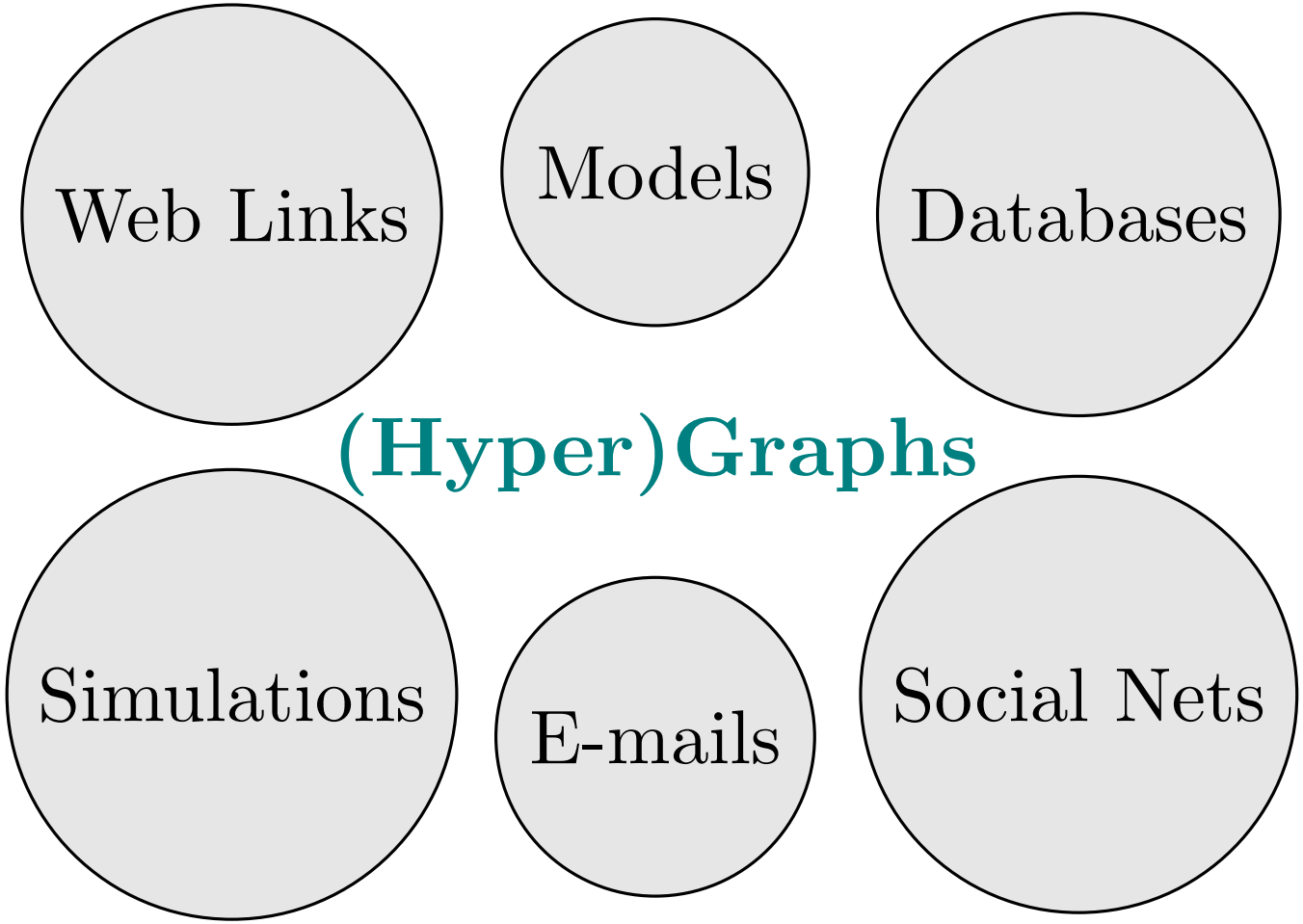
FREIGHT: Fast Streaming Hypergraph Partitioning

Kamal Eyubov, Heidelberg University
Marcelo Fonseca Faraj, Heidelberg University
Christian Schulz, Heidelberg University

Introduction

Ubiquity of (Hyper)Graphs



Introduction

Processing Large-Scale (Hyper)Graphs

Split workload among processors

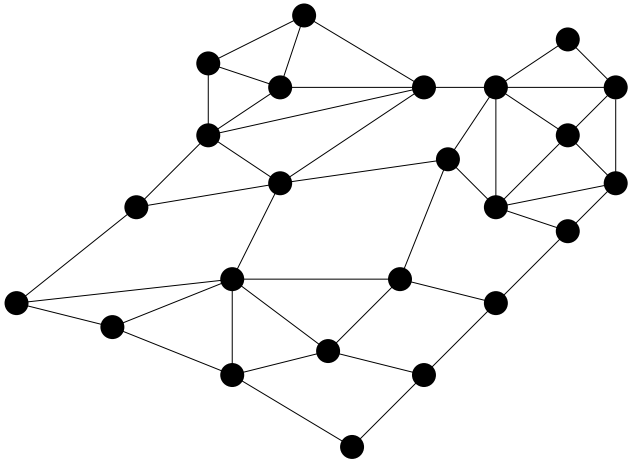
- In roughly equal blocks
- Cutting as little as possible

Graphs and Hypergraphs

Let $n = |V|, m = |E|$

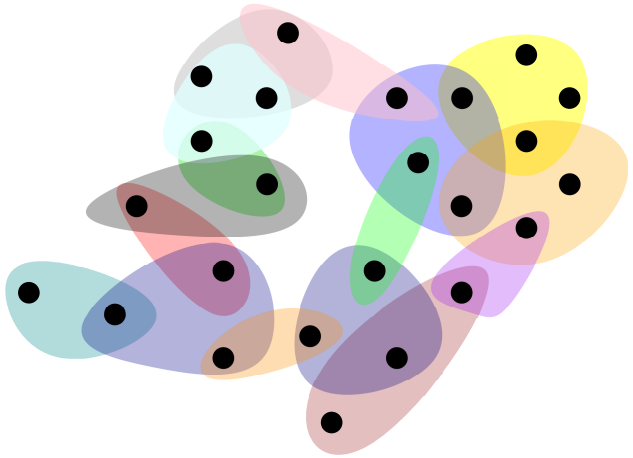
Graph $G = (V, E)$

- Edges connect two nodes
- Memory required to store it: $O(n + m)$



Hypergraphs $H = (V, E)$

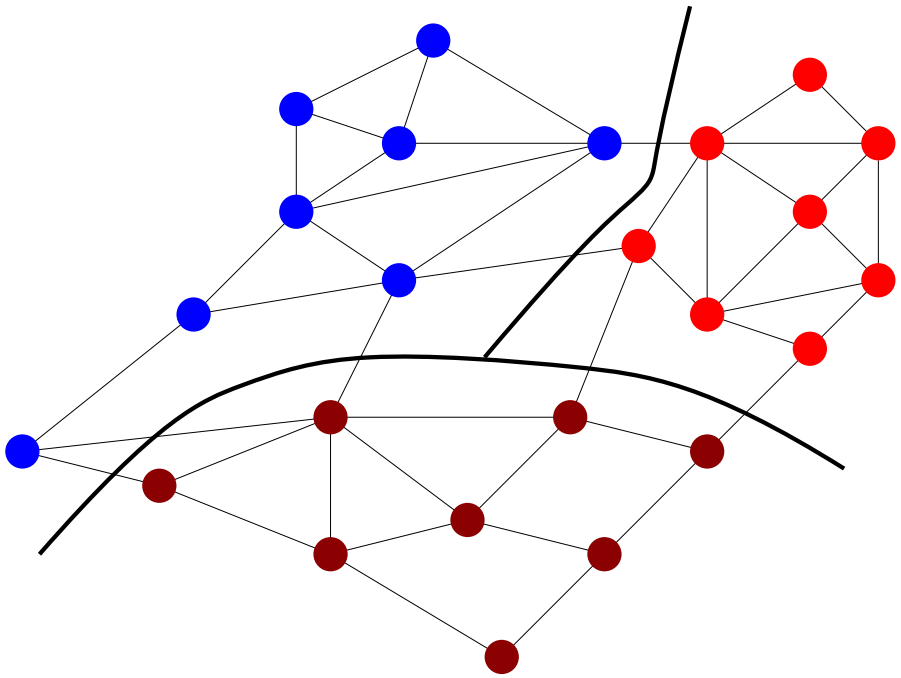
- Nets connect two or more nodes
- We say a net e has $|e|$ pins
- Memory required to store it: $O\left(n + \sum_{e \in E} |e|\right)$



Graph Partitioning (GP)

Partition nodes of graph $G = (V, E)$ s.t.

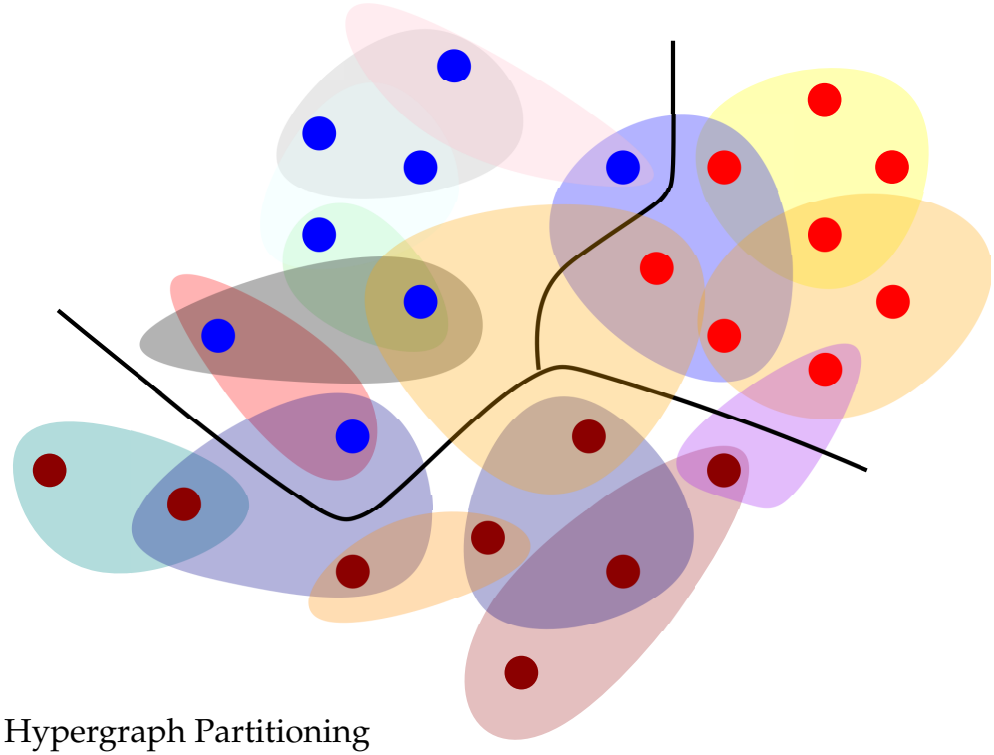
- Fixed amount k of disjoint blocks
- ϵ-Balanced blocks: Cardinality of each block $\leq \frac{1+\epsilon}{k} n$
- Objective: minimize edge-cut: $|Cut|$



Hypergraph Partitioning (HGP)

Partition nodes of hypergraph $H = (V, E)$ s.t.

