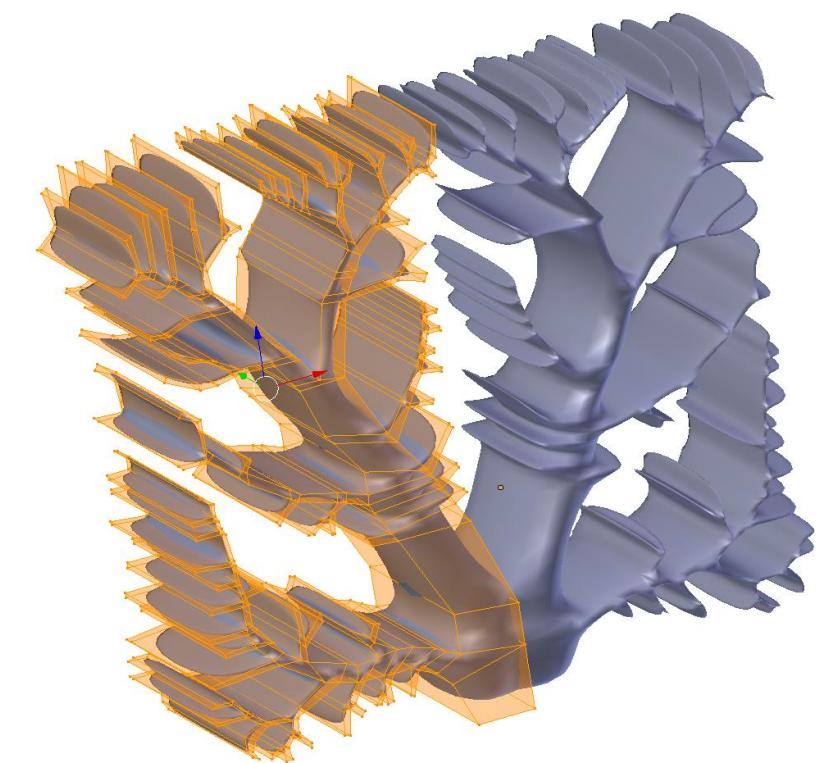
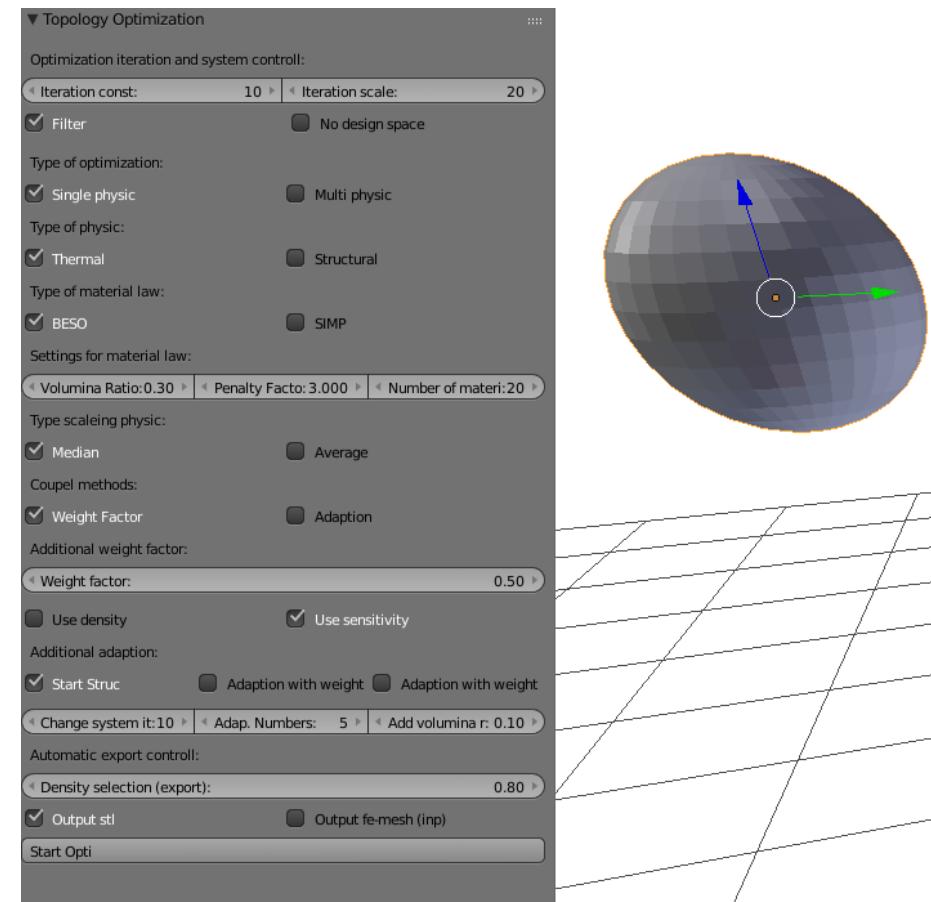


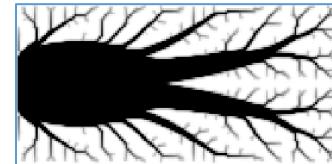
**Entwicklung einer Applikation
für eine automatisierte
multiphysikalische
Topologieoptimierung und
Geometrierückführung**



Gliederung



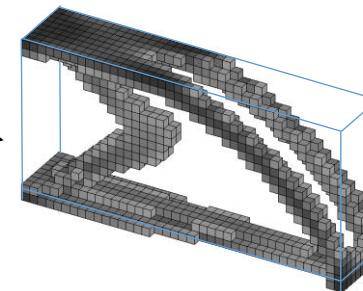
Entwicklung einer Optimierungsstrategie



Umsetzung in eine Applikation

Rückführung der Geometrie

Ausblick/Fazit



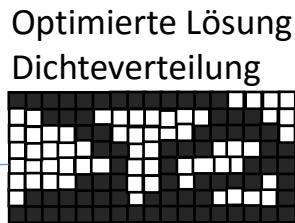
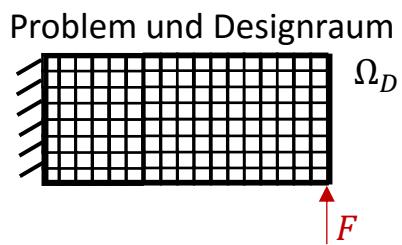
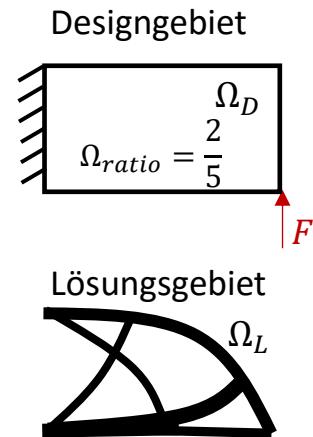
Formulierung des Optimierungsproblems (Steifigkeit)

Minimierung der Dehnungsenergie: $\min(E_{den})$

Designvariable (normierte Dichte): $\rho_e = \frac{\rho_{elem}}{\rho_0}$

Gleichungsrestriktionen:

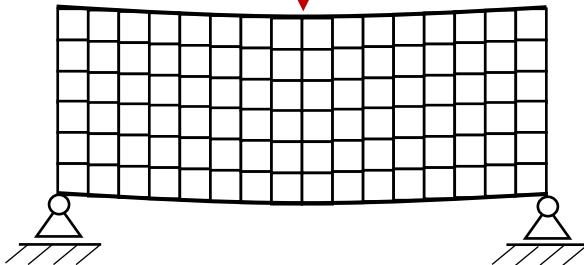
$$\begin{aligned}\frac{\Omega_L(\rho)}{\Omega_D} - \Omega_{Ratio} &= 0 \\ 0 - \rho_e &\leq 0 \\ \rho_e - 1 &\leq 0\end{aligned}$$



Vorgehensweise der Topologieoptimierung

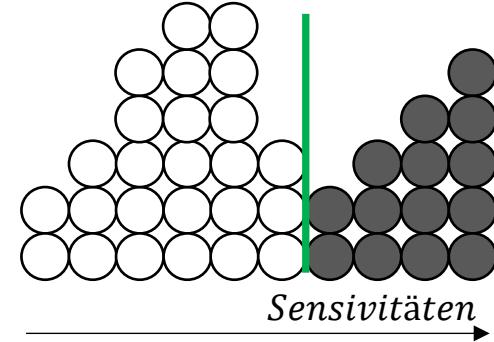
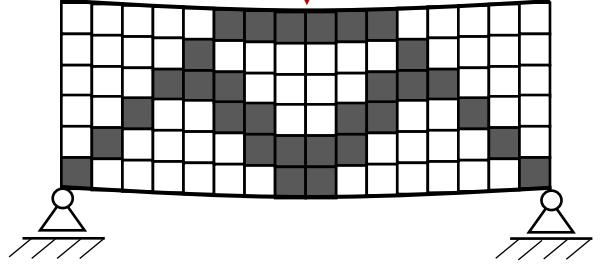
1. Iteration

$$K_e u_e = f_e$$



n. Iteration

$$K_e u_e = f_e$$



Initialisierung des Problems

Finite Elemente Analysis

Sensitivitätsanalyse

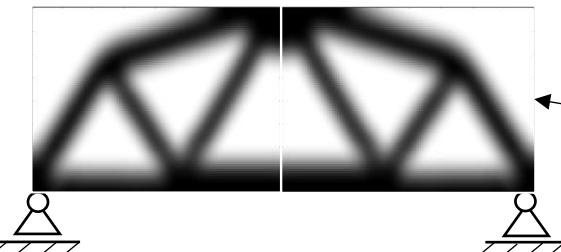
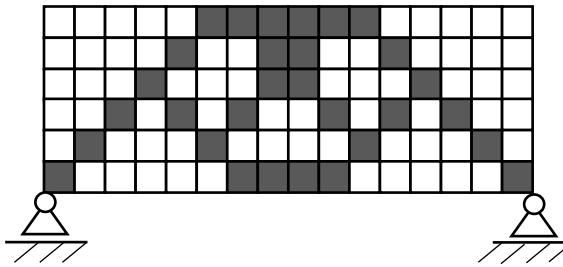
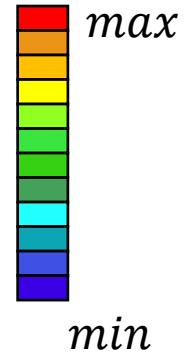
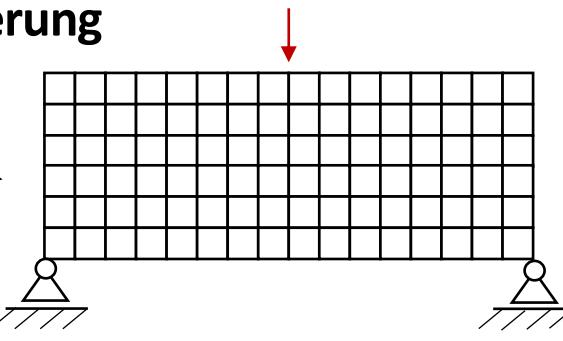
Optimierung durch „method of moving asymptotes“

