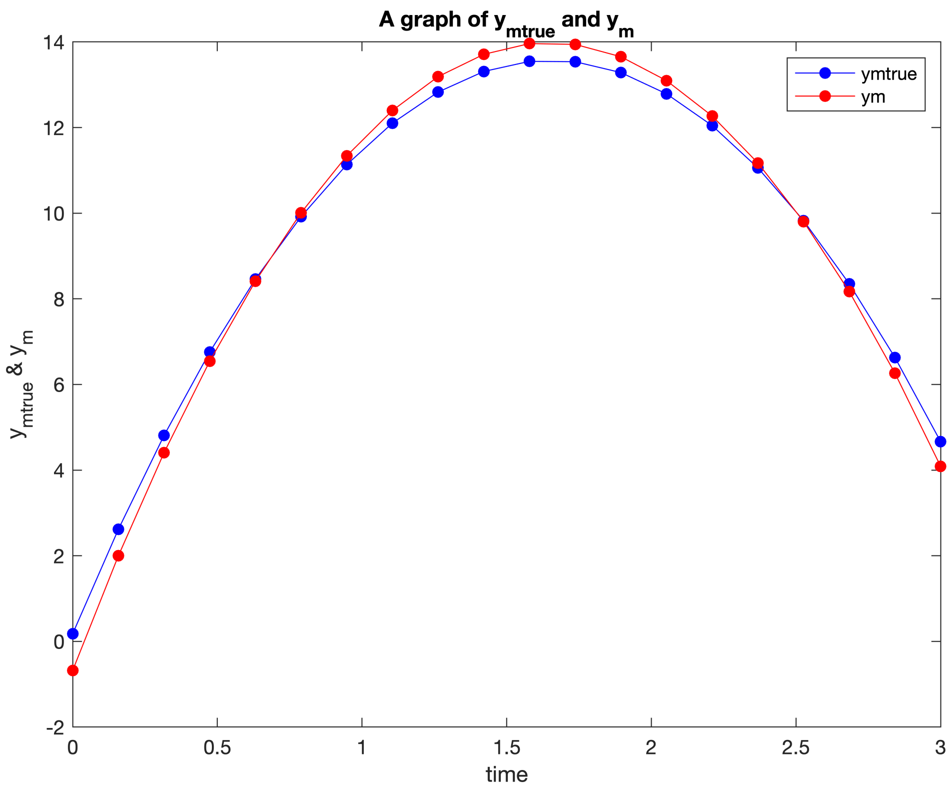
Solve Gm = d for m and plot y(t) using m and mtrue

```
m = G\d
```

```
m = 3×1
   -0.6776
   17.8078
   10.8129
```

```
%function
y = @(t,m1,m2,m3) m1 + m2*t - 0.5*m3*t.^2;

%using mtrue
ymtrue = y(tj, mtrue(1),  mtrue(2), mtrue(3));
%using m
ym = y(tj, m(1),  m(2), m(3));

%plot y
figure(2)
```

```
plot(tj,ymtrue,'b.-','MarkerSize',20)
hold on
plot(tj, ym,'r.-','MarkerSize',20)
title('A graph of y_{mtrue} and y_{m}')
legend('ymtrue','ym')
xlabel('time');
ylabel('y_{mtrue} & y_{m}')
```



Discuss the difference between the trajectories.

There is a discrepancy between the trajectories: ymtrue (blue) and ym (red) obtained using mtrue and m values respectively. This is due to the noisy data ,d, used to obtain m and the forward model (y(t)) used to obtain y which is approximate, hence even the trajectory wont fit the the observed trajectory (ytrue) due to the distorted values of m. Therefore for what ever values of the m, the trajectories will never fit each other but will always have the same trend.