Problem Chosen	2023 MCM/ICM Summary Sheet	Team Control Number
E		2322666

A Light Pollution Risk Level Metric System and Evaluation of Possible Policies with Case Analysis

Summary

After performing data analysis and modeling, we developed a light pollution risk level metric system and evaluated three possible strategies to tackle the excessive artificial light with case analysis.

Firstly, we divided the metric into four components: Economic, Ecological, Traffic and Health metric. We then suggested a list of parameters that affects the metric components. To evaluate the effects, we used **Analytic Hierarchy Process (AHP)** to assign the weightings and **Technique for Order Preference by Similarity to Ideal Solution (TOPSIS)** to express the relationships between parameters and the metric. Then, we used AHP again to assign weightings to each metric and completed the light pollution risk level metric.

Secondly, we applied the metric on four types of location, namely: Tokyo as urban location, Gainesville as suburban location, Borwan as rural location, and Yellow Stone as protected area. By inputting the parameters of each country, the result showed that the decreasing light pollution risk level order is Tokyo, Gainesville, Yellow Stone, and Borwan, which fits the real life situation. We also found out that the population density is the most determining factor in our metric.

Thirdly, we suggested three possible strategies to tackle light pollution, namely: **Adding Light Shades, Pigouvion Tax, and Smart Light Grid System**. We built a **utility model** for each strategy by considering how the strategies affect the parameters in our metric and the cost functions of the strategies. To illustrate how the government implements the strategies, we used **First Order Condition (FOC)** to simulate their optimal actions.

Last but not least, we decided to choose the two extreme locations, Tokyo and Yellow Stone as our case analysis. To increase the flexibility of our model, we considered two cases: short run and long run. To decide which is the best strategy for each location in long run, we used **Barycentric Form Lagrange Interpolation** to illustrate the optimal utility of each strategy against time. After analysing the data, we concluded Pigouvion Tax is the best strategy for Tokyo in short run, and Smart light grid system for long run. For Yellow Stone, adding light shades is the best strategy for both short run and long run. We explained the detailed benefits of Smart light grid system for Tokyo in long run with a flyer.

Contents

1 Introduction

1.1 Problem Statement

The light pollution issue affects our lives in several channels, which include altering our view of the night sky, affecting our health and safety, and distorting the ecosystem. Intervention strategies are needed to be implemented to mitigate the negative effects of light pollution. Despite the fact that artificial light may cause severe negative impacts on our health, safety, and environment, it brings benefit to society as the same time. For example, artificial light allows shopping malls and restaurants to operate at night, which benefits the economy. Thus, a balance between usage of artificial light and prevention of light pollution needs to be struck.

1.2 Our Goal

Based on our understanding of the problem, we set the following goals:

- Develop a metric to identify the light pollution risk level of a location based on various obtainable parameters.
- Apply out metric on a protected land, a rural community, a suburban community and an urban community to evaluate the light pollution risk levels of these locations respectively.
- Find three possible intervention strategies to address light pollution and evaluate the impacts of the actions on the effects of light pollution.
- Choose two of our locations and determine which strategy is the most effective for each chose location by evaluating the impact of the strategy on the risk level for the location.

1.3 Our thinking

We illustrate our thinking by using flowcharts:

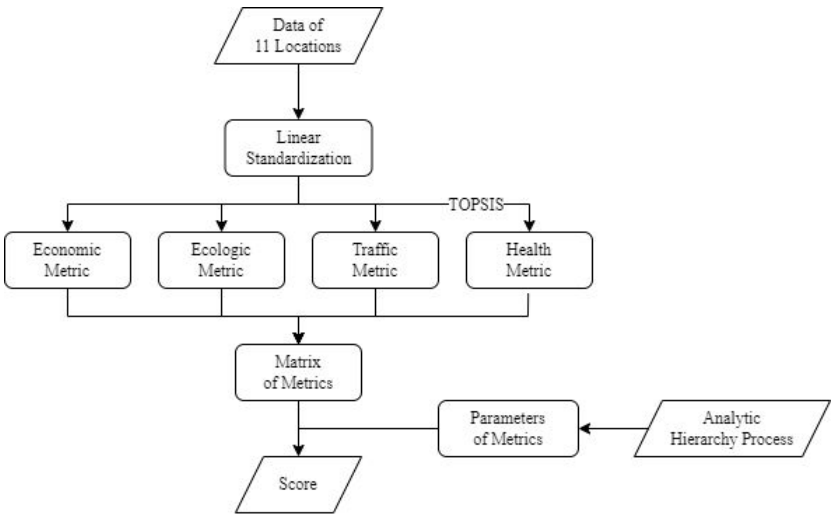

Figure 1: Flowchart of construction of light pollution risk level metric

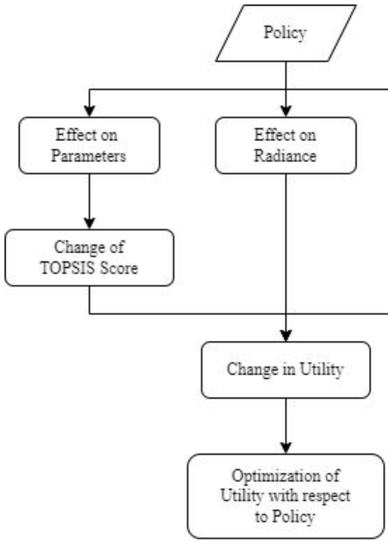

Figure 1: Flowchart of evaluation and implementatoin of policies

2 Assumptions and Justifications

Assumption 1: The minimum population of a location is 100.
Justification 1: To avoid overvaluation of the effects of population and it is reasonable to assume that there is a certain number of people live in protected lands since protected creatures need to be monitored by conservators.

Assumption 2: For each location, it only belongs to a single type of location.
Justification 2: To avoid complex evaluation of inter effects between different types of location.

Assumption 3: Radiance is the only parameter of excessive light.
Justification 3: Other parameters describing excessive light do not add a significant information entropy.

Assumption 4: Cars are distributed according to population.
Justification 4: Data of number of cars of each locations are unobtainable.