Update der Designvariablen

Lösungsgebiet konvergiert

Ja

Ausgabe des Ergebnisses



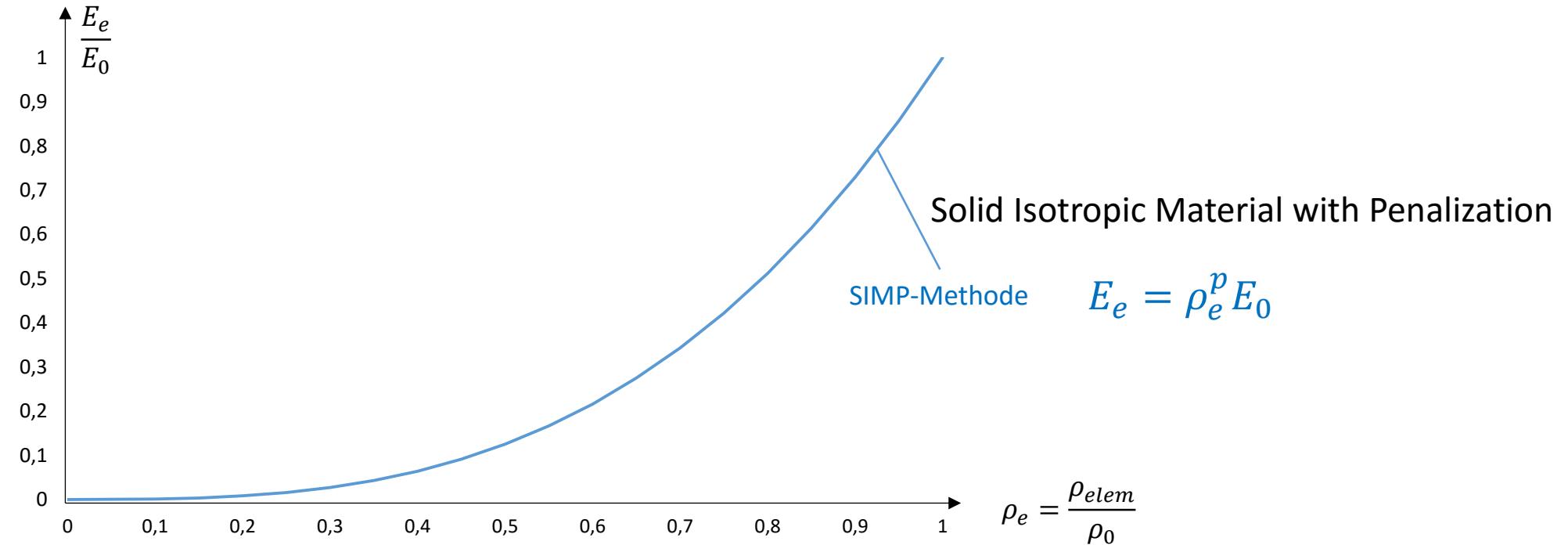
Entwicklung einer Optimierungsstrategie

Umsetzung in eine Applikation

Rückführung der Geometrie

Ausblick/Fazit

Materialmodell



$$E_0 = 70000 \text{ MPa}$$



$$E_e = 8,77 \text{ MPa}$$



$$E_e = 664,77 \text{ MPa}$$



$$E_e = 21000 \text{ MPa}$$



$$E_e = 70000 \text{ MPa}$$

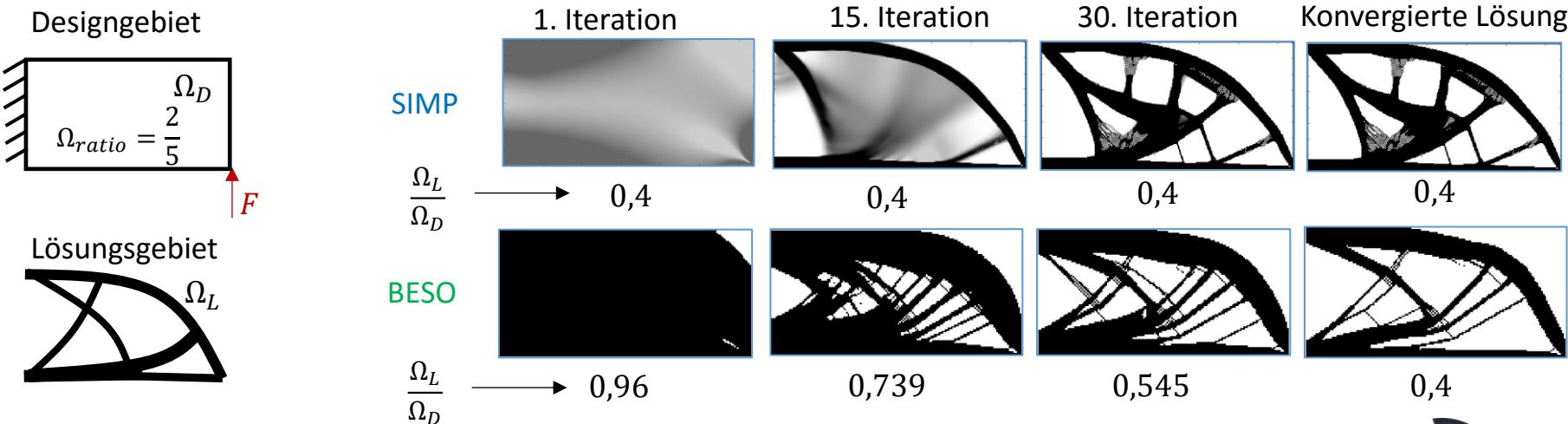
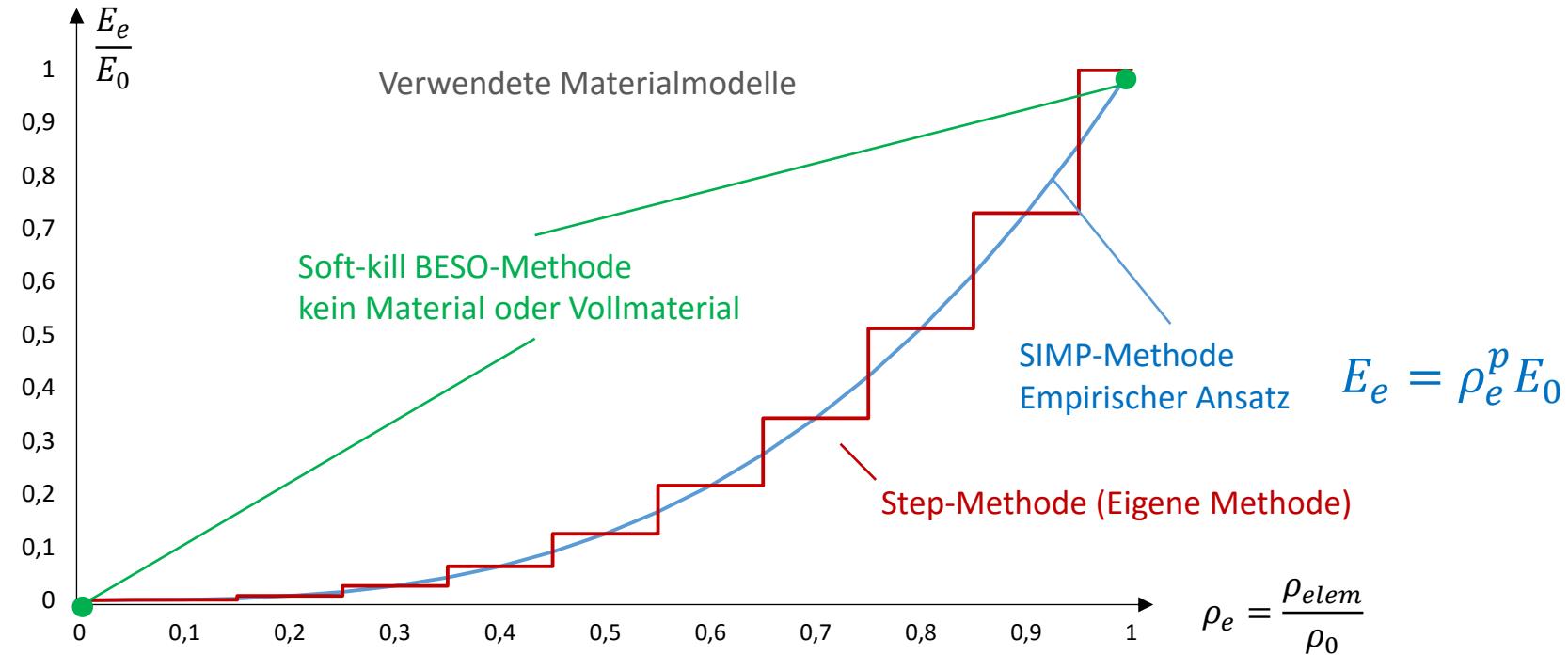
Entwicklung einer
Optimierungsstrategie

Umsetzung in eine Applikation

Rückführung der Geometrie

Ausblick/Fazit

Materialmodelle



Entwicklung einer
Optimierungsstrategie

Umsetzung in eine Applikation

Rückführung der Geometrie

Ausblick/Fazit



Herleitung mittels Lagrange

$$\min \frac{1}{2} \mathbf{u}^T \mathbf{K} \mathbf{u}$$

$$\frac{\Omega_L(\rho)}{\Omega_D} - \Omega_{Ratio} = 0$$

$$\begin{aligned}\rho^L - \rho_e &\leq 0 \\ \rho_e - \rho^U &\leq 0\end{aligned}$$

$$L = \frac{1}{2} \mathbf{u}^T \mathbf{K} \mathbf{u} + \Lambda \left(\frac{\Omega_L}{\Omega_D} - \Omega_{Ratio} \right) + \sum_{i=e}^n \lambda_e (\rho^L - \rho_e) + \sum_{i=e}^n \gamma_e (\rho_e - \rho^U)$$

$$\frac{\partial L}{\partial \rho_e} = \frac{1}{2} \left(\frac{\partial \mathbf{u}_e^T}{\partial \rho_e} \mathbf{K}_e \mathbf{u}_e + \mathbf{u}_e^T \frac{\partial \mathbf{K}_e}{\partial \rho_e} \mathbf{u}_e + \mathbf{u}_e^T \mathbf{K}_e \frac{\partial \mathbf{u}_e}{\partial \rho_e} \right) + \Lambda V_e - \lambda_e + \gamma_e = 0$$

$$\frac{1}{2} \left(\frac{\partial \mathbf{u}_e^T}{\partial \rho_e} \mathbf{K}_e \mathbf{u}_e + \mathbf{u}_e^T \frac{\partial \mathbf{K}_e}{\partial \rho_e} \mathbf{u}_e + \mathbf{u}_e^T \mathbf{K}_e \frac{\partial \mathbf{u}_e}{\partial \rho_e} \right) = -\frac{1}{2} \mathbf{u}^T \frac{\partial \mathbf{K}}{\partial \rho_e} \mathbf{u}$$

$$\frac{\partial L}{\partial \rho_e} = -\frac{1}{2} \mathbf{u}^T \frac{\partial \mathbf{K}}{\partial \rho_e} \mathbf{u} + \Lambda V_e - \lambda_e + \gamma_e = 0$$

$$G_i = 1 - \frac{\lambda_e}{\Lambda V_e} + \frac{\gamma_e}{\Lambda V_e}$$

$$G_i = \frac{\frac{1}{2} \mathbf{u}^T \frac{\partial \mathbf{K}}{\partial \rho_e} \mathbf{u}}{2 \Lambda V_e}$$

$$\begin{aligned}G_i &= 1; \rho_e \neq \rho^L; \rho_e \neq \rho^U & \longrightarrow & \rho_e^{i+1} = [G_e^i]^\gamma \rho_e^i = [G_e^i]^{0,5} \rho_e^i \\ G_i &\leq 1; \rho_e = \rho^L \\ G_i &\geq 1; \rho_e = \rho^U\end{aligned}$$

$$\mathbf{K}_e \mathbf{u}_e = \mathbf{f}_e$$

$$\mathbf{K} \frac{\partial \mathbf{u}}{\partial \rho_e} + \frac{\partial \mathbf{K}}{\partial \rho_e} \mathbf{u} = \frac{\partial \mathbf{f}}{\partial \rho_e} = 0$$

$$\mathbf{K} \frac{\partial \mathbf{u}}{\partial \rho_e} = -\frac{\partial \mathbf{K}}{\partial \rho_e} \mathbf{u}$$

$$\mathbf{K}_e = \rho_e^p \mathbf{K}_{0,e}$$

$$\frac{\partial \mathbf{K}_e}{\partial \rho_e} = p \cdot \rho_e^{p-1} \mathbf{K}_{0,e}$$

Entwicklung einer
Optimierungsstrategie

Umsetzung in eine Applikation

Rückführung der Geometrie

Ausblick/Fazit



Steuerung der Designvariablen

Zielfunktion

$$E_{den,e} = \frac{1}{2} \frac{\mathbf{u}_e^T \mathbf{K}_e \mathbf{u}_e}{V_e}$$

$$\frac{\partial E_{den,e}}{\partial \rho_e} = \frac{1}{2V_e} \left(\frac{\partial \mathbf{u}_e^T}{\partial \rho_e} \mathbf{K}_e \mathbf{u}_e + \mathbf{u}_e^T \frac{\partial \mathbf{K}_e}{\partial \rho_e} \mathbf{u}_e + \mathbf{u}_e^T \mathbf{K}_e \frac{\partial \mathbf{u}_e}{\partial \rho_e} \right) = -\frac{1}{2V_e} \mathbf{u}_e^T \frac{\partial \mathbf{K}_e}{\partial \rho_e} \mathbf{u}_e$$

Systemantwort

$$\mathbf{K}_e \mathbf{u}_e = \mathbf{f}_e$$

$$\mathbf{K}_e \frac{\partial \mathbf{u}_e}{\partial \rho_e} + \frac{\partial \mathbf{K}_e}{\partial \rho_e} \mathbf{u}_e = \frac{\partial \mathbf{f}_e}{\partial \rho_e}$$

$$\mathbf{K}_e \frac{\partial \mathbf{u}_e}{\partial \rho_e} + \frac{\partial \mathbf{K}_e}{\partial \rho_e} \mathbf{u}_e = 0$$

$$\mathbf{K}_e \frac{\partial \mathbf{u}_e}{\partial \rho_e} = -\frac{\partial \mathbf{K}_e}{\partial \rho_e} \mathbf{u}_e$$

$\mathbf{K}_e \rightarrow$ Verteiltes Material

Materialgesetz

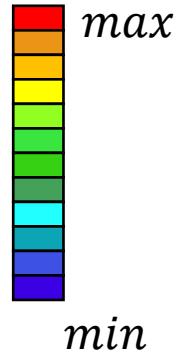
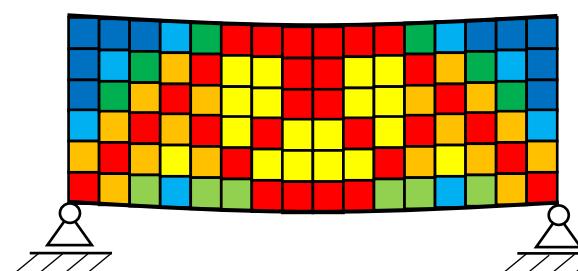
$$\mathbf{E}_e = \rho_e^p \mathbf{E}_{0,e}$$



$$\mathbf{K}_e = \rho_e^p \mathbf{K}_{0,e}$$

$$\frac{\partial \mathbf{K}_e}{\partial \rho_e} = p \cdot \rho_e^{p-1} \mathbf{K}_{0,e}$$

