

Содержание

1 Setup & Scripts	1
1.1 CMake	1
1.2 wipe.sh	2
2 G++ builtins	2
3 Bugs	2
4 Geometry	3
4.1 Пересечение прямых	3
5 Numbers	3
6 Graphs	4
6.1 Weighted matroid intersection	4
7 Push-free segment tree	6
8 Number theory	7
8.1 Chinese remainder theorem without overflows	7
8.2 Integer points under a rational line	8
9 Suffix Automaton	8
10 Palindromic Tree	9
11 Smth added at last moment	10
11.1 Dominator Tree	10
11.2 Suffix Array	12
11.3 Fast LCS	14
11.4 Fast Subset Convolution	16
12 Karatsuba	17

1 Setup & Scripts

1.1 CMake

```

1 cmake_minimum_required(VERSION 3.14)
2 project(olymp)
3
4 set(CMAKE_CXX_STANDARD 17)
5 add_compile_definitions(LOCAL)
6 #set(CMAKE_CXX_FLAGS "${CMAKE_CXX_FLAGS} -fsanitize=undefined -fno-
    ↪ sanitize-recover")
7 #sanitizers: address, leak, thread, undefined, memory
8
9 add_executable(olymp f.cpp)

```

1.2 wipe.sh

```

1 touch {a..l}.cpp
2
3 for file in *.cpp ; do
4     cat template.cpp > $file ;
5 done

```

2 G++ builtins

- `__builtin_popcount(x)` — количество единичных бит в двоичном представлении 32-битного (знакового или беззнакового) целого числа.
- `__builtin_popcountll(x)` — то же самое для 64-битных типов.
- `__builtin_ctz(x)` — количество нулей на конце двоичного представления 32-битного целого числа. Например, для 5 вернётся 0, для $272 = 256 + 16$ — 4 и т. д. Может не работать для нуля (вообще не стоит вызывать для $x = 0$, по-моему это и упасть может).
- `__builtin_ctzll(x)` — то же самое для 64-битных типов.
- `__builtin_clz(x)` — количество нулей в начале двоичного представления 32-битного целого числа. Например, для 2^{31} или -2^{31} вернётся 0, для 1 — 31 и т. д. Тоже не надо вызывать с $x = 0$.
- `__builtin_clzll(x)` — то же самое для 64-битных типов.
- `bitset<N>._Find_first()` — номер первой позиции с единицей в битсете или его размер (то есть N), если на всех позициях нули.
- `bitset<N>._Find_next(x)` — номер первой позиции с единицей среди позиций с номерами строго больше x ; если такой нет, то N .

3 Bugs

- `powmod :`)
- Всегда чекать Куна дважды, особенно на количество итераций
- `uniform_int_distribution` от одного параметра
- `for (char c : "NEWS")`
- Порядок верхних и нижних границ в случае, когда задача двумерна $t - b \neq b - t$
- `static` с мультитестами
- `set` со своим компаратором склеивает элементы
- Два вектора с соответствующими элементами, сортируем один, а элементы второго ссылаются на чужие. Предлагается лечить заведением структуры с компаратором на каждый чих. В целом, для этого можно написать навороченную хрень на шаблонах.

- В графе с вершинами степени не больше одного надо писать выделение цикла полностью, срезать угол на какой-нибудь тупой меморизации, потому что кажется, что он может выглядеть только одним или несколькими какими-нибудь специальными способами, не получится, а дебажить сложно.
- Структуры, основанные на указателях не стоит хранить в векторах.