- Fixed amount k of disjoint blocks
- Cardinality of each block $\leq \frac{1+\epsilon}{k} n$
- Common objectives minimized: (λ = # of blocks connected by e)
  - Cut-Net: $|Cut|$
  - Connectivity: $\sum_{e \in \text{Cut}} (\lambda - 1)$

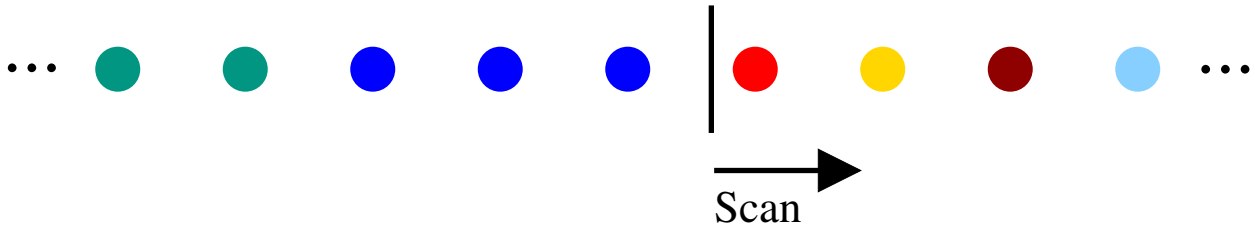


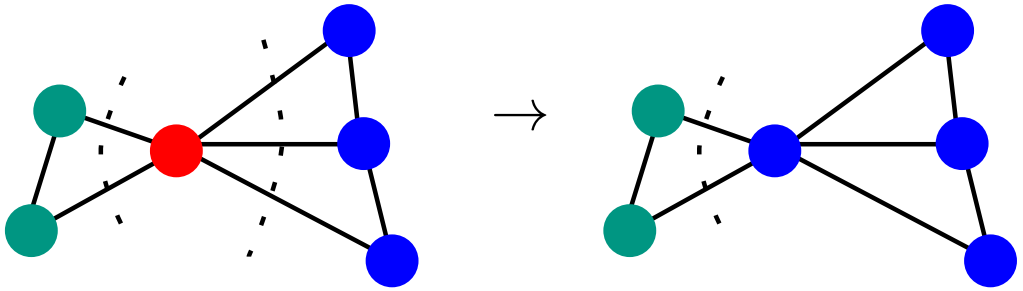
Streaming (H)GP

Computational model: one-pass partitioning

- Nodes arrive one at a time with their (hyper)edges
- Each node is assigned on the fly
- Assignments to blocks are permanent
-

Memory Budged in	Streaming	In-Memory
Graphs	$\mathcal{O}(n+k)$	$\mathcal{O}(n+m+k)$
Hypergraphs	$\mathcal{O}(mk)$	$\mathcal{O}\left(n+\sum_{e\in E}\lvert e\rvert+k\right)$





The Case for Streaming (H)GP

Many real-world use cases

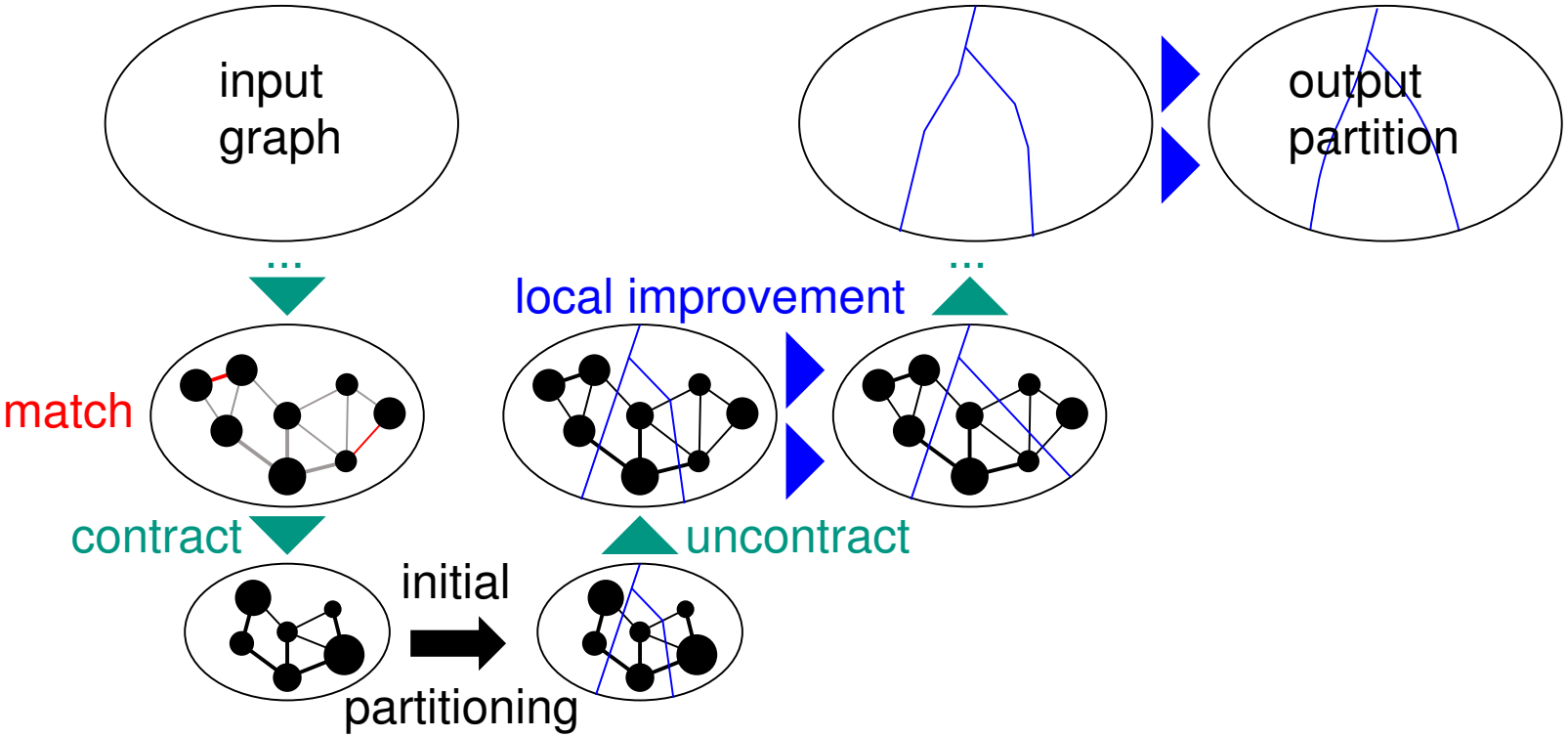
- (Hyper)Graph systems based on load-compute-store
  - Spark
  - Giraph
- Systems with (hyper)graph-as-a-stream computations
  - Flink
- Prepartitioning step for distributed (hyper)graph partitioning
- Partitioning huge (hyper)graphs using modest machines

Related Work

Related Work

In-Memory GP

Multilevel Graph Partitioning: THE Traditional Approach

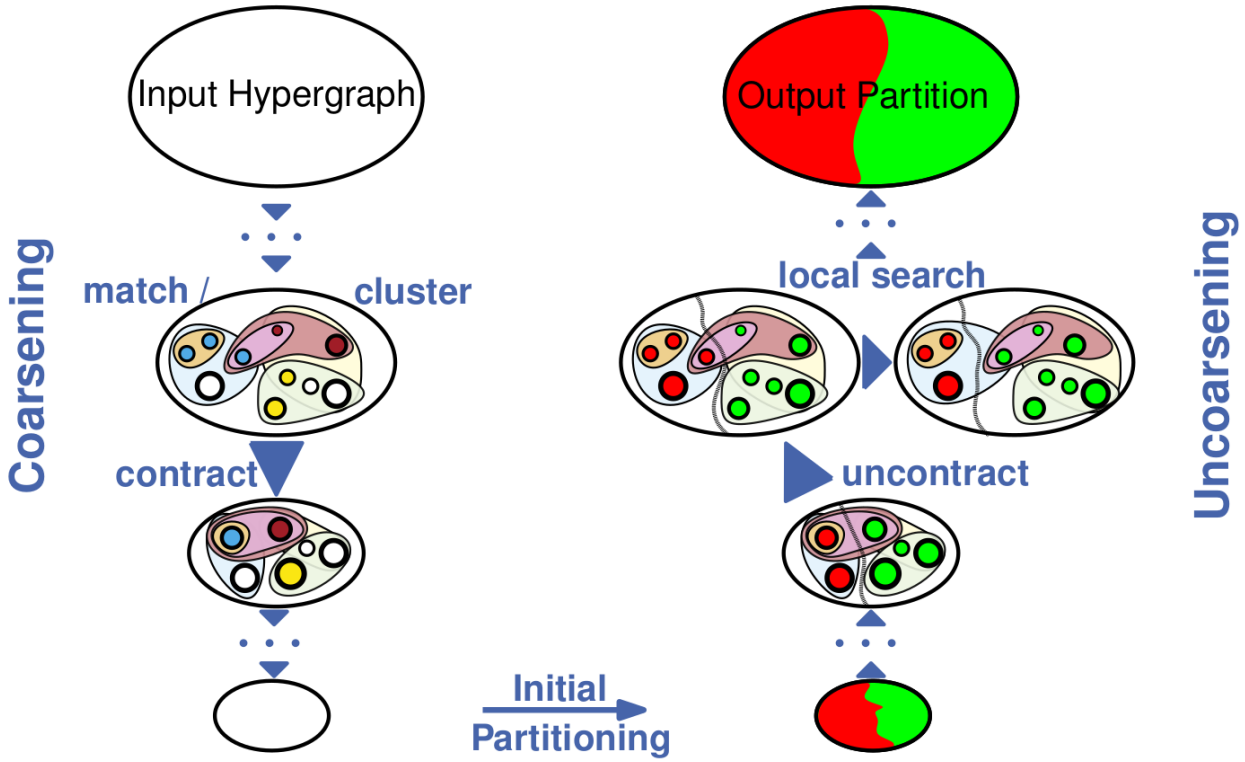


Some in existing systems:
Chaco, (mt)-Metis, Scotch, Jostle, DiBaP, (mt-)KaHIP, KaMinPar, …

