16 Suppose that the eigen values of matrix A are 1, 2, 4. The determinant of $(A^{-1})^T$ is ______.

Sol Determinant of a matrix is the product of its eigen values.

Determinant of a matrix is same as its transpose.

Determinant of a matrix is reciprocal to its inverse.

eigen values of matrix A are 1, 2, 4.

$|A| = 1 \times 2 \times 4 = 8$.

Determinant of inverse of $A = \det(A^{-1})$

$= \frac{1}{|A|}$

$= \frac{1}{8} = 0.125$

$\therefore |A^{-1}| = |A^{-1}|^T = 0.125$

17. Consider the following 2x2 matrix A where two elements are unknown and are marked by a and b. The eigen values of this matrix are -1 and 7. What are the values of a and b?

$A = \begin{pmatrix} 1 & 4 \\ b & a \end{pmatrix}$

$\lambda_1 = -1 \qquad \lambda_2 = 7.$

$1 + a = \lambda_1 + \lambda_2$

$1 + a = -1 + 7$

$\boxed{a = 5}$

$|A| = \begin{vmatrix} 1 & 4 \\ b & a \end{vmatrix}$

$(1 \times a) - 4b = \lambda_1 \times \lambda_2$

$a - 4b = (-1) \times (7)$

$5 - 4b = -7$

$\boxed{b = 3}$

In the LU decomposition of the matrix $\begin{bmatrix} 2 & 2 \\ 4 & 9 \end{bmatrix}$, if the diagonal elements of U are both 1, then the lower diagonal entry l_{22} of L is.

Soln Given $LU = \begin{bmatrix} 2 & 2 \\ 4 & 9 \end{bmatrix}$.

$$\begin{bmatrix} x & 0 \\ z & y \end{bmatrix} \begin{bmatrix} 1 & p \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 4 & 9 \end{bmatrix}$$

$$\begin{bmatrix} x & xp \\ z & zp+y \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 4 & 9 \end{bmatrix}.$$

lower diagonal entry $l_{22} = y$.

$x = 2, \quad xp = 2$

$\therefore p = 1$

$z = 4$

$zp + y = 9$

$(4)(1) + y = 9$

$y = 5$

lower diagonal entry $l_{22} = 5$

19) The largest of the two eigenvalues of the matrix $\begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}$ is ______

product of eigen values = det of the matrix

$$\begin{vmatrix} 4 & 5 \\ 2 & 1 \end{vmatrix} = -6 = \lambda_1 \lambda_2$$

Sum of the eigen values = trace of A

$\lambda_1 + \lambda_2 = 4+1$

$\lambda_1 + \lambda_2 = 5$

$\lambda_2 = 5 - \lambda_1$

$\lambda_1 \lambda_2 = -6$

$\lambda_1 (5-\lambda_1) = -6$

$\lambda_1^2 - 5\lambda_1 - 6 = 0$

$\lambda_1 = 6$ (or) $\lambda_1 = -1$

Larger of two eigen value is 6

80) Perform the following operations on the matrix.

$$\begin{bmatrix} 3 & 4 & 45 \\ 7 & 9 & 105 \\ 13 & 2 & 195 \end{bmatrix}$$

(i) Add the third row to the second row.

(ii) Subtract the third column from the first column.

~~(iii)~~ The determinant of the resultant matrix is ______

Sol

$$\begin{bmatrix} 3 & 4 & 45 \\ 7 & 9 & 105 \\ 13 & 2 & 195 \end{bmatrix} \rightarrow$$

$$\begin{bmatrix} 3 & 4 & 15 \\ 20 & 11 & 300 \\ 13 & 2 & 195 \end{bmatrix}$$

$\frac{195}{13}$

$$\begin{bmatrix} 3 & 4 & 12 \\ 20 & 11 & 280 \\ 13 & 2 & 182 \end{bmatrix}$$

As elementry row operations and clms does not change the determinant, we will ignore the instruction given.

Also we can see that the last column is multiple of first colum.

$$\begin{bmatrix} 3 & 4 & 45 \\ 7 & 9 & 105 \\ 13 & 2 & 195 \end{bmatrix} \xrightarrow{C_3 \to C_3 - 15C_1} \begin{bmatrix} 3 & 4 & 0 \\ 7 & 9 & 0 \\ 13 & 2 & 0 \end{bmatrix}$$

$|A| = 0$.

