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GATE 2025 - MATHEMATICS

Linear Algebra

1. Consider the system of linear equations $x + 2y + z = 5$; $2x + ay + 4z = 12$; $2x + 4y + 6z = b$ the values of a and b such that there exists a non-trivial null space and the system admits infinite solutions are

   A. $a = 8, b = 14$

   B. $a = 4, b = 12$

   C. $a = 8, b = 12$

   D. $a = 4, b = 14$

2. The matrix $\begin{bmatrix} 1 & \alpha \\ 8 & 3 \end{bmatrix}$ where $a > 0$ has a negative eigenvalue of a is greater than

   A. $^{3}/_{8}$
   B. $^{1}/_{8}$
   C. $^{1}/_{4}$
   D. $^{1}/_{5}$

3. Consider the matrix $\begin{pmatrix} 1 & k \\ 2 & 1 \end{pmatrix}$, where k is a positive real number. Which of the following vectors is / are eigenvector(s) of this matrix?

   A. $\begin{pmatrix} 1 \\ -\sqrt{\frac{2}{k}} \end{pmatrix}$ B. $\begin{pmatrix} 1 \\ \sqrt{\frac{2}{k}} \end{pmatrix}$ C. $\begin{pmatrix} \sqrt{2k} \\ 1 \end{pmatrix}$ D. $\begin{pmatrix} \sqrt{2k} \\ -1 \end{pmatrix}$

4. If $A = \begin{bmatrix} 10 & 2K + 5 \\ 3K - 3 & K + 5 \end{bmatrix}$ is a symmetric matrix, the value of K is ________

   A. 8
   B. 5
   C. -0.4
   D. $\frac{1+\sqrt{1561}}{12}$

5. The system of linear equations in real (x, y) given by $(x\ y)\begin{bmatrix} 2 & 5 - 2\alpha \\ \alpha & 1 \end{bmatrix} = (0\ 0)$ involves a real parameter α and has infinitely many non-trivial solutions for special

value(s) of α. Which one or more among the following options is/are non-trivial solution(s) of (x, y) for such special value(s) of α?

A. $x = 2, y = -2$

B. $x = -1, y = 4$

C. $x = 1, y = 1$

D. $x = 4, y = -2$

6. A is a 3×5 real matrix of rank 2. For the set of homogeneous equations $= 0$, where 0 is a zero vector and x is a vector of unknown variables, which of the following is/are true?

   A. The given set of equations will have a unique solution

   B. The given set of equations will be satisfied by a zero vector of appropriate size.

   C. The given set of equations will have infinitely many solutions.

   D. The given set of equations will have many but a finite number of solutions.

7. If the sum and product of eigenvalues of a 2 × 2 real matrix $\begin{bmatrix} 3 & P \\ P & Q \end{bmatrix}$ are 4 and −1 respectively, then $|p|$ is (in integer)-------

8. The state equation of a second order system is $\dot{x}(t) = Ax(t), x(0)$ is the initial condition. Suppose λ_1 and λ_1 are two distinct eigenvalues of A and v_1 and v_2 are the corresponding eigenvectors. For constants α_1 and α_2. The solution $x(t)$, of the state equation is

   A. $\sum_{i=1}^{2} \alpha_i e^{\lambda_i t} v_i$ B. $\sum_{i=1}^{2} \alpha_i e^{2\lambda_i t} v_i$ C. $\sum_{i=1}^{2} \alpha_i e^{3\lambda_i t} v_i$ D. $\sum_{i=1}^{2} \alpha_i e^{4\lambda_i t} v_i$

9. Consider matrix $A = \begin{pmatrix} k & 2k \\ k^2 - k & k^2 \end{pmatrix}$ and vector $X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$. The number of distinct real values of k for which the equation $AX = 0$ has infinitely many solutions is ____.

