

R16

Maths for coding

Basic Concepts of Logs

$$\log_a b^m = m \log_a b \quad \log_2 2 = 1$$

$$\log_{\underline{a}} \underline{a} = 1 \quad \log_a m + \log_a n = \underline{\log_a (m n)}$$

204. Count Primes

Medium

Topics

Companies

Hint

Given an integer n , return the number of prime numbers that are strictly less than n .

Example 1:

Input: $n = 10$

Output: 4

Explanation: There are 4 prime numbers less than 10, they are 2, 3, 5, 7.

Example 2:

Input: $n = 0$

Output: 0

Example 3:

Input: $n = 1$

Output: 0

Constraints:

$0 \leq n \leq 5 * 10^6$

array, True
Sieve \rightarrow count

$n \rightarrow 1, n$

1 - - - - - n \rightarrow (2) Prime

\rightarrow (2 - - - $n-1$) \rightarrow Not Prime

36 → 1 2 3 4 6 9 12 18 36

$$< \sqrt{36} = \textcircled{6} >$$

2 _ _ _ \sqrt{n}

Pairs

Sqrt(n)

Sieve

F	F	T	T	F	T	F	T	F	F	F	T	F	T
0	1	2	3	4	5	6	7	8	9	10	11	12	13

→

$$\begin{aligned} 2 * 2 &= 4 \\ 3 * 3 &= 9 \end{aligned}$$

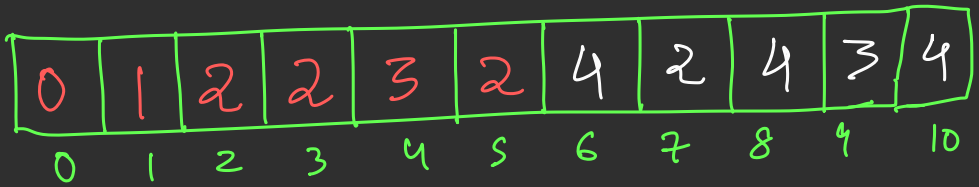
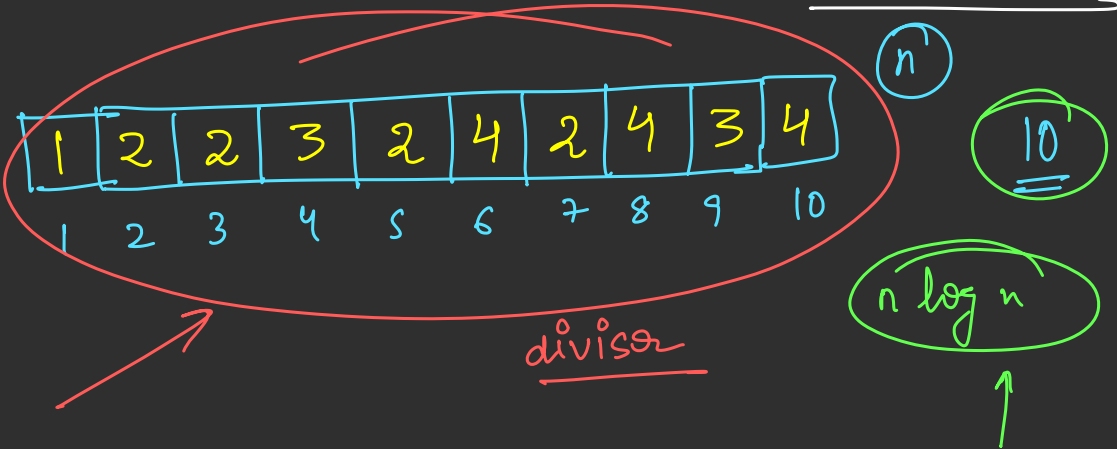
5 →

$$\begin{aligned} \textcircled{10} &\rightarrow \underline{2} * 5 \\ \textcircled{15} &\rightarrow \underline{3} * 5 \\ 20 &\rightarrow \underline{2 * 2 * 5} \end{aligned}$$

$\textcircled{5} \rightarrow 5 * 5 = \underline{25}$

Different versions of sieve of Eratosthenes

- Divisor Sieve \longrightarrow count divisors
- Sum of divisor Sieve \longrightarrow sum of divisors



```

for (int i = 1; i <= n; i++)
{
    for (int j = i; j <= n; j = j + i)
    {
        divisor[j]++;
    }
}

