Student Information

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Answer 1

a.

Set of rational numbers inside the open interval $(-1, 0)$ is countably infinite.
We can arrange number in a format:

$$\frac{-1}{2}, \frac{-1}{3}, \frac{-2}{3}, \frac{-1}{4}, \frac{-3}{4}, \frac{-1}{5}, \frac{-2}{5}, \frac{-3}{5}, \frac{-4}{5}...$$

In this format, it is clear to see that every rational number between $(-1, 0)$ will appear in this list. Duplicates are removed.

b.

Given definition, L is a regular language. Because regular languages are closed under concatenation, all elements of L^+ are regular because concatenation of any two word that are in language L is regular. D is countably finite.

c.

Lets say A is a set of all languages with given alphabet and L is the set of all regular languages on the given alphabet. And $A \setminus L$ denotes set of languages that are not recognized by Finite Automaton.

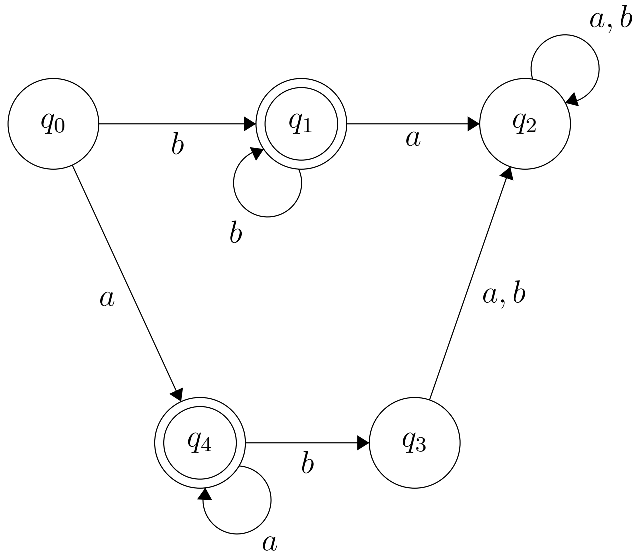
Proof by contradiction:

Assume $A \setminus L$ is countable. Since L is countable, $(A \setminus L) \cup L$ is countable. But, A is a subset of $(A \setminus L) \cup L$, and it should be countable, but it is not. It is a contradiction, thus our assumption is wrong. Therefore, they are infinitely uncountable.

Answer 2

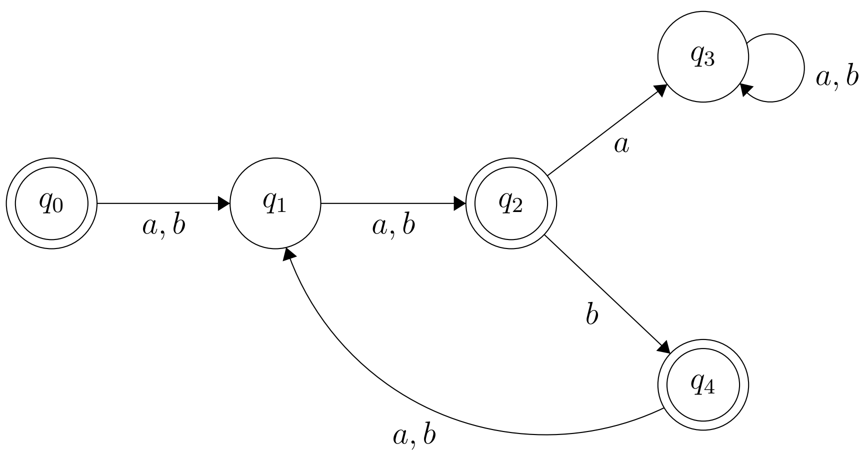
a.

q_0 is initial state.



