CENG 222

Statistical Methods for Computer Engineering

Spring '2016-2017

Assignment 2

Student Information

Full Name : Koray Can Yurtseven
Id Number : 2099547

Answer 3.15

a.

At least one hardware failure means that $P\{X \geq 1\}$, which is $1 - P(0)$.

$$P\{X \geq 1\} = 1 - P_{(X,Y)}(0,0) = 1 - 0.52 = 0.48$$

b.

Let P_X and P_Y be pmf of X and Y respectively. Adding the joint probabilities along the rows to find P_X and along the coloumns to find P_Y, we have;

$$P_X(0) = 0.72, P_X(1) = 0.23, P_X(2) = 0.05$$
$$and$$
$$P_Y(0) = 0.76, P_Y(1) = 0.17, P_Y(2) = 0.07$$

Since

$$P_{(X,Y)}(0,0) = 0.52 \neq P_X(0)P_Y(0) = (0.72)(0.76) = 0.5472$$

we found a counterexample, so X and Y are not independent.

Answer 3.32

Let X be the number of crashed computers. This is the number of "success" (crashed computers) out of 4000 "trials" (computers), with the probability of success 1/800. Since $n = 4000$, which is larger than 30 and $p = 1/800$, which is smaller than 0.05, we can use Poisson approximation to Binomial.

λ = np = 5. From Table A3,

a.

Less than 10 computer crashed means that $P\{X < 10\}$ =$F(9)$ = 0.968

b.

Exactly 10 computer crashed means that $P\{X < 10\}$ = $F(10)$ - $F(9)$ = 0.986-0.968 = 0.018

Answer 3.35

Let T be the event(thunderstorm) and X be the Poisson number of accidents yesterday. By Bayes Rule, using Table A3;

$$P\{T|X = 7\} = \frac{P\{X = 7|T\}P\{T\}}{P\{X = 7|T\}P\{T\} + P\{X = 7|T_2\}P\{T_2\}} = \frac{(0.0901)(0.6)}{(0.0901)(0.6) + (0.0596)(0.4)} = 0.6940$$

where,

$P\{X = 7|T\} = 0.0901$ Poisson(10) distribution (during a thunderstorm)

$P\{X = 7|T_2\} = 0.0596$ Poisson(4) distribution (not during a thunderstorm).

Answer 4.4

a.

Find K from the condition $\int f(x)dx$=1:

$$\int f(x)dx = \int_0^{10} (K - x/50)dx = Kx - \frac{x^2}{2.50}\Big|_0^{10} = 10K - 1 = 1$$

Solving for K, we get $K = 0.2$.

b.

$$P\{X < 5\} = \int_0^5 (0.2 - x/50)dx = 0.2x - \frac{x^2}{2.50}\Big|_0^5 = 1 - 0.25 = 0.75$$

c.

$$E(X) = \int xf(x)dx = \int_0^{10} x(0.2 - x/50)dx = \frac{0.2x^2}{2} - \frac{x^3}{3.50}\Big|_0^{10} = 10 - \frac{20}{3} = 3\frac{1}{3}years$$

Answer 4.10

Let A be the event that the first specialist is working on the order. We know that $P(A) = 0.6$.
Let X be the amount of time (in hours) the order takes to be competed.
X given A is exponential with rate 3, and X given not A is exponential with rate 2.

$$P\{X > x|A\} = 1 - P\{X \leq x|A\} = 1 - (1 - e^{-3x}) = e^{-3x}$$
$$P\{X > x|\overline{\text{A}}\} = 1 - P\{X \leq x|\overline{\text{A}}\} = 1 - (1 - e^{-2x}) = e^{-2x}$$

30 minutes is 0.5 hours.
P{**First specialist is working on the order given order is not ready after 0.5hours** }

$$\begin{aligned}
&= P\{A|X > 0.5\} \\
&= \frac{P\{A \cap X > 0.5\}}{P\{X > 0.5\}} \\
&= \frac{P\{X > 0.5|A\}P\{A\}}{P\{A\}P\{X > 0.5|A\} + P\{X > 0.5|\overline{\text{A}}\}P\{\overline{\text{A}}\}} \\
&= \frac{0.6 * e^{-3*0.5}}{0.6 * e^{-3*0.5} + 0.4 * e^{-2*0.5}} \\
&= 0.4764
\end{aligned}$$