10. Consider the 5 x 5 matrix A = $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 1 & 2 & 3 & 4 \\ 4 & 5 & 1 & 2 & 3 \\ 3 & 4 & 5 & 1 & 2 \\ 2 & 3 & 4 & 5 & 1 \end{bmatrix}$. It is given that A has only one real eigenvalue. Then the real eigenvalue of A is

    A. -2.5 B. 0 C. 15 D. 25

11. The rank of the matrix M = $\begin{pmatrix} 5 & 10 & 10 \\ 1 & 0 & 2 \\ 3 & 6 & 6 \end{pmatrix}$ is

    A. 0 B. 1 C. 2 D. 3

12. The rank of the matrix $\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$ is __________

13. The value of x for which the matrix $A = \begin{bmatrix} 3 & 2 & 4 \\ 9 & 7 & 13 \\ -6 & -4 & -9 + x \end{bmatrix}$ has zero as an eigenvalue is…………

14. The matrix $\begin{bmatrix} a & 0 & 3 & 7 \\ 2 & 5 & 1 & 3 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & b \end{bmatrix}$ has $\det(A) = 100$ and trace(A)=14. The value of $|a-b|$ is ______

15. The value of p such that the vector $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ is an eigenvector of the matrix $\begin{pmatrix} 4 & 1 & 2 \\ p & 2 & 1 \\ 14 & -4 & 10 \end{pmatrix}$ is ________

16. Consider a system of linear equations: $x - 2y + 3z = -1$, $x - 3y + 4z = 1$ and -$2x + 4y - 6z = k$. the value of k for which the system has infinitely many solutions is ___

17. The determinant of matrix A is 5 and the determinant of matrix B is 40. The determinant of matrix AB is ________.

18. The maximum value of the determinant among all 2 × 2 real symmetric matrices with trace 14 is __________

19. The minimum eigenvalue of the following matrix A is ----- , where $A = \begin{pmatrix} 3 & 5 & 2 \\ 5 & 12 & 7 \\ 2 & 7 & 5 \end{pmatrix}$

    A. 0 B. 1 C. 2 D. 3

20. Which one of the following is NOT true for a square matrix A?
    A. If A is upper triangular, the eigenvalues of A are the diagonal elements of it
    B. If A is real symmetric, the eigenvalues of A are always real and positive.
    C. If A is real, the eigenvalues of A and A^T are always the same
    D. If all the principal minors of A are positive, all the eigenvalues of A are also positive.

21. Consider an $n \times n$ matrix A and a non-zero $n \times 1$ vector p. Their product $Ap = \alpha^2 p$ where $\alpha \in R$ and $\alpha \notin \{-1,0,1\}$ based on the given information, the Eigen value of A^2 is:

    A. α
    B. α^2
    C. $\sqrt{\alpha}$
    D. α^4

22. Multiplication of real valued square matrices of same dimension is
    A. Associative
    B. Commutative
    C. Always positive definite
    D. Not always possible to commute

23. A matrix P is decomposed into its symmetric part S and skew symmetric part V. If $S = \begin{pmatrix} -4 & 4 & 2 \\ 4 & 3 & 7/2 \\ 2 & 7/2 & 2 \end{pmatrix}$, $V = \begin{pmatrix} 0 & -2 & 3 \\ 2 & 0 & 7/2 \\ -3 & -7/2 & 0 \end{pmatrix}$, then matrix P is

A. $\begin{pmatrix} -4 & 6 & -1 \\ 2 & 3 & 0 \\ 5 & 7 & 2 \end{pmatrix}$

B. $\begin{pmatrix} -4 & 2 & 5 \\ 6 & 3 & 7 \\ -1 & 0 & 2 \end{pmatrix}$

C. $\begin{pmatrix} 4 & -6 & 1 \\ -\ 2 & -3 & 0 \\ -5 & -7 & -2 \end{pmatrix}$

D. $\begin{pmatrix} -2 & 9/2 & -1 \\ -1 & 81/4 & 11 \\ -2 & 45/2 & 73/4 \end{pmatrix}$

24. Let I be a 100 dimensional identity matrix and E be the set of its distinct (no value appears more than once in E) real Eigen values. The number of elements in E is ___

