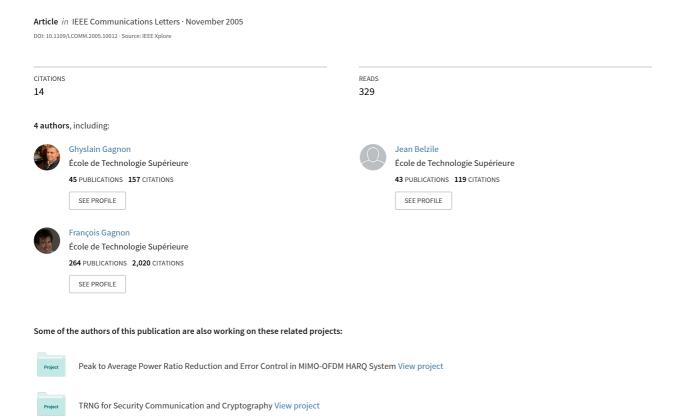
# A simple and fast carrier recovery algorithm for high-order QAM



# A Simple and Fast Carrier Recovery Algorithm for High-Order QAM

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Abstract—We propose a new carrier recovery algorithm for high-order Quadrature Amplitude Modulation (QAM). The proposed solution relies on two phase detectors, combined with a novel track and hold algorithm for faster frequency acquisition. Compared to previous works, a tenfold improvement or better is achieved in terms of acquisition speed, while keeping a low residual phase noise.

Index Terms—Carrier recovery, phase-frequency detector, phase noise, quadrature amplitude modulation.

## I. INTRODUCTION

AST carrier frequency acquisition is still a designer's challenge when it comes to blind, high-order Quadrature Amplitude Modulation (QAM). Simple decision-directed phase error estimators provide limited performances for frequency acquisition, since the phase error range is very low for high-power symbols, e.g.  $\pm 3.7^{\circ}$  for corner symbols in 256 QAM. This limitation is usually overcome by using the "four-corners" technique, which consists in using only the high-power symbols for phase detection and treating the square QAM constellation as a QPSK signal [1]. This method can be combined with a track and hold algorithm and act as a frequency discriminator [2]–[4].

This paper presents an improved carrier recovery algorithm for high-order QAM. The proposed architecture is based on the work by Matsuo and Namiki [2], relying on a coarse frequency acquisition mode and a fine phase tracking mode, selected by a lock detector module. In [2], only the four inner symbols are used for phase error estimation when in the coarse mode. The probability of occurrence of these symbols falls dramatically with higher-order QAM. In this paper, the proposed solution (as in [3]–[5]) uses more symbols for phase estimation. Furthermore, a novel track and hold algorithm for frequency discrimination allows us to achieve a tenfold improvement over previous works in acquisition speed.

#### II. SYSTEM MODEL

Assuming perfect timing synchronization and proper gain control, the receiver observes the sampled signal

$$r(n) = m(n)e^{j(\omega nT + \theta)} + v_c(n), \quad n = 1, 2, \dots$$
 (1)

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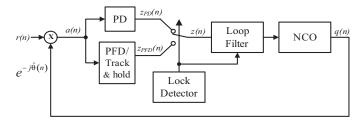


Fig. 1. Proposed carrier recovery loop.

where m(n) is the  $n^{\rm th}$  transmitted complex QAM symbol,  $\omega$  and  $\theta$  are the carrier frequency and phase, T is the symbol period and  $v_c(n)$  is a zero-mean complex Gaussian noise component. The numerically controlled oscillator (NCO) generates the signal

$$g(n) = e^{-j\hat{\theta}(n)}, \quad \hat{\theta}(n) = \hat{\theta}(n-1) + \hat{\omega}(n)T \tag{2}$$

with  $\hat{\omega}(n)$  being the estimated frequency at the  $n^{\rm th}$  symbol. Therefore, the input of the phase detectors is:

$$a(n) = m(n)e^{j\theta_e(n)} + v(n), \theta_e(n) = \omega nT + \theta - \hat{\theta}(n) \quad (3)$$

where  $\theta_e(n)$  is the residual phase error and v(n) is complex Gaussian noise with the same power as  $v_c(n)$ . A hard decision is made on a(n) to generate the detected complex symbol  $\hat{m}(n)$ , part of the QAM constellation. Both signals a(n) and  $\hat{m}(n)$  are used by the carrier recovery loop to estimate the phase error  $\theta_e(n)$  and consequently generate  $\hat{\theta}(n+1)$ .