Related Work

In-Memory HGP

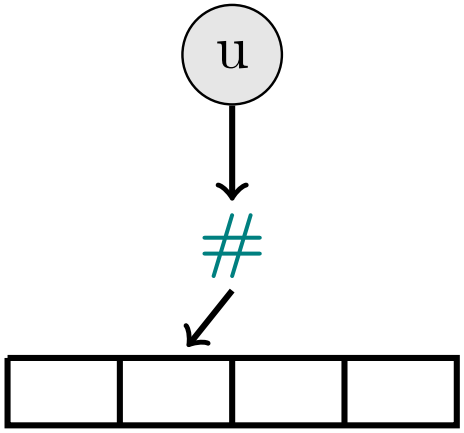
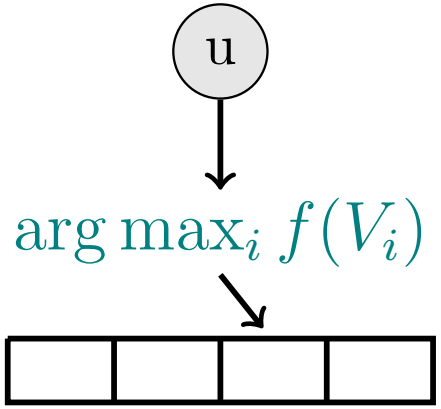
Multilevel Hypergraph Partitioning: THE Traditional Approach



Some in existing systems:
(mt-)KaHyPar, hMetis, PaToH, …

Related Work

Streaming GP

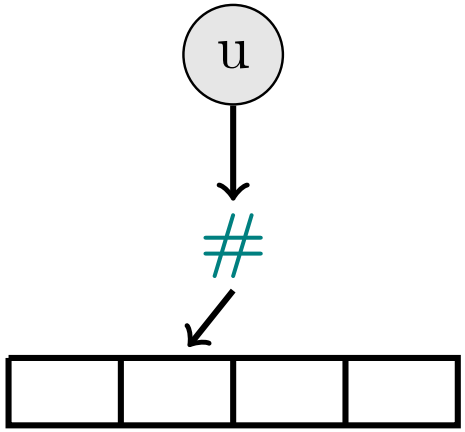
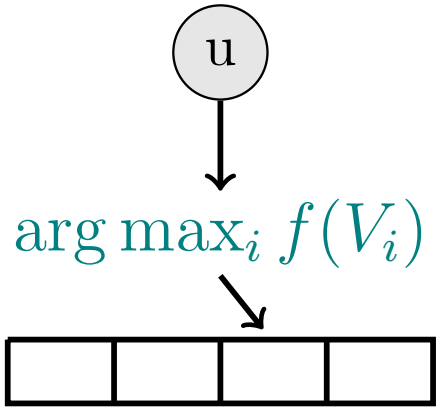
Algorithm	Hashing	Fennel
Scheme		

$$f(V_i) = |V_i \cap N(u)| - \alpha * \gamma * |V_i|^{\gamma-1}$$

Intuition of $f(V_i)$: consider cut and balance on the fly

Related Work

Streaming GP

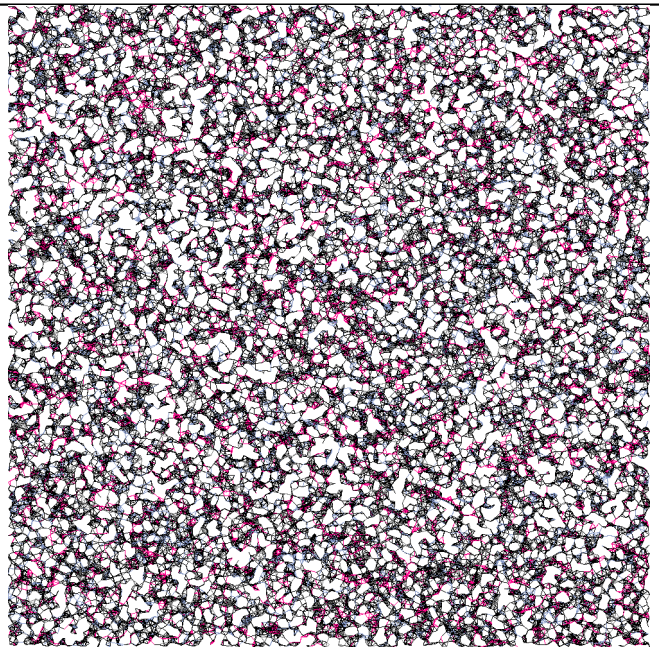
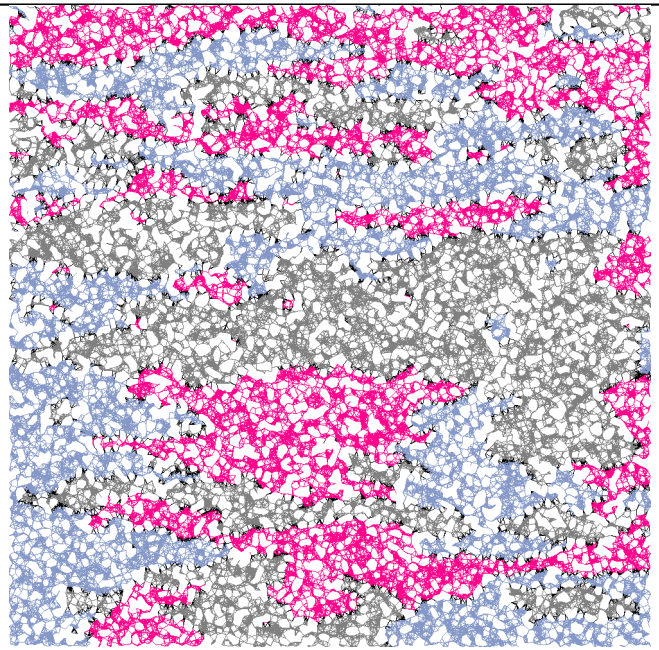
Algorithm	Hashing	Fennel
Scheme		
Memory	$O(k)$	$O(n+k)$
Running Time	$O(n)$	$O(nk+m)$

$$f(V_i) = |V_i \cap N(u)| - \alpha * \gamma * |V_i|^{\gamma-1}$$

Intuition of $f(V_i)$: consider cut and balance on the fly

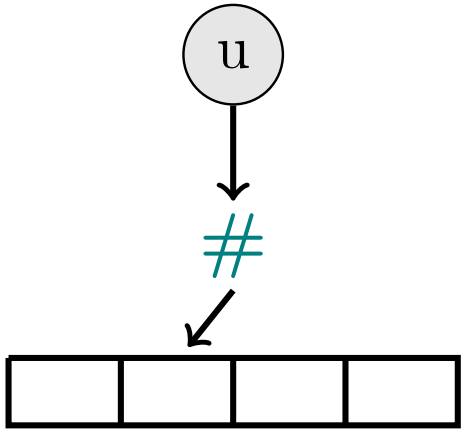
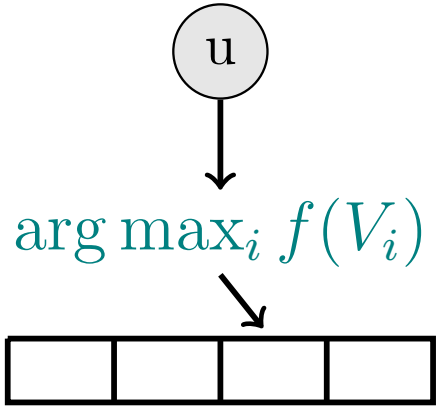
Related Work

Streaming GP

Algorithm	Hashing	Fennel
Quality		
Memory	$O(k)$	$O(n+k)$
Running Time	$O(n)$	$O(nk+m)$

Related Work

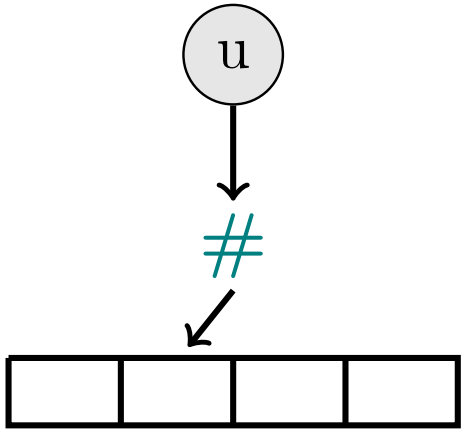
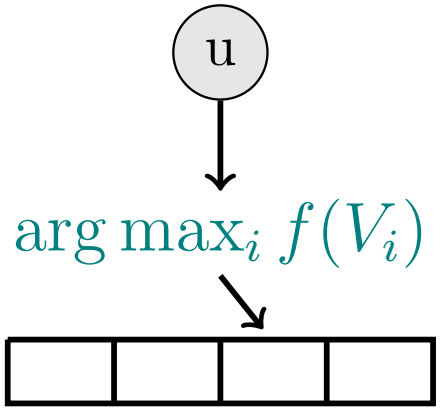
Streaming HGP

Algorithm	Hashing	Min-Max
Scheme		

$$f(V_i) = \sum_{e \in Nets(u)} |V_i \cap e \neq \emptyset|$$

Related Work

Streaming HGP

Algorithm	Hashing	Min-Max
Scheme		
Memory	$O(k)$	$O(mk)$
Running Time	$O(n)$	$O(n + k \sum_{e \in E} \lvert e \rvert)$

$$f(V_i) = \sum_{e \in Nets(u)} |V_i \cap e \neq \emptyset|$$

Related Work

Streaming HGP

Taşyaran et al. (2021) propose **improved versions** of Min-Max

Faster implementation with same quality: (same complexity)

- Min-Max-N2P: maps nets → blocks rather than blocks → nets

Lightweight versions of Min-Max-N2P w/ worse quality:

- Min-Max-Lℓ: uses upper-bound ℓ for memory consumption per net,
- Min-Max-BF: uses Bloom filters for membership queries
- Min-Max-MH: uses hashing to replace some connectivity info

Buffered streaming algorithms based on Min-Max-N2P:

- REF: buffers every incoming vertex but only reassigns those that may improve connectivity
- REF_RLX: buffers all nodes and reassigns all nodes in the buffer
- REF_RLX_SV: only buffers nodes with small net sets and reassigns all nodes in the buffer.