25. Consider the matrix $P = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ the number of distinct eigenvalues of P is

A. 0
B. 1
C. 2
D. 3

26. The set of equations $x + y + z = 1$; $ax - ay + 3y = 5$; $5x - 3y + az = 6$ has infinite solutions, if a =

A. – 3

B. **3**

C. 4

D. $\mathbf{-4}$

27. In matrix equation [A] [X]=[R], $[A] = \begin{bmatrix} 4 & 8 & 4 \\ 8 & 16 & -4 \\ 4 & -4 & 15 \end{bmatrix}$, $[X] = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$ and $[R] = \begin{bmatrix} 32 \\ 16 \\ 64 \end{bmatrix}$. One of the eigenvalues of matrix [A] is

A. 4

B. 8

C. 15

D. 16

28. The transformation matrix for mirroring a point in x-y plane about the line $y = x$ is given by

A. $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

B. $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

C. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

D. $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

29. The rank of the matrix $\begin{bmatrix} -4 & 1 & -1 \\ -1 & -1 & -1 \\ 7 & -3 & 1 \end{bmatrix}$ is

A. 1

B. 2

C. 3

D. 4

30. The problem of maximizing $z = x_1 - x_2$ subject to constraints $x_1 + x_2 \leq 10, x_1 \geq 0, x_2 \geq 0$ and $x_2 \leq 5$ has

A. No solution

B. One solution

C. Two solutions

D. More than two solutions

31. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 1 \end{bmatrix}$ then $\det(A^{-1})$ is __________ (correct to two decimal places).

32. The Product of the Eigen values of the matrix P is $\begin{pmatrix} 2 & 0 & 1 \\ 4 & -3 & 3 \\ 0 & 2 & -1 \end{pmatrix}$

A. – 6

B. 2

C. 6

D. -2

33. Consider the matrix P = $\begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$. Which one of the following statements about P is INCORRECT?

34. The determinant of a 2×2 matrix is 50. If one Eigen value of the matrix is 10, the other Eigen value is ________

35. Consider the matrix $A = \begin{bmatrix} 50 & 70 \\ 70 & 80 \end{bmatrix}$ whose eigen vectors corresponding to Eigen values λ1 and λ2 are $X1 = \begin{bmatrix} 70 \\ \lambda 1 - 50 \end{bmatrix}$ and $X2 = \begin{bmatrix} \lambda 2 - 80 \\ 70 \end{bmatrix}$ respectively. The value of $X1^T X2$ is ______

36. The solution to the system of equations $\begin{bmatrix} 2 & 5 \\ -4 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{pmatrix} 2 \\ -30 \end{pmatrix}$ is

37. The condition for which the eigenvalues of the matrix A = $\begin{pmatrix} 2 & 1 \\ 1 & \text{k} \end{pmatrix}$ are positive is

38. A real square matrix A is called skew-symmetric if
   A. $A^T = A$
   B. $A^T = A^{-1}$
   C. $A^T = -A$
   D. $A^T = A + A^{-1}$

39. If any two columns of a determinant P = $\begin{bmatrix} 4 & 7 & 8 \\ 3 & 1 & 5 \\ 9 & 6 & 2 \end{bmatrix}$ are interchanged, which one of the following statements regarding the value of the determinant is correct?
   A. Absolute value remains unchanged but sign will change.
   B. Both absolute value and sign will change.
   C. Absolute value will change but sign will not change.
   D. Both Absolute value and sign value will remain unchanged.