```

$n + \frac{n}{2} + \frac{n}{3} + \frac{n}{4} + \frac{n}{5} + \dots + \frac{n}{n}$

$n \log n$

0	1	3	4	7	6	12	8	15		
0	1	2	3	4	5	6	7	8	9	10

0	1	3	4	3	1	6	1	3	4	3
0	1	2	3	4	5	6	7	8	9	10

```
for (int i = 1; i <= n; i++)
```

```
{
```

```
    for (int j = i; j <= n; j = j + i)
```

```
    {
```

```
        divisor[j] += i;
```

```
    }
```

→ Sum of divisor

DIVSUM - Divisor Summation

#number-theory

Given a natural number n ($1 \leq n \leq 500000$), please output the summation of all its proper divisors.

Definition: A proper divisor of a natural number is the divisor that is strictly less than the number.

e.g. number 20 has 5 proper divisors: 1, 2, 4, 5, 10, and the divisor summation is: $1 + 2 + 4 + 5 + 10 = 22$.

Input

An integer stating the number of test cases (equal to about 200000), and that many lines follow, each containing one integer between 1 and 500000 inclusive.

Output

One integer each line: the divisor summation of the integer given respectively.

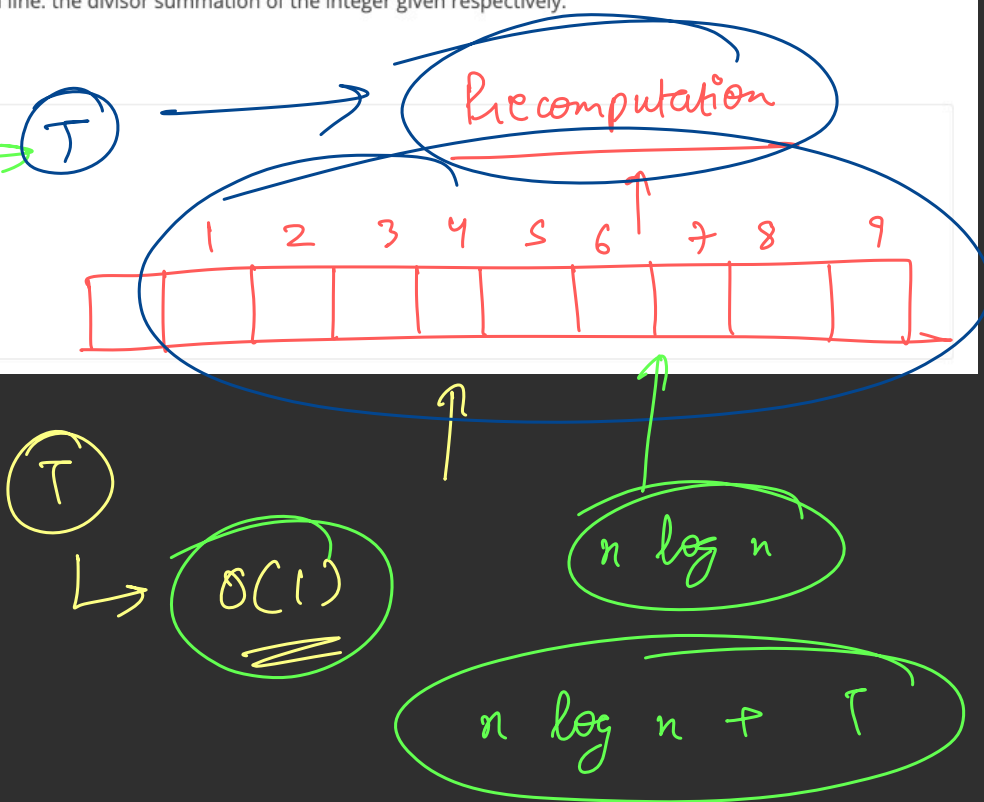
Example

Sample Input:

3
2
10
20

Sample Output:

1
8
22



1390. Four Divisors

Medium

Topics

Companies

Hint

Given an integer array `nums`, return the sum of divisors of the integers in that array that have exactly four divisors. If there is no such integer in the array, return `0`.