Related Work

(H)GP

You are referred to [1] for extensive references.

[1] Çatalyürek et al. (2023). More Recent Advances in (Hyper)Graph Partitioning. *ACM Computing Surveys*.

FREIGHT: Fast stREamInG Hypergraph parTitioning

FREIGHT

Mathematical Definition

FREIGHT is inspired in Fennel.

- Fennel (for graphs)

$$d = \underset{i,\ |V_i|<L_{\max}}{\operatorname{argmax}} \left\{ |V_i \cap N(v)| - \alpha * \gamma * |V_i|^{\gamma-1} \right\} \tag{1}$$

- FREIGHT (for hypergraphs):

$$d = \underset{i,\ |V_i|<L_{\max}}{\operatorname{argmax}} \left\{ |I^i_{obj}(v)| - \alpha * \gamma * |V_i|^{\gamma-1} \right\} \tag{2}$$

FREIGHT

Mathematical Definition

FREIGHT is inspired in Fennel.

- Fennel (for graphs)

$$d = \underset{i,\ |V_i|<L_{\max}}{\operatorname{argmax}} \left\{ |V_i \cap N(v)| - \alpha * \gamma * |V_i|^{\gamma-1} \right\} \tag{1}$$

- FREIGHT (for hypergraphs):

$$d = \underset{i,\ |V_i|<L_{\max}}{\operatorname{argmax}} \left\{ |I^i_{obj}(v)| - \alpha * \gamma * |V_i|^{\gamma-1} \right\} \tag{2}$$

Factor to penalize block imbalance is identical

FREIGHT

Mathematical Definition

FREIGHT is inspired in Fennel.

- Fennel (for graphs)

$$d = \underset{i,\ |V_i|<L_{\max}}{\operatorname{argmax}} \left\{ |V_i \cap N(v)| - \alpha * \gamma * |V_i|^{\gamma-1} \right\} \tag{1}$$

- FREIGHT (for hypergraphs):

$$d = \underset{i,\ |V_i|<L_{\max}}{\operatorname{argmax}} \left\{ |I^i_{obj}(v)| - \alpha * \gamma * |V_i|^{\gamma-1} \right\} \tag{2}$$

Factor to penalize block imbalance is identical

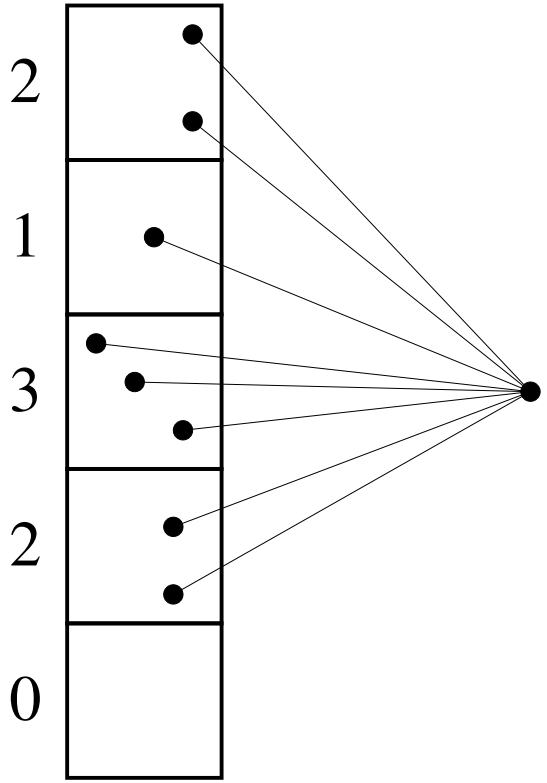
Factor to minimize edge-cut is adapted to minimize the intended metric: cut-net or connectivity.

FREIGHT

Mathematical Definition

- Fennel (for graphs):

$$d = \underset{i,\ |V_i|<L_{\max}}{\operatorname{argmax}} \left\{ |V_i \cap N(v)| - \alpha * \gamma * |V_i|^{\gamma-1} \right\} \tag{1}$$



FREIGHT

Mathematical Definition

- FREIGHT (for hypergraphs):

$$d = \underset{i,\ |V_i| < L_{\max}}{\operatorname{argmax}} \left\{ |I^i_{obj}(v)| - \alpha * \gamma * |V_i|^{\gamma - 1} \right\} \tag{2}$$

Keep d_e: block to which latest pin of net e was assigned.

Connectivity: $I^i_{obj}(v) = I^i_{con}(v) = \{e \in Nets(v) : d_e = i\}$

Cut-Net: $I^i_{obj}(v) = I^i_{cut}(v) = I^i_{con}(v) \setminus E'$

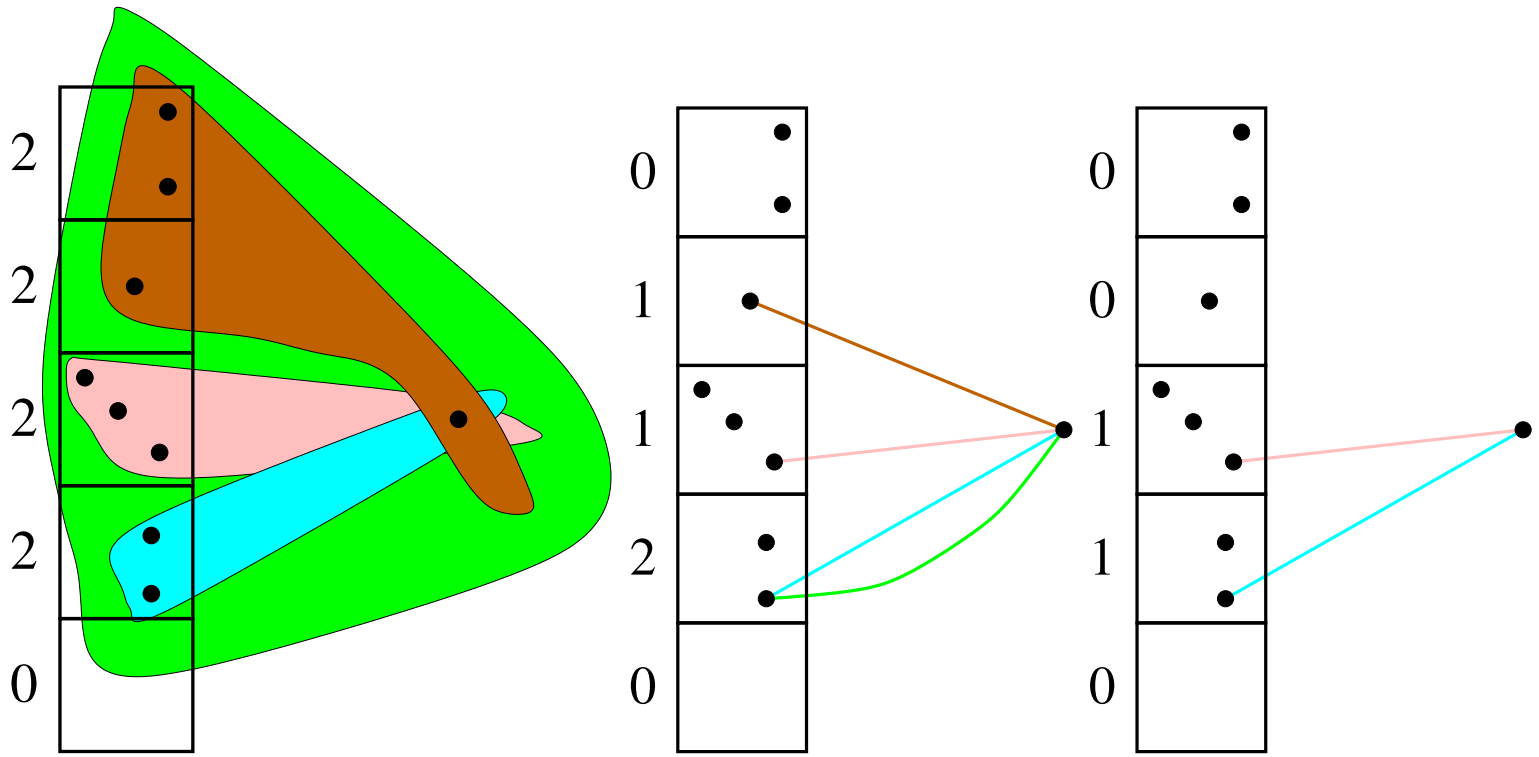
(E': set of already cut nets)

FREIGHT

Mathematical Definition

- FREIGHT (for hypergraphs):

$$d = \underset{i,\ |V_i|<L_{\max}}{\operatorname{argmax}} \left\{ |I^i_{obj}(v)| - \alpha * \gamma * |V_i|^{\gamma-1} \right\} \tag{2}$$



FREIGHT

Memory Usage

- FREIGHT (for hypergraphs):

$$d = \underset{i,\ |V_i| < L_{\max}}{\operatorname{argmax}} \left\{ |I^i_{obj}(v)| - \alpha * \gamma * |V_i|^{\gamma-1} \right\} \tag{2}$$

FREIGHT uses $O(m+k)$ memory.

