

CENG 223

Discrete Computational Structures

Fall '2016-2017

Take Home Exam 4

Due date: 2 January 2017, Monday, 23:59

Question 1

How many relations are there on a set with n elements that are

- a) Symmetric ?
- b) Antisymmetric ?
- c) Reflexive and Symmetric ?
- d) Neither reflexive nor Irreflexive ?

No partial credit will be given for this question. You have to show your detailed solutions. Otherwise, you won't get any points.

Question 2

Prove or disprove the following:

- a) For any relation R on a set A , if R is symmetric, then $R = R^{-1}$.
- b) If R is any relation on a set A , then $R \circ R^{-1}$ is a transitive relation on A .
- c) If R is any relation on a set A , R is transitive if and only if $R \circ R \subseteq R$.

Question 3

- a) Give the recurrence relation P_n for calculating the number of ways to arrange n pairs of parantheses?
- b) Generate two questions which has the recurrence relation in the question (a). Give detailed intuitive explanation why it is so.

(one hint: path finding from one point to another in an $n \times n$ grid)

No partial credit will be given for this question. You have to show your detailed solutions. Otherwise, you won't get any points.

Question 4

Suppose that you will have lab exam in a particular lab, and everybody has a particular place (think of the seating plan we prepare for you). But unfortunately one of your genius friends, let it be G , who is first to arrive forgot his assigned inek computer, and chooses an inek machine randomly. After, successively arriving students will sit their assigned inek if it is not occupied, if occupied he/she has to choose randomly too. Assume you are last to arrive and n students are participating for the class, then generate a recurrence relation for the probability that you will take the lab exam in your originally assigned inek.

Question 5

Generate and solve (using generating functions) the recurrence relations for the following, include intuitive explanations:

- a) for the number ternary strings that do not contain two consecutive 0s or two consecutive 1s.
- b) for the number ternary strings that contains two consecutive 0s, 1s or 2s.

Question 6

Construct a recurrence relation for the number of ways to cover a $3 \times n$ chessboard using 1×2 tile pieces?

1 Regulations

1. You have to write your answers to the provided sections of the template answer file given. Other than that, you cannot change the provided template answer file. If a latex structure you want to use cannot be compiled with the included packages in the template file, that means you should not use it.
2. Do not write any other stuff, e.g. question definitions, to answers' sections. Only write your answers. Otherwise, you will get 0 from that question.
3. **Late Submission: Not Allowed**
4. **Cheating: We have zero tolerance policy for cheating.** People involved in cheating will be punished according to the university regulations.
5. **Newsgroup:** You must follow the newsgroup (news.ceng.metu.edu.tr) for discussions and possible updates on a daily basis.
6. **Evaluation:** Your latex file will be converted to pdf and evaluated by course assistants. The .tex file will be checked for plagiarism automatically using "black-box" technique and manually by assistants, so make sure to obey the specifications.

2 Submission

Submission will be done via COW. Download the given template file, "the4.tex", when you finish your exam upload the .tex file with the same name to COW.

Note: You cannot submit any other files. Don't forget to make sure your .tex file is successfully compiled in Inek machines using the command below.

```
$ pdflatex the4.tex
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