Assumption 5: Biomass is directly proportional to amount of animals.
Justification 5: Data of amount of animals are unobtainable.

3 Variable Description

Variable	Meaning	First appearance page
E_1	Economic metric	4
E_2	Ecological metric	4
E_3	Safety metric	4
E_4	Health metric	4
GDP	GDP of location	5
P_d	Population density of location	5
P_e	Price of electricity of location	5
Bio_m	Biomass of location	5
Bio_{div}	Biodiversity of location	5
Bio_f	Functional diversity of species	5
car	No. of cars of a location	6
σ_B	Standard deviation of radiance	6
H	Health care index of location	7
R	Risk Level of Light Pollution	9
B	Radiance of light of location	12
P_{Gov}	Willingness to pay for reduction of R of government	12
L_{shade}	Light Shaded by lampshade	12
T	Pigouvion Tax	12
L_{store}	Light Stored by Smart Grid	12

4 Light Pollution Risk Level Evaluation System

According to current research **City Light or Star Bright: A Review of Urban Light Pollution, Impacts, and Planning Implications**[1], the effect of light pollution can be categorized into **Economical**, **Ecological**, **Safety** and **Health** sectors. We set the light pollution risk level metric based on these four categories.

Category	Notation	Description
Economics	E_1	The shock of light pollution to the Economy
Ecological	E_2	The impact of light pollution on Nocturnal & Migration Animals
Traffic	E_3	The danger that light pollution brings to the traffic
Health	E_4	The health impact of nighttime light pollution

To evaluate each metrics based on different parameters, we have applied different weightings to each parameters by using **Analytic Hierarchy Process (AHP)** and **Technique for Order Preference by Similarity to Ideal Solution (TOPSIS)**.

4.1 Metric for Economical Sector

For Economical Sector, we consider three factors: GDP (GDP), population density (P_d), and price of electricity (P_e). Firstly, a high GDP indicates a high level of development of a location. Thus, the technological level of that location will be high as well, which indicates a better utilization of radiance. Secondly, a high P_d indicates that the level of development of the location is high. For example, the P_d of Macau 12.8k per km^2, whose development level is high due to the high concentration of specialists. Yet, for the example of Kolkata, whose having P_d as 27k per km^2. The P_d is high because of the bad population control. Thus, the P_d has a weak relationship with the economic metric. Thirdly, a high P_e indicates a high cost of using light, which significantly hinders the benefit of light. By using AHP, we assigned weightings $(0.4, 0.2, 0.4)$ to GDP, P_d, and P_e.

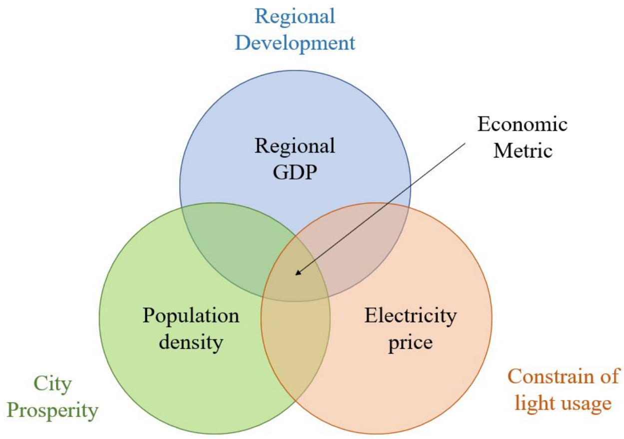

Figure 3: Illustration of Economic Metric

4.2 Metric for Ecological Sector

For Ecological Sector, we consider three factors: Biomass (Bio_m), species diversity (Bio_{div}), and functional diversity of species (Bio_f). Firstly, a higher Bio_m means that a higher number of animals will be affected by the excessive light, which is a direct factor. Secondly, a higher Bio_{div} increases the probability of having a large number of nocturnal animals, which are severely affected by light pollution, yet it is a relatively uncertain effect. Thirdly, a higher Bio_f indicates that a higher probability of having a large number of nocturnal animals, yet the condition of having different functional of species is much looser. Thus, its weighting is the lowest among three. By using AHP, we assigned $(0.6, 0.3, 0.1)$ to Bio_m, Bio_d, and Bio_f.

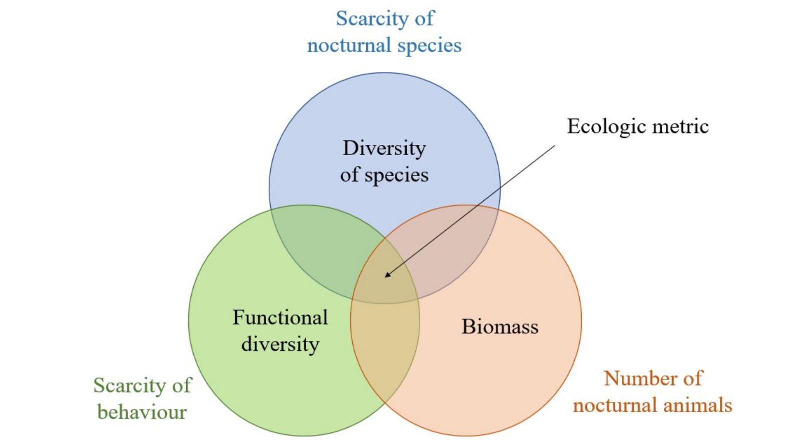

Figure 4: Illustration of Ecological Metric

4.3 Metric for Traffic Sector

For Traffic Sector, we consider three factors: number of cars car, σ_B, and P_d. Firstly, a higher car indicates that a higher probability of having a car accident. Note that it is a direct effect on the metric. Secondly, a higher σ_B indicates that a higher concentration of radiance, which increases the probability of car accidents which are caused by glare. Yet, this is only a proportion of all accidents, thus it weighting will be lower than the former. Thirdly, a higher P_d may indicate a higher probability of having a car accident, but it is an indirect effect since with good city planning can offset the increase in probability. By using AHP, we assigned $(0.5, 0.3, 0.2)$ as the weighting of car, σ_B, and P_d respectively.