40. At least one Eigen value of a singular matrix is
   A. Positive
   B. Zero
   C. Negative
   D. Imaginary

41. For a given matrix P = $\begin{bmatrix} 4+3i & -i \\ i & 4-3i \end{bmatrix}$, where i = $\sqrt{-1}$, the inverse of matrix P is
   A. $\frac{1}{24} \begin{bmatrix} 4-3i & i \\ -i & 4+3i \end{bmatrix}$
   B. $\frac{1}{25} \begin{bmatrix} i & 4-3i \\ 4+3i & -i \end{bmatrix}$
   C. $\frac{1}{24} \begin{bmatrix} 4+3i & -i \\ i & 4-3i \end{bmatrix}$

D. $\frac{1}{25}\begin{bmatrix} 4+3i & -i \\ i & 4-3i \end{bmatrix}$

42. The lowest eigenvalue of the $2 \times\ 2$ matrix $\begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}$ is ________

43. Given that the determinant of the matrix $\begin{pmatrix} 1 & 3 & 0 \\ 2 & 6 & 4 \\ -1 & 0 & 2 \end{pmatrix}$ is -12, the determinant of the matrix $\begin{pmatrix} 2 & 6 & 0 \\ 4 & 12 & 8 \\ -2 & 0 & 4 \end{pmatrix}$ is

44. The matrix form of the linear system $\frac{dx}{dt} = 3x - 5y$ and $\frac{dy}{dt}\ =\ 4x\ + 8y$ is

A. $\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 & -5 \\ 4 & 8 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$

B. $\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 & 8 \\ 4 & -5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$

C. $\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 & -5 \\ 3 & 8 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$

D. $\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 & 8 \\ 3 & -5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$

45. One of the eigenvectors of the matrix $\begin{pmatrix} -5 & 2 \\ -9 & 6 \end{pmatrix}$ is

A. $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$

B. $\begin{pmatrix} -2 \\ 9 \end{pmatrix}$

C. $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$

D. $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

46. The eigenvalues of a symmetric matrix are all
A. Complex with non-zero positive imaginary part
B. Complex with non-zero negative imaginary par.
C. Real
D. Pure imaginary

47. Choose the CORRECT set of functions, which are linearly dependent
A. $sinx, sin^2x\ and\ cos^2x$
B. $sinx\ , cosx\ and\ tanx$
C. $cos^2x, sin^2x\ and\ cos^2x$
D. $cos^2x\ , sinx\ and\ cosx$

48. For the matrix A = $\begin{pmatrix} 5 & 3 \\ 1 & 3 \end{pmatrix}$, one of the normalized Eigen vector is given by

A. $\begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix}$

B. $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix}$

C. $\begin{pmatrix} \frac{3}{\sqrt{10}} \\ \frac{-1}{\sqrt{10}} \end{pmatrix}$

D. $\begin{pmatrix} \frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{pmatrix}$

49. $x + 2y + z = 4$; $2x + y + 2z = 5$; $x - y + z = 1$. The system of algebraic equations given above has

A. A unique solution of $x = 1$, $y = 1$ and $z = 1$

B. Only the two solutions of $(x = 1, y = 1, z = 1)$ and $(x = 2, y = 1, z = 0)$

C. Infinite number of solutions

D. No feasible solution

50. Eigen values of a real symmetric matrix are always

A. Positive

B. Negative

C. Real

D. Complex

51. Consider the following system of equations:$2x_1 + x_2 + x_3 = 0$; $x_2 - x_3 = 0$; $x_1 + x_2 = 0$. This system has

A. A unique solution

B. No solution

C. Five solutions

D. Infinite number of solutions

52. One of the Eigen vectors of the matrix A = $\begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}$ is

A. $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$

B. $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$

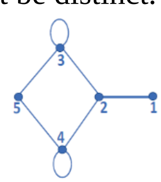
C. $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$