4 Geometry

4.1 Пересечение прямых

$$AB = A - B; CD = C - D$$

$$(A \times B \cdot CD.x - C \times D \cdot AB.x : A \times B \cdot CD.y - C \times D \cdot AB.y : AB \times CD)$$

5 Numbers

- A lot of divisors
 - $\leq 20 : d(12) = 6$
 - $\leq 50 : d(48) = 10$
 - $\leq 100 : d(60) = 12$
 - $\leq 1000 : d(840) = 32$
 - $\leq 10^4 : d(9240) = 64$
 - $\leq 10^5 : d(83160) = 128$
 - $\leq 10^6 : d(720720) = 240$
 - $\leq 10^7 : d(8648640) = 448$
 - $\leq 10^8 : d(91891800) = 768$
 - $\leq 10^9 : d(931170240) = 1344$
 - $\leq 10^{11} : d(97772875200) = 4032$
 - $\leq 10^{12} : d(963761198400) = 6720$
 - $\leq 10^{15} : d(866421317361600) = 26880$
 - $\leq 10^{18} : d(897612484786617600) = 103680$
- Numeric integration
 - simple: $F(0)$
 - simpson: $\frac{F(-1)+4 \cdot F(0)+F(1)}{6}$
 - runge2: $\frac{F(-\sqrt{\frac{1}{3}})+F(\sqrt{\frac{1}{3}})}{2}$
 - runge3: $\frac{F(-\sqrt{\frac{3}{5}}) \cdot 5 + F(0) \cdot 8 + F(\sqrt{\frac{3}{5}}) \cdot 5}{18}$

6 Graphs

6.1 Weighted matroid intersection

```

1 // here we use T = __int128 to store the independent set
2 // calling expand k times to an empty set finds the maximum
3 // cost of the set with size exactly k,
4 // that is independent in blue and red matroids
5 // ver is the number of the elements in the matroid,
6 // e[i].w is the cost of the i-th element
7 // first return value is new independent set
8 // second return value is difference between
9 // new and old costs
10 // oracle(set, red) and oracle(set, blue) check whether
11 // or not the set lies in red or blue matroid respectively
12
13 auto expand = [&] (T cur_set) → pair<T, int>
14 {
15     vector<int> in(ver);
16     for (int i = 0; i < ver; i++)
17         in[i] = ((cur_set >> i) & 1);
18
19     const int red = 1;
20     const int blue = 2;
21
22     vector<vector<int>> g(ver);
23     for (int i = 0; i < ver; i++)
24     for (int j = 0; j < ver; j++)
25     {
26         T swp_mask = (cur_set ^ (T(1) << i) ^ (T(1) << j));
27         if (!in[i] && in[j])
28         {
29             if (oracle(swp_mask, red))
30                 g[i].push_back(j);
31             if (oracle(swp_mask, blue))
32                 g[j].push_back(i);
33         }
34     }
35
36     vector<int> from, to;
37     for (int i = 0; i < ver; i++) if (!in[i])
38     {
39         T add_mask = cur_set ^ (T(1) << i);
40         if (oracle(add_mask, blue))
41             from.push_back(i);
42         if (oracle(add_mask, red))
43             to.push_back(i);
44     }
45
46     auto get_cost = [&] (int x)

```

```
47     {
48         const int cost = (!in[x] ? e[x].w : -e[x].w);
49         return (ver + 1) * cost - 1;
50     };
51
52     const int inf = int(1e9);
53     vector<int> dist(ver, -inf), prev(ver, -1);
54     for (int x : from)
55         dist[x] = get_cost(x);
56
57     queue<int> q;
58
59     vector<int> used(ver);
60     for (int x : from)
61     {
62         q.push(x);
63         used[x] = 1;
64     }
65
66     while (!q.empty())
67     {
68         int cur = q.front(); used[cur] = 0; q.pop();
69
70         for (int to : g[cur])
71         {
72             int cost = get_cost(to);
73             if (dist[to] < dist[cur] + cost)
74             {
75                 dist[to] = dist[cur] + cost;
76                 prev[to] = cur;
77                 if (!used[to])
78                 {
79                     used[to] = 1;
80                     q.push(to);
81                 }
82             }
83         }
84     }
85
86     int best = -inf, where = -1;
87     for (int x : to)
88     {
89         if (dist[x] > best)
90         {
91             best = dist[x];
92             where = x;
93         }
94     }
95
96     if (best == -inf)
```

```

97         return pair<T, int>(cur_set, best);
98
99     while (where  $\neq$  -1)
100     {
101         cur_set ^= (T(1) << where);
102         where = prev[where];
103     }
104
105     while (best % (ver + 1))
106         best++;
107     best /= (ver + 1);
108
109     assert(oracle(cur_set, red) && oracle(cur_set, blue));
110     return pair<T, int>(cur_set, best);
111 };