Example 1:

Input: `nums = [21,4,7]`

Output: 32

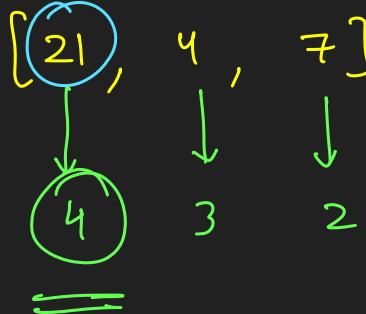
Explanation:

21 has 4 divisors: 1, 3, 7, 21

4 has 3 divisors: 1, 2, 4

7 has 2 divisors: 1, 7

The answer is the sum of divisors of 21 only.



Example 2:

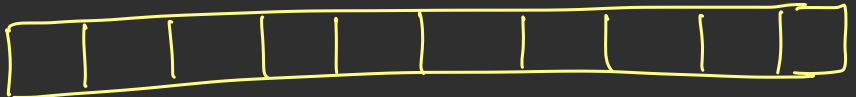
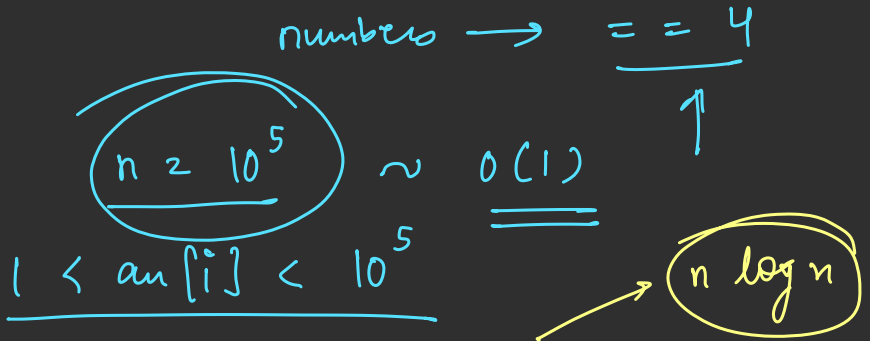
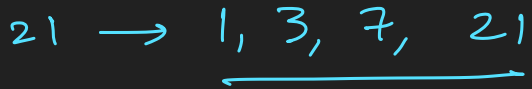
Input: `nums = [21,21]`

Output: 64

Example 3:

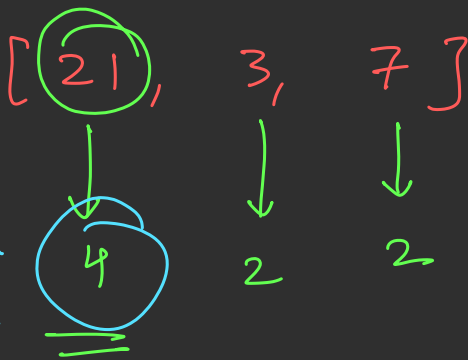
Input: `nums = [1,2,3,4,5]`

Output: 0

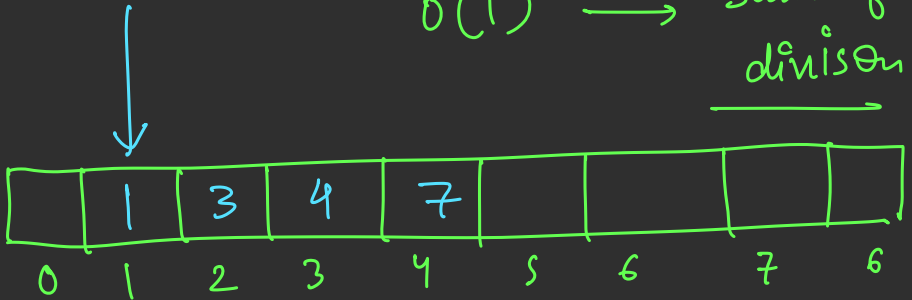


divisor

pre-compute



$O(1) \rightarrow$ sum of divisor



$divisor[an[i]] = 4$

sum += $sd[an[i]]$

$n \log n$

divisor [21]

$= 4$

↓
 $sd[21]$

1688. Count of Matches in Tournament

Solved

Easy Topics Companies Hint

You are given an integer n , the number of teams in a tournament that has strange rules:

- If the current number of teams is **even**, each team gets paired with another team. A total of $n / 2$ matches are played, and $n / 2$ teams advance to the next round.
- If the current number of teams is **odd**, one team randomly advances in the tournament, and the rest gets paired. A total of $(n - 1) / 2$ matches are played, and $(n - 1) / 2 + 1$ teams advance to the next round.