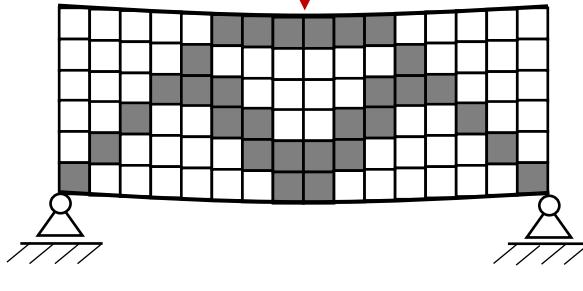
$$\frac{\partial E_{den,e}}{\partial \rho_e} = -p \cdot \rho_e^{p-1} \frac{1}{2} \frac{\mathbf{u}_e^T \mathbf{K}_{0,e} \mathbf{u}_e}{V_e}$$



Ermittlung und Einsatz der Sensitivitäten (BESO)

$$\frac{\partial E_{den,e}}{\partial \rho_e} = -p \cdot \rho_e^{p-1} \frac{1}{2} \frac{\mathbf{u}_e^T \mathbf{K}_{0,e} \mathbf{u}_e}{V_e}$$

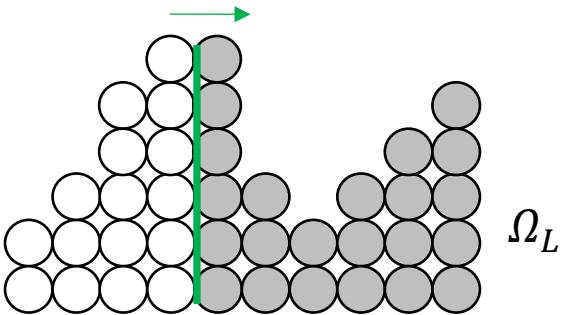
$$\mathbf{K}_e \mathbf{u}_e = \mathbf{f}_e \rightarrow \mathbf{u}_e$$



\mathbf{u}_e

ρ_e

$$\frac{\partial E_{den,e}}{\partial \rho_e}$$



Ω_L

$$\frac{\partial E_{den,e}}{\partial \rho_e}$$

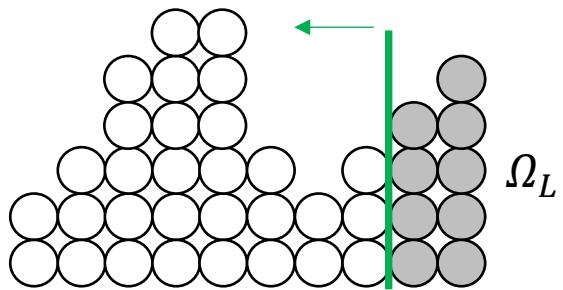
max



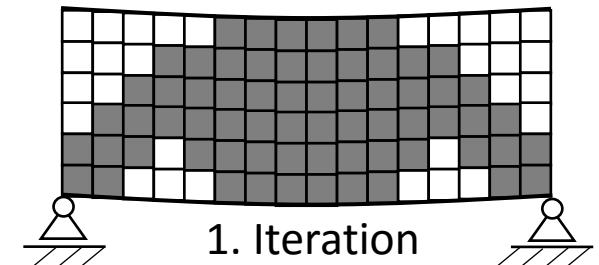
min

$$\mathbf{K}_{0,e} \mathbf{u}_e = \mathbf{f}_e$$

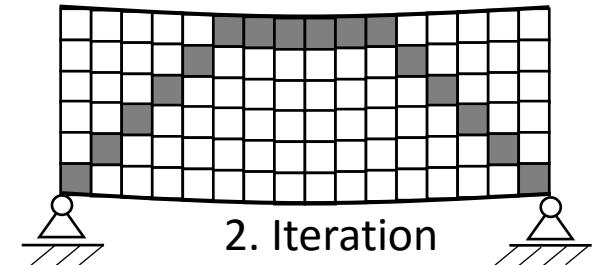
$$-p \cdot \rho_e^{p-1} \frac{1}{2} \frac{\mathbf{u}_e^T \mathbf{K}_{0,e} \mathbf{u}_e}{V_e}$$



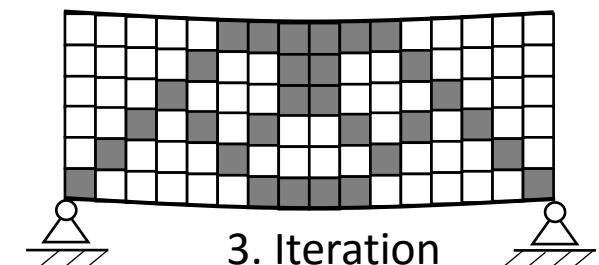
Ω_L



1. Iteration



2. Iteration



3. Iteration

Entwicklung einer
Optimierungsstrategie

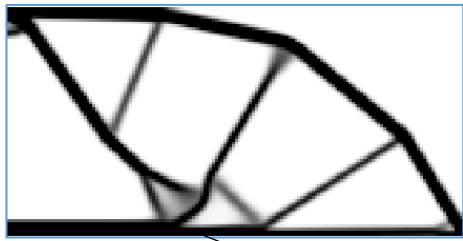
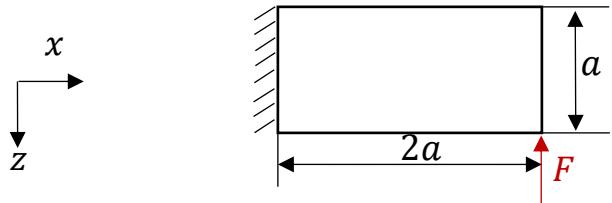
Umsetzung in eine Applikation

Rückführung der Geometrie

Ausblick/Fazit

2D-Einzeloptimierung

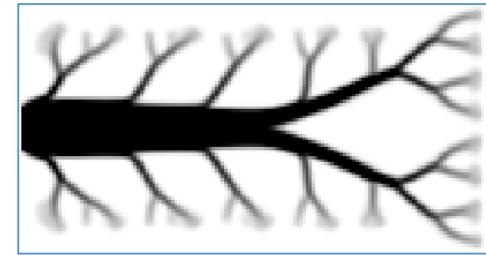
Statischer Lastfall



Wärmeübertragung (Wärmeleitung)

$$\begin{array}{c} n\nabla T = 0 \\ T = T_1 \bullet \\ n\nabla T = 0 \\ \dot{Q} \\ 2a \\ n\nabla T = 0 \\ n\nabla T = 0 \end{array}$$

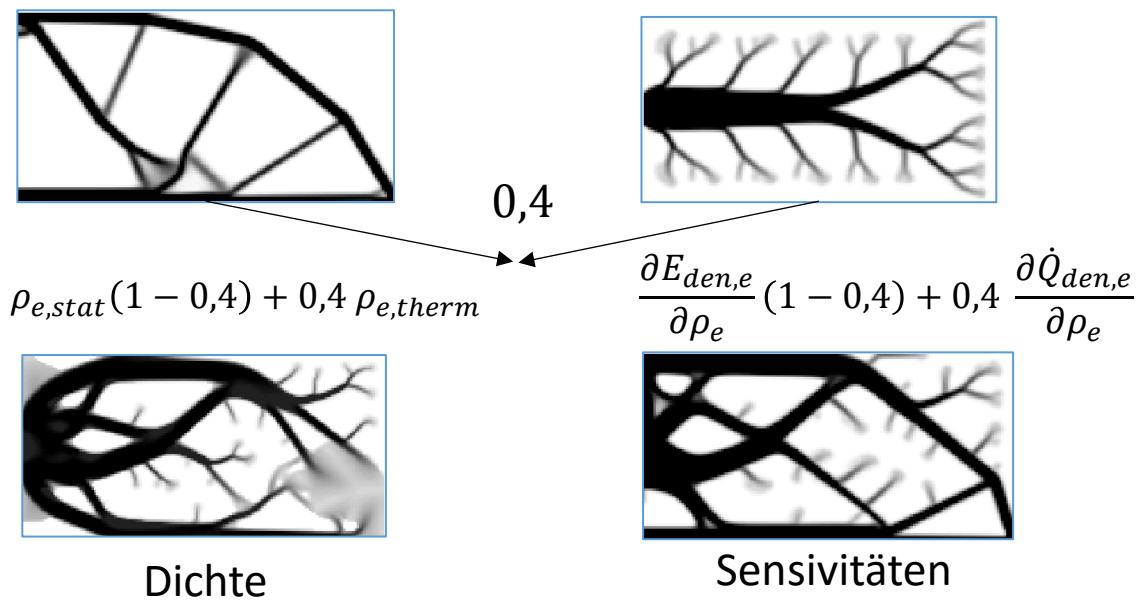
A diagram of a rectangular domain of width $2a$ and height a . The boundary conditions are $n\nabla T = 0$ on all four sides. A point $T = T_1$ is marked inside the domain. A heat source \dot{Q} is located at the center of the top edge. The equation $n\nabla T = 0$ is repeated at the bottom boundary.



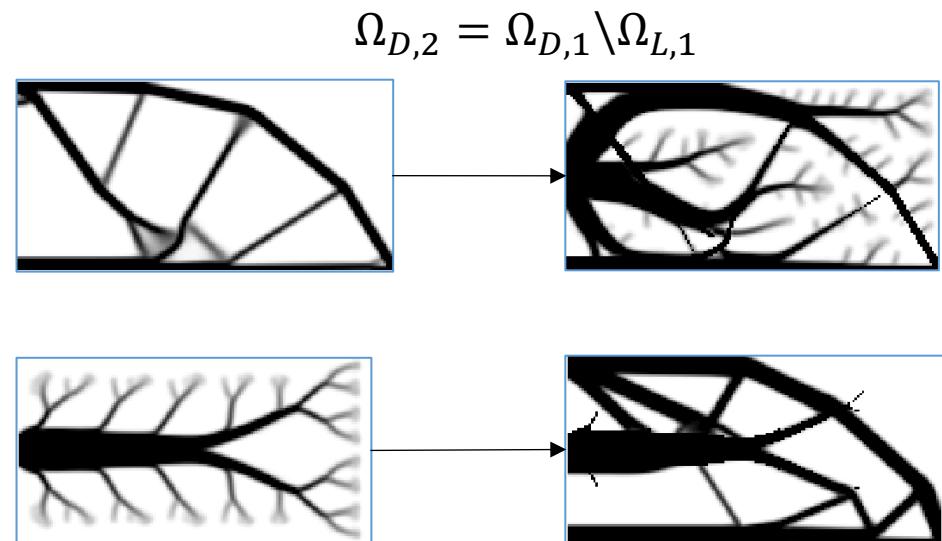
?

2D-Koppelung (Erfolgreiche Varianten)

Gewichtungsfaktoren



Adaption



Adaption mit Gewichtung



Entwicklung einer
Optimierungsstrategie

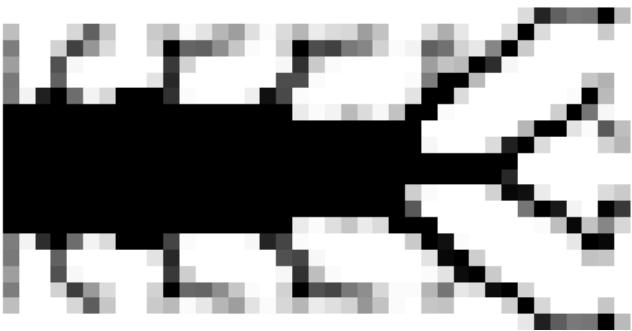
Umsetzung in eine Applikation

Rückführung der Geometrie

Ausblick/Fazit

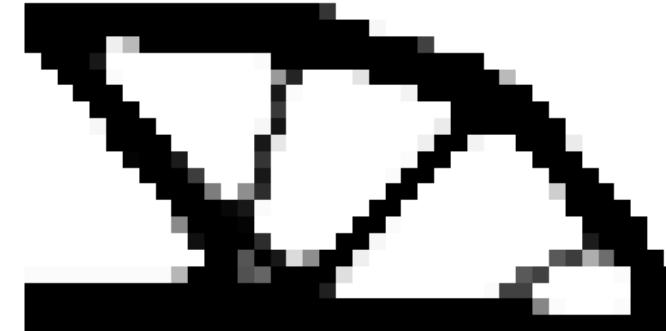
Beispielrechnung (Gewichtung mit 0,5)

Thermik-Lösung



40x20 Elemente

Mechanik-Lösung



40x20 Elemente

Topologie Multi- Physik

1) 3D-Multi-Physik Inputdecks

2) 2D-Multi-Physik Beispiele

1) 2) 3) 4)

1) 2) Strategie

Om_L / Om_D 0.4

p 3

1)

a) Thermal

b) Struktur

c) Sensivitäten

Mat Sets 20

skal.: n_max 50

konst: n_max 20

2) 3)

e_x 40

e_y 20

r_min 2

d) Adaption

e) Startentwurf Struktur

f) Adaption Gewichtung

g) Iterative Adaption

1) Mittelwert

1) SIMP

1) Nicht zu optimierende Bereiche aktiv

Topologie Beispiele

3) 2D-Struktur Beispiel

4) 3D-Struktur Beispiel

d,f,g) n_change 20

d,f,g) Om_LAD / Om_D 0.2

g) n_ad,max 6

c,f) g_struct: 0.5

d,f) g) Grenzdichte: 0.9

Inputdeck eines Solvers

Definition des Modellaufbaus

*Node KnotenID mit Knotenkoordinaten

```

1, -35., 15., 0.
2, -35., 15., 5.
3, -30., 15., 5.
4, -30., 15., 0.
5, -35., 20., 0.
6, -35., 20., 5.
7, -30., 20., 5.
8, -30., 20., 0.
9, -25., 15., 5.
10, -25., 15., 0.
11, -25., 20., 0.
12, -25., 20., 5.

