Appendix

```
load ifk.mat

[m,n] = size(d);
a = 0; b = 1;

n = m;

sig  = 1e-4;

dx = (b-a)/m;
g = @(x,y) x*exp(-x*y);

x = [];
for j = 1:n
    x = [x  a + (dx/2) + (j-1)*dx]; %form x
end
y = x;

G = zeros(m,n);

for i = 1:m
    for j = 1:n
        G(i,j) = g(x(j),y(i)).*dx; %form G
    end
end

% Get the singular values for the system matrix
[U,S,V] = svd(G);

Si = diag(S);

i = 4;
mdagger = 0;
for j = 1:i
    mdagger = mdagger + (U(:,j)'*d/Si(j))*V(:,j);
end
mdagger;

Sii = Si(1:i-1);
mest = 0;
for j = 1:i-1
    mest = mest + (U(:,j)'*d/Sii(j))*V(:,j);
end
mest;

%true model
mtrue = @(x) exp(-10*(x-0.2).^2) + 0.4*exp(-10*(x-0.9).^2);

mtrue(x);
```

N0.1

```
chi = @(m) (norm(d - G*m,2)^2)/sig^2; %function Chi-square obs

chi_tsvd_i = chi(mdagger) %TSVD_i
chi_tsvd_i_1 = chi(mest) %TSVD_i-1
chi_mtrue = chi(mtrue(x)') %true estimates
expected_value = m-n
```

N0.2

```
alpha = [1e-1 1e-3 1e-5];
fp = [];
for i = 1:3
   fp= [fp filtered(alpha(i),Si,U,d,m)];
end

figure(1)
semilogy(fp(1:20),'-*'); hold on
semilogy(fp(21:40),'-o'); hold on
semilogy(fp(41:60),'-^')
legend('fP_{\alpha} = 10^{-1}','fP_{\alpha} = 10^{-3}','fP_{\alpha} = 10^{-5}')
xlabel('i'); ylabel('Magnitude of the filtered Picard ratios')
title('Magnitude of the filtered Picard ratios')
```

Discuss how they compare to the unfiltered Picard ratios in the Ch3: Discrete ill-posed problems individual activity.

```
Picard = [];
for i = 1:m
   Picard = [Picard abs((U(:,i)'*d)/Si(i))];
end

figure(3)
semilogy(Picard,'-.'); hold on
semilogy(fp(1:20),'-*'); hold on
semilogy(fp(21:40),'-o'); hold on
semilogy(fp(41:60),'-^'); grid on
legend('Unfiltered','fP_{\alpha} = 10^{-1}','fP_{\alpha} = 10^{-3}','fP_{\alpha} =
10^{-5}','Location','eastoutside')
ylabel('Magnitude of Picard ratios'); xlabel('i');
title('Magnitude of Picard ratios');
```

N0.3 Plot the estimates for each value of α on the same graph.

```
m = @(alpha) (G'*G + (alpha^2)*eye(m))\(G'*d);
m1 = m(alpha(1));
m2 = m(alpha(2));
m3 = m(alpha(3));

figure(2)
plot(m1,'-*'); hold on
plot(m2,'-o'); hold on
plot(m3,'-^')
```

```
legend('m_{\alpha} = 10^{-1}','m_{\alpha} = 10^{-3}','m_{\alpha} = 10^{-5}')
xlabel('i'); ylabel('Parameter Estimates')
title('Parameter estimates')
```

Discuss the accuracy of the model parameter estimates with different values of α, and relate them to the Picard ratios in 2.

```
figure(4)
plot(mtrue(x)','-.'); hold on
plot(m1,'-*'); hold on
plot(m2,'-o'); hold on
plot(m3,'-^')
legend('m_{true}','m_{\alpha} = 10^{-1}','m_{\alpha} = 10^{-3}','m_{\alpha} = 10^{-5}')
xlabel('i'); ylabel('Parameter Estimates')
title('Parameter estimates')
```

Report the values for each value of .

```
residual_a1 = chi(m1) % for alpha = 1e-1
residual_a2 = chi(m2) % for alpha = 1e-3
residual_a3 = chi(m3)  %for alpha = 1e-5


function [fp] = filtered(alpha,Si,U,d,m)
    fp = [];
    for i = 1:m
        fi = (Si(i)^2)/(Si(i)^2 + alpha^2);
        fp = [fp abs(fi*U(:,i)'*d/Si(i))];
    end
end
```