- $O(k)$ to keep block cardinalities $|V_i|$, $1 \leq i \leq k$
- $O(m)$ to keep one assignment d_e per edge.

This is more memory than streaming GP.

- $O(n+k)$ used by the state-of-the-art.

This is less memory than streaming HGP.

- $O(mk)$ used by the state-of-the-art.

FREIGHT

Efficient Implementation

- FREIGHT (for hypergraphs):

$$d = \underset{i,\ |V_i|<L_{\max}}{\operatorname{argmax}} \left\{ |I^i_{obj}(v)| - \alpha * \gamma * |V_i|^{\gamma-1} \right\} \tag{2}$$

For every loaded vertex v, FREIGHT splits Equation (2) by separating blocks V_i for which $|V_i| < L_{\max}$ into two sets: S_1 and S_2.

- Set S_1: Blocks V_i where $|I^i_{obj}(v)| > 0$.
- Set S_2: Blocks V_i where $|I^i_{obj}(v)| = 0$.

$$d = \underset{i \in S_1}{\operatorname{argmax}} \left\{ |I^i_{obj}(v)| - \alpha * \gamma * |V_i|^{\gamma-1} \right\} \tag{3}$$

$$d = \underset{i \in S_2}{\operatorname{argmax}} \left\{ |I^i_{obj}(v)| - \alpha * \gamma * |V_i|^{\gamma-1} \right\} = \underset{i \in S_2}{\operatorname{argmin}} |V_i| \tag{4}$$

FREIGHT

Efficient Implementation

$$d = \underset{i \in S_1}{\operatorname{argmax}} \left\{ |I^i_{obj}(v)| - \alpha * \gamma * |V_i|^{\gamma-1} \right\} \tag{3}$$

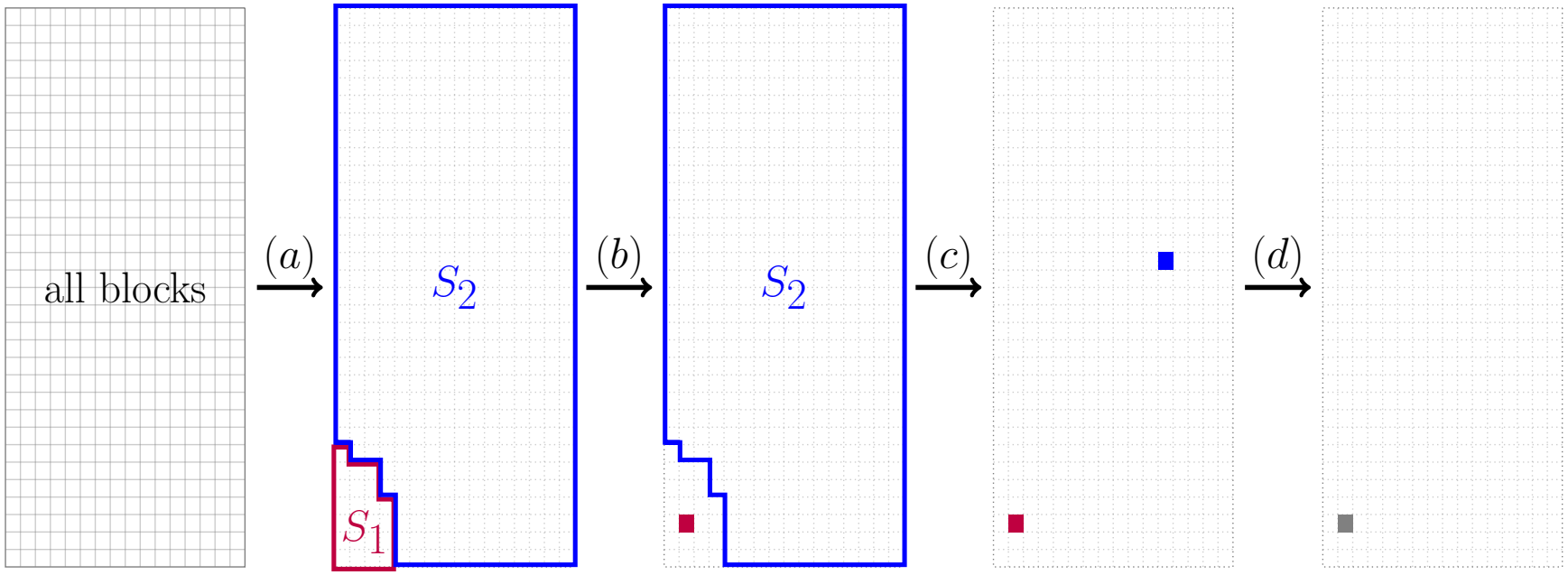
$$d = \underset{i \in S_2}{\operatorname{argmax}} \left\{ |I^i_{obj}(v)| - \alpha * \gamma * |V_i|^{\gamma-1} \right\} = \underset{i \in S_2}{\operatorname{argmin}} |V_i| \tag{4}$$

FREIGHT

Efficient Implementation

$$d = \underset{i \in S_1}{\operatorname{argmax}} \left\{ |I^i_{obj}(v)| - \alpha * \gamma * |V_i|^{\gamma-1} \right\} \tag{3}$$

$$d = \underset{i \in S_2}{\operatorname{argmax}} \left\{ |I^i_{obj}(v)| - \alpha * \gamma * |V_i|^{\gamma-1} \right\} = \underset{i \in S_2}{\operatorname{argmin}} |V_i| \tag{4}$$



FREIGHT

Efficient Implementation

$$d = \operatorname*{argmax}_{i \in S_1} \left\{ |I^i_{obj}(v)| - \alpha * \gamma * |V_i|^{\gamma-1} \right\} \tag{3}$$

$$d = \operatorname*{argmax}_{i \in S_2} \left\{ |I^i_{obj}(v)| - \alpha * \gamma * |V_i|^{\gamma-1} \right\} = \operatorname*{argmin}_{i \in S_2} |V_i| \tag{4}$$

Theorem

Equation (3) can be solved in time $O(|I(v)|)$ ($|I(v)|$ is the degree of v).

Proof.

- The terms $|I^i_{obj}(v)|$ in Equation (3) can be computed by iterating through the nets of v at a cost of $O(|I(v)|)$ and determining their status as cut, unassigned, or assigned to a block.
- The calculation of the factors $-\alpha * \gamma * |V_i|^{\gamma-1}$ in Equation (3) can be done in time $O(|S_1|) = O(|I(v)|)$, thus completing the proof.

□

FREIGHT

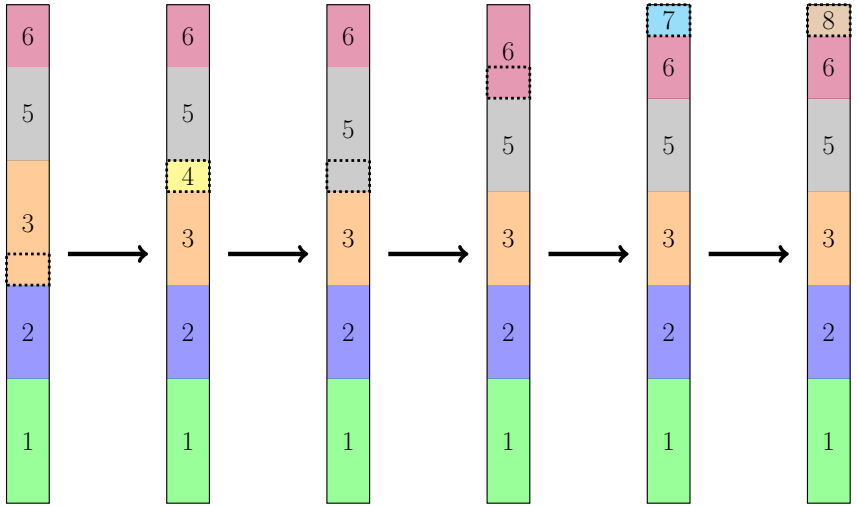
Efficient Implementation

$$d = \underset{i \in S_1}{\operatorname{argmax}} \left\{ |I^i_{obj}(v)| - \alpha * \gamma * |V_i|^{\gamma-1} \right\} \tag{3}$$

$$d = \underset{i \in S_2}{\operatorname{argmax}} \left\{ |I^i_{obj}(v)| - \alpha * \gamma * |V_i|^{\gamma-1} \right\} = \underset{i \in S_2}{\operatorname{argmin}} |V_i| \tag{4}$$

Theorem

Our proposed data structure keeps the blocks sorted in ascending order of cardinality at cost $O(1)$ per vertex.



FREIGHT

Efficient Implementation

$$d = \underset{i \in S_1}{\operatorname{argmax}} \left\{ |I^i_{obj}(v)| - \alpha * \gamma * |V_i|^{\gamma-1} \right\} \tag{3}$$

$$d = \underset{i \in S_2}{\operatorname{argmax}} \left\{ |I^i_{obj}(v)| - \alpha * \gamma * |V_i|^{\gamma-1} \right\} = \underset{i \in S_2}{\operatorname{argmin}} |V_i| \tag{4}$$