```

*Element, type=C3D8R

```
1, 1, 2, 3, 4, 5, 6, 7, 8
```

```
2, 4, 3, 9, 10, 8, 7, 12, 11
```

*Nset, nset=Set-1

```
1, 2, 5, 6
```

*Nset, nset=Set-2

```
9, 10, 11, 12
```

*Material, name=Aluminium

*Elastic

```
70000., 0.33
```

Elementtyp: Hexaeder

ElementID, KnotenID's

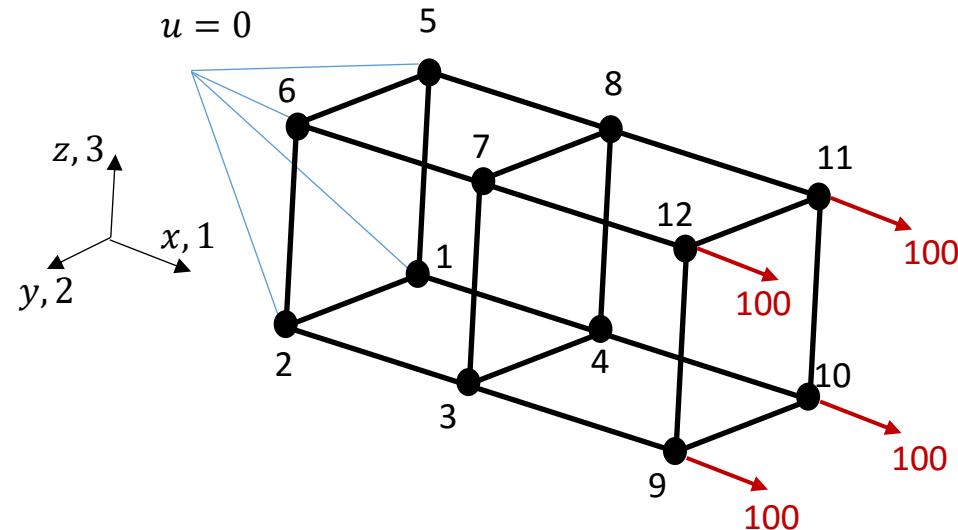
Knotenset

Knoten im Set

Materialdefinition

Elastizität

E-Modul, Querkont.



Definition eines Steps (Lastgeschichte)

*Step, name=Step-1, nlgeom=NO

*Static

```
1., 1., 1e-05, 1.
```

*Boundary

```
Set-1, 1, 0
```

```
Set-1, 2, 0
```

```
Set-1, 3, 0
```

*Cload

```
Set-2, 1, 100.
```

*End Step

Statischer Lastfall
Inkrementsteuerung

Randbedingung

Feste Einspannung
in 1,2,3-Richtung

Konzentrierte Last

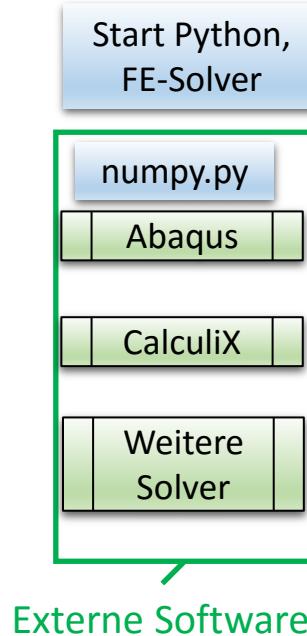
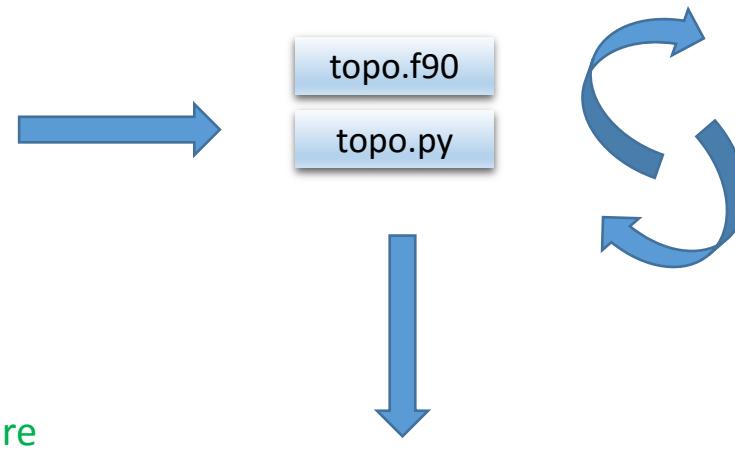
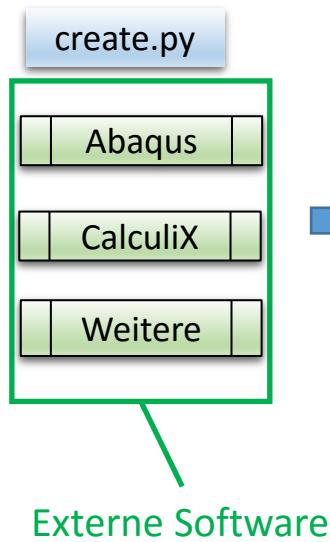
100 N in 1-Richtung

Aufbau der Applikation

Modellierung

Steuerung und Optimierung

Gleichungslöser



Ergebnis:
Materialverteilung

Entwicklung einer
Optimierungsstrategie

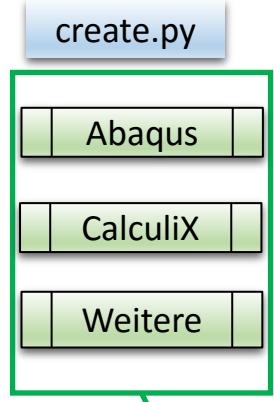
Umsetzung in eine Applikation

Rückführung der Geometrie

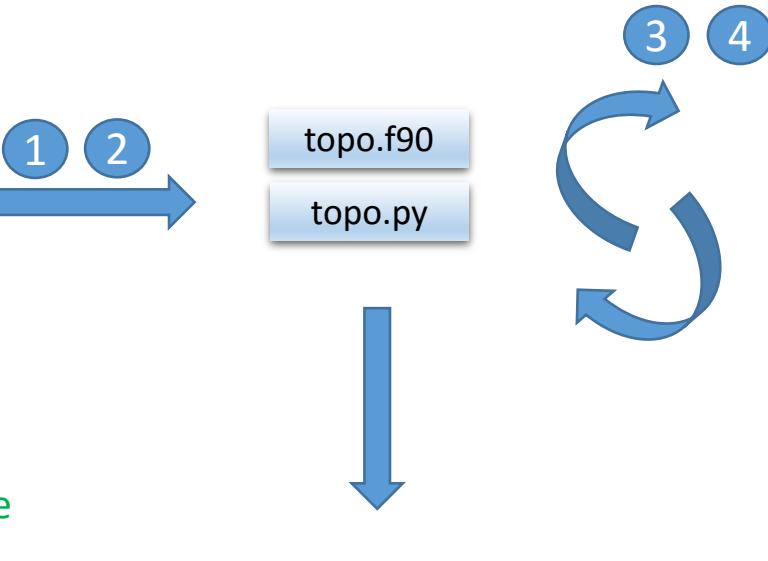
Ausblick/Fazit

Aufbau der Applikation

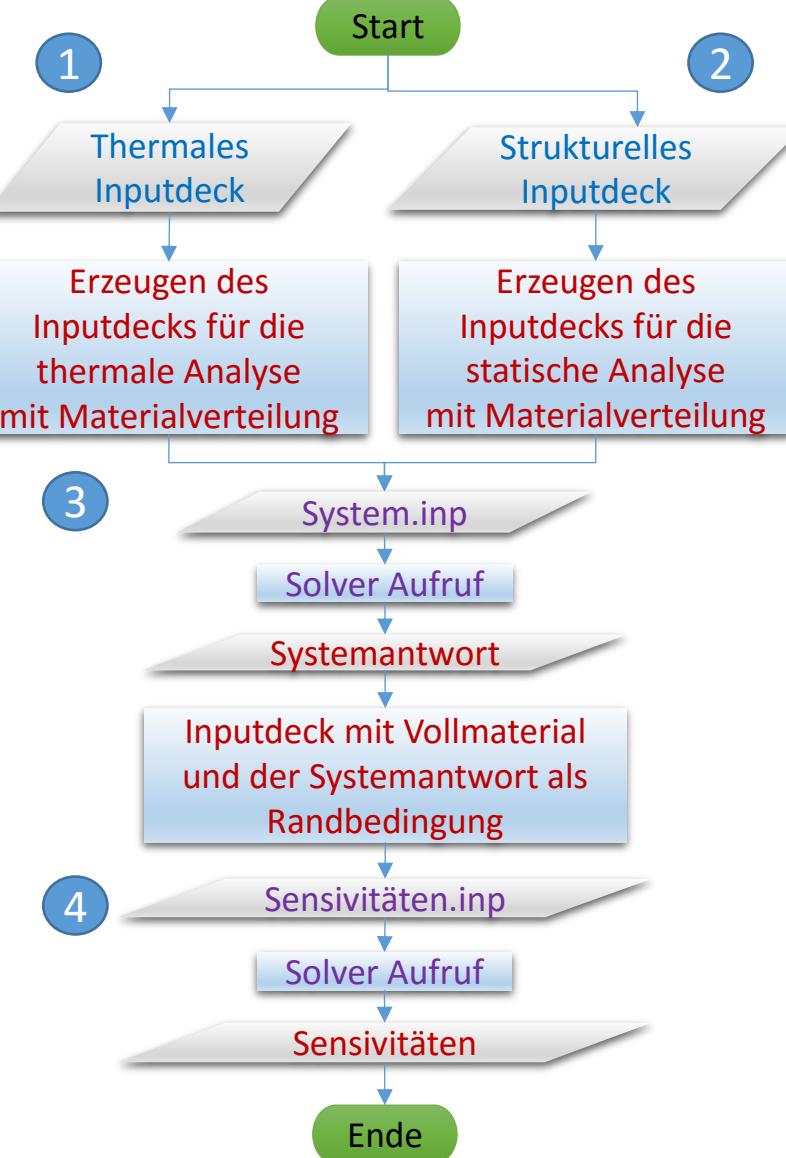
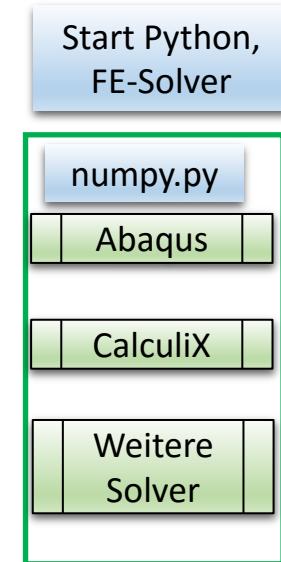
Modellierung