D. $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

53. The product of all Eigen values of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ is

A. -1 B. 0 C. 1 D. 2

54. Let A be any $n \times n$ matrix, where $m > n$. Which of the following statements is/are TRUE about the system of linear equations $Ax = 0$?
   A. There exist at least $m - n$ linearly independent
   B. There exist $m - n$ linearly independent vectors such that every solution is a linear combinations of these vectors.
   C. There exist a non-zero solution in which at least $m - n$ variables are 0
   D. There exists a solution in which at least n variables are non-zero.
55. Let A be an $n \times n$ matrix over the set of all real numbers R. Let B be a matrix obtained from A by swapping two rows. Which of the following statements is/are TRUE?
   A. The determinant of B is the negative of the determinant of A
   B. If A is invertible, then B is also invertible
   C. If a is symmetric, then b is also symmetric
   D. If the trace of a is zero, then the trace of b is also zero
56. Let $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \\ 3 & 4 & 1 & 2 \\ 2 & 3 & 4 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{bmatrix}$ Let $det(A)$ and $det(B)$ denote the determinants of the matrices A and B, respectively. Which one of the options given below is TRUE?
   A. $\det(A) = \det(B)$
   B. $\det(B) = -\det(A)$
   C. $\det(A) = 0$
   D. $\det(AB) = \det(A) + \det(B)$
57. Let $\lambda_1, \lambda_2, \lambda_3, \lambda_4,$ and λ_5 be the five eigenvalues of A. Note that these eigenvalues need not be distinct.



The value of $\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 =$__________ .

58. Consider the following two statements with respect to the matrices $A_{m\times n}, B_{n\times m}, C_{n\times n}$ and $D_{n\times n}$ Statement 1: $tr(AB) = tr(BA)$
Statement 2: $tr(CD) = tr(DC)$
where $tr()$ represents the trace of a matrix. Which one of the following holds?
   A. Statement 1 is correct and Statement 2 is wrong.

B. Statement 1 is wrong and Statement 2 is correct.
C. Both Statement 1 and Statement 2 are correct.
D. Both Statement 1 and Statement 2 are wrong.

59. Consider solving the following system of simultaneous equations using LU decomposition. $x_1 + x_2 - 2x_3 = 4; x_1 + 3x_2 - x_3 = 7; 2x_1 + x_2 - 5x_3 = 7$ where L and U are denoted as $L = \begin{pmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{pmatrix}$ and $U = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{pmatrix}$ Which one of the following is the correct combination of values for L_{32} , U_{33}, and x_1?
A. $L_{32} = 2, U_{33} = -\frac{1}{2}, x_1 = -1$
B. $L_{32} = 2, U_{33} = 2, x_1 = -1$
C. $L_{32} = -\frac{1}{2}, U_{33} = 2, x_1 = 0$
D. $L_{32} = -\frac{1}{2}, U_{33} = -\frac{1}{2}, x_1 = 0$

60. Which of the following is/are the eigenvector(s) for the matrix given below?

$$\begin{bmatrix} -9 & -6 & -2 & -4 \\ -8 & -6 & -3 & -1 \\ 20 & 15 & 8 & 5 \\ 32 & 21 & 7 & 12 \end{bmatrix}$$

A. $\begin{pmatrix} -1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$ B. $\begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$ C. $\begin{pmatrix} -1 \\ 0 \\ 2 \\ 2 \end{pmatrix}$ D. $\begin{pmatrix} 0 \\ 1 \\ -3 \\ 0 \end{pmatrix}$

61. Consider the following matrix $\begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$ the largest eigenvalue of the above matrix is ____________.

62. Suppose that P is a 4×5 matrix such that every solution of the equation matrix such that every solution of the equation $P_x = 0$ is a scalar multiple of $[2\ 5\ 4\ 3\ 1]^T$. The rank of p is ______.

63. Let A and B be two $n \times n$ matrices over real numbers. Let $rank(M)$ and $det(M)$ denote the rank and determinant of a matrix M, respectively. Consider the following statements
$I.\ rank(AB) = rank(A)rank(b)$
II. $\det(AB) = \det(A)\det(B)$
$III.\ rank(A + B) \le rank(A) + rank(B)$
$IV.\ \det(A + B) \le \det(A) + \det(B)$
Which of the above statements are TRUE?
A. I and II only B. I and IV only C. II and III only D. III and IV only

64. Let X be a square matrix. Consider the following two statements on X.

I. X is invertible.
II. Determinant of X is non-zero.
Which one of the following is TRUE?

A. I implies II: II does not imply I
B. II implies I; I does not imply II
C. I does not imply II; II does not imply I
D. I and II are equivalent statements.