#### III. PROPOSED CARRIER RECOVERY LOOP

The proposed carrier recovery algorithm uses two different phase detectors to generate z(n), the estimate of the phase error. Initially in the coarse mode, a phase-frequency detector (PFD) tracks the frequency error. Once locked, the phase error estimation is provided by a simple decision-directed phase detector (PD) for fine phase tracking.

## A. The Phase-Frequency Detector

The PFD is based on Kim and Choi's PD algorithm [5], with a new track and hold algorithm for improved frequency detection. The signal magnitude |a(n)| is compared to two threshold values  $\tau_1$  and  $\tau_2$ . Only high-energy symbols and the four inner symbols are used to provide information on the phase error. Unlike in [5], when symbols are discarded, i.e. when  $\tau_1 < |a(n)| < \tau_2$ , the phase error estimate is held to its

previous value. The phase error estimate, before the proposed track and hold, is

$$p(n) = \begin{cases} p(n-1), & \text{if } \tau_1 < |a(n)| < \tau_2 \\ \text{Im} \left\{ \frac{a(n)}{d(n)} \right\}, & \text{otherwise} \end{cases}$$
(4)

with [4]

$$d(n) = \frac{|a(n)|}{\sqrt{2}}(\operatorname{sgn}(\operatorname{Re}\{a(n)\}) + j\operatorname{sgn}(\operatorname{Im}\{a(n)\}))$$
 (5)

where sgn(x) = 1 when x < 0 and sgn(x) = 1 when  $x \ge 0$ .

When the "corner symbols" or the "inner symbols" of the constellation are received, the phase error estimate is proportional to their distance from the  $45^{\circ}$  frontier of their respective quadrant. By setting  $\tau_2$  to a very high value, the proposed algorithm can also be used with non-square constellations, but with limited performances.

In presence of a relatively large frequency offset, the rapid constellation rotation will give place to alternate changes of the phase error polarity, resulting in a mean phase error near zero. To overcome this problem, previous works [2]–[4] used saturation to keep the mean value of the phase detector away from zero in presence of a frequency offset. In this paper, we propose an improved track and hold algorithm that further increases the mean value of the PFD output in presence of a frequency offset, as shown in Fig. 2. The new track and hold algorithm is described by

$$\begin{split} z_{\scriptscriptstyle PFD}(n) &= \\ \left\{ \begin{array}{ll} p(n), & \text{if } |p(n) - z_{\scriptscriptstyle PFD}(n-1)| < \frac{\pi}{4} \\ \frac{\pi}{4} \mathrm{sgn}(z_{\scriptscriptstyle PFD}(n-1)), & \text{otherwise} \end{array} \right. \end{split} \tag{6}$$

## B. The Phase Detector

When phase-locked, the phase error estimate z(n) is provided by the PD. The PD does not discard symbols for phase estimation and is thus much less noisy for high-order constellations. The PD algorithm used in this paper is the simple decision-directed phase detector [4]

$$z_{PD}(n) = \operatorname{Im}\left\{\frac{a(n)}{\hat{m}(n)}\right\} \tag{7}$$

#### C. The Lock Detector

The lock detector monitors the signals a(n) and  $\hat{m}(n)$  to determine whether or not the constellation is phase-locked. The lock detector generates the binary signal

$$y(n) = \begin{cases} 1, & \text{if } |a(n) - \hat{m}(n)| < \lambda \\ 0, & \text{otherwise} \end{cases}$$
 (8)

In presence of a frequency error, the received signal points will not distribute closely around the expected points [2]. Hence, when phase-locked, y(n) will be equal to 1 in majority. Every  $N_{LD}$  symbols, the lock detector circuit compares the average value of y(n) to a threshold value  $\beta$  to determine if the loop must use the PFD (coarse) or the PD (fine). Even though the parameters  $N_{LD}$ ,  $\lambda$  and  $\beta$  play a role in the probability of erroneous lock-detection (and ultimately the acquisition time), simulations showed that the system is of limited sensitivity to these parameters.