```

7 Push-free segment tree

```

1 class pushfreesegtree
2 {
3     vector<modulo◇> pushed, unpushed;
4
5     modulo◇ add(int l, int r, int cl, int cr, int v, const modulo
6         ↪ ◇ &x)
7     {
8         if (r ≤ cl || cr ≤ l)
9             return 0;
10        if (l ≤ cl && cr ≤ r)
11        {
12            unpushed[v] += x;
13
14            return x * (cr - cl);
15        }
16
17        int ct = (cl + cr) / 2;
18
19        auto tmp = add(l, r, cl, ct, 2 * v, x) + add(l, r, ct, cr,
20            ↪ 2 * v + 1, x);
21
22        pushed[v] += tmp;
23
24        return tmp;
25    }
26
27    modulo◇ sum(int l, int r, int cl, int cr, int v)
28    {
29        if (r ≤ cl || cr ≤ l)
30            return 0;
31        if (l ≤ cl && cr ≤ r)

```

```

31         return pushed[v] + unpushed[v] * (cr - cl);
32
33     int ct = (cl + cr) / 2;
34
35     return sum(l, r, cl, ct, 2 * v) + unpushed[v] * (min(r, cr)
    ↪      - max(l, cl)) + sum(l, r, ct, cr, 2 * v + 1);
36 }
37
38 public:
39     pushfreesegetree(int n) : pushed(2 * up(n)), unpushed(2 * up(n))
40     {}
41
42
43     modulo◇ sum(int l, int r)
44     {
45         return sum(l, r, 0, pushed.size() / 2, 1);
46     }
47
48
49     void add(int l, int r, const modulo◇ &x)
50     {
51         add(l, r, 0, pushed.size() / 2, 1, x);
52     }
53 };

```

8 Number theory

8.1 Chinese remainder theorem without overflows

```

1 // Replace T with an appropriate type!
2 using T = long long;
3
4 // Finds x, y such that ax + by = gcd(a, b).
5 T gcdext (T a, T b, T &x, T &y)
6 {
7     if (b == 0)
8     {
9         x = 1, y = 0;
10        return a;
11    }
12
13    T res = gcdext (b, a % b, y, x);
14    y -= x * (a / b);
15    return res;
16 }
17
18 // Returns true if system x = r1 (mod m1), x = r2 (mod m2) has
    ↪ solutions
19 // false otherwise. In first case we know exactly that x = r (mod m
    ↪ )

```

```

20
21 bool crt (T r1, T m1, T r2, T m2, T &r, T &m)
22 {
23     if (m2 > m1)
24     {
25         swap(r1, r2);
26         swap(m1, m2);
27     }
28
29     T g = __gcd(m1, m2);
30     if ((r2 - r1) % g  $\neq$  0)
31         return false;
32
33     T c1, c2;
34     auto nrem = gcdext(m1 / g, m2 / g, c1, c2);
35     assert(nrem == 1);
36     assert(c1 * (m1 / g) + c2 * (m2 / g) == 1);
37     T a = c1;
38     a *= (r2 - r1) / g;
39     a %= (m2 / g);
40     m = m1 / g * m2;
41     r = a * m1 + r1;
42     r = r % m;
43     if (r < 0)
44         r += m;
45
46     assert(r % m1 == r1 && r % m2 == r2);
47     return true;
48 }

```

8.2 Integer points under a rational line

```

1 // integer (x, y) :  $0 \leq x < n$ ,  $0 < y \leq (kx + b) / d$ 
2 // (real division)
3 // In other words,  $\sum_{x=0}^{n-1} [(kx+b)/d]$ 
4 ll trapezoid (ll n, ll k, ll b, ll d)
5 {
6     if (k == 0)
7         return (b / d) * n;
8     if (k  $\geq$  d || b  $\geq$  d)
9         return (k / d) * n * (n - 1) / 2 + (b / d) * n + trapezoid(
10              $\rightarrow$  n, k % d, b % d, d);
11     return trapezoid((k * n + b) / d, d, (k * n + b) % d, k);