65. Consider the following matrix:

$$R = \begin{bmatrix} 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \\ 1 & 4 & 16 & 64 \\ 1 & 5 & 25 & 125 \end{bmatrix}$$

The absolute value of the product of Eigen value of R is ____.

66. Consider a matrix P whose only eigenvectors are the multiples of $\binom{1}{4}$. Consider the following statements.
I. P does not have an inverse
II. P has a repeated eigenvalue
III. P cannot be diagonalized
Which one of the following options is correct?
A. Only I and III are necessarily true
B. Only II is necessarily true
C. Only I and II are necessarily true
D. Only II and III are necessarily true

67. Consider a matrix $A = uv^T$ where$u = \binom{1}{2}, v = \binom{1}{1}$ Note that v^T denotes the transpose of v. The largest eigenvalue of A is _____.

68. Let $C_1, \dots, C_n$ be scalars, not all zero, such that $\sum_{i=1}^{n} c_i a_i = 0$ where a_i are column vectors in R^n consider the set of linear equations $Ax = b$ where $A = [a_1 \dots a_n]$ and $b = \sum_{i=1}^{n} a_i$. The set of equations has
A. A unique solution at $x = J_n$ where J_n denotes a n-dimensional vector of all 1
B. Infinitely many solutions
C. Finitely many solutions
D. No solution

69. Let u and v be two vectors in R^2 whose Euclidean norms satisfy $||u|| = 2||v||$. What is the value of α such that $w = u = \alpha v$ bisects the angle between u and v?
A. 2 B. ½ C. -1 D. -1/2

70. Let A be $n \times n$ real valued square symmetric matrix of rank 2 with $\sum_{i=1}^{n} \sum_{j=1}^{n} {A_{ij}}^2 = 50$, Consider the following statements

I. One eigenvalue must be in [-5,5]

II. The eigenvalue with the largest magnitude must be strictly greater than 5

Which of the above statements about eigenvalues of A is/are necessarily CORRECT?

A. Both I and II
B. I only
C. II only
D. Neither I nor II

71. Let $P = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$ and $Q = \begin{bmatrix} -1 & -2 & -1 \\ 6 & 12 & 6 \\ 5 & 10 & 5 \end{bmatrix}$ be two matrices. Then the rank of $P + Q$ is ________.

72. Consider a quadratic equation $x^2 - 13x + 36 = 0$ with coefficients in a base b. The solutions of this equation in the same base b are $x = 5$ and $x = 6$. Then $b =$ __.

73. If the characteristic polynomial of a 3×3 matrix M over R (the set of real numbers) is $\lambda^3 - 4\lambda^2 + a\lambda \in R$, and one eigenvalue of M is 2, then the largest among the absolute values of the eigenvalues of M is ______.

74. Two eigenvalues of a 3×3 real matrix P are $(2 + \sqrt{-1})$ and 3. The determinant of P is ___.

75. Consider the systems, each consisting of m linear equations in n variables.

I. If $m < n$, then all such systems have a solution

II. If $m > n$, then none of these systems has a solution

III. If $m = n$, then there exists a system which has a solution which one of the following is CORRECT?

A. I, II and III are true
B. Only II and III are true
C. Only III is true
D. None of them is true

76. Suppose that the eigenvalues of matrix A are 1,2,4. The determinant of $(A^{-1})^T$ is____.

77. Consider the following 2×2 matrix A where two elements are unknown and are marked by a and b. The eigenvalues of this matrix are -1 and 7. What are the values of a and b?

$A = \begin{pmatrix} 1 & 4 \\ b & a \end{pmatrix}$

A. $a = 6, b = 4$ B. $a = 4, b = 6$ C. $a = 3, b = 5$ D. $a = 5, b = 3$

78. In the LU decomposition of the matrix $\begin{bmatrix} 2 & 2 \\ 4 & 9 \end{bmatrix}$, if the diagonal elements of U are both 1, then the lower diagonal entry l_{22} of L is ______.