Return the number of matches played in the tournament until a winner is decided.

Example 1:

Input: $n = 7$

Output: 6

Explanation: Details of the tournament:

- 1st Round: Teams = 7, Matches = 3, and 4 teams advance.
 - 2nd Round: Teams = 4, Matches = 2, and 2 teams advance.
 - 3rd Round: Teams = 2, Matches = 1, and 1 team is declared the winner.
- Total number of matches = $3 + 2 + 1 = 6$.

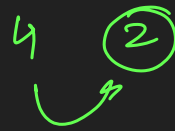
Example 2:

Input: $n = 14$

Output: 13

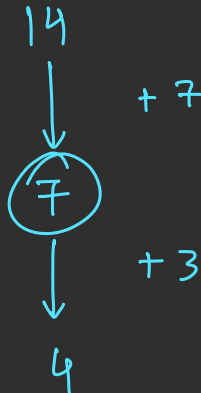
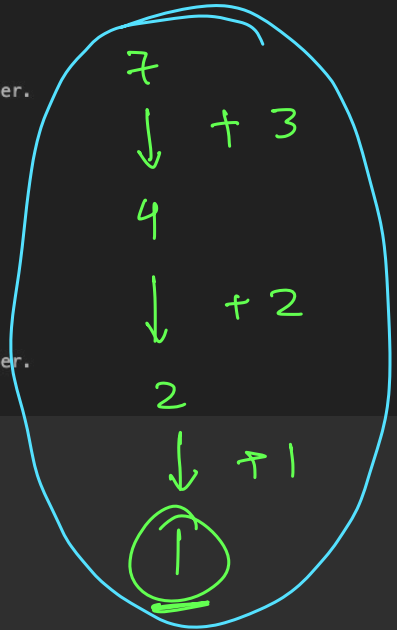
Explanation: Details of the tournament:

- 1st Round: Teams = 14, Matches = 7, and 7 teams advance.
 - 2nd Round: Teams = 7, Matches = 3, and 4 teams advance.
 - 3rd Round: Teams = 4, Matches = 2, and 2 teams advance.
 - 4th Round: Teams = 2, Matches = 1, and 1 team is declared the winner.
- Total number of matches = $7 + 3 + 2 + 1 = 13$.



$$n = 7$$

$$\frac{7}{2} = 3 + 1$$



$$\frac{n}{2} + \frac{n}{4}$$

7



14



while (n > 1)

{

even $r = n/2$

$n = n/2$

odd $r = n/2$

$n = n/2 + 1$

}

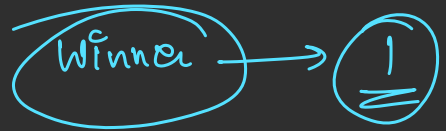
$\log n$

n

↓
 $n/2$

↓
 $n/4$

↓
 $n/8$



matches = (n-1)

O(1)

$$\underline{5 \% 4 = 1}$$

Modular arithmetic

$$n \% (4)$$

↓
0, 1, 2, 3

$$\rightarrow \underline{10^9 + 7} \sim \text{Prime}$$

sum $\underbrace{1 \dots N}$

$$\underline{N = 10^8}$$

$$\frac{N * (N + 1)}{2}$$

$$\sim \frac{10^8 * (10^8 + 1)}{2}$$

(int)

$$\sim \underline{10^{16} / 10^{15}}$$

$$\underline{10^9 + 7}$$

$$\begin{matrix} a \% M - \\ b \% M \end{matrix}$$

< 0

Properties

$$\rightarrow (a + b) \% M$$

$$= (a \% M + b \% M) \% M$$

$$\rightarrow (a - b) \% M$$

$$= (a \% M - b \% M + M) \% M$$

$$\frac{-5 \% 4}{\downarrow} = \underline{\underline{3 \% 4}}$$

$$\underline{10^9 \div 7}$$

$$\frac{-5 + 4 \div 4}{\downarrow} \\ 3 \% 4$$

Product % M

$$\underline{(a * b) \% M}$$

$$\hookrightarrow \frac{(a \% M * b \% M) \% M}{\uparrow \quad \uparrow}$$

$$\underline{(a/b) \% M} =$$

$$\left((a \% M) * \left(\underline{\underline{b^{-1} \% M}} \right) \right) \% M$$

Number