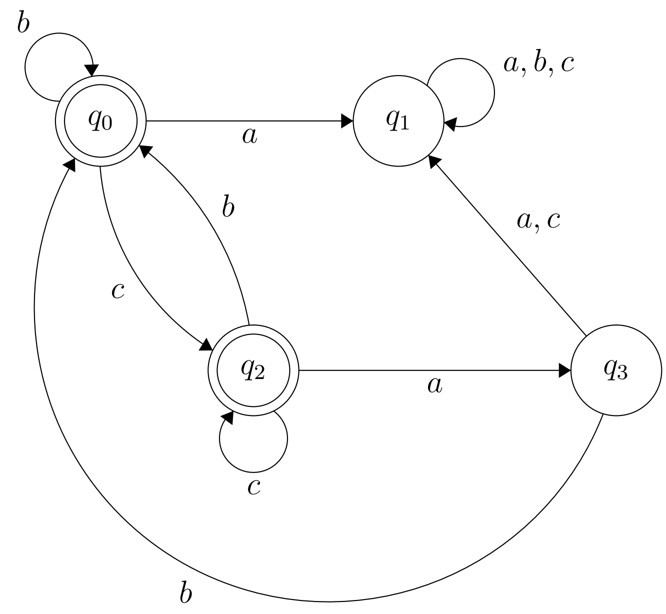
b.

q_0 is initial state.

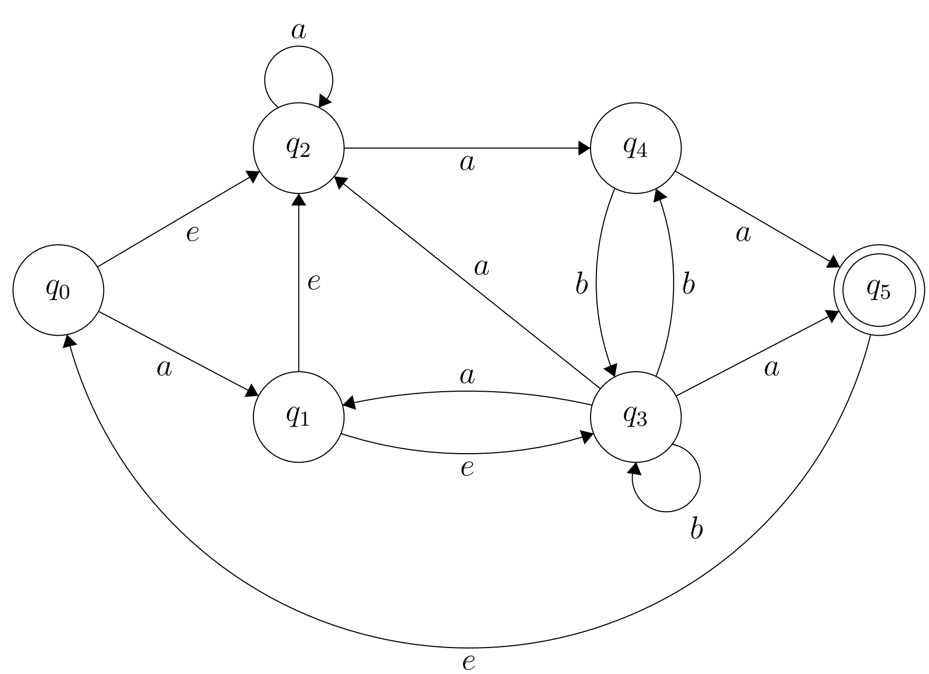


c.

q_0 is initial state.



Answer 3



a.

It is clear to see that a word must end with a. Since w does not end with a, it is not in the language.