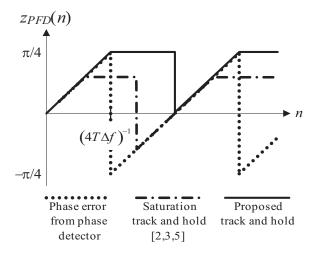


Fig. 2. Track and hold algorithms for a carrier frequency offset  $\Delta f$ . The mean value of  $z_{PFD}(n)$  is higher using the proposed track and hold algorithm in presence of a frequency offset while having the same gain in the linear range

# D. The Loop Filter

The loop filter is a simple digital first order lead-lag filter. Applying standard  $2^{\rm nd}$  order PLL equations [6], the residual phase noise at the PLL output can be predicted:

$$\theta_{e-rms} \simeq \frac{180}{\pi} \sqrt{\frac{\omega_n T(\zeta + 1/(4\zeta))}{SNR_i}}$$
 (9)

where  $\zeta$  and  $\omega_n$  are respectively the loop damping factor and natural frequency.  $SNR_i$  is the signal to noise ratio at the loop input. Because the phase detector is decision-directed, the system must operate at reasonably low symbol error rates for eq. (9) to remain valid.

Since the carrier recovery loop uses two distinct modes for frequency acquisition and tracking, the loop filter can have an increased bandwidth when in the coarse mode to speed up acquisition, simply by using two sets of filter constants, selected by the lock detector [2].

# IV. SIMULATION RESULTS

The proposed carrier recovery loop has been simulated. Results are compared to simulation results using Matsuo and Namiki's algorithm [2]. Though the system in [2] suggests an analog Voltage-Controlled Oscillator (VCO), for ease of comparison and without effect on acquisition performances, the simulations were performed using the ideal NCO of Fig. 1.

The published results of Ouyang and Wang's [4] are also reported. Their work demonstrated the superiority of their algorithm over Kim and Choi's [5], using specific system parameters.

As in [4], the symbol rate is set to 5.056941 MHz, for a 64 QAM constellation with a signal-to-noise ratio  $SNR_i = 30$  dB. The threshold values to discard symbols in the PFD are set to  $\tau_1 = 2.3$  and  $\tau_2 = 8.1$ . Hence, the 12 most powerful and the 4 less powerful symbols of the constellation are considered for phase error estimation in the PFD, as suggested in [4] and [2] respectively.

Based on previous simulation results, for both the proposed algorithm and the Matsuo algorithm, the value of  $\lambda$  is set

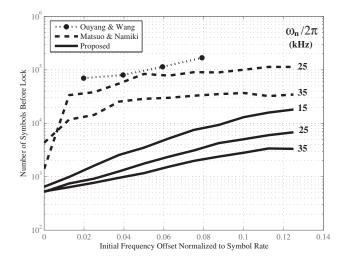


Fig. 3. Acquisition performances of the carrier recovery loop for 64 QAM. The loop natural frequency during the coarse acquisition mode is also shown. There are no notions of natural frequency in the algorithm from Ouyang & Wang. The results are taken directly from [4], assuming their parameters were optimally set for these simulation conditions. For the proposed algorithm and  $\omega_n/2\pi=15$  kHz, only one set of filter coefficients was used, thus trading acquisition speed for simpler implementation while retaining low residual phase noise  $(0.29^{\rm o}~{\rm rms})$ .

to 0.7,  $\beta$  is set to 0.6 and  $N_{LD}$  is set to 256 symbols. In presence of a carrier frequency offset, it can be shown that using  $\lambda=0.7$ , the 64 QAM constellation exhibits a probability  $P(y(n)=1)\approx 0.4$ . A false lock-detection occurs when  $\bar{y}>\beta$ , where  $\bar{y}$  is the average value of y(n) over the last  $N_{LD}$  symbols. The false lock-detection probability is easily found using binomial coefficients. For instance, using  $\beta=0.6$  and  $N_{LD}=256$  symbols, the false lock-detection probability is less than  $10^{-10}$ . Conversely, once locked and using the same parameters, the false unlock-detection probability  $P(\bar{y}<\beta)$  is practically 0 when operating under reasonable bit error rates.