(81) For a given matrix A if V is the eigen vector corresponding to the eigen value λ, then

$$AV = \lambda V.$$

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} -4\alpha \\ 2\alpha \\ 1\alpha \end{bmatrix} = \begin{bmatrix} -4\alpha \\ 2\alpha \\ 1\alpha \end{bmatrix}$$

$$= 1 \cdot \begin{bmatrix} -4\alpha \\ 2\alpha \\ 1\alpha \end{bmatrix}$$

$\therefore \{\alpha(-4,2,1) \mid \alpha \neq 0, \alpha \in R\}$ are the corrponding eigen vectors

82) Consider the following system of equation
$3x + 2y = 1$ — (1)
$4x + 7z = 1$ — (2)
$x + y + z = 3$ — (3)
$x - 2y + 7z = 0$ — (4)
The No of soln of this system

Sol Add (1) + (2).
$7x + 2y + 7z = 1$
$x + y + z = 3.$
$x - 2y + 7z = 0.$
Rank of Aug Matc = Rank of coeff Matrix = No of variables = 3.
∴ No of solns = 1

Alitor
$$\begin{vmatrix} 7 & 2 & 7 \\ 1 & 1 & 1 \\ 1 & -2 & 7 \end{vmatrix} \neq 0.$$

~~R rank = 3.~~
Ran Augment matrix = Rank of coef Matrix = 3
Hence unique sol

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83) The value of the dot product of the eigen vectors corresponding to any pair of different eigen values of a 4 by 4 symmetric positive definite matrix is

Soln
$Au = \lambda_1 u$ — (1)
$AV = \lambda_2 V$ — (2)
On xplying eqn (1) with V^T, we get
$V^T A u = V^T \lambda_1 u.$
$(V^T A) u = V^T \lambda_1 u.$
$(A^T V)^T u = V^T \lambda_1 u.$
$(AV)^T u = V^T \lambda_1 u$
[Since A is a symmet matrix we can write $A^T = A$
Using eqn (2).
$AV = \lambda_2 V$
$(\lambda_2 V)^T u = V^T \lambda_1 u.$
$\lambda_2 V^T u = \lambda_1 V^T u.$
(as λ_1 is a constant we can write $V^T \lambda_1 = \lambda V^T$)
$\lambda_2 V^T u - \lambda_1 V^T u = 0$
$(\lambda_2 - \lambda_1) V^T u = 0$
$\lambda_2 - \lambda_1 = 0$ (or) $V^T u = 0.$
But $\lambda_1 \neq \lambda_2$ ∴ $V^T u = 0$
The dot product of the eigen vectors u and V,

84) If the matrix A is such that

$$A = \begin{bmatrix} 2 \\ -4 \\ 7 \end{bmatrix} \begin{bmatrix} 1 & 9 & 5 \end{bmatrix}$$

Then $\det(A) =$

Soln

$$A = \begin{bmatrix} 2 \\ -4 \\ 7 \end{bmatrix} \begin{bmatrix} 1 & 9 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 18 & 10 \\ -4 & -36 & -20 \\ 7 & 63 & 35 \end{bmatrix}$$

$$|A| = 0$$

85) The product of the non zero eigen values of the matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$ is.

Let $A = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$

$R_5 \to R_5 - R_1$

$R_3 \to R_3 - R_2$

$R_4 \to R_4 - R_2$

$$A \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank $(A) = 2$

No of non zero eigen value $\leq$ rank(A)

$\therefore$ There are 2 (or) less non zero eigen values.

The characteristic eqn of A is $|A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & 0 & 0 & 0 & 1 \\ 0 & 1-\lambda & 1 & 1 & 0 \\ 0 & 1 & 1-\lambda & 1 & 0 \\ 0 & 1 & 1 & 1-\lambda & 0 \\ 1 & 0 & 0 & 0 & 1-\lambda \end{vmatrix} =$$

$R_1 \to R_1 + R_5$

$$\begin{vmatrix} 2-\lambda & 0 & 0 & 0 & 2-\lambda \\ 0 & 1-\lambda & 1 & 1 & 0 \\ 0 & 1 & 1-\lambda & 1 & 0 \\ 0 & 1 & 1 & 1-\lambda & 0 \\ 1 & 0 & 0 & 0 & 1-\lambda \end{vmatrix} =$$

$$(2-\lambda) \begin{vmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1-\lambda & 1 & 1 & 0 \\ 0 & 1 & 1-\lambda & 1 & 0 \\ 0 & 1 & 1 & 1-\lambda & 0 \\ 1 & 0 & 0 & 0 & 1-\lambda \end{vmatrix} =$$

$R_2 \to R_2 + R_3 + R_4$.

$$(2-\lambda)\begin{vmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 3-\lambda & 3-\lambda & 3-\lambda & 0 \\ 0 & 1 & 1-\lambda & 1 & 0 \\ 0 & 1 & 1 & 1-\lambda & 0 \\ 1 & 0 & 0 & 0 & 1-\lambda \end{vmatrix}$$

$$(2-\lambda)(3-\lambda)\begin{vmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1-\lambda & 1 & 0 \\ 0 & 1 & 1 & 1-\lambda & 0 \\ 0 & 0 & 0 & 0 & 1-\lambda \end{vmatrix}$$

$\therefore \lambda = 2$ (or) $\lambda = 3$

Product of eigen valu $= 2 \times 3 = 6$

86) Ans:- option 1.