Theorem

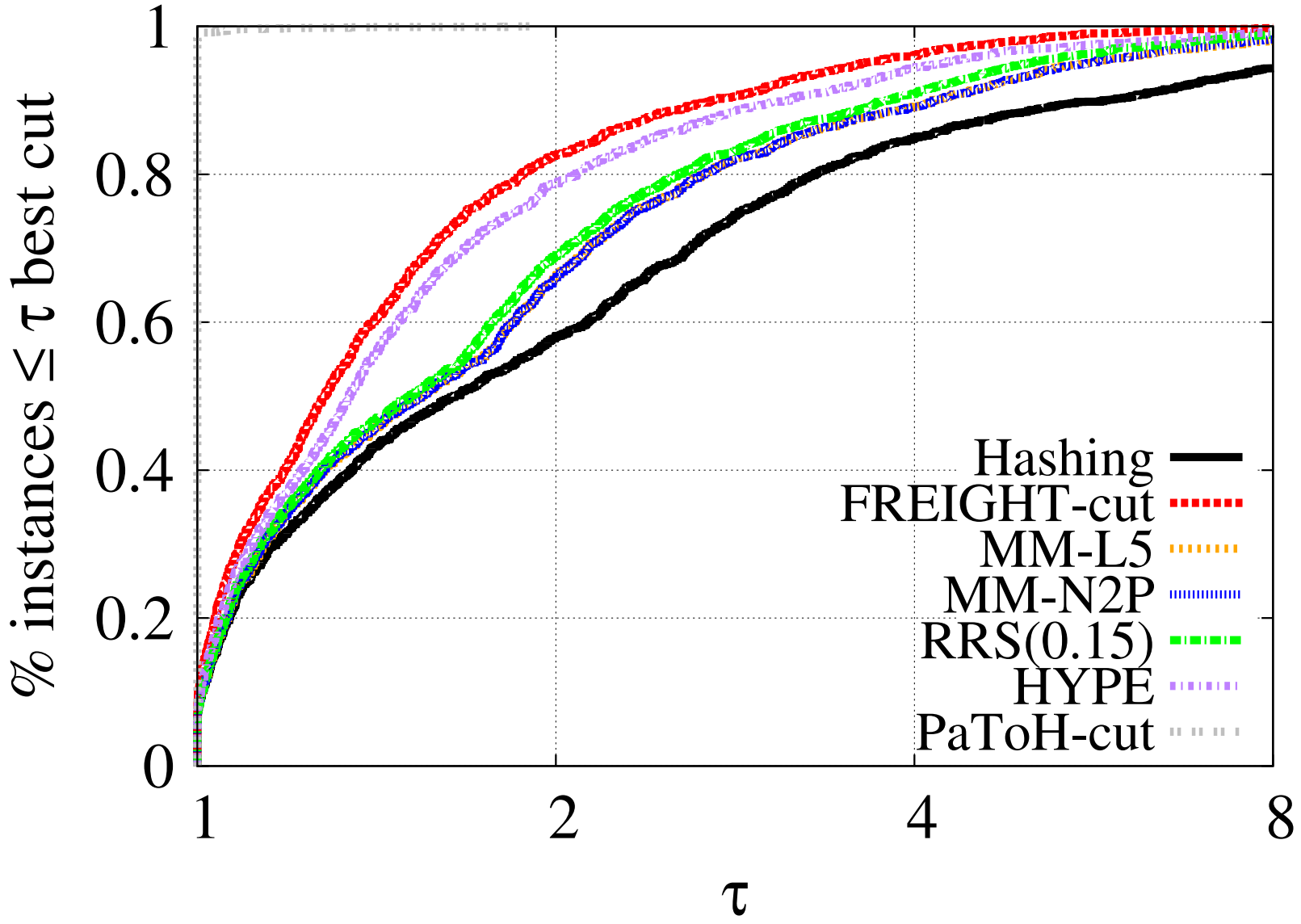
By utilizing our proposed data structure, solving Equation (4) or demonstrating that any solution for Equation (3) is also a solution for Equation (2) can be accomplished in $O(1)$ time.

Corollary

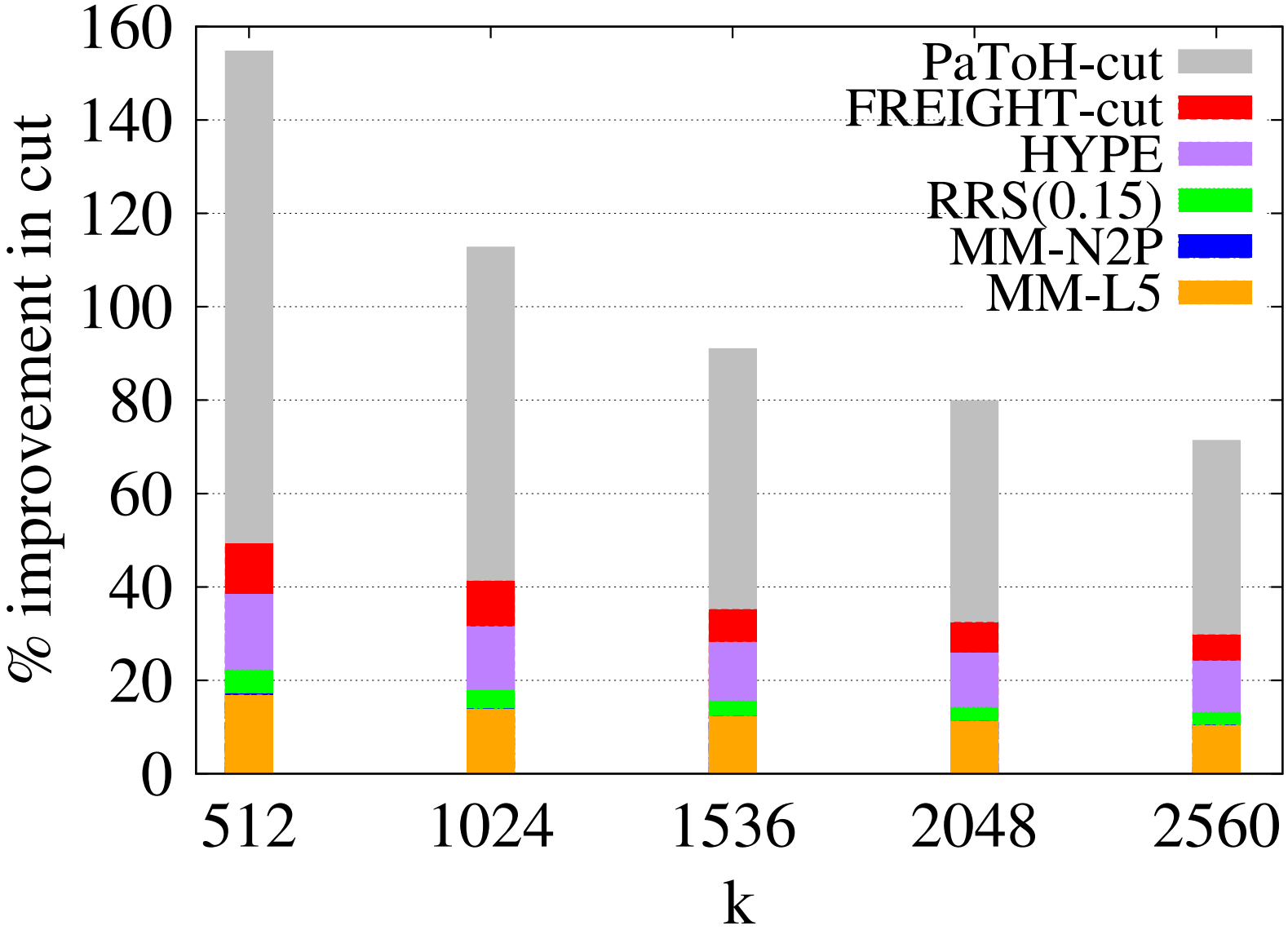
The overall complexity of `FREIGHT` *is* $O\left(\sum_{e \in E} |e| + n\right)$.