Fig. 3 shows the acquisition time performances, given in symbols, for various initial carrier frequency offsets  $\Delta f$  and several loop natural frequencies with  $\zeta=0.7$ . Lock is considered achieved when the rms value of the phase error  $\theta_e(n)$  over the last 256 symbols is less than 1°rms. This condition could only be satisfied when toggled in the fine tracking mode, thus confirming the effectiveness of the lock detector. It should be noted that the same technique is used to determine the lock time for Ouyang and Wang's published results, but using the last 2000 symbols. Since in their case, the lock time is well over  $10^4$  symbols, this bias is negligible.

For initial frequency offsets near or greater than 1/8T, the algorithm can converge to 1/4T. At that point, the constellation rotates 1/4 turn at each symbol, not creating

any phase error. After some time, this "false-lock" situation can be identified by management software in the radio. Once detected, it is possible to instantly hop the carrier frequency by  $\pm 1/4T$  until the constellation stops rotating. For that reason, the system was also considered locked when it converged to 1/4T.

Each curve shown on Fig. 3 is obtained by averaging the results of 100-run simulations (40 for Ouyang and Wang [4]). From Fig. 3, one can see that using the same loop bandwidths, the proposed method is at least 10 times faster than the Matsuo algorithm for any frequency offset greater than 0. Furthermore, it can be seen that acquisition speed does not always improve when increasing the value of  $\omega_n$ , notably for low offset frequencies, where high values of  $\omega_n$  makes this randomly sampled feedback system approach instability.

In the case of  $\omega_n/2\pi=15$  kHz, only one set of coefficients have been used in the loop filter for a less complex hardware implementation. This value of  $\omega_n$  showed good acquisition performances, with excellent residual phase noise, without the need of two different loop bandwidths. In this configuration, the mean value of the residual phase noise throughout simulations was  $0.29^{\rm o}$ rms, which concords with (9):  $0.25^{\rm o}$ rms. Ouyang and Wang's work present a value of  $0.5^{\rm o}$ rms for residual phase noise. For dual-loop bandwidth test cases, the residual phase noise is irrelevant since the loop bandwidth in the fine mode can be made arbitrarily small.

The algorithm also showed good acquisition performances using 256 and 1024 QAM. For instance, using 256 QAM and  $\omega_n/2\pi=15$  kHz, it locked within  $3x10^4$  symbols for a normalized initial frequency offset of 0.1. The results are not plotted for the sake of brevity.

#### REFERENCES

- J. R. Treichler, M. G. Larimore, and J. C. Harp, "Practical blind demodulators for high-order QAM signals," in *Proc. of the IEEE*, vol. 86, pp. 1907-1926, Oct. 1998.
- [2] Y. Matsuo and J. Namiki, "Carrier recovery systems for arbitrarily mapped APK signals," *IEEE Trans. Commun.*, vol. 30, pp. 2385-2390, Oct. 1982.
- [3] H. Sari and S. Moridi, "New phase and frequency detectors for carrier recovery in PSK and QAM systems," *IEEE Trans. Commun.*, vol. 36, pp. 1035-1043, Sept. 1988.
- [4] Y. Ouyang and C.-L. Wang, "A new carrier recovery loop for high-order quadrature amplitude modulation," in *Proc. IEEE Global Telecommuni*cations Conference (GLOBECOM'02), vol. 1, pp. 478-482, Nov. 2002.
- [5] K.-Y. Kim and H.-J. Choi, "Design of carrier recovery algorithm for high-order QAM with large frequency acquisition range," in *Proc. IEEE International Conference on Communications (ICC'01)*, vol. 4, pp. 1016-1020, June 2001.
- [6] F. M. Gardner, Phaselock Techniques, Second Ed. New York: J. Wiley and Sons, 1979.