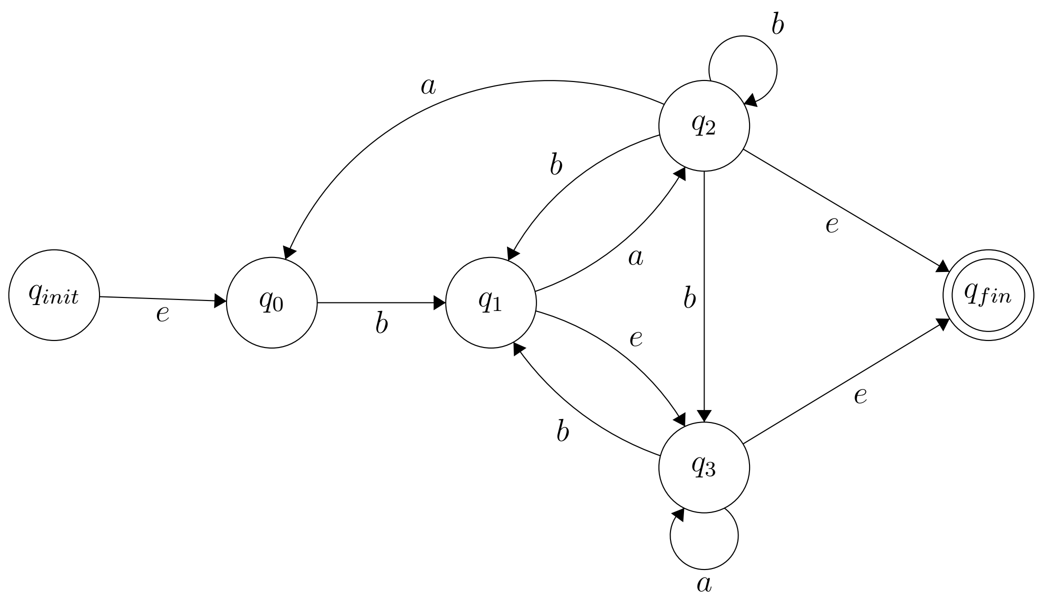
b.

We can find a path from our initial state to final state. The path is:

$$(q_0, a, q_1), (q_1, e, q_3), (q_3, b, q_3), (q_3, a, q_1), (q_1, e, q_3), (q_3, b, q_3), (q_3, a, q_5)$$

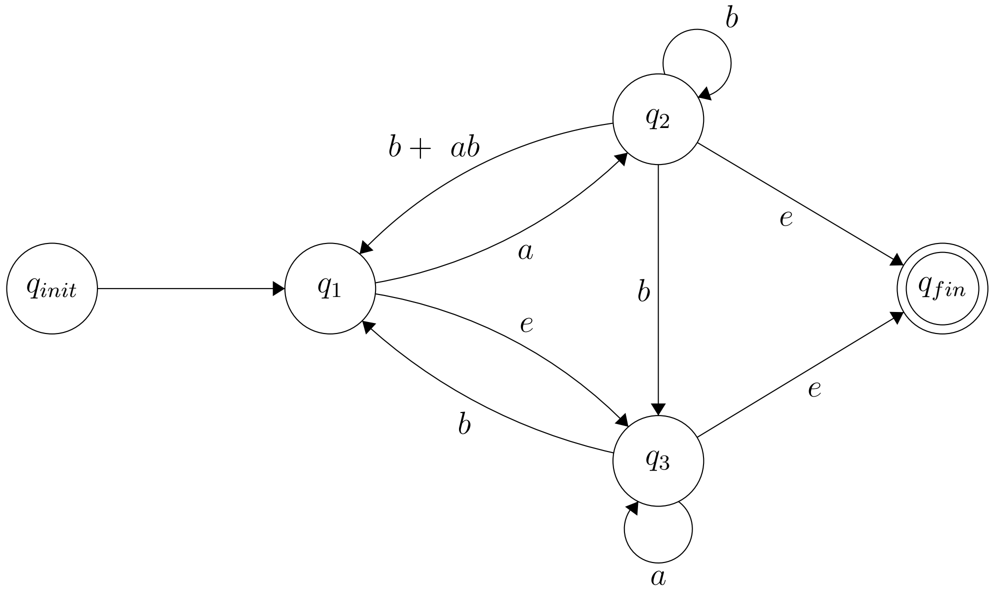
Answer 4

a.