Experimental Results

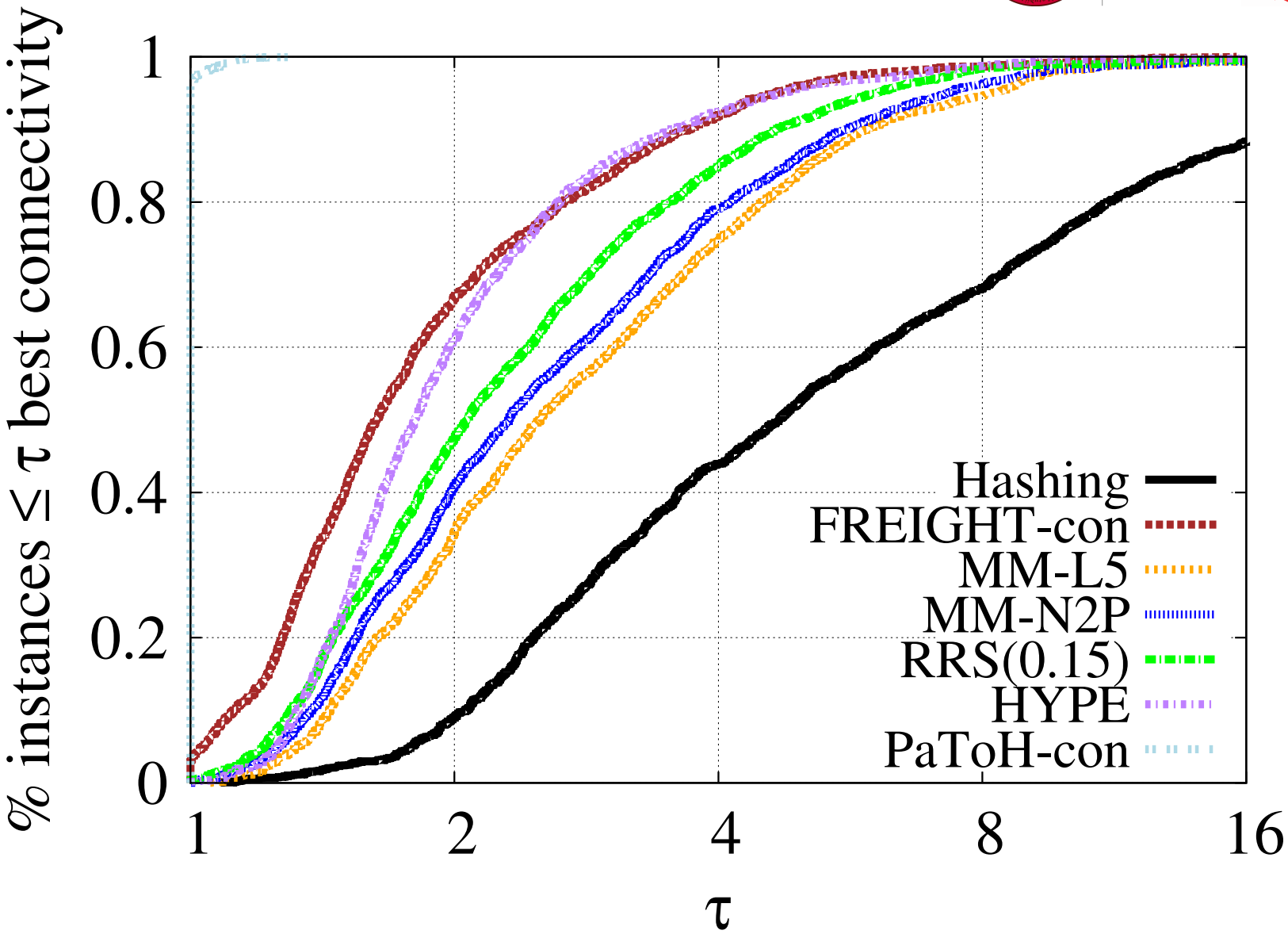
Experimental Results



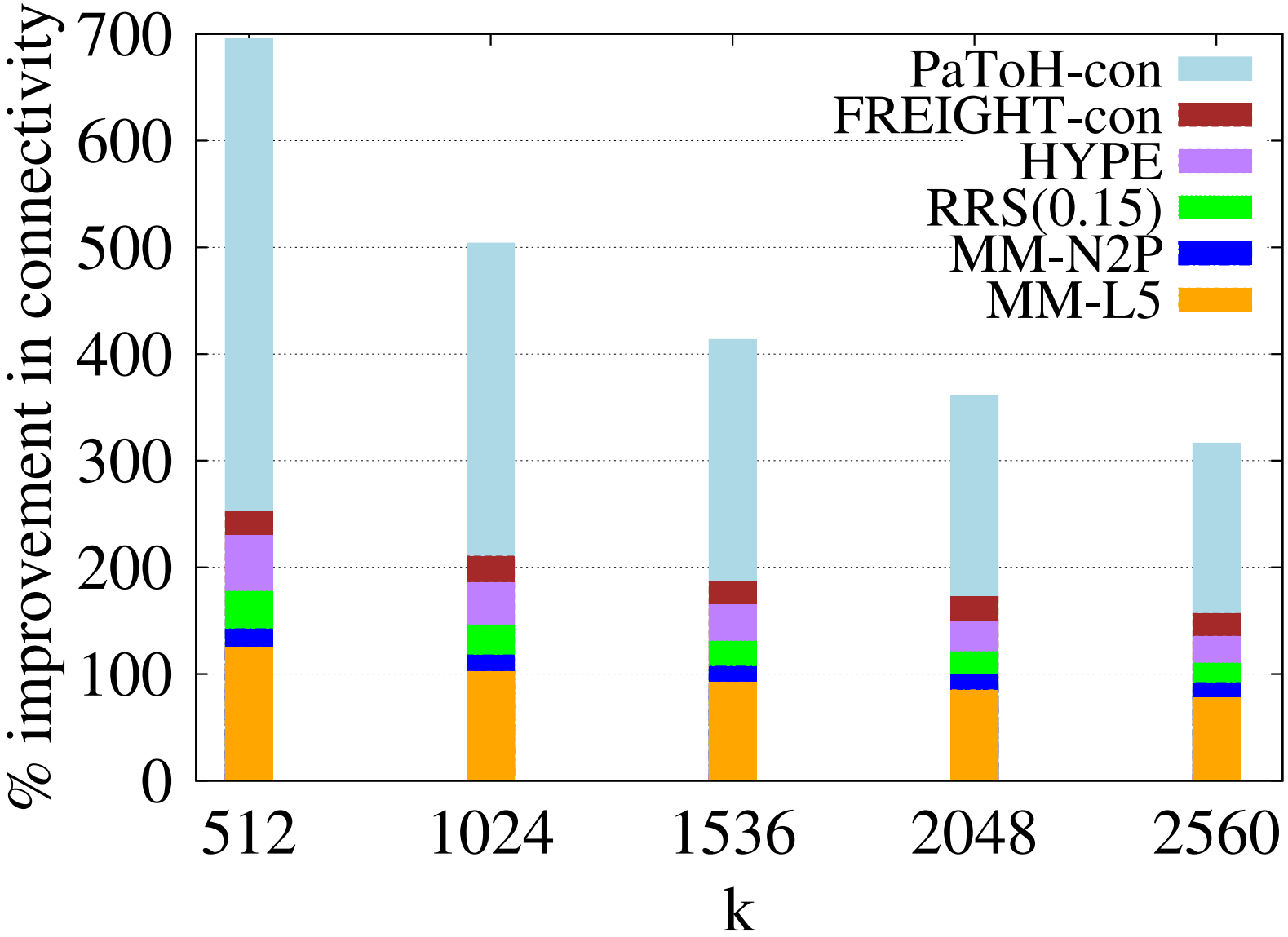
Experimental Results



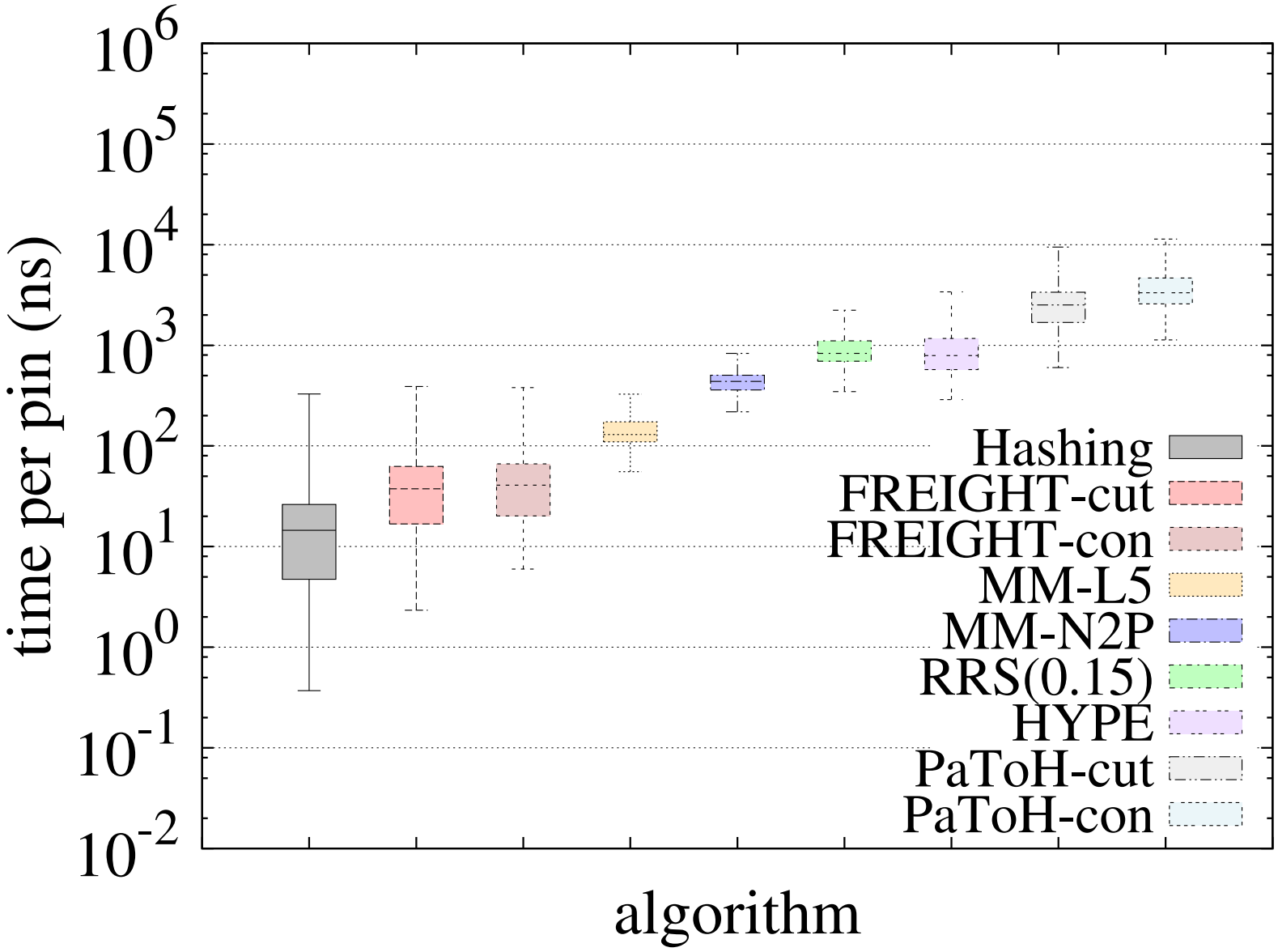
Experimental Results



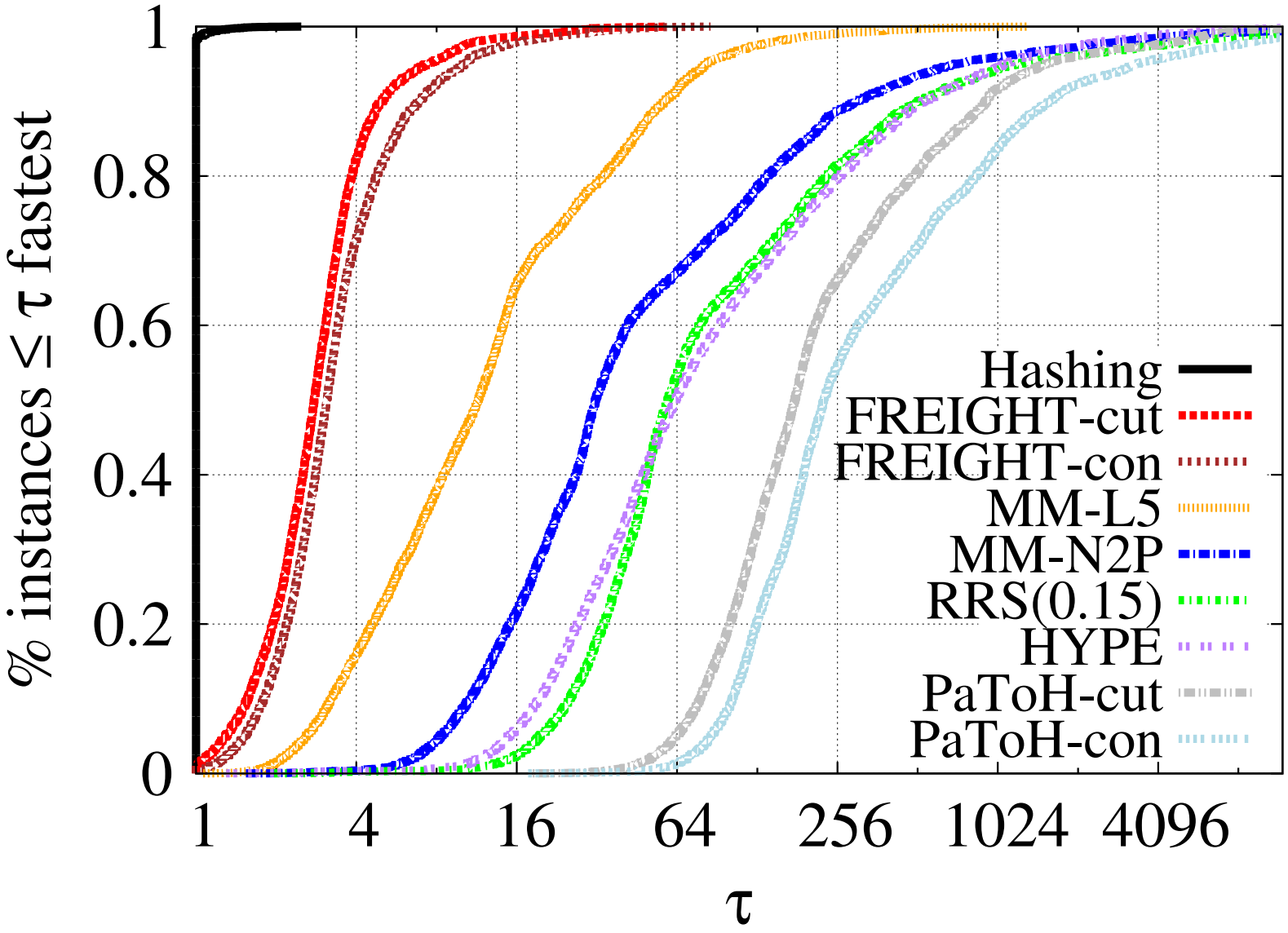
Experimental Results



Experimental Results



Experimental Results



Conclusion

FREIGHT

Conclusion

It is highly efficient and effective in theory and in practice.

It utilizes optimized data structure
$\Rightarrow$ linear running time w.r.t. pin-count.

Linear memory consumption w.r.t. the numbers of nets and blocks.

Empirically competitive with the speed of `Hashing`
$\Rightarrow$ at most four times difference in runtime for 80% of instances

Outperforms all existing streaming algorithms and even the in-memory algorithm `HYPE` in both cut-net and connectivity.

We just released FREIGHT [MIT License]!
`https://github.com/KaHIP/FREIGHT`

FREIGHT: Fast Streaming Hypergraph Partitioning

Kamal Eyubov, Heidelberg University
Marcelo Fonseca Faraj, Heidelberg University
Christian Schulz, Heidelberg University

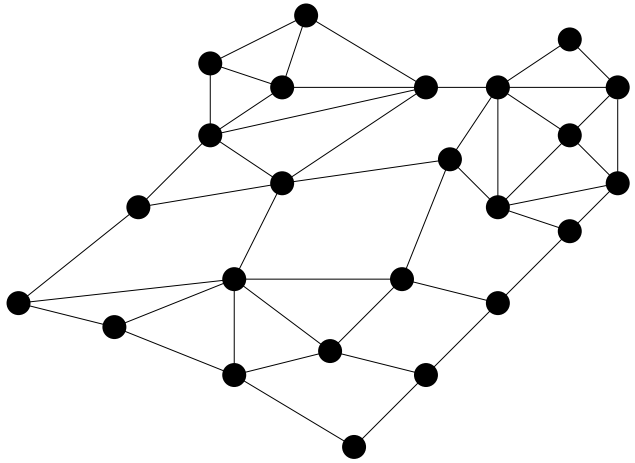