79. The larger of the two eigenvalues of the matrix $\begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}$ is ______.

80. Perform the following operations on the matrix $\begin{bmatrix} 3 & 4 & 45 \\ 7 & 9 & 105 \\ 13 & 2 & 195 \end{bmatrix}$

(i)Add the third row to the second row

(ii)Subtract the third column from the first column.

The determinant of the resultant matrix is _______.

81. In the given matrix $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}$, one of the eigenvalues is 1. The eigenvectors corresponding to the eigenvalue 1 are.............

A. $\{\alpha(4,2,1) | \alpha \neq 0,\ \alpha \in R\}$

B. $\{\alpha(-4,2,1) | \alpha \neq 0,\ \alpha \in R\}$

C. $\{\alpha(\sqrt{2},0,1) | \alpha \neq 0,\ \alpha \in R\}$

D. $\{\alpha(-\sqrt{2},0,1) | \alpha \neq 0,\ \alpha \in R\}$

82. Consider the following system of equations:

$3x + 2y = 1;\ 4x + 7z = 1;\ x + y + z = 3;\ x - 2y + 7z = 0;$

The number of solutions for this system is ________.

83. The value of the dot product of the eigenvectors corresponding to any pair of different eigenvalues of a $4\ by\ 4$ symmetric positive definite matrix is ___________

84. If the matrix A is such that $A = \begin{bmatrix} 2 \\ -4 \\ 7 \end{bmatrix} \begin{bmatrix} 1 & 9 & 5 \end{bmatrix}$ then the determinant of A is equal to _____

85. The product of the non-zero eigenvalues of the matrix $\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$ is _

86. Which one of the following statements is TRUE about every $n \times n$ matrix with only real eigen values?

A. If the trace of the matrix is positive and the determinant of the matrix is negative, at least one of its eigenvalues is negative.

B. If the trace of the matrix is positive, all its eigenvalues are positive.

C. If the determinant of the matrix is positive, all its eigenvalues are positive.

D. If the product of the trace and determinant of the matrix is positive, all its eigenvalues are positive.

87. Which one of the following does **NOT** equal $\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$?

A. $\begin{vmatrix} 1 & x(x+1) & x+1 \\ 1 & y(y+1) & y+1 \\ 1 & z(z+1) & z+1 \end{vmatrix}$

B. $\begin{vmatrix} 1 & (x+1) & x^2+1 \\ 1 & (y+1) & y^2+1 \\ 1 & (z+1) & z^2+1 \end{vmatrix}$

C. $\begin{vmatrix} 0 & x-y & x^2-y^2 \\ 0 & y-z & y^2-z^2 \\ 1 & z & z^2 \end{vmatrix}$

D. $\begin{vmatrix} 2 & x+y & x^2+y^2 \\ 2 & y+z & y^2+z^2 \\ 1 & z & z^2 \end{vmatrix}$

88. Let A be the 2×2 matrix with elements $a_{11} = a_{12} = a_{21} = +1$ and $a_{22} = -1$ the eigenvalues of the matrix A^{19} are

A. 1024 and -1024
B. $1024\sqrt{2}$ and $-1024\sqrt{2}$
C. $4\sqrt{2}$ and $-4\sqrt{2}$
D. $512\sqrt{2}$ and $-512\sqrt{2}$

86. Consider a 3 x 3 matrix A whose $(i,, j)$-th element, $a_{i,j} = (i - j)^3$. Then the matrix A will be

a) symmetric b) skew symmetric c) unitary d) null

87. Consider a matrix $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & -2 \\ 0 & 1 & 1 \end{pmatrix}$. The matrix A satisfies the equation $6A - 1 = A2 + cA + dI$, where C & D are scalars and I is the identity matrix. Then $(c + d)$ is equal to

a) 5 b) 17 c) -6 d) 11

88. Let A be a 10×10 matrix such that A^5 is a null matrix, and let I be the 10×10 identity matrix. The determinant of $A + I$ is ________.