```

9 Suffix Automaton

```

1 struct Vx{
2     static const int AL = 26;
3     int len, suf;
4     int next[AL];

```



```

5     Vx(){}
6     Vx(int l, int s):len(l), suf(s){}
7 };
8
9 struct SA{
10     static const int MAX_LEN = 1e5 + 100, MAX_V = 2 * MAX_LEN;
11     int last, vcnt;
12     Vx v[MAX_V];
13
14     SA(){
15         vcnt = 1;
16         last = newV(0, 0); // root = vertex with number 1
17     }
18     int newV(int len, int suf){
19         v[vcnt] = Vx(len, suf);
20         return vcnt++;
21     }
22
23     int add(char ch){
24         int p = last, c = ch - 'a';
25         last = newV(v[last].len + 1, 0);
26         while(p && !v[p].next[c]) //added p &&
27             v[p].next[c] = last, p = v[p].suf;
28         if(!p)
29             v[last].suf = 1;
30         else{
31             int q = v[p].next[c];
32             if (v[q].len == v[p].len + 1)
33                 v[last].suf = q;
34             else{
35                 int r = newV(v[p].len + 1, v[q].suf);
36                 v[last].suf = v[q].suf = r;
37                 memcpy(v[r].next, v[q].next, sizeof(v[r].next));
38                 while(p && v[p].next[c] == q)
39                     v[p].next[c] = r, p = v[p].suf;
40             }
41         }
42         return last;
43     }
44 };

```

10 Palindromic Tree

```

1 auto pal_tree(const string &str)
2 {
3     const int rooteven = str.size();
4     const int rootodd = str.size() + 1;
5
6     vector<int> siz(str.size() + 2, 1), link(str.size() + 2,
    ↪ rooteven), type(str.size());

```

```

7
8     vector<vector<int>> mem(str.size() + 2, vector<int>(26, -1));
9     siz[rooteven] = 0;
10    siz[rootodd] = -1;
11    link[rooteven] = rootodd;
12    link[rootodd] = rootodd;
13
14    for (int i = 0; i < str.size(); i++)
15    {
16        auto it = str[i] - 'a';
17
18        auto get = [&](int p) {
19            while (!(i - siz[p] - 1 ≥ 0 && str[i - siz[p] - 1] =
20                ↪ str[i]))
21                p = link[p];
22
23            return p;
24        };
25
26        auto prev = get(i == 0 ? rootodd : type[i - 1]);
27
28        if (mem[prev][it] == -1)
29        {
30            mem[prev][it] = i;
31            siz[i] = siz[prev] + 2;
32
33            auto l = get(link[prev]);
34
35            link[i] = mem[l][it];
36            if (link[i] == i)
37                link[i] = rooteven;
38
39            type[i] = mem[prev][it];
40        }
41
42    return make_tuple(type, link, siz, mem);
43 }

```

11 Smth added at last moment

11.1 Dominator Tree

```

1 struct dom_tree {
2     vvi g, rg, tree, bucket;
3     vi sdom, par, dom, dsu, label, in, order, tin, tout;
4     int T = 0, root = 0, n = 0;
5
6     void dfs_tm (int x) {
7         in[x] = T;