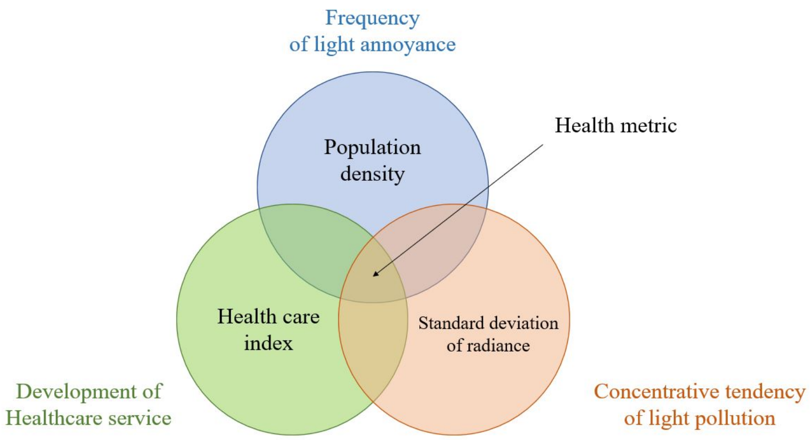

Figure 5: Illustration of Health Metric

4.4 Metric for Health Sector

For Health Sector, we consider three factors: P_d, standard deviation of radiance (σ_B), and health care index (H). Firstly, a higher P_d means that a higher number of people are affected by pollution at night, which means that the quality of sleeping is hindered. Note that P_d has a direct effect on the metric. Secondly, a higher σ_B means that the concentration of radiance is higher, which may severely affect quality of sleeping. Yet, there is still a small possibility that the lights concentrate at a point which is far from residential area. Thus, its weighting will be slightly less than that of P_d. Thirdly, a higher H can offset part of the negative impact since it can improve overall health of people. Yet, its effect is not targeted on quality of sleeping, thus its weighting will be slightly less than that of P_d. By using AHP, we assigned $(0.4, 0.3, 0.3)$ to P_d, σ_B, and H.

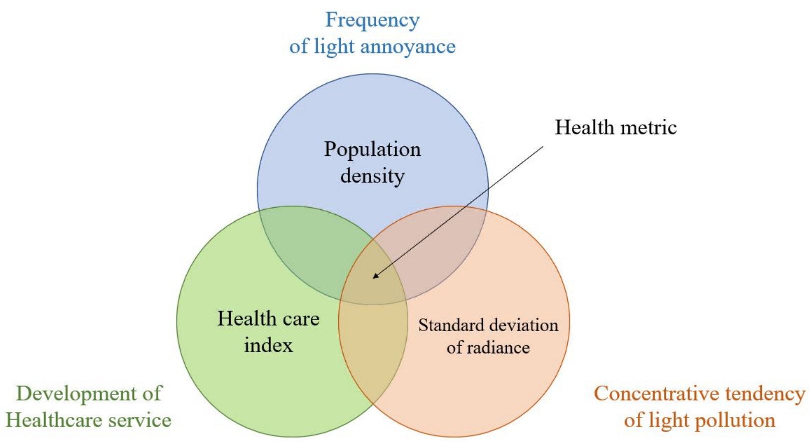

Figure 6: Illustration of Health Metric

4.5 Metric for overall Light Pollution Risk Level

The following table shows the weightings of each parameters. Note that $\hat{p}$ indicates that the parameter is linearly standardized, where

$$\hat{p} = \frac{p - p_{min}}{p_{max} - p_{min}}.$$

$\check{p}$ indicates that the parameter is inversely linearly standardized, where

$$\check{p} = \frac{p_{max} - p}{p_{max} - p_{min}}.$$

Then, we got the following table:

metric	parameters & weightings		
E_1	$\hat{GDP}$	$\hat{P_d}$	$\check{P_e}$
	0.4	0.2	0.4
E_2	$\hat{Bio_m}$	$\hat{Bio_{div}}$	$\hat{Bio_f}$
	0.6	0.3	0.1
E_3	$\hat{car}$	$\hat{\sigma_B}$	$\hat{P_d}$
	0.5	0.3	0.2
E_4	$\hat{P_d}$	$\hat{\sigma_B}$	$\check{H}$
	0.4	0.3	0.3

Table 1: Weightings of each standardized parameters used in TOPSIS

Each metric can be calculated by applying TOPSIS:

$$E_1 = \sqrt{0.4(\hat{GDP})^2 + 0.2(\hat{P_d})^2 + 0.4(\check{P_e})^2}$$

$$E_2 = \sqrt{0.6(\hat{Bio_m})^2 + 0.3(\hat{Bio_{div}})^2 + 0.1(\hat{Bio_f})^2}$$

$$E_3 = \sqrt{0.5(\hat{car})^2 + 0.3(\hat{\sigma_B})^2 + 0.1(\hat{P_d})^2}$$

$$E_4 = \sqrt{0.4(\hat{P_d})^2 + 0.3(\hat{\sigma_B})^2 + 0.3(\check{H})^2}$$

As we have completed the feature scaling for Economical, Ecological, Safety and Health Sector, we can calculate the overall effect metric by constructing linear weighted average of each sector. Firstly, economic metric is the only metric among the four which can offset the risk level to a certain extent. Yet, we are inclined to reduce the excessive artificial light. Thus, its weighting should be relatively low. Secondly, given the fact that animals are much sensitive to light pollution than human, the weighting of ecological metric should be higher than the traffic metric and the health metric. By using AHP, we assigned weights $(-0.2, 0.4, 0.2, 0.2)$ to (E_1, E_2, E_3, E_4) respectively.

metric	parameters & weightings			weightings of metrics
E_1	$\hat{GDP}$	$\hat{P_d}$	$\check{P_e}$	-0.2
	0.4	0.2	0.4	
E_2	$\hat{Bio_m}$	$\hat{Bio_{div}}$	$\hat{Bio_f}$	0.4
	0.6	0.3	0.1	
E_3	$\hat{car}$	$\hat{\sigma_B}$	$\hat{P_d}$	0.2
	0.5	0.3	0.2	
E_4	$\hat{P_d}$	$\hat{\sigma_B}$	$\check{H}$	0.2
	0.4	0.3	0.3	