89. The number of purely real elements in a lower triangular representation of the 3×3 matrix, obtained through the given decomposition is _________.

$$\begin{pmatrix} 2 & 3 & 3 \\ 3 & 2 & 1 \\ 3 & 1 & 7 \end{pmatrix} = \begin{pmatrix} a_{11} & 0 & 0 \\ a_{12} & a_{22} & 0 \\ a_{13} & a_{23} & a_{33} \end{pmatrix} \begin{pmatrix} a_{11} & 0 & 0 \\ a_{12} & a_{22} & 0 \\ a_{13} & a_{23} & a_{33} \end{pmatrix}^T$$

a) 5 b) 6 c) 8 d)9

90. Consider a 2×2 matrix $M = [v_1\ v_2]$, where v_1 and v_2 are the column vectors. Suppose $M^{-1} = \begin{bmatrix} {u_1}^T \\ {u_2}^T \end{bmatrix}$, where ${u_1}^T$ and ${u_2}^T$ are the row vectors. Consider the following statements:

Statement 1: ${u_1}^T v_1 = 1$ and ${u_2}^T v_2 = 1$

Statement 2: ${u_1}^T v_2 = 0$ and ${u_2}^T v_1 = 0$

Which of the following options is correct?

- **a)** Statement 1 is true and statement 2 is false
- **b)** Statement 2 is true and statement 1 is false
- **c)** Both the statements are true.
- **d)** Both the statements are false.

91. M is a 2 x2 matrix with eigenvalues 4 and 9. The Eigen values of M^2 are

a) 4 and 9 b) 2 and 3 c) -2 and -3 d) 16 and 81

92. The rank of the matrix $M = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ is ________

93. Consider a non-singular 2 x 2 square matrix A. If trace(A) = 4 and trace (A^2) = 5, the determinant of the matrix A is -------(up to 1 decimal place).

94. Let $A = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$ and $= A^3 - A^2 - 4A + 5I$, where I is the 3×3 identity matrix . The determinant of B is ---------(up to 1 decimal place)

95. The matrix $A = \begin{bmatrix} \frac{3}{2} & 0 & \frac{1}{2} \\ 0 & -1 & 0 \\ \frac{1}{2} & 0 & \frac{3}{2} \end{bmatrix}$ has three distinct eigen values and one of its eigen vectors is $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$. which one of the following can be another eigenvector of A?

a) $\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$ b) $\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$ c) $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ d) $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

96. The eigenvalues of the matrix given below are $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -3 & -4 \end{pmatrix}$

a) (0,-1,-3) b) (0,-2,-3) c) (0,2,3) d) (0,1,3)

97. Let the eigenvalues of a 2×2 matrix A be 1, -2 with eigenvectors x_1 and x_2 respectively. Then the eigenvalues and eigenvectors of the matrix $A^2 - 3A + 4I$ would, respectively, be

a) 2,14 ; x_1 , x_2 **b)** 2,14 ; $x_1 + x_2$, $x_1 - x_2$ **c)** 2,0 ; x_1 , x_2 **d)** 2,0;$x_1 + x_2$,$x_1 - x_2$

98. Let A be a 4 × 3 real matrix with rank 2. Which one of the following statement is TRUE?

a) Rank of A^TA is less than 2 b) Rank of A^TA is equal to 2 c) Rank of A^TA is greater than 2 d) Rank of A^TA can be any number between 1 and 2

99. Consider a 3×3 matrix with every element being equal to 1. Its only non-zero eigenvalue is ____

100. Let $P = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$. Consider the set S of all vectors $\begin{pmatrix} x \\ y \end{pmatrix}$ such that $a^2 + b^2 = 1$ where $\begin{pmatrix} a \\ b \end{pmatrix} = P\begin{pmatrix} x \\ y \end{pmatrix}$ then S is

a) a circle of radius $\sqrt{10}$ **b)** a circle of radius $\frac{1}{\sqrt{10}}$ **c)** an ellipse with major axis along $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

d) an ellipse with minor axis along $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$