```

```

8     order[T] = x;
9     label[T] = T, sdom[T] = T, dsu[T] = T, dom[T] = T;
10    T++;
11    for (int to : g[x]) {
12        if (in[to] == -1) {
13            dfs_tm(to);
14            par[in[to]] = in[x];
15        }
16        rg[in[to]].pb(in[x]);
17    }
18 }
19
20 void dfs_tree (int v, int p) {
21     tin[v] = T++;
22     for (int dest : tree[v]) {
23         if (dest != p) {
24             dfs_tree(dest, v);
25         }
26     }
27     tout[v] = T;
28 }
29
30 dom_tree (const vvi &g_, int root_) {
31     g = g_;
32     n = sz(g);
33     assert(0 ≤ root && root < n);
34     in.assign(n, -1);
35     rg.resize(n);
36     order = sdom = par = dom = dsu = label = vi(n);
37     root = root_;
38     bucket.resize(n);
39     tree.resize(n);
40
41     dfs_tm(root);
42
43     for (int i = n - 1; i ≥ 0; i--) {
44         for (int j : rg[i])
45             sdom[i] = min(sdom[i], sdom[find(j)]);
46         if (i > 0)
47             bucket[sdom[i]].pb(i);
48
49         for (int w : bucket[i]) {
50             int v = find(w);
51             dom[w] = (sdom[v] == sdom[w] ? sdom[w] : v);
52         }
53
54         if (i > 0)
55             unite(par[i], i);
56     }
57 }

```

```

58     for (int i = 1; i < n; i++) {
59         if (dom[i] ≠ sdom[i])
60             dom[i] = dom[dom[i]];
61         tree[order[i]].pb(order[dom[i]]);
62         tree[order[dom[i]]].pb(order[i]);
63     }
64
65     T = 0;
66     tin = tout = vi(n);
67     dfs_tree(root, -1);
68 }
69
70 void unite (int u, int v) {
71     dsu[v] = u;
72 }
73
74 int find (int u, int x = 0) {
75     if (u = dsu[u])
76         return (x ? -1 : u);
77     int v = find(dsu[u], x + 1);
78     if (v = -1)
79         return u;
80     if (sdom[label[dsu[u]]] < sdom[label[u]])
81         label[u] = label[dsu[u]];
82     dsu[u] = v;
83     return (x ? v : label[u]);
84 }
85
86 bool dominated_by (int v, int by_what) {
87     return tin[by_what] ≤ tin[v] && tout[v] ≤ tout[by_what];
88 }
89 };

```

11.2 Suffix Array

```

1  namespace suff_arr {
2
3  const int MAXN = 2e5 + 10;
4
5  string s;
6  int n;
7  int p[MAXN];
8  int lcp[MAXN];
9  int pos[MAXN];
10 int c[MAXN];
11
12 void print() {
13     #ifndef LOCAL
14         return;
15     #endif
16     eprintf("p:\n");

```

```
17     forn(i, sz(s)) {
18         eprintf("i=%d -- %d: %s, lcp=%d, c=%d\n", i, p[i], s.substr
           ↪ (p[i], sz(s) - p[i]).data(), lcp[i], c[p[i]]);
19     }
20     eprintf("\n");
21 }
22
23 void build(const string& s_) {
24     static int cnt[MAXN];
25     static int np[MAXN];
26     static int nc[MAXN];
27
28     s = s_;
29     n = sz(s);
30
31     memset (cnt, 0, sizeof cnt);
32     for (char ch : s) {
33         ++cnt[int(ch)];
34     }
35     forn(i, 256) {
36         cnt[i + 1] += cnt[i];
37     }
38     forn(i, sz(s)) {
39         p[--cnt[int(s[i])]] = i;
40     }
41
42     int cls = 1;
43     c[p[0]] = cls - 1;
44     for (int i = 1; i < n; ++i) {
45         if (s[p[i]] != s[p[i - 1]]) {
46             ++cls;
47         }
48         c[p[i]] = cls - 1;
49     }
50
51     for (int len = 1; len ≤ n; len *= 2) {
52         memset (cnt, 0, sizeof(int) * cls);
53         forn(i, n) {
54             ++cnt[c[i]];
55         }
56         forn(i, cls - 1) {
57             cnt[i + 1] += cnt[i];
58         }
59         ford(i, n) {
60             const int j = p[i];
61             int j2 = (j - len + n) % n;
62             np[--cnt[c[j2]]] = j2;
63         }
64         memcpy(p, np, sizeof(int) * n);
65     }
```

```

66     cls = 1;
67     nc[p[0]] = cls - 1;
68     for (int i = 1; i < n; ++i) {
69         if (c[p[i]] ≠ c[p[i - 1]] || c[(p[i] + len) % n] ≠ c
            ↪ [(p[i - 1] + len) % n]) {
70             ++cls;
71         }
72         nc[p[i]] = cls - 1;
73     }
74     memcpy(c, nc, sizeof(int) * n);
75 }
76
77 forn(i, n) {
78     pos[p[i]] = i;
79 }
80
81 int pref = 0;
82 forn(i, n) {
83     int pi = pos[i];
84     if (pi == n - 1) {
85         continue;
86     }
87     int j = p[pi + 1];
88     while (i + pref < n && j + pref < n && s[i + pref] = s[j +
        ↪ pref]) {
89         ++pref;
90     }
91     lcp[pi] = pref;
92     pref = max(0, pref - 1);
93 }
94
95 //     print();
96 }
97
98 };