Steuerung und Optimierung



Gleichungslöser



Entwicklung einer
Optimierungsstrategie

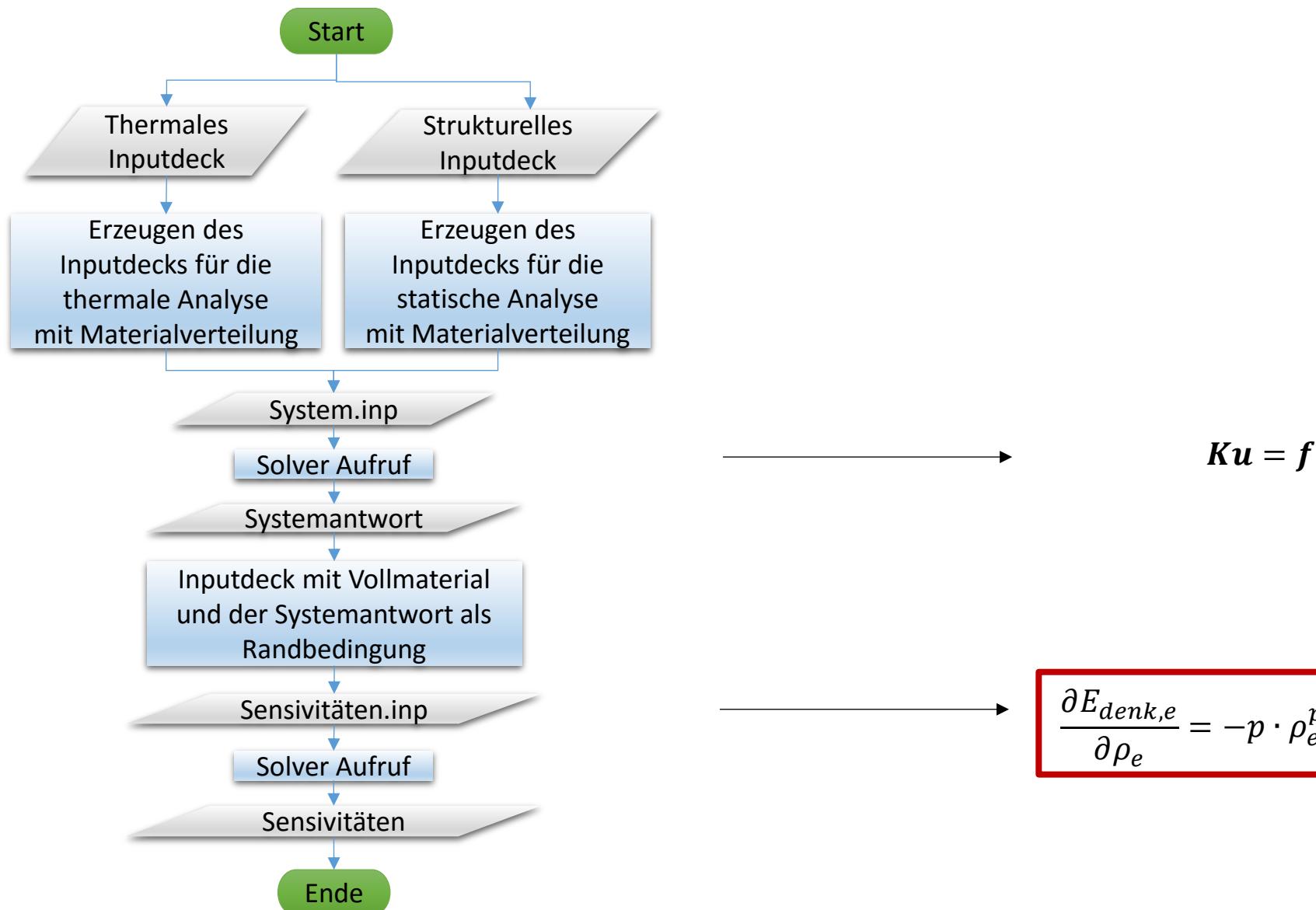
Umsetzung in eine Applikation

Rückführung der Geometrie

Ausblick/Fazit



Ablaufdiagramm einer Iteration



```
...
*ELSET, ELSET = matSet1
9,10,15,20,17, ...
*ELSET, ELSET = matSet2
11,21,31,41,51
*ELSET, ELSET = matSet3
16,27,12,14, ...

*Material, name = TopoMat_1
*Elastic
8.75, 0.33
*Material, name = TopoMat_2
*Elastic
8750.0, 0.33
*Material, name = TopoMat_3
*Elastic
70000.0, 0.33

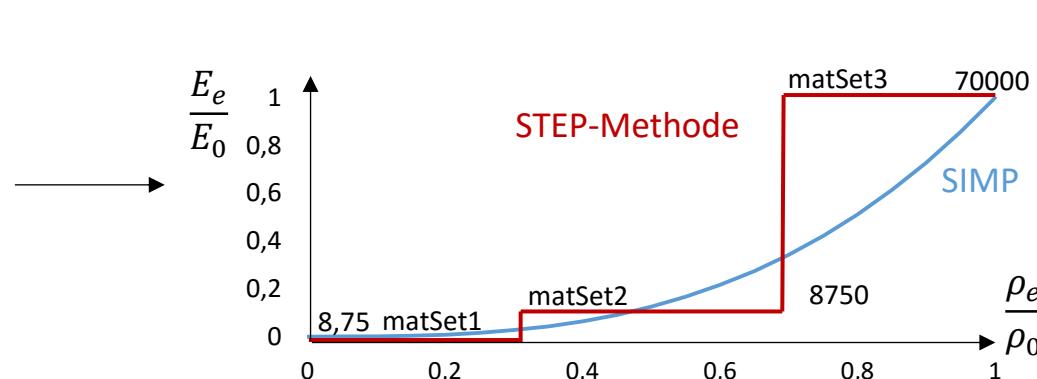
*Solid Section, ELSET=matSet1,
material=TopoMat_1
*Solid Section, ELSET=matSet2,
material=TopoMat_2
*Solid Section, ELSET=matSet3,
material=TopoMat_3

...
*Node Print, NSET = Nall
U
```

$$K_e \mathbf{u}_e = f_e$$

Programmiertechnische Umsetzung

9	10	15	20
31	41	51	17
14	11	21	
16	27	12	



Verschiebungen
als Randbedingung

Beispiel Ergebnis Knoten 1
 1, 1,1,-2.777958
 1, 2,2,8.364013
 1, 3,3,0.2099042

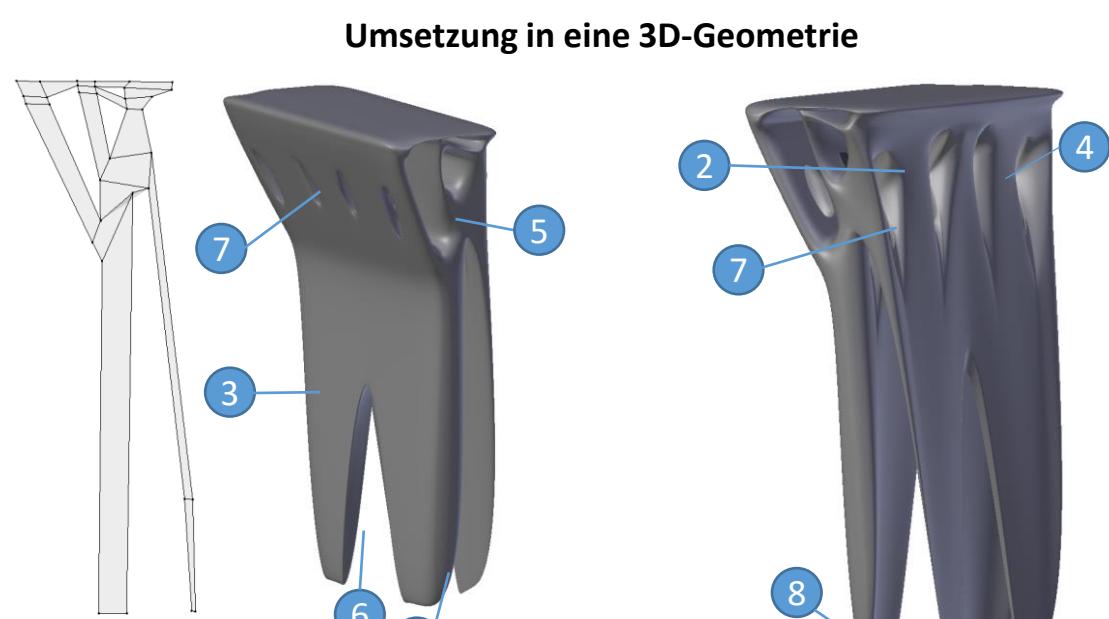
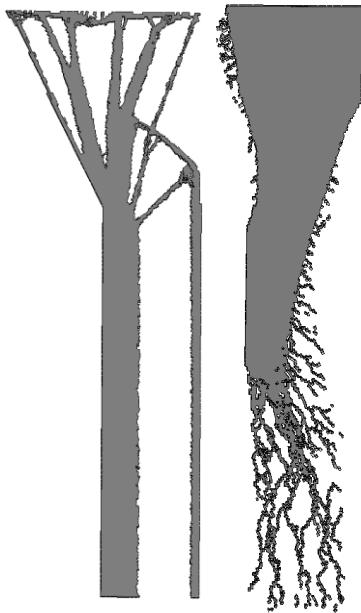
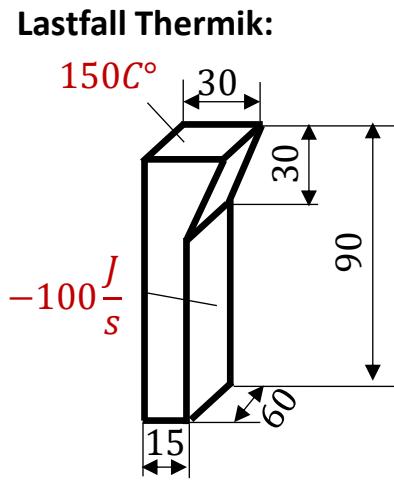
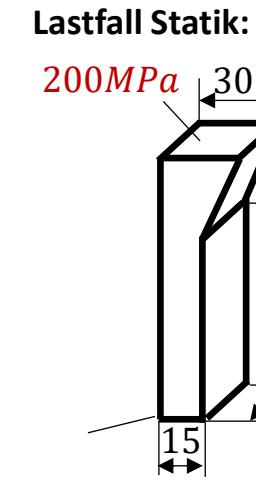
$$\frac{\partial E_{den,e}}{\partial \rho_e} = -p \cdot \rho_e^{p-1} \frac{1}{2} \frac{\mathbf{u}_e^T K_{0,e} \mathbf{u}_e}{V_e}$$

...
*ELSET, ELSET = Eall
9,10,15,20,17, ...
16,27,12,14, ...

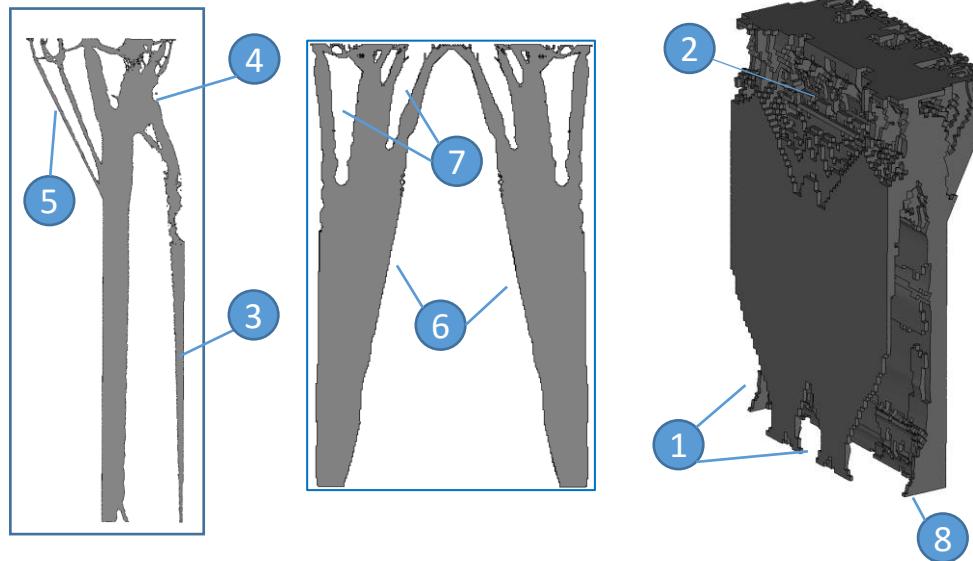
*Material, name = Aluminium
*Elastic
70000.0, 0.33
*Solid Section, ELSET=Eall,
material=Aluminium

...
*Boundary
1, 1,1,-2.777958
1, 2,2,8.364013
1, 3,3,0.2099042
2, 1,1,-1.688803
2, 2,2,8.597918
2, 3,3,-0.08850997
...
*EL File,ELSET=Eall
ENER

Generierung der 3D-Lösung



Gewichtungsfaktoren 0,5



- 1 Ausprägungen und Stützen (Block)
- 2 Beule und Verformung innen, außen Erhebungen (Block)
- 8 Wölbung, Stütze (Block)
- 4 6 Ausprägung (Scheibe)
- 5 3 7 Stützelemente (Scheibe)