Table 2: Weightings of each standardized parameters and metrics

The ratio of consistency index (CI) and the random index (RI) is 6.6% which means our weight is fully acceptable (less than 10%). The total risk level metric (R) can be given by

$$R = \sum_{i=1}^{4} \alpha_i \cdot E_i = -0.2 \cdot E_1 + 0.4 \cdot E_2 + 0.2 \cdot E_3 + 0.2 \cdot E_4$$

5 Risk Level Case Analysis

As we have developed our analysis metric, we could use it to evaluate the risk level of different areas. We select four different areas with distinct population density compared to their own country population density. They are:

- **Urban: Tokyo, Japan**,
- **Suburban: Gainsville, GA, USA**,
- **Rural: Barwon, NSW, Australia**,
- **Protected Area: Yellow Stone, WY, USA**,

We also chose other 7 locations to help us normalized the data of each location in order to calculate the standardized score for each location. Our sample range from South America to Europe, including all 7 continents and different environment. The locations we chose for this purpose are **Sydney (Australia)**, **La Paz (Bolivia)**, **Denver (USA)**, **Cambridge (UK)**, **Hoffenheim (Germany)**, **Amazonas (Brazil)**, and **Masai Mara (Kenya)**

5.1 Urban Community: Tokyo, Japan

Tokyo is generally considered as an urban city. In fact, it is one of the most populous and densely populated cities in the world, with a population of over 13 million people and a metropolitan area population of over 37 million people.Tokyo is known for its modern architecture, bustling streets, and diverse cultural offerings, as well as its efficient public transportation system and advanced technology. It is a major economic and cultural center not just for Japan, but for the Asia-Pacific region as a whole. Here are the data of Tokyo:

metric	**data of parameters**		
E_1	GDP	P_d	P_e
	2055698	4.352	0.251
E_2	Bio_m	Bio_{div}	Bio_f
	80	187	5.6
E_3	car	σ_B	P_d
	2.323	28.6	4.352
E_4	P_d	σ_B	H
	4.352	28.6	80.3

Table 3: Data of parameters of Tokyo

By applying our metric, the risk level of light pollution (R) of Tokyo is 0.249.

5.2 Suburban Community: Gainesville, GA, USA

Gainesville, Georgia is generally considered to be a suburban city. While it has a small downtown area with some commercial and institutional uses, much of the city consists of suburban neighborhoods with single-family homes, shopping centers, and office parks. Gainesville, Georgia is located about 50 miles north of Atlanta, and many people who work in Atlanta choose to live in Gainesville and commute to the city. The city has experienced significant growth in recent years, with new housing developments and commercial centers being built on the outskirts of the city. Overall, Gainesville, Georgia has a suburban character, with a mix of residential, commercial, and industrial land uses. Here are the data of Gainesville:

metric	data of parameters		
E_1	GDP	P_d	P_e
	8355	0.099	0.177
E_2	Bio_m	Bio_{div}	Bio_f
	110	248	5.8
E_3	car	σ_B	P_d
	3.516E-3	12.5	0.099
E_4	P_d	σ_B	H
	0.099	12.5	68.6

Table 4: Data of parameters of Gainesville

By applying our metric, R of Gainesville is 0.124.

5.3 Rural Community: Barwon, NSW, Australia

Barwon is a region located in the north-western part of the state of New South Wales (NSW), Australia, and is generally considered to be a rural area. Barwon covers a vast area of approximately 44,000 square kilometers and is known for its large rural properties, farming and grazing land, and natural resources such as minerals, gas, and oil. The region has a small population, and most of the settlements in Barwon are small towns or villages, with a focus on agricultural and mining activities. Here are the data of Barwon:

metric	data of parameters		
E_1	GDP	P_d	P_e
	5280	2.170E-4	0.223
E_2	Bio_m	Bio_{div}	Bio_f
	80	221	2.7
E_3	car	σ_B	P_d
	6.046E-3	0.41	2.170E-4
E_4	P_d	σ_B	H
	2.170E-4	0.41	75.3

Table 5: Data of parameters of Barwon

By applying our metric, R of Barwon is 0.010.

5.4 Protected Area: Yellow Stone, WY, USA

Yellowstone National Park is a protected area located primarily in the state of Wyoming in the United States. Established in 1872, it was the first national park in the United States and one of the first in the world. Yellowstone is known for its stunning natural features, including geysers, hot springs, canyons, and rivers, as well as its diverse wildlife, such as grizzly bears, wolves, bison, and elk. The park covers over 2 million acres and is managed by the National Park Service with the goal of preserving its natural and cultural resources for the enjoyment of present and future generations. The park is subject to special regulations designed to protect its natural features and wildlife, and visitors are required to follow certain rules and guidelines while in the park to minimize their impact on the environment. Here are the data of Yellow Stone:

metric	**data of parameters**		
E_1	GDP	P_d	P_e
	0	1.177E-4	0.148
E_2	Bio_m	Bio_{div}	Bio_f
	190	226	7.3
E_3	car	σ_B	P_d
	0	0	1.177E-4
E_4	P_d	σ_B	H
	1.177E-4	0	68.6

Table 6: Data of parameters of Yellow Stone

By applying our metric, R of Yellow Stone is 0.102.

5.5 Conclusion of Risk Level Case Analysis

By inputting the data into our metric, we got the following result:

Location	Tokyo	Gainesville	Barwon	Yellow Stone
Type	Urban	Suburban	Rural	Protected Area
R	0.249	0.124	0.010	0.102
Risk Level Ranking	1	2	4	3

Table 7: Risk Level Analysis of each Locations

The result of our metric application is satisfactory. Firstly, Tokyo has the highest R since it has the highest population density, which indicates that the frequency of light annoyance is significantly high. Secondly, Gainesville has the second highest R since its population density is relatively high, yet not as high as that of Tokyo. Thirdly, Borwan has the lowest R since its population density and biomass are relatively low, thus the potential bad impacts on humans and animals are the smallest. Last but not least, Yellow Stone has the second lowest R since its population density is very low. In addition, its biomass is not as high as an expected protected area's since Yellow Stone is famous for its natural features but not a significantly high biomass or biodiversity. Thus, the potential bad impacts on humans and animals are small.