```

11.3 Fast LCS

```

1 // assumes that strings consist of lowercase latin letters
2 const int M = ((int)1e5 + 64) / 32 * 32;
3 // maximum value of m
4 using bs = bitset<M>;
5 using uint = unsigned int;
6 const ll bnd = (1LL << 32);
7
8 // WARNING: invokes undefined behaviour of modifying ans through
    ↪ pointer to another data type (uint)
9 // seems to work, but be wary
10 bs sum (const bs &bl, const bs &br)
11 {
12     const int steps = M / 32;

```

```
13     const uint* l = (uint*)&bl;
14     const uint* r = (uint*)&br;
15
16     bs ans;
17     uint* res = (uint*)&ans;
18
19     int carry = 0;
20     for (i, steps)
21     {
22         ll cur = ll(*l++) + ll(*r++) + carry;
23         carry = (cur ≥ bnd);
24         cur = (cur ≥ bnd ? cur - bnd : cur);
25         *res++ = uint(cur);
26     }
27
28     return ans;
29 }
30
31 int fast_lcs (const string &s, const string &t)
32 {
33     const int m = sz(t);
34     const int let = 26;
35
36     vector<bs> has(let);
37     vector<bs> rev = has;
38
39     for (i, m)
40     {
41         const int pos = t[i] - 'a';
42         has[pos].set(i);
43         for (j, let) if (j ≠ pos)
44             rev[j].set(i);
45     }
46
47     bs row;
48     for (i, m)
49         row.set(i);
50
51     int cnt = 0;
52     for (char ch : s)
53     {
54         const int pos = ch - 'a';
55
56         bs next = sum(row, row & has[pos]) | (row & rev[pos]);
57         cnt += next[m];
58         next[m] = 0;
59
60         row = next;
61     }
62 }
```

```

63     return cnt;
64 }

```

11.4 Fast Subset Convolution

```

1 // algorithm itself starts here
2 void mobius (int* a, int n, int sign)
3 {
4     for (i, n)
5     {
6         int free = ((1 << n) - 1) ^ (1 << i);
7         for (int mask = free; mask > 0; mask = ((mask - 1) & free))
8             (sign == +1 ? add : sub)(a[mask ^ (1 << i)], a[mask]);
9         add(a[1 << i], a[0]);
10    }
11 }
12
13 // maximum number of bits allowed
14 const int B = 20;
15
16 vi fast_conv (vi a, vi b)
17 {
18     assert(!a.empty());
19     const int bits = __builtin_ctz(sz(a));
20     assert(sz(a) == (1 << bits) && sz(a) == sz(b));
21
22     static int trans_a[B + 1][1 << B];
23     static int trans_b[B + 1][1 << B];
24     static int trans_res[B + 1][1 << B];
25
26     for (cnt, bits + 1)
27     {
28         for (auto cur : {trans_a, trans_b, trans_res})
29             fill(cur[cnt], cur[cnt] + (1 << bits), 0);
30     }
31
32     for (mask, 1 << bits)
33     {
34         const int cnt = __builtin_popcount(mask);
35         trans_a[cnt][mask] = a[mask];
36         trans_b[cnt][mask] = b[mask];
37     }
38
39     for (cnt, bits + 1)
40     {
41         mobius(trans_a[cnt], bits, +1);
42         mobius(trans_b[cnt], bits, +1);
43     }
44
45     // Not really a valid ranked mobius transform! But algorithm
    ↪ works anyway

