Algorithms – Assignment 1

1) Show directly that $f(n) = n^2 + 3n^3 \in O(n^3)$ and $f(n) = n^2 + 3n^3 \in \Omega(n^3)$.

When $n > 1$, $n^2 > n$ and $n^3 > n^2$ is always validated. $n^2 + 3n^3 \leq n^3 + 3n^3$.

Because $4n^3 \leq C \times n^3$ is correct, $n^2 + 3n^3$ could be $O(n^3)$. Pair (C = 4, k = 1)

Likewise, $n^2 + 3n^3 \geq C \times n^3$ is also true. Pair (C = 3, k = 1).

$n^2 + 3n^3 - 3n^3 \geq 3n^3 - 3n^3$ is equal to $n^2 \geq 0$ so $n^2 + 3n^3$ could also be $\Omega(n^3)$.

Consequently, $n^2 + 3n^3$ could be $O(n^3)$, $\Omega(n^3)$, and $\theta(n^3)$.

$n^2+3n^3 \in O(n^3)$

$|f(n)| \leq C|g(n)|$

$n>1$, $n^2>n$, $n^3>n^2$

$\therefore n^2+3n^3 \leq n^3+3n^3$

Since $4n^3 \leq \underset{C=4}{C} \cdot n^3 \Rightarrow n^2+3n^3 \in O(n^3)$

$n^2+3n^3 \in \Omega(n^3)$

$|f(n)| \geq C|g(n)|$

$n^2+3n^3 \geq \underset{C=3}{C} \cdot n^3$

$n^2+3n^3 \geq 3n^3 \Rightarrow n^2 \geq 0$

$\therefore n^2+3n^2 \in \Omega(n^3)$

Since $f(n) \in O(n^3)$ and $f(n) \in \Omega(n^3)$, $f(n)$ also $\theta(n^3)$.

2) Using the definitions of O and Ω, show that $6n^2 + 20n \in O(n^3)$, but $6n^2 + 20n \notin \Omega(n^3)$.

When $n > 1$, $n^2 > n$ and $n^3 > n^2$ is always validated. Thus $6n^2 + 20n \leq 6n^2 + 20n^2 \leq 6n^3 + 20n^3$.

As the definition of Big O is $|f(x)| \leq C\,|g(x)|$, $6n^3 + 20n^3 \leq C \times n^3$ is accepted.

That means $6n^2 + 20n$ is consequently $O(n^3)$. Pair (C = 26, k = 1)

However, it is logically impossible for n^3 to be $f(n)$'s lower bounds because $n > 1$, $n^3 > n^2$.

Therefore, $6n^2 + 20n \ngeq C \times n^3$ and $6n^2 + 20n$ could not be $\Omega(n^3)$. It could be $\Omega(n^2)$.

As a result, $6n^2 + 20n$ could be $O(n^3)$, but not $\Omega(n^3)$ and $\theta(n^3)$.

$6n^2+20n \in O(n^3)$

$n>1, n^2>n \ , n^3>n^2$

$\therefore \ 6n^2+20n \le 6n^2+20n^2 \le 6n^3+20n^3.$

$26n^3 \le C \cdot n^3.$ (O)

As a result $6n^2+20n \Rightarrow O(n^3).$

However when $n>1$, $n^3>n^2.$

$6n^2+20n$ would be smaller than $C \times n^3.$

as the size of input increasing.

As a result $6n^2+20n \notin \Omega(n^3)$

as long as $\theta(n^3)$.

[3~5] According to the definition of asymptotic notation, we should abstract away some details of the function. In other words, we need to focus more on what affects more to time complexity as dealing with more elements of input.

3) The answer: **(e)**

The most influential variable in this polynomial is $3n^2$.

4) The answer: **(c)**

The most influential variable in this polynomial is $6n$ from $2n + 4n$.

5) The answer: **(f)**

The most influential variable in this polynomial is 2^n.