Location	Sydney	La Paz	Denver	Cambridge	Hoffenheim	Amazonas	Masai Mara
Type	Urban	Urban	Urban	Urban	Suburban	Rural	Protected
R	0.296	0.503	0.154	0.077	0.198	0.410	0.117

Table 8: Risk Level Analysis of Other Locations

Besides the chosen locations, we also attached the Risk score of locations that we used to standardize the score. Our model performs quite well in accurately addressing the score for each area that can explained be reasonably. For example, the high population density, high biodiversity/mass and low economics development make the score of La Paz very high. On the other hand, Masai Mara has a low risk level because of its big area and low population/ animal density.

6 Policy Tool for Light Pollution

With our analysis above, all those sample areas are still somehow problematic with its air pollution situation. In order to improve the current situation better in terms of less risk, we develop a utility function $U(B)$ for the reduction of risk level in terms of the radiance B, the risk function R and a parameter P_{Gov} which measures how much the government is willing to pay for the additional unit reduction of R. Also, the implementation of policy will charge its own cost, so the total utility function will be the utility function for reducing risk minus the cost function (All in unit of utility).

$$U(B) = P_{Gov} \cdot B \sum_{i=1}^{4} \alpha_i \cdot E_i - C = R \cdot B \cdot P_{Gov} - C$$

Our policy aims to make the radiance into optimized form which maximizes the total utility function. As a result, our optimal B^* satisfy the equation:

$$\frac{\partial U(B^*)}{\partial B} = 0$$

We could achieve the optimal B^* by using our selected policy: **Adding Shades**, **Pigouvian Tax**, and **Smart Light Grid System**. However, each policy may affect other variables and have different cost functions. The non-coefficient parameters in each of our strategies are light shaded by lampshade (L_{shade}), tax (T) ,and light stored by smart grid (L_{store}). The table below illustrates each policy and its details:

Policy	Influencing Variables	Policy parameters
Adding Shades	B, σ_B	L_{shade}
Pigouvian Tax	B, P_e, H	T
Smart Grid System	B_i, P_e	L_{store}

Table 9: Influence on parameters caused by policies

6.1 Adding Shades

Light shading is a technique of controlling the direction of outdoor lighting so that light is directed only where it is needed, rather than being dispersed in all directions.[2] By shading outdoor lights, the light is directed downward or towards a specific area, which can reduce light pollution and minimize the amount of light that is wasted. Light shading can help reduce the amount of light that is dispersed in all directions, which can minimize the amount of light that is wasted, reduce skyglow, protect wildlife and improve safety by reducing glare and shadows. The effect of light shadings can be represented by equations below:

$$\begin{cases} B = B_{initial} - \alpha \cdot L_{shade} \\ \sigma_B = \sigma_{initial} - \beta \cdot L_{shade}. \end{cases}$$

Note that the cost function is in linear since we assume adding each light shading is causing the same cost. We model the cost function as:

$$C(L_{shade}) = \gamma \cdot L_{shade}$$

The equation below capture how government will use light shadings to solve radiance problem:

$$\begin{cases} \dfrac{\partial U(L^*_{shade})}{\partial L_{shade}} = \dfrac{\partial E_1}{\partial L_{shade}} + \dfrac{\partial E_2}{\partial L_{shade}} + \dfrac{\partial E_3}{\partial L_{shade}} + \dfrac{\partial E_4}{\partial L_{shade}} = 0 \\ \\ \dfrac{\partial^2 U}{\partial B^2} < 0 \end{cases}$$

6.2 Pigouvian Tax

Pigouvian tax is a type of tax that is intended to correct for the negative externalities associated with certain goods within a market. In the case of lighting, a Pigouvian tax could be applied to encourage more efficient and responsible use of lighting and to discourage the wasteful or excessive use of light.

In our model, Pigouvian Tax is a proportional tax to the energy price P_e. And due to our model, radiance B is determined by demand and supply model. Because the taxation is always positive, so the demand will be smaller or equal to supply, which means the radiance B is always determined by the supply model. Besides, the revenue for taxation can be used to fund the health institution, and thus increase the health index. The effect of taxation can be represented by equations below:

$$\begin{cases} P_e = P_{initial} + T \\ B = B_{initial} - \beta \cdot T \\ H = H_{initial} + \gamma \cdot T \end{cases}$$

The cost of taxation is measured by the lost of consumption expectation in the future. Inspired by the macroeconomics model **IS-LM**[3], we model the loss of consumption use the following equation:

$$C(T) = \alpha + \beta T + \gamma T^2$$

The optimal amount of taxation at a given point is to maximize the total utility function which is the utility for risk reducing minus cost of taxation. The equation below capture how government will use Pigouvian tax to solve radiance problem:

$$\begin{cases} \dfrac{\partial U(T^*)}{\partial T} = \left(\dfrac{\partial E_1}{\partial T} + \dfrac{\partial E_2}{\partial T} + \dfrac{\partial E_3}{\partial T} + \dfrac{\partial E_4}{\partial T}\right) \cdot B'(T) - \beta T - 2\gamma T^2 = 0 \\ \\ \dfrac{\partial^2 U}{\partial T^2} < 0 \end{cases}$$

6.3 Smart Light Grid System

A smart light grid system is an advanced network of lighting infrastructure that uses digital communication technology and advanced controls to improve energy efficiency and reduce costs. This system allows for the remote monitoring and control of lighting fixtures, as well as the ability to adjust radiance and color temperature based on different factors such as time of day, weather conditions, and occupancy. Smart light grid systems can also incorporate sensors and other data-gathering devices to provide real-time data on energy usage, occupancy levels, and other factors, which can then be used to optimize energy usage and improve the performance of the lighting system.