Entwicklung einer
Optimierungsstrategie

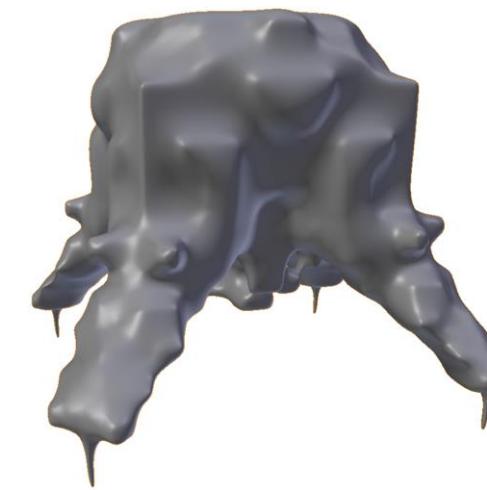
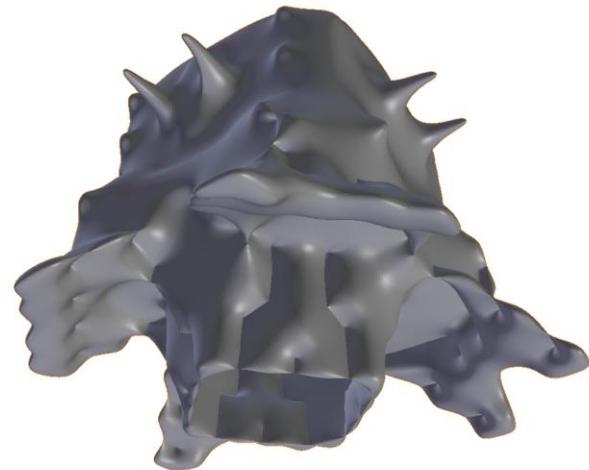
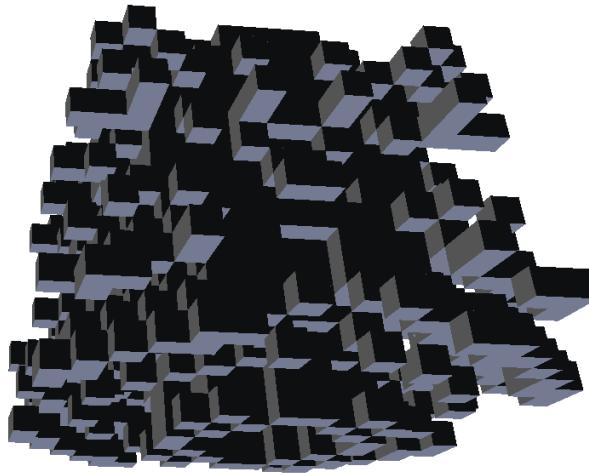
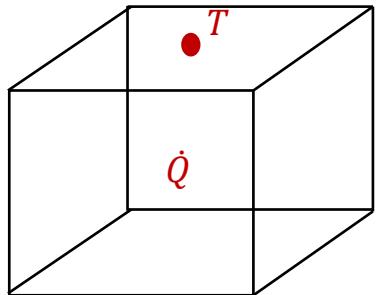
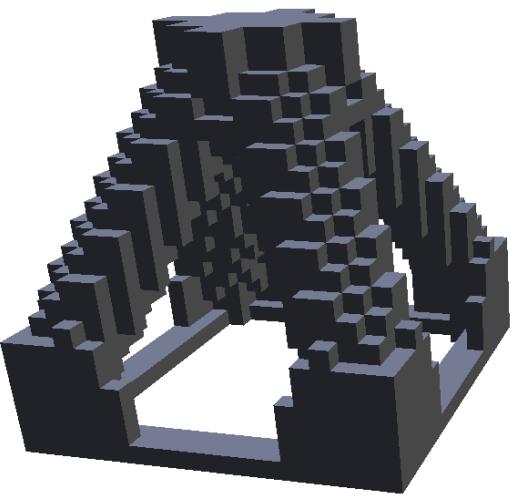
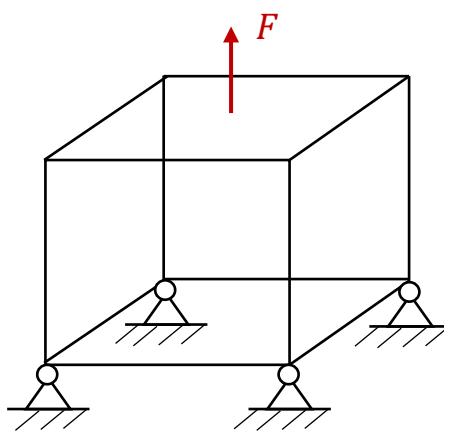
Umsetzung in eine Applikation

Rückführung der Geometrie

Ausblick/Fazit

Generierung der 3D-Lösung

Gewichtung



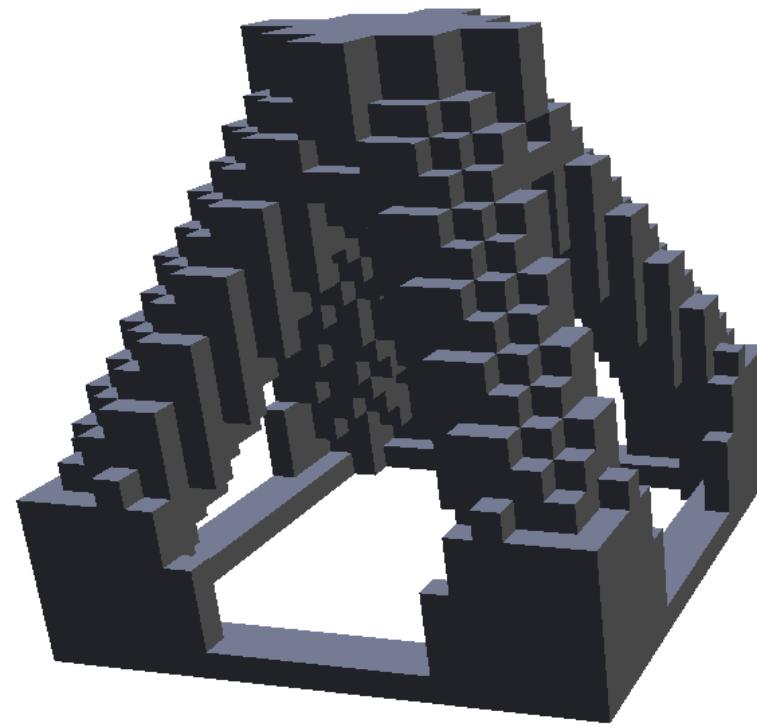
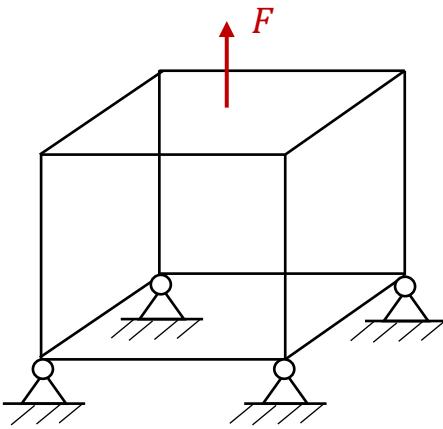
Entwicklung einer
Optimierungsstrategie

Umsetzung in eine Applikation

Rückführung der Geometrie

Ausblick/Fazit

Glättung und Optimierung in Blender



Diskrete Lösungsstruktur

Entwicklung einer
Optimierungsstrategie

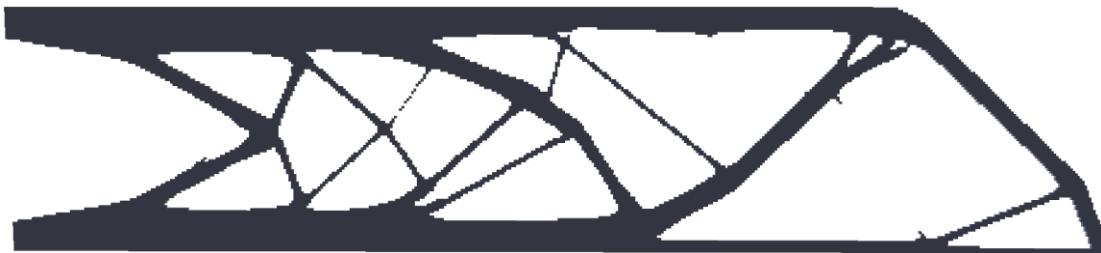
Umsetzung in eine Applikation

Rückführung der Geometrie

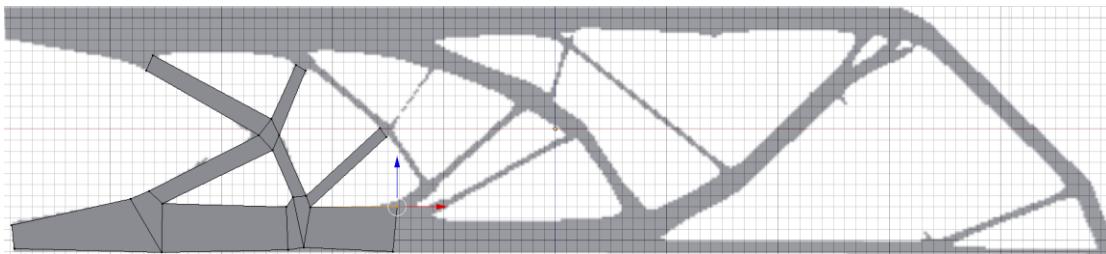
Ausblick/Fazit

Ergebnisrückführung (Nachzeichnen)

1. Optimierung



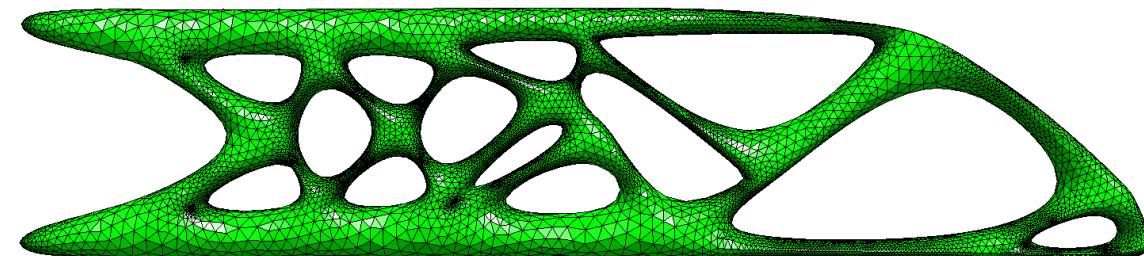
Nachzeichnung



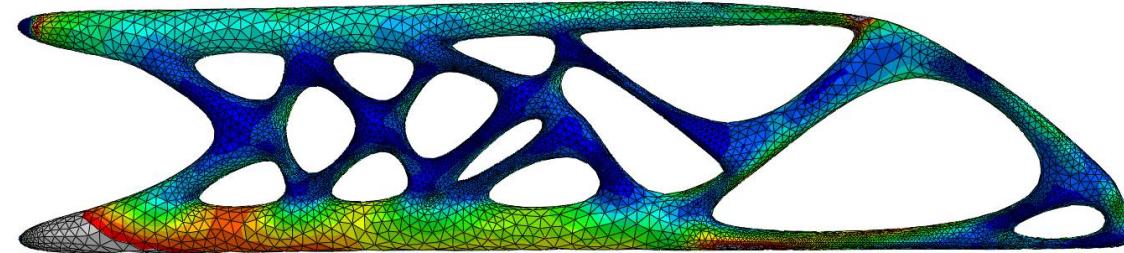
3D-Kontur



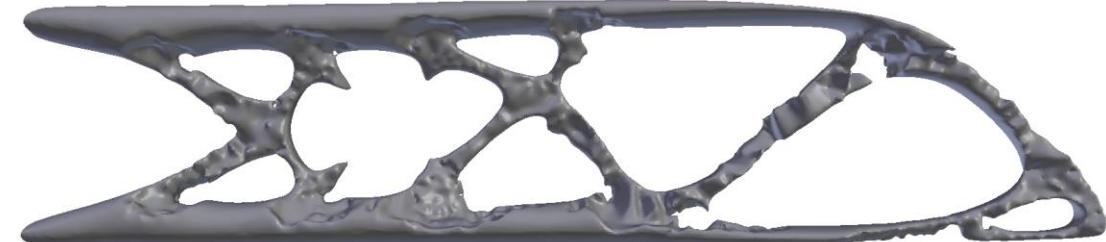
FE-Netz



Auswertung



2. Optimierung



Entwicklung einer
Optimierungsstrategie

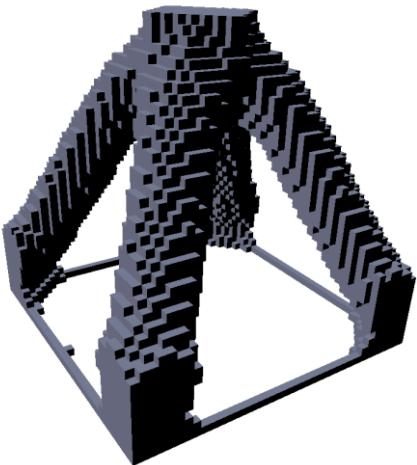
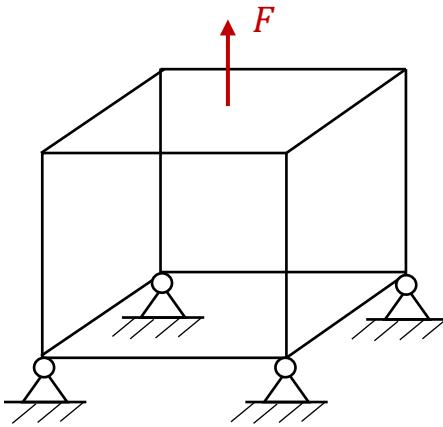
Umsetzung in eine Applikation

Rückführung der Geometrie

Ausblick/Fazit

Ergebnisrückführung (Automatisch)

Optimierungsergebnis



Add Modifier

Mode: Smooth
Octree Depth: 4 Scale: 0.900
Smooth Shading
Remove Disconnected Pieces
Threshold: 1.000

Subsurf
Mode: Catmull-Clark Subdivisions: View: 1 Render: 2 Options: Subdivide UVs Optimal Display Use OpenSubdiv

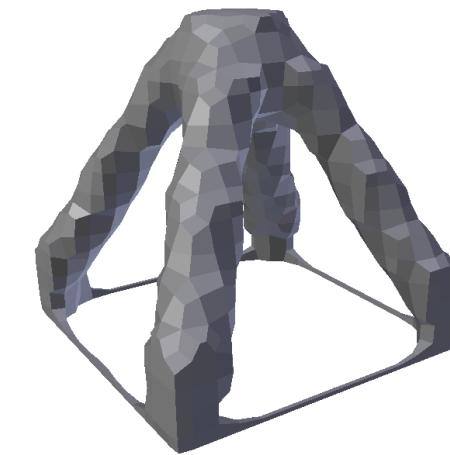
Remesh.001
Mode: Smooth
Octree Depth: 7 Scale: 0.900
Smooth Shading Remove Disconnected Pieces Threshold: 1.000

Subsurf.0
Mode: Catmull-Clark Subdivisions: View: 2 Render: 2 Options: Subdivide UVs Optimal Display Use OpenSubdiv

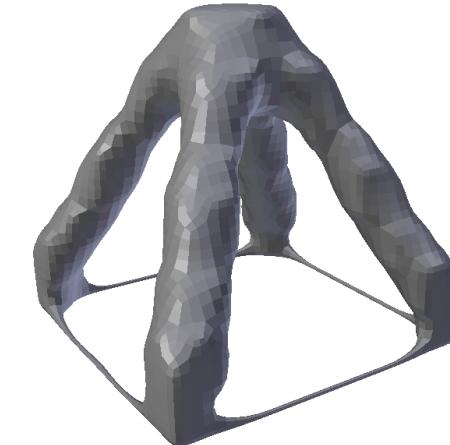
Apply Copy

This panel shows the modifier stack for the torso mesh. It includes a Remesh modifier (Octree Depth 4) followed by a Subsurf modifier (Catmull-Clark, Subdivisions 2). Below it is another Remesh modifier (Octree Depth 7) followed by another Subsurf modifier (Catmull-Clark, Subdivisions 2). The 'Smooth' mode is selected for both remeshing steps.

