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CENG 222 THE 1

2.17)

Let $P\{x\}$, $P\{y\}$, $P\{z\}$ denotes probabilities of part received from supplier X, Y, Z

$P\{x\} = 0.24$
$P\{Y\} = 0.36$
$P\{z\} = 0.40$

Also $P\{D|X\}$, $P\{D|Y\}$, $P\{D|Z\}$ denotes probabilities of defective parts in given supplier X, Y, Z

$P\{D|X\} = 0.05$
$P\{D|Y\} = 0.10$
$P\{D|Z\} = 0.06$

We want to know $P\{Z|D\}$.

$$P\{Z|D\} = \frac{P\{D|Z\}.P\{Z\}}{P\{D\}}$$

since they are mutually exclusive and exhaustive

$$P\{D\} = P\{D|X\}.P\{x\} + P\{D|Y\}.P\{Y\} + P\{D|Z\}.P\{Z\}$$

$$P\{Z|D\} = \frac{P\{D|Z\}.P\{Z\}}{P\{D|X\}.P\{x\} + P\{D|Y\}.P\{Y\} + P\{D|Z\}.P\{Z\}}$$

$$= \frac{0.06 \times 0.40}{0.05 \times 0.24 + 0.10 \times 0.36 + 0.06 \times 0.40} = \frac{0.024}{0.072} = \boxed{\frac{1}{3}}$$

2.29)

We have to find the probabilities of finding the keyword exactly n times, and subtract from 1 the probabilities for n=1 and 0.

If N is the number of dictionaries where the keyword is found, then

$$P(N=n) = \frac{\binom{5}{n}\binom{4}{4-n}}{\binom{9}{4}}$$

$\binom{5}{n}$ → we have to choose n dictionaries from 5 with the keyword

$\binom{4}{4-n}$ → 4-n from 4 without

$\binom{9}{4}$ → 4 from 9 overall.

for n=0 $P\{0\} = \frac{\binom{5}{0}\binom{4}{4}}{\binom{9}{4}} = \frac{1.1}{126} = \frac{1}{126}$

for n=1 $P\{1\} = \frac{\binom{5}{1}\binom{4}{3}}{\binom{9}{4}} = \frac{5.4}{126} = \frac{20}{126}$

$$1 - \left(\frac{1}{126} + \frac{20}{126}\right) = \frac{105}{126} = \frac{35}{42} = \boxed{\frac{5}{6}}$$

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3.6) There are $\binom{6}{2}$ ways of selecting two of the six blocks at random. We want to have 1 error and 1 error free block. Which represents as $\binom{1}{1}\binom{5}{1}$. Thus, the probability that one of them contains an error is:

$$\frac{\binom{1}{1}\binom{5}{1}}{\binom{6}{2}} = \frac{5}{15} = \frac{1}{3}$$

So, X has probability mass function.

$$f_X(x) = \begin{cases} \frac{1}{3} & \text{for } x=1 \\ \frac{2}{3} & \text{for } x=0 \end{cases}$$

These are only two cases because, there exists only one error and, any two blocks of code chosen at random must either contain the error or not.

Therefore:

$$E(x) = \sum_{x=0}^{1} x f_X(x) = 0 \cdot \frac{2}{3} + 1 \cdot \frac{1}{3} = \boxed{\frac{1}{3}}$$

3.7)

We must first calculate the probabilities of games.

$P(0) = P(X_1=0 \text{ and } X_2=0) = P(0).P(0) = 0.4 \times 0.4 = 0.16$

$P(1) = P(X_1=0 \text{ and } X_2=1) \text{ or } P(X_1=1 \text{ and } X_2=0) = 0.4 \times 0.4 + 0.4 \times 0.4 = 0.32$

$P(2) = P(X_1=0 \text{ and } X_2=2) \text{ or } P(X_1=1 \text{ and } X_2=1) \text{ or } P(X_1=2 \text{ and } X_2=0) = 0.4 \times 0.2 + 0.4 \times 0.4 + 0.2 \times 0.4 = 0.32$

$P(3) = P(X_1=1 \text{ and } X_2=2) \text{ or } P(X_1=2 \text{ and } X_2=1) = 0.4 \times 0.2 + 0.2 \times 0.4 = 0.16$

$P(4) = P(X_1=2 \text{ and } X_2=2) = 0.2 \times 0.2 = 0.04$

$$\mu = E(X) = \sum_{x=0}^{4} x f_X(x) = 0 \times 0.16 + 1 \times 0.32 + 2 \times 0.32 + 3 \times 0.16 + 4 \times 0.04 = \boxed{1.60}$$

x	P(x)	x - μ
0	0.16	0-1.6 = -1.6
1	0.32	1-1.6 = -0.6
2	0.32	2-1.6 = 0.4
3	0.16	3-1.6 = 1.4
4	0.04	4-1.6 = 2.4

$$\text{Variance} = \sigma^2 = \sum_{x=0}^{4} (x-\mu)^2 P(x)$$

$$= (-1.6)^2.(0.16) + (-0.6)^2.(0.32) + (0.4)^2.(0.32) + (1.4)^2.(0.16) + (2.4)^2.(0.04)$$

$$Var(Y) = \boxed{1.12}$$