Since we can redistribute the radiance for each metric by using the smart grid, our utility function changes from one variable of B to 4 variables of B_i. We can rewrite the utility function as:

$$U(B_1, B_2, B_3, B_4) = P_{Gov} \sum_{i=1}^{4} \alpha_i \cdot E_i \cdot B_i - C(B)$$

The effect of the smart grid can be represented by the equation below:

$$P_e = P_{initial} - \alpha$$

Since we assume the cost of adding each light stored by smart grid is the same, the cost function is linear shown as below:

$$C(L_{store}) = \beta + \gamma \cdot L_{store}$$

The equation below captures how the government will use smart grid system to solve radiance problem:

$$\begin{cases} \dfrac{\partial U}{\partial B_1} = \dfrac{\partial U}{\partial B_2} = \dfrac{\partial U}{\partial B_3} = \dfrac{\partial U}{\partial B_4} = 0 \\ \\ \dfrac{\partial^2 U}{\partial {B_1}^2}, \dfrac{\partial^2 U}{\partial {B_2}^2}, \dfrac{\partial^2 U}{\partial {B_3}^2}, \dfrac{\partial^2 U}{\partial {B_4}^2} < 0 \end{cases}$$

7 Short Run & Long Run Policy Analysis

We have chosen Tokyo and Yellow Stone as our locations to implement our strategies since they are the ”most extreme cases”, which can show how diverse and flexible our strategies are. We use our model to show the optimal policy for both areas in different time span which are **Short Run** (instant) and **Long Run** (4 years).

Different from short run, our long run model captures the change of some variables in our metric with respect to time instead of seeing them as given. In the long run, we assume the government will change its optimal policy **every 3 months** to cope with the change of input variables. Also, for the **Smart Grid Policy** in the long run, we assume they will have a **time lag of two years** which means it will take the government two years to build the smart grid system in order to use it to achieve the optimal radiance. The expression for the variable that will change with time is listed below.

Variable	Population Density	Car Number	GDP	Biomass
Abbr.	P_d	car	GDP	Bio_m
Change Rate	Linear	Second-Order	Linear	Linear

Table 10: Change rate of different variables along time

Also, some variables (such as sensitivity of government’s reaction to the risk level P_{Gov}) within the utility function may also vary a lot in different countries or regions. For our analysis model of Tokyo and Yellow Stone, our assumption for those parameters are based on several past research like **Reducing the ecological consequences of night-time light pollution: options and developments**[4] and **Light Pollution: Economic Valuation Methods and a Market Solution**[5].

7.1 Short Run Optimal Policy for Tokyo & Yellow Stone

Our Short Run policy measures the instant optimal instant solution to maximize the total utility. Due to the time lag of Smart Grid System, it produces no utility at the moment when it is implemented. As the government only think about the utility given in the instant moment, the **Smart Grid System will not be consider in the short run**. Then, we just compare the utility generated by the optimal taxation $U(T^*)$ or the utility generated by the optimal shading ratio$U(L^*_{shade})$. The short run policy will be the one that **generate higher utility** compared to the other.

Short Run Policy for Tokyo

We used the First Order Condition and parameter for cost function / utility function we discussed previously to determine the optimal taxation T^* and the optimal light shade L^*_{shade}. The result of our calculation is below:

$$\begin{cases} U(T^*) = 0.812 \ \text{, where} \quad T^* = 4.94\% \\ U(L^*_{shade}) = 0.694 \ \text{, where} \quad L^*_{shade} = 34.4 \end{cases}$$

As a result, the utility generated by tax is higher than the utility generated by the shading. Tokyo should choose **Pigouvian taxation** as the short run optimal policy.

Short Run Policy for Yellow Stone

Similarly, by utilizing the First Order Condition and parameter we talked about already, we can decide the ideal taxation rate T^* and the ideal light shade L^*_{shade}. The result of our calculation is underneath:

$$\begin{cases} U(T^*) = 0.401 \ \text{, where} \quad T^* = 6.88\% \\ U(L^*_{shade}) = 1.092 \ \text{, where} \quad L^*_{shade} = 42.3\% \end{cases}$$

Hence, the optimal short run policy for Yellow stone is **Adding Light Shades**.

7.2 Long Run Optimal Policy for Tokyo & Yellow Stone

For long run, our model assume government implements the optimal policy for every 3 months. We calculate $T*$ and L^*_{shade} every 3 months and evaluate B^*_i after 2 years from the initial time where the construction of smart grid finishes. After calculating the optimal policy at each 3 months, we use **Barycentric Form Lagrange Interpolation**[6] to get our final optimal utility line given by implementing optimized policy each 3 months. The calculation of such interpolation line can be represented by a function $L(x)$ that is dependent on the weight function w_j and the displacement form function $l(x)$. The detailed calculation process can be found below:

$$l(x) = \Pi_m(x - x_m)$$

$$w_j = \Pi_{m \neq j}(x_j - x_m)^{-1}$$

$$L(x) = l(x) \sum_{j=0}^{k} \frac{w_j}{x - x_j} y_j$$

Long Run Policy for Tokyo

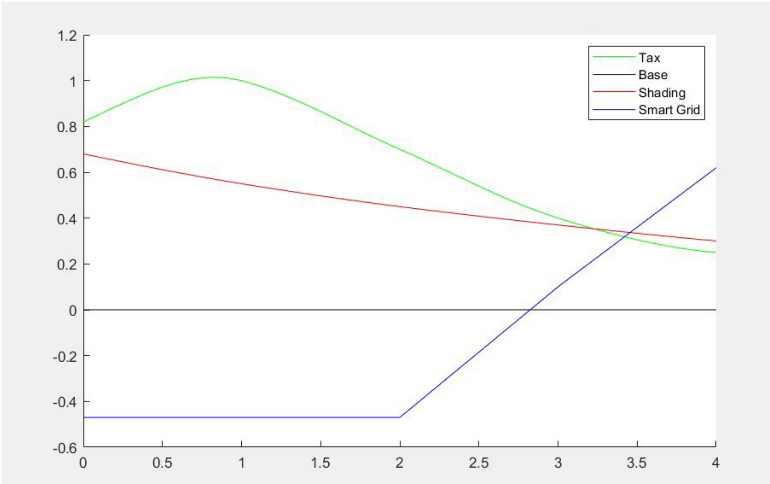

Figure 7: Graph of utility that brought by each strategies to Tokyo against time

From the graph above, we can see that in short run, tax is the best strategy for Tokyo since tax can cause a instantaneous effect without causing much cost in short run. Yet, in long run, the cost of taxation will be significantly high since it causes a loss of consumption continuously. For shading, since for each addition of shading, the cost increases linearly, which limits the increase in utility in long run. Notice that the utility level of Smart Grid is relatively low since the grid requires certain time and a large fix cost to develop. Yet, in long term, the utility level of the grid will surpass the other strategies after 3.5 years since the development of the grid will cause a constant increase in utility. On top of that, the grid can reduce light pollution without significantly reducing the use of light, which suits the interest of Tokyo, an urban location.

Thus, **Smart Light Grid System** is the best strategy for Tokyo in long run.

Long Run Policy for Yellow Stone

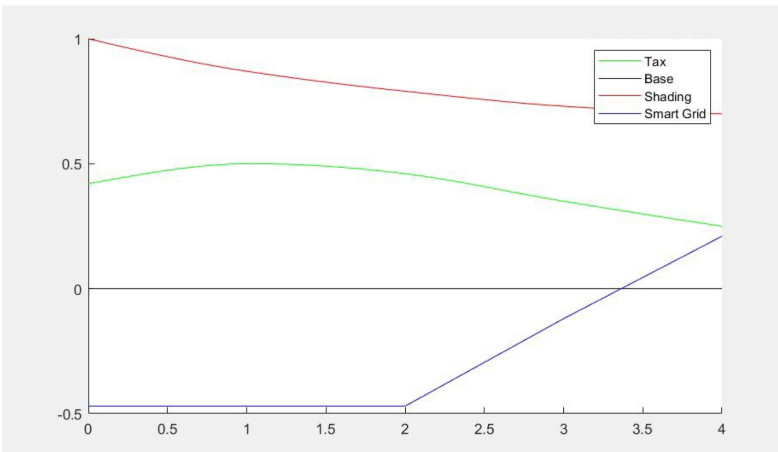

Figure 8: Graph of utility that brought by each strategies to Yellow Stone against time

From the graph above, we can notice that shading has a high utility in both short run and long run. This can be explained by the **pertinence** of shadings since it effectively lowered the radiance in a sharp way and improve the quality of life of wildlife. This is not going to cause a significant loss to Yellow Stone since the demand for light is low at the location. In contrast, the smart grid lacks pertinence in this case thus the utility level is lower than that of shadings. The effectiveness of tax is low since the population of Yellow Stone is low, which indicates that the number of tax payers is low.

Thus, **Adding Light Shades** is the best strategy for Yellow Stone in long run.

7.3 Sensitivity Analysis

To test our sensitivity of our model, we decrease the exogenous factor biodiversity to simulate the real life situation the biodiversity of a location will change along the time. The change in the result of long term can be visualised by the graphs:

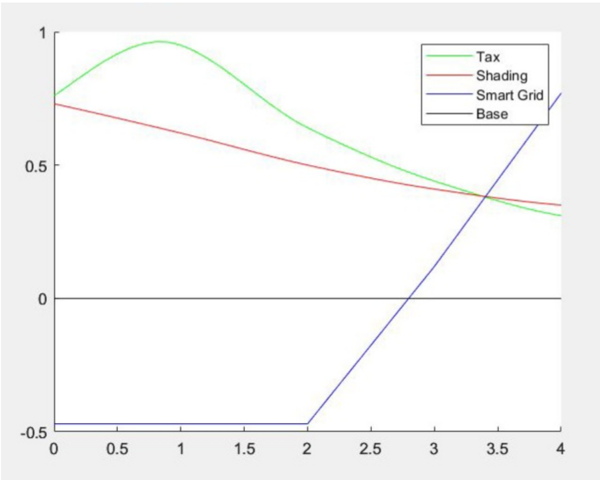

Figure 9: Sensitivity analysis of long run strategies implemented on Tokyo

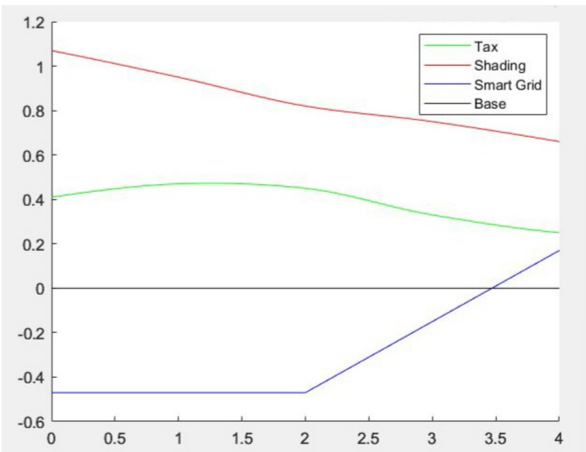

Figure 10: Sensitivity analysis of long run strategies implemented on Yellow Stone

From the graphs above, we notice that by slightly decreasing the biodiversity, the effectiveness of taxation decreases since the most beneficial group for the strategy is the wildlife, which means taxation is a process of **transfer of welfare from humans to wildlife**. Thus, when the biodiversity decreases, the effectiveness of taxation decreases. On top of that, the effectiveness of shadings is lowered since it is strategy that has pertinence on improving the quality of life of wildlife. When biodiversity decreases, its effectiveness drops.

Although the effectiveness of strategies has changed, this does not cause any changes on our preferences of strategies for the locations since the locations still retains their strong characteristic. This suits the real life situation since small change of parameter will not change the whole characteristics of the location.

The sensitivity analysis of our model demonstrates that our model has a significant and reasonable reaction to changes of exogenous parameter.

