Team Reference

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1 Setup & Scripts

1.1 CMake

```
cmake_minimum_required(VERSION 3.14)
project(olymp)

set(CMAKE_CXX_STANDARD 17)
add_compile_definitions(LOCAL)
#set(CMAKE_CXX_FLAGS "${CMAKE_CXX_FLAGS} -fsanitize=undefined -fno-
   ↪ sanitize-recover")
#sanitizers: address, leak, thread, undefined, memory

add_executable(olymp f.cpp)
```

1.2 wipe.sh

```
touch {a..l}.cpp

for file in ?.cpp ; do
    cat template.cpp > $file ;
done
```

2 Bugs

- powmod :)
- Всегда чекать Куна дважды, особенно на количество итераций
- uniform_int_distribution от одного параметра
- for (char c : "NEWS")
- Порядок верхних и нижних границ в случае, когда задача двумерна $t - b \neq b - t$
- static с мультитестами

3 Geometry

3.1 Пересечение прямых

$$AB = A - B; CD = C - D$$

$$(A \times B \cdot CD.x - C \times D \cdot AB.x : A \times B \cdot CD.y - C \times D \cdot AB.y : AB \times CD)$$

4 Numbers

- A lot of divisors
  - $\leq 20 : d(12) = 6$
  - $\leq 50 : d(48) = 10$
  - $\leq 100 : d(60) = 12$
  - $\leq 1000 : d(840) = 32$
  - $\leq 10^4 : d(9240) = 64$
  - $\leq 10^5 : d(83160) = 128$
  - $\leq 10^6 : d(720720) = 240$
  - $\leq 10^7 : d(8648640) = 448$
  - $\leq 10^8 : d(91891800) = 768$
  - $\leq 10^9 : d(931170240) = 1344$
  - $\leq 10^{11} : d(97772875200) = 4032$
  - $\leq 10^{12} : d(963761198400) = 6720$
  - $\leq 10^{15} : d(866421317361600) = 26880$
  - $\leq 10^{18} : d(897612484786617600) = 103680$
- Numeric integration
  - simple: $F(0)$
  - simpson: $\frac{F(-1)+4 \cdot F(0)+F(1)}{6}$
  - runge2: $\frac{F(-\sqrt{\frac{1}{3}})+F(\sqrt{\frac{1}{3}})}{2}$
  - runge3: $\frac{F(-\sqrt{\frac{3}{5}}) \cdot 5+F(0) \cdot 8+F(\sqrt{\frac{3}{5}}) \cdot 5}{18}$

5 Graphs

5.1 Weighted matroid intersection

```
// here we use T = __int128 to store the independent set
// calling expand k times to an empty set finds the maximum
// cost of the set with size exactly k,
// that is independent in blue and red matroids
// ver is the number of the elements in the matroid,
// e[i].w is the cost of the i-th element
```

```
// first return value is new independent set
// second return value is difference between
// new and old costs
// oracle(set, red) and oracle(set, blue) check whether
// or not the set lies in red or blue matroid respectively

auto expand = [&] (T cur_set) → pair<T, int>
{
    vector<int> in(ver);
    for (int i = 0; i < ver; i++)
        in[i] = ((cur_set >> i) & 1);

    const int red = 1;
    const int blue = 2;

    vector<vector<int>> g(ver);
    for (int i = 0; i < ver; i++)
    for (int j = 0; j < ver; j++)
    {
        T swp_mask = (cur_set ^ (T(1) << i) ^ (T(1) << j));
        if (!in[i] && in[j])
        {
            if (oracle(swp_mask, red))
                g[i].push_back(j);
            if (oracle(swp_mask, blue))
                g[j].push_back(i);
        }
    }

    vector<int> from, to;
    for (int i = 0; i < ver; i++) if (!in[i])
    {
        T add_mask = cur_set ^ (T(1) << i);
        if (oracle(add_mask, blue))
            from.push_back(i);
        if (oracle(add_mask, red))
            to.push_back(i);
    }

    auto get_cost = [&] (int x)
    {
        const int cost = (!in[x] ? e[x].w : -e[x].w);
        return (ver + 1) * cost - 1;
    };

    const int inf = int(1e9);
    vector<int> dist(ver, -inf), prev(ver, -1);
    for (int x : from)
        dist[x] = get_cost(x);
```

```
queue<int> q;

vector<int> used(ver);
for (int x : from)
{
    q.push(x);
    used[x] = 1;
}

while (!q.empty())
{
    int cur = q.front(); used[cur] = 0; q.pop();

    for (int to : g[cur])
    {
        int cost = get_cost(to);
        if (dist[to] < dist[cur] + cost)
        {
            dist[to] = dist[cur] + cost;
            prev[to] = cur;
            if (!used[to])
            {
                used[to] = 1;
                q.push(to);
            }
        }
    }
}

int best = -inf, where = -1;
for (int x : to)
{
    if (dist[x] > best)
    {
        best = dist[x];
        where = x;
    }
}

if (best == -inf)
    return pair<T, int>(cur_set, best);

while (where != -1)
{
    cur_set ^= (T(1) << where);
    where = prev[where];
}

while (best % (ver + 1))
    best++;
```

```
    best /= (ver + 1);

    assert(oracle(cur_set, red) && oracle(cur_set, blue));
    return pair<T, int>(cur_set, best);
};
```

6 Push-free segment tree

```
class pushfreesegtree
{
    vector<modulo<>> pushed, unpushed;

    modulo<> add(int l, int r, int cl, int cr, int v, const modulo
       ↪ <> &x)
    {
        if (r <= cl || cr <= l)
            return 0;
        if (l <= cl && cr <= r)
        {
            unpushed[v] += x;

            return x * (cr - cl);
        }

        int ct = (cl + cr) / 2;

        auto tmp = add(l, r, cl, ct, 2 * v, x) + add(l, r, ct, cr,
           ↪ 2 * v + 1, x);

        pushed[v] += tmp;

        return tmp;
    }


    modulo<> sum(int l, int r, int cl, int cr, int v)
    {
        if (r <= cl || cr <= l)
            return 0;
        if (l <= cl && cr <= r)
            return pushed[v] + unpushed[v] * (cr - cl);

        int ct = (cl + cr) / 2;

        return sum(l, r, cl, ct, 2 * v) + unpushed[v] * (min(r, cr)
           ↪  - max(l, cl)) + sum(l, r, ct, cr, 2 * v + 1);
    }

public:
    pushfreesegtree(int n) : pushed(2 * up(n)), unpushed(2 * up(n))
```

```
    {}


    modulo<> sum(int l, int r)
    {
        return sum(l, r, 0, pushed.size() / 2, 1);
    }


    void add(int l, int r, const modulo<> &x)
    {
        add(l, r, 0, pushed.size() / 2, 1, x);
    }
};
```

7 Number theory

7.1 Chinese remainder theorem without overflows

```
// Replace T with an appropriate type!
using T = long long;

// Finds x, y such that ax + by = gcd(a, b).
T gcdext (T a, T b, T &x, T &y)
{
    if (b == 0)
    {
        x = 1, y = 0;
        return a;
    }

    T res = gcdext (b, a % b, y, x);
    y -= x * (a / b);
    return res;
}

// Returns true if system x = r1 (mod m1), x = r2 (mod m2) has
   ↪ solutions
// false otherwise. In first case we know exactly that x = r (mod m
   ↪ )

bool crt (T r1, T m1, T r2, T m2, T &r, T &m)
{
    if (m2 > m1)
    {
        swap(r1, r2);
        swap(m1, m2);
    }

    T g = __gcd(m1, m2);