Graphs and Hypergraphs

Backup Slide

Let $n = |V|, m = |E|, c : V \to \mathbf{R}_{>0}, \omega : E \to \mathbf{R}_{>0}$

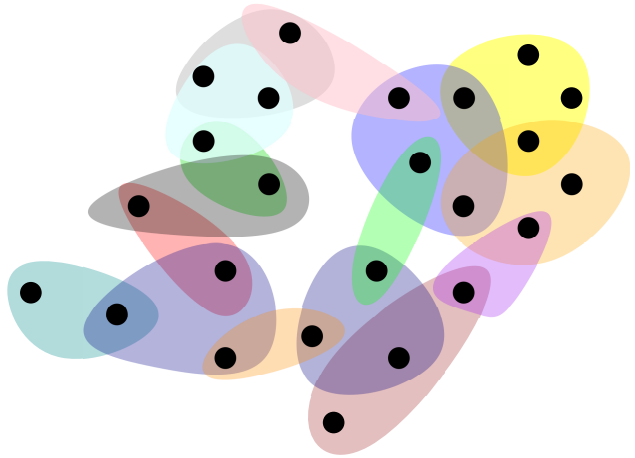
Graph $G = (V, E)$

- Edges connect two nodes
- Memory required to store it: $O(n + m)$



Hypergraphs $H = (V, E)$

- Nets connect two or more nodes
- We say a net e has $|e|$ pins
- Memory required to store it: $O\left(n + \sum_{e \in E} |e|\right)$

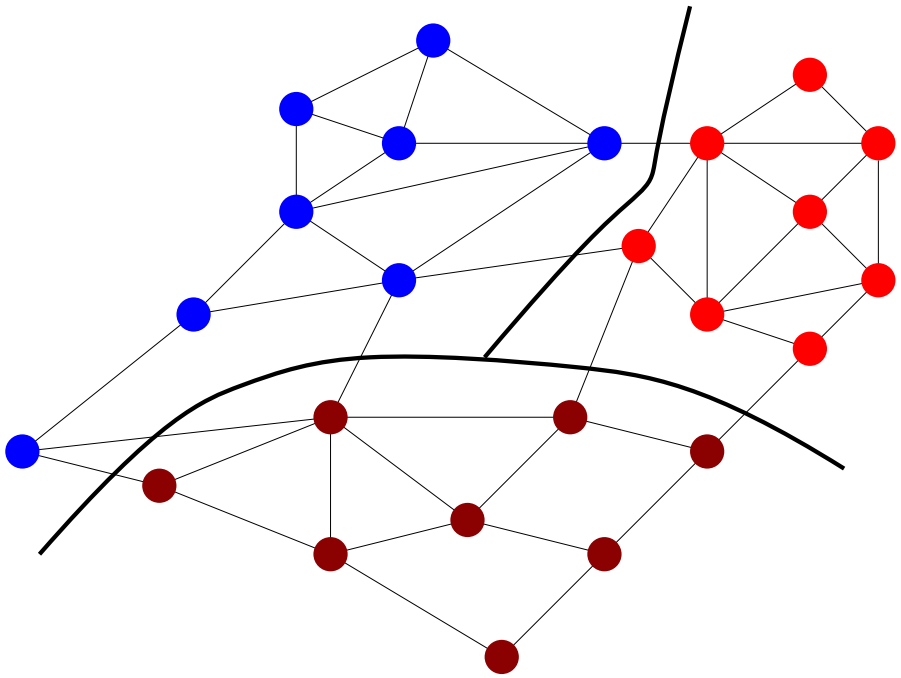


Graph Partitioning (GP)

Backup Slide

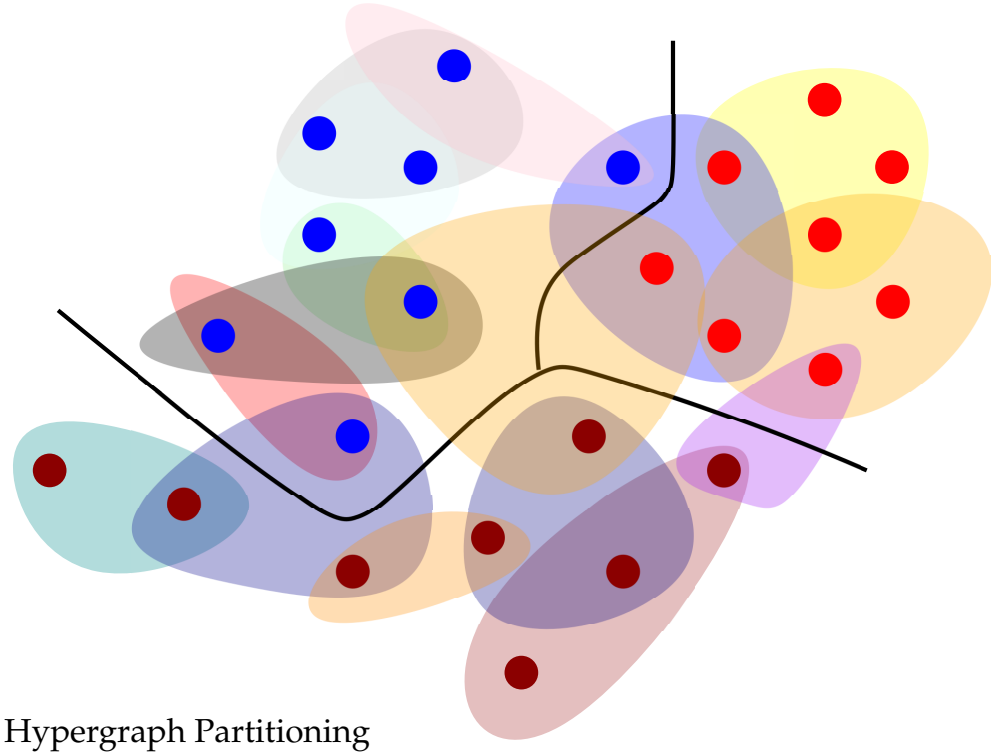
Partition nodes of graph $G = (V, E)$ s.t.

- Fixed amount k of disjoint blocks
- ϵ-Balanced blocks: Weight of each block $\leq \frac{1+\epsilon}{k} c(V)$
- Objective: minimize edge-cut: $\sum_{e \in \text{Cut}} \omega(e)$



Hypergraph Partitioning (HGP)

Backup Slide

Partition nodes of hypergraph $H = (V, E)$ s.t.

- Fixed amount k of disjoint blocks
- Weight of each block $\leq \frac{1+\epsilon}{k} c(V)$
- Common objectives minimized: (λ = # of blocks connected by e)
  - Cut-Net: $\sum_{e \in \text{Cut}} \omega(e)$
  - Connectivity: $\sum_{e \in \text{Cut}} (\lambda - 1)\omega(e)$