8 Flyer

Say Goodbye to Tokyo's Light Pollution with a Smart Light Grid System

Are you tired of Tokyo's bright and constantly glowing skyline? Do you want to see more stars at night and reduce your carbon footprint? The solution is here! The Smart Light Grid System for Tokyo is the answer to all your problems.

What is the Smart Light Grid System?

A smart light grid system is an advanced network of lighting infrastructure that uses digital communication technology and advanced controls to improve energy efficiency and reduce costs. This system allows for the remote monitoring and control of lighting fixtures, as well as the ability to adjust radiance and color temperature based on different factors such as time of day, weather conditions, and occupancy.

How does it work?

A smart light grid system uses digital communication technology to connect and control the lighting fixtures. This allows for remote control and monitoring of the lighting system, as well as the ability to collect data and optimize energy usage. They can also incorporate sensors to detect occupancy, ambient light levels, and other factors. This allows the system to adjust lighting levels in real-time, based on the needs of the environment and the users. Smart light grid systems can even collect and analyze data on energy usage, occupancy levels, and other factors. This data can then be used to optimize energy usage and improve the performance of the lighting system.

What are the benefits?

Reduced light pollution, allowing for better visibility of stars at night and improving the quality of life for residents. Lowered energy waste and carbon emissions, promoting sustainability and reducing the impact of climate change. Improved energy management, leading to cost savings and more efficient use of resources. Join us in our mission to make Tokyo a more sustainable and livable city. Contact us today to learn more about the Smart Light Grid System and how you can contribute to a better future for Tokyo.

Isn't it so costly?

Yes, indeed for short run. Yet, after about 3.5 years, the benefits generated by smart grid system can cover the fixed cost and have a long term sustainable development. Smart light grid system can improve reliability and resiliency. The advanced monitoring and control systems used in a smart light grid can identify and address issues before they become major problems, reducing the likelihood of power outages or other disruptions. Additionally, the system can reroute power around damaged or overloaded infrastructure, ensuring that power continues to flow to critical areas.

9 Evaluation

9.1 Strength

- Compared with statistical and machine learning methods that require a large scale of computational power, our model is significantly more time-saving and cost-saving to solve. Since the whole architecture of our metric and optimization is composed of algebraic equations, the optimal solution of magnitude of policy application can also be expressed as an algebraic analytic solution, which is both precise and consistent with different data sets.
- The metric of our model is scientific and inclusive because it is constructed with TOPSIS method that takes into account a large variety of factors about light pollution, with a carefully designed set of parameters from AHP method.
- The inherent property of TOPSIS method, ranking, makes the metric very comparable between different locations at different time instants, making evaluation of future trends and study of regional imbalance of light pollution very feasible.
- Since a large variety of policy can be modelled by a variable that directly influence some parameters of our model, application of our model on future research of other policies is open and feasible.

9.2 Weakness

- Due to the constraint of time and informational source, the data used for our analysis come from a variety of sources, which affects the precision of the model.
- The parameters within the system is mostly derived from AHP method and other logical derivations. Further improvements of precision can be made by replacing the estimation of parameter with results from statistical and experimental studies.

9.3 Conclusion & Future Work

In conclusion, our model successfully developed a reliable metric to estimate light pollution risk and thus utilized it to get the optimal policy plan in both short run and long run accurately. However, there are still some spaces for future improvement of this model:

- More data for different areas is needed to calculate more accurate estimation of standardized metric score.
- We could use some advanced mathematical tools such as optimal control theory to estimate best combination of policies over long run.
- We could use machine learning methods such as BP Neural Network to estimate the parameter such as Health Index in a dynamic motion.

10 Bibliography

References

[1] City Light or Star Bright: A Review of Urban Light Pollution, Impacts, and Planning Implications
https://journals.sagepub.com/doi/pdf/10.1177/0885412220986421

[2] An investigation into the risk of night light pollution in a glazed office building: The effect of shading solutions
https://www.researchgate.net/publication/327772416_An_investigation_into_the_risk_of_night_light_pollution_in_a_glazed_office_building_The_effect_of_shading_solutions

[3] IS-LM model with a general consumption tax
https://ro.ecu.edu.au/ecuworks/6979/

[4] Reducing the ecological consequences of night-time light pollution: options and developments
https://besjournals.onlinelibrary.wiley.com/doi/full/10.1111/j.1365-2664.2012.02212.x

[5] Light Pollution: Economic Valuation Methods and a Market Solution
https://www.atlantis-press.com/proceedings/esfct-22/125980551

[6] Barycentric Lagrange Interpolation
https://people.maths.ox.ac.uk/trefethen/barycentric.pdf

[7] Population Data
https://www.worldometers.info/population/largest-cities-in-the-world/

[8] Biodiversity Data
https://mol.org/patterns/richnessrarity

[9] Biomass Data
https://www.esa.int/Applications/Observing_the_Earth/Space_for_our_climate/New_biomass_map_to_take_stock_of_the_world_s_carbon

[10] Functional Diversity Data
https://mol.org/patterns/facets?taxa=mammals&pattern=fd

[11] Health Index Data
https://www.numbeo.com/health-care/rankings_by_country.jsp

11 Appendix

Program: Calculation of TOPSIS score of each location.

```
import numpy as np
from getData import getData, standardize
from topsis import topsis
from AHP import AHP
from showData import printRegions

# Initialization of data
regions = getData()
standardize(regions)

# TOPSIS analysis of four sub-metrics
econ_dataset = []
econ_coeffs = [0.2, 0.4, 0.4]
for region in regions:
    data = [region.den, - region.price, region.GDP]
    econ_dataset.append(data)
econ_topsis = topsis(econ_dataset, econ_coeffs)
# Analysis of other sub-metrics are similar

results = np.array([econ_topsis, ecol_topsis, hlth_topsis,
↪   traf_topsis])

# Apply AHP results
scores = AHP() @ results
for i in range(len(scores)):
    print(regions[i].name + ': ' + '{:.3f}'.format(scores[i]))

# Formatted output
printRegions(regions)
```