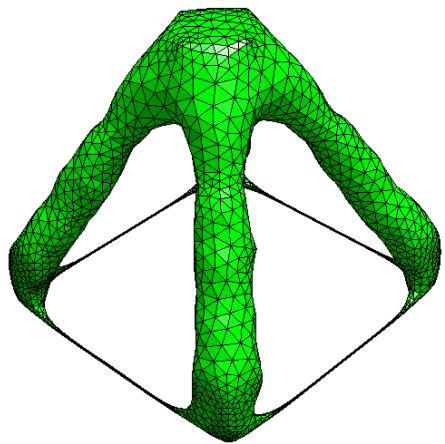
Neuvernetzung



Netzverfeinerung



Netzverfeinerung und Neuvernetzung



Entwicklung einer Optimierungsstrategie

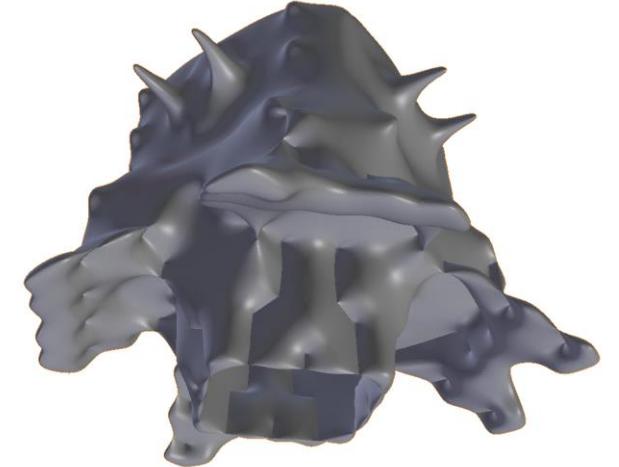
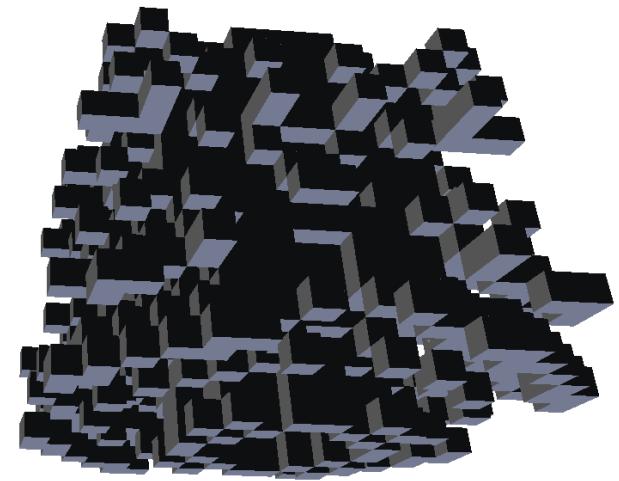
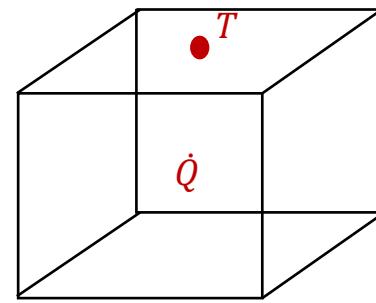
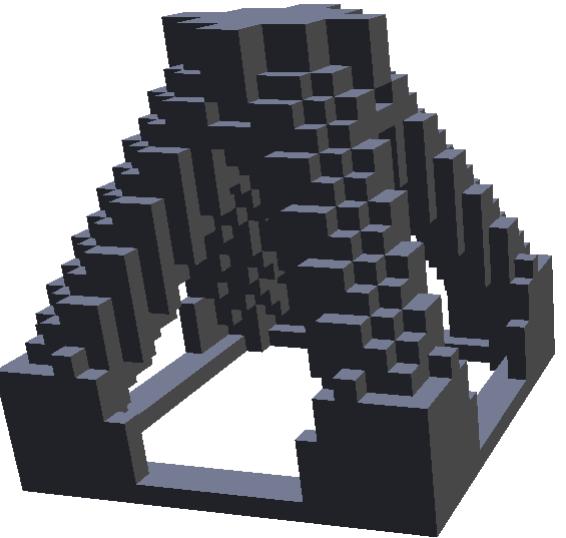
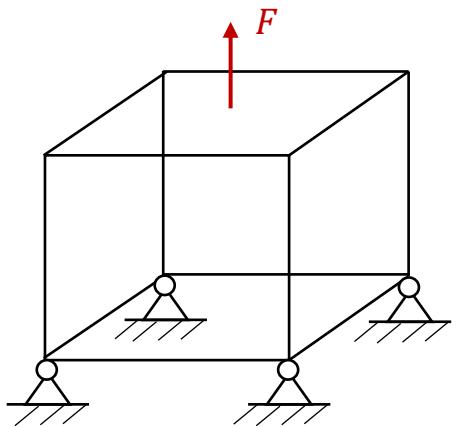
Umsetzung in eine Applikation

Rückführung der Geometrie

Ausblick/Fazit

Generierung der 3D-Lösung und automatische Geometrieglättung

Diskrete Lösungsstruktur



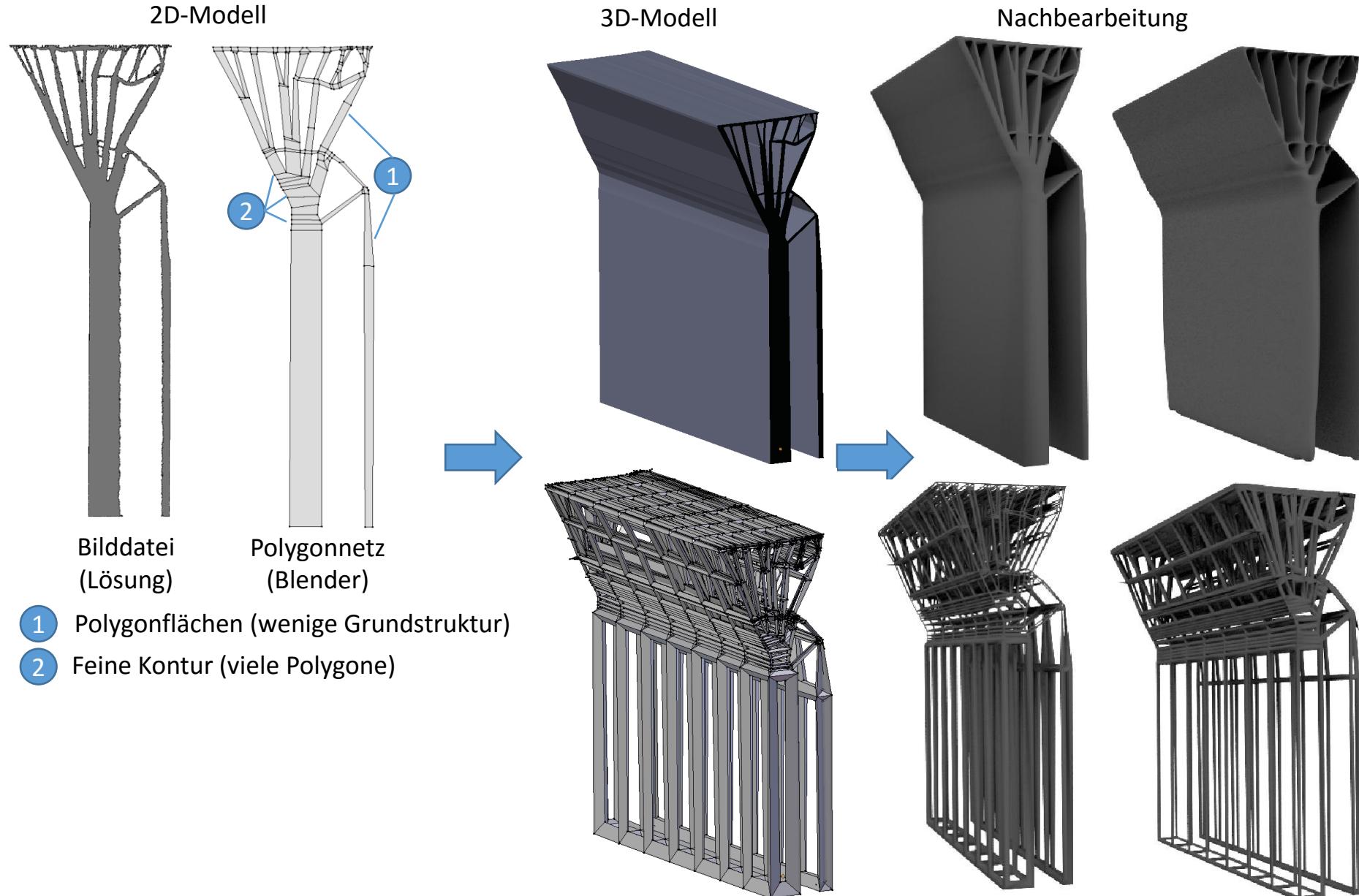
Entwicklung einer
Optimierungsstrategie

Umsetzung in eine Applikation

Rückführung der Geometrie

Ausblick/Fazit

Manuelle Geometrieerstellung (Nachzeichnen)