Trace is the sum of all diagonal elements of a square matrix.

The determinant of a matrix = Product of Eigen values

In case, where the trace is positive, and determinant is negative, then at least one eigen value of the matrix or an odd number of eigen values has to be negative because the product of eigen values of a given matrix is equal to the determinant of a given matrix.

Hence, to have the determinant negative, atleast one eigen value has to be negative but reverse may (or) may not be true.

87) which one of the following does not equal

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = ?$$

Ans option 1.

Sol:

$$\begin{vmatrix} 1 & x(x+1) & x+1 \\ 1 & y(y+1) & y+1 \\ 1 & z(z+1) & z+1 \end{vmatrix} = \begin{vmatrix} 1 & x^2+x & x+1 \\ 1 & y^2+y & y+1 \\ 1 & z^2+z & z+1 \end{vmatrix}$$

$C_3 \to C_3 - C_1$

$$\begin{vmatrix} 1 & x^2+x & x \\ 1 & y^2+y & y \\ 1 & z^2+z & z \end{vmatrix}$$

$C_2 \to C_2 - C_3$

$$\begin{vmatrix} 1 & x^2 & x \\ 1 & y^2 & y \\ 1 & z^2 & z \end{vmatrix}$$

since $\begin{vmatrix} 1 & x^2 & x \\ 1 & y^2 & y \\ 1 & z^2 & z \end{vmatrix} = -\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$

$\therefore$ switching two rows (or) clms causes the determinant to switch again.

$$\begin{vmatrix} 1 & x^2 & x \\ 1 & y^2 & y \\ 1 & z^2 & z \end{vmatrix} \neq \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$$

$\therefore$ option 1 is crt

88) Let A be the 2×2 matrix with elements $a_{11}=a_{12}=a_{21}=1$ and $a_{22}=-1$. Then the eigen values of the matrix A^{19} are

Soln

$a_{11}=a_{12}=a_{21}=1$

$a_{22}=-1$

$$\therefore A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Chare eqn:

$|A-\lambda I|=0$

$$\begin{vmatrix} 1-\lambda & 1 \\ 1 & -1-\lambda \end{vmatrix} = 0.$$

$-(1-\lambda)(1+\lambda)-1=0$

$\lambda^2=2$

$\lambda=\pm(2)^{1/2}$

$\therefore$ eigen values are

$\left((2)^{1/2}\right)^{19} = 2^9\times2^{1/2}$

$=512\sqrt{2}$

$\left((-2)^{1/2}\right)^{19} = -512\sqrt{2}$

(89) - Consider a 3X3 matrix A whose $(i, j)^{th}$ element $a_{ij} = (i-j)^3$. Then the matrix A will be

Sn

Ans: Skew Symmetric

Given $A = [a_{ij}]_{3\times3}$.

$a_{ij} = (i-j)^3$

To know about the main diagonal put $i=j$

$\therefore$ for $i=j$

$\Rightarrow a_{ij} = (i-i)^3 = 0 \ \forall i$

For remaining elements.

$i \neq j$

$\Rightarrow a_{ij} = (i-j)^3 = (-(j-i))^3$

$= -(j-i)^3 = -a_{ji}$

$\therefore$ Both the above conditions are satisfied

90. Consider a matrix A

$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & -2 \\ 0 & 1 & 1 \end{pmatrix}$. The matrix A satisfies the eqn

$\cancel{6A-cd}\ 6A^{-1} = \cancel{2}A^2 + cA + dI$

where c & d are scalars

I is the iden. Matr.

Then $c+d = \ldots$

Let $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -2 \\ 0 & 1 & 1 \end{bmatrix}$

$|B - A I| = 0$.

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -2 \\ 0 & 1 & 1 \end{bmatrix} - A\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{vmatrix} 1-A & 0 & 0 \\ 0 & 4-A & -2 \\ 0 & 1 & 1-A \end{vmatrix} = 0$

$(1-A)[(4-A)(1-A)+2] = 0$

$A^3 - 6A^2 + 11A = 6$

$A^2 - 6A + 11 = 6A^{-1}$ ①

Given $6A^{-1} = A^2 + cA + d$ ②

Compare ① and ②.

$c = -6 \quad d = 11$

$\boxed{c + d = 5}$