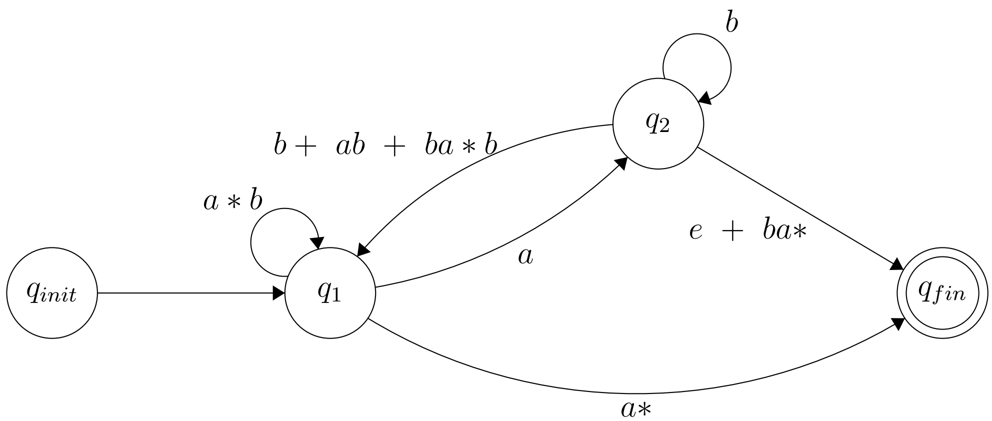


b.

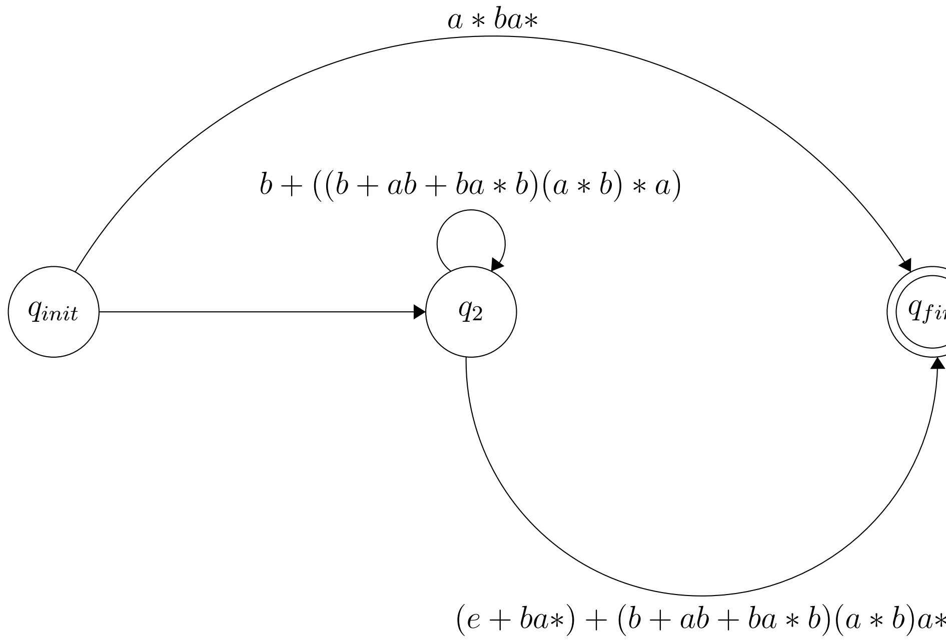
First, delete q_0.



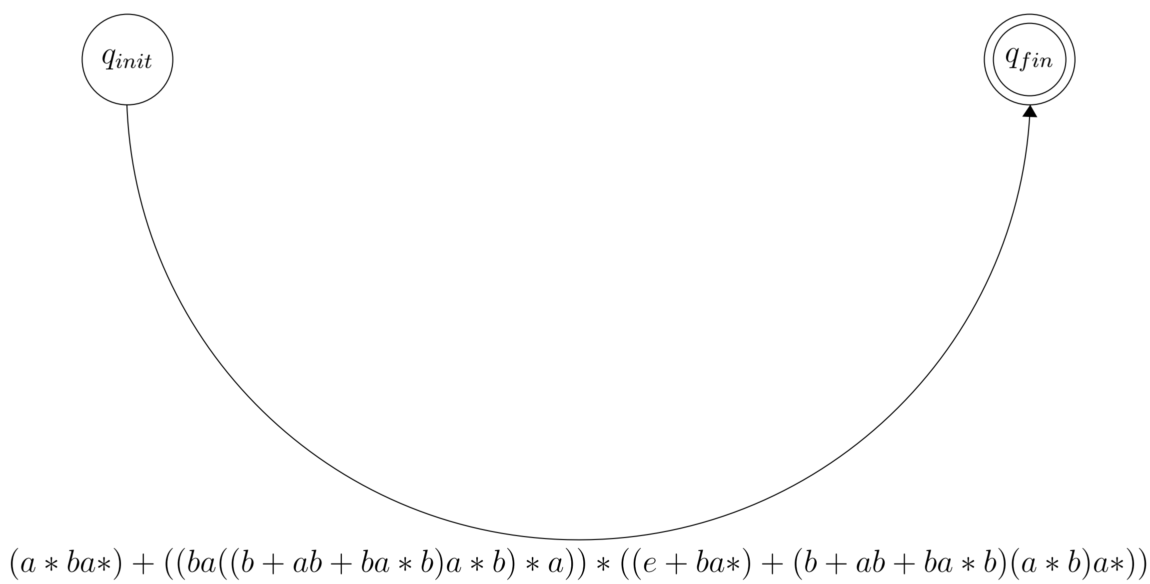
Then, we delete q_3.