Entwicklung einer
Optimierungsstrategie

Umsetzung in eine Applikation

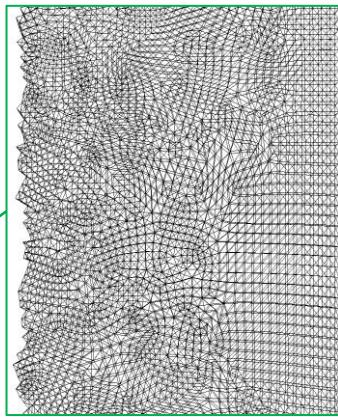
Rückführung der Geometrie

Ausblick/Fazit

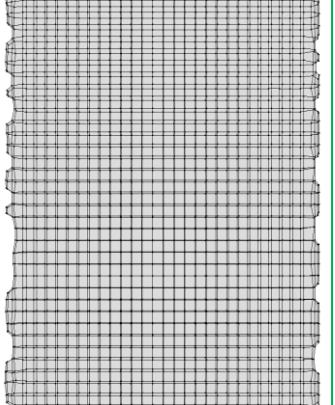
Automatisierte Geometrieerstellung



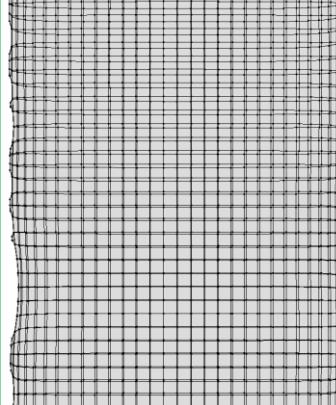
FE-Netz



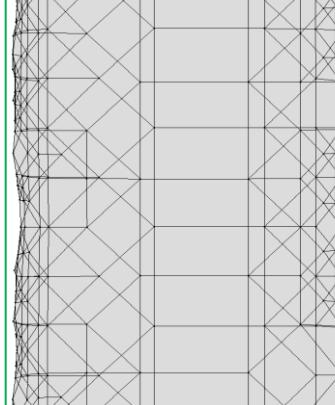
Neuvernetzung



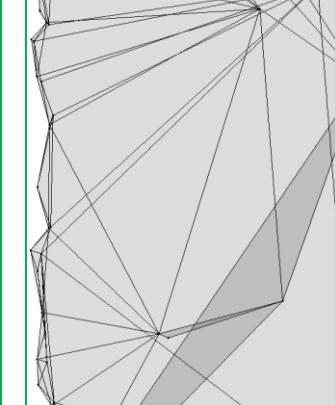
Randglättung



Netzvereinfachung



Netzstrukturierung



Entwicklung einer
Optimierungsstrategie

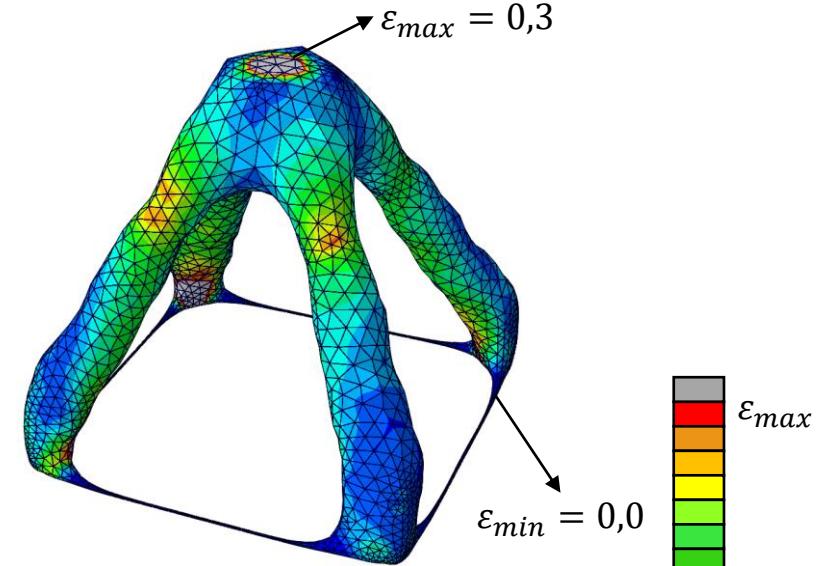
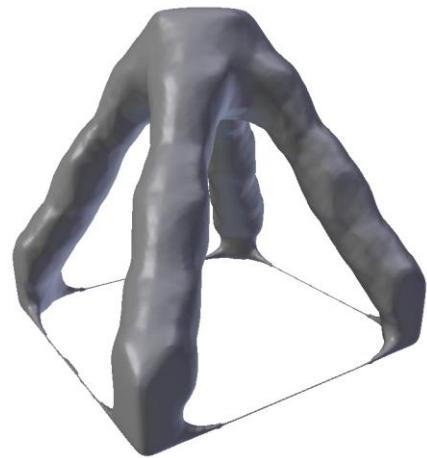
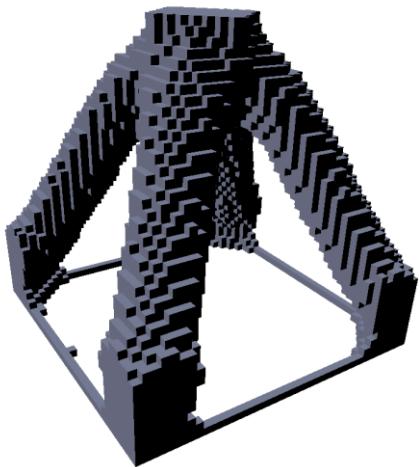
Umsetzung in eine Applikation

Rückführung der Geometrie

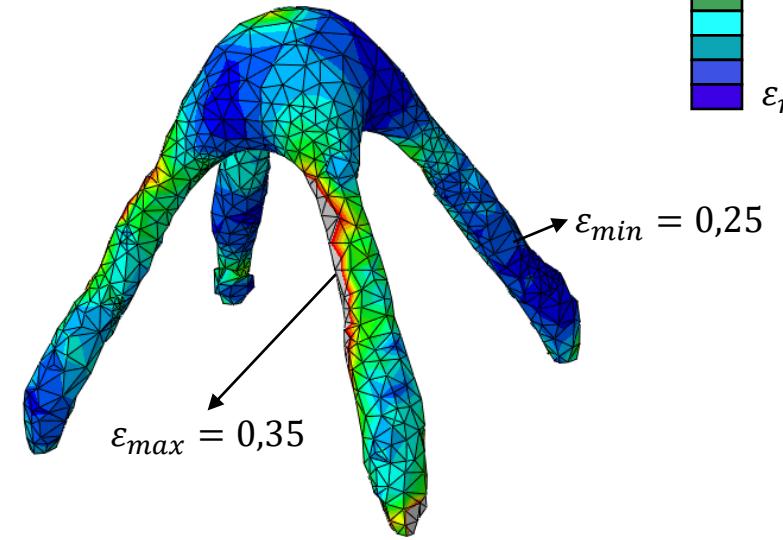
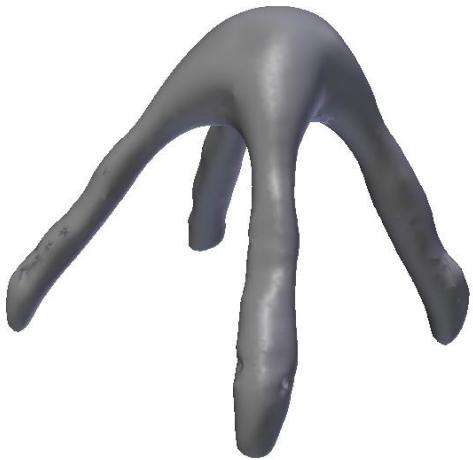
Ausblick/Fazit

Auswertung des Ergebnisses

1. Optimierung



2. Optimierung



Entwicklung einer
Optimierungsstrategie

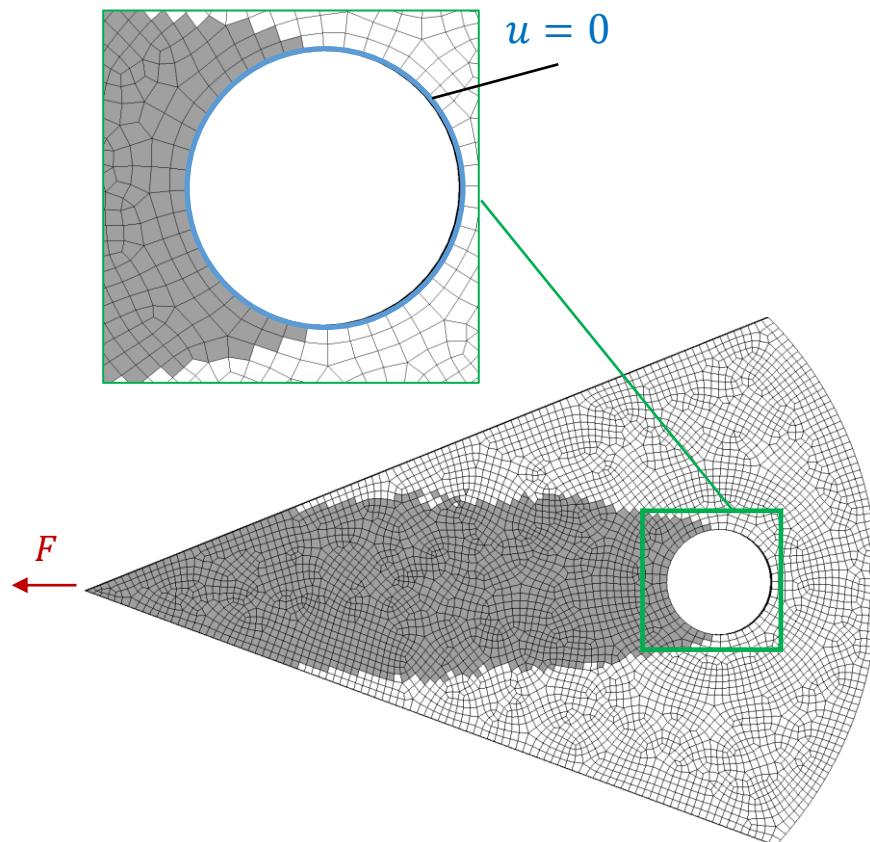
Umsetzung in eine Applikation

Rückführung der Geometrie

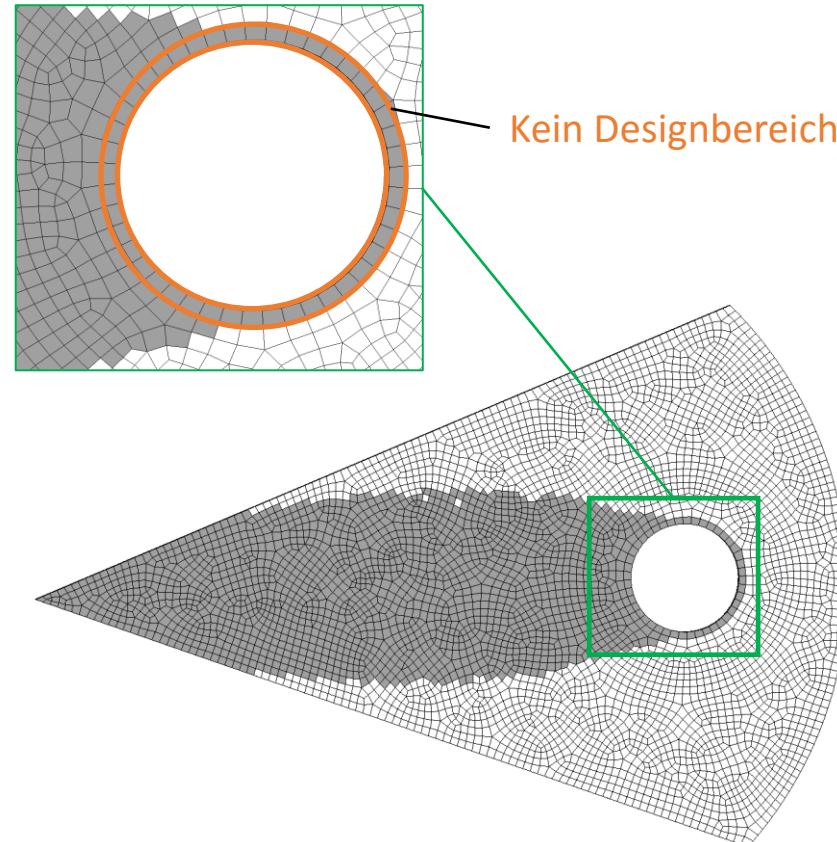
Ausblick/Fazit

Lösungsraummodifikation

Ohne Einschränkung



Mit Einschränkung



Entwicklung einer
Optimierungsstrategie

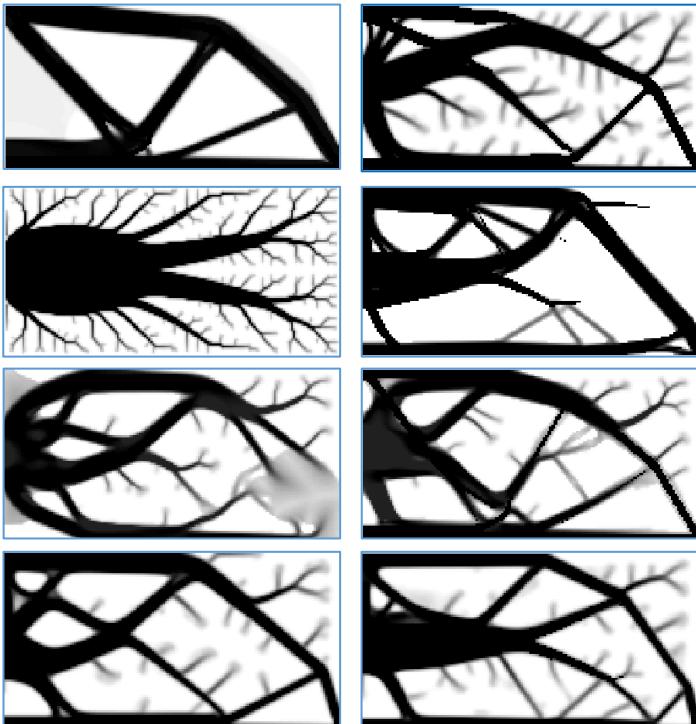
Umsetzung in eine Applikation

Rückführung der Geometrie

Ausblick/Fazit

Optimierung

- Viele Varianten und Parameter
- Einige sinnlose Konturen
- Passende Ergebnisse
- Vergleichbar mit kommerzieller Software



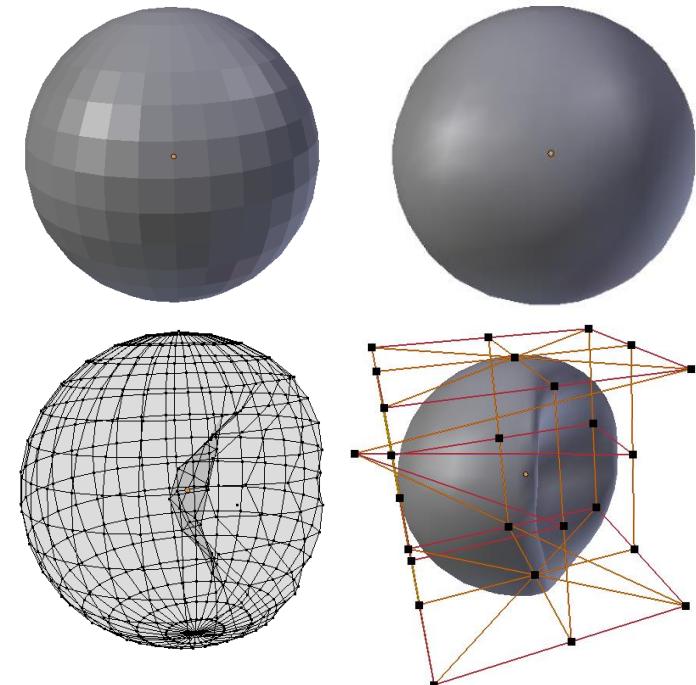
Fazit

Software

- Stabilität
- Fehlender Support
- Keine Lizenzkosten
- Vorsprung über kommerzieller Software
- OpenSource über GitHub
- Fehlerverbesserung über Pull-Request
- Einbindung von Blender

Geometrie

- Keine exakte Geometrie
- Fehlende CAD-Bearbeitung (Außnahme z.B. AutoCAD)
- Einfache Modellierung
- Automatisierte Nachbesserung
- Direkter FE-Netz Import
- Komplexe Formen



Entwicklung einer
Optimierungsstrategie

Umsetzung in eine Applikation

Rückführung der Geometrie

Ausblick/Fazit