```



```

46
47     forn (i, bits + 1) forn (j, bits - i + 1) forn (mask, 1 << bits
    ↪ )
48         add(trans_res[i + j][mask], mult(trans_a[i][mask], trans_b[
    ↪ j][mask]));
49
50     forn (cnt, bits + 1)
51         mobius(trans_res[cnt], bits, -1);
52
53     forn (mask, 1 << bits)
54     {
55         const int cnt = __builtin_popcount(mask);
56         a[mask] = trans_res[cnt][mask];
57     }
58
59     return a;
60 }

```

12 Karatsuba

```

1 // functon Karatsuba (and stupid as well) computes c += a * b, not
    ↪ c = a * b
2
3 using hvect = vector<modulo◇>::iterator;
4 using hcvect = vector<modulo◇>::const_iterator;
5
6
7 void add(hvect abegin, hcvect aend, hvect ans)
8 {
9     for (auto it = abegin; it ≠ aend; ++it, ++ans)
10         *ans += *it;
11 }
12
13
14 void sub(hvect abegin, hcvect aend, hvect ans)
15 {
16     for (auto it = abegin; it ≠ aend; ++it, ++ans)
17         *ans -= *it;
18 }
19
20
21 void stupid(int siz, hcvect abegin, hcvect bbegin, hvect ans)
22 {
23     for (auto a = abegin; a ≠ abegin + siz; ++a, ans -= (siz - 1))
24         for (auto b = bbegin; b ≠ bbegin + siz; ++b, ++ans)
25             *ans += *a * *b;
26 }
27
28
29 void Karatsuba(size_t siz, hcvect abegin, hcvect bbegin, hvect ans,

```

```

    ↪ hvect small, hvect big, hvect sum)
30 {
31     assert((siz & (siz - 1)) == 0);
32
33     if (siz ≤ 32)
34     {
35         stupid(siz, abegin, bbegin, ans);
36
37         return;
38     }
39
40     auto amid = abegin + siz / 2, aend = abegin + siz;
41     auto bmid = bbegin + siz / 2, bend = bbegin + siz;
42     auto smid = sum + siz / 2, send = sum + siz;
43
44     fill(small, small + siz, 0);
45     Karatsuba(siz / 2, abegin, bbegin, small, small + siz, big +
    ↪ siz, sum);
46     fill(big, big + siz, 0);
47     Karatsuba(siz / 2, amid, bmid, big, small + siz, big + siz, sum
    ↪ );
48
49     copy(abegin, amid, sum);
50     add(amid, aend, sum);
51     copy(bbegin, bmid, sum + siz / 2);
52     add(bmid, bend, sum + siz / 2);
53
54     Karatsuba(siz / 2, sum, smid, ans + siz / 2, small + siz, big +
    ↪ siz, send);
55
56     add(small, small + siz, ans);
57     sub(small, small + siz, ans + siz / 2);
58     add(big, big + siz, ans + siz);
59     sub(big, big + siz, ans + siz / 2);
60 }
61
62
63 void mult(vector<modulo◇> a, vector<modulo◇> b, vector<modulo◇>
    ↪ &c)
64 {
65     a.resize(up(max(a.size(), b.size())), 0);
66     b.resize(a.size(), 0);
67
68     c.resize(max(c.size(), a.size() * 2), 0);
69
70     vector<modulo◇> small(2 * a.size());
71     auto big = small;
72     auto sum = small;
73
74     Karatsuba(a.size(), a.begin(), b.begin(), c.begin(), small.

```

```
75     ↪ begin(), big.begin(), sum.begin());  
    }
```