Then, we delete q_1.



Finally, we delete q_2



Answer 5

a.

q_0 is initial.

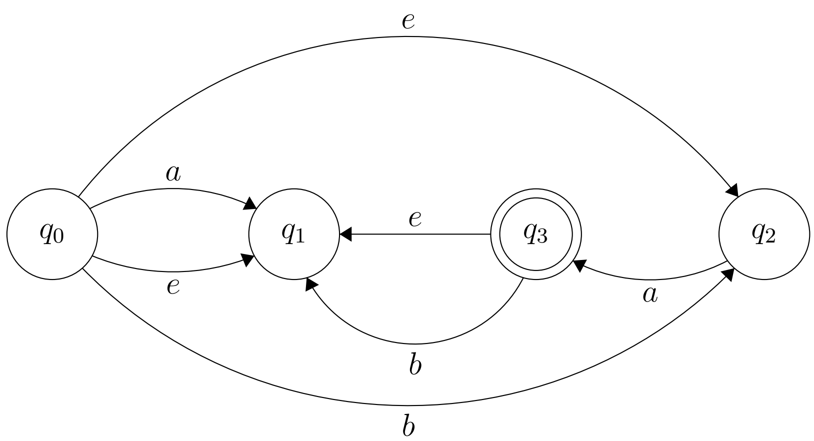


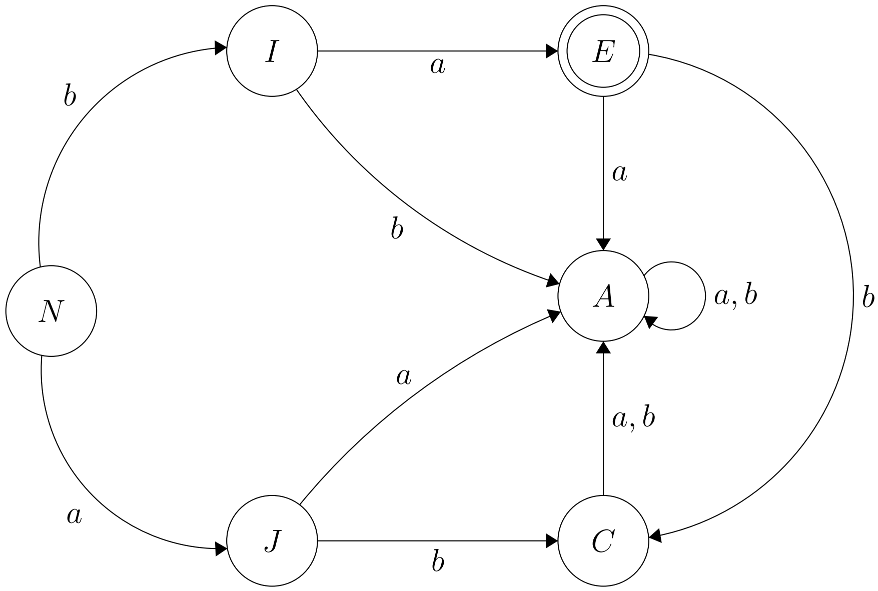
Table is:

My names	States	a	b
A	emptyset	emptyset	emptyset
B	q0	q1	q2
C	q1	emptyset	emptyset
D	q2	q3	emptyset
E	q3	emptset	q1
F	q0,q1	q1	q2
G	q0,q2	q1,q3	q2
H	q0,q3	q1	q1,q2
I	q1,q2	q3	emptyset
J	q1,q3	emptyset	q1
K	q2,q3	q3	q1
L	q0,q1,q2	q1,q3	q2
M	q0,q1,q3	q1	q1,q2
N	q0,q2,q3	q1,q3	q1,q2
O	q1,q2,q3	q3	q1
P	q0,q1,q2,q3	q1,q3	q1,q2

Lets start with N, because it has one of the most states. Table for N is in below:

Name	a	b
N	J	I
J	A	C
I	E	A
A	A	A
E	A	C
C	A	A

And the diagram is:



b.

Answer 6

Let L_1 and L_2 denote two reguar languages. Then, the following is also regular:

$$L_1{}' = \{x \in \Sigma^* | x \notin L_1\}$$

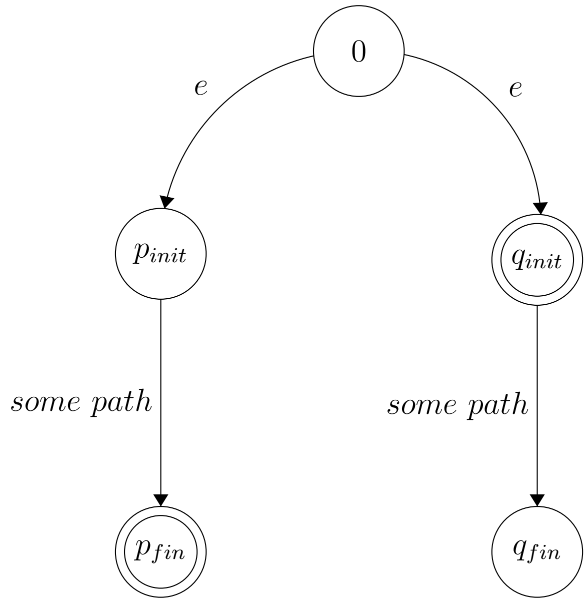
Meaning that, $L_1{}'$ is compliment of L_1. We can also use union property of language to obtain:

$$L_1 \setminus L_2 = L_1 \cap L_2' = (L_1' \cup L_2)'$$

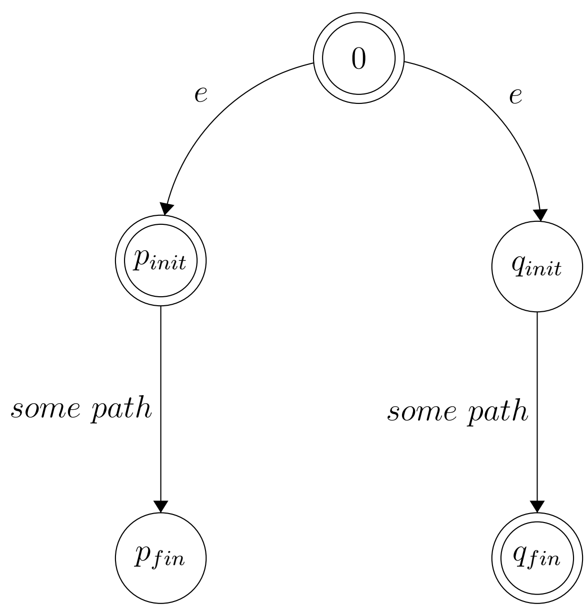
Now, let L1 is the language of the left figure and L2 is the language of the right figure.

By complementing L_1, we obtain:

Using union property, we obtain:



And, by complementing the figure above, we can construct the difference operation on regular languages.



Answer 7

a.

If a language L is regular, then there exists a number $n \geq 1$ s.t. every string uwv in L with $|w| \geq n$ can be written in the form $w = xyz$ s.t. $|y| \geq 1$ and $|xy| \leq n$ and $xy^iz \in L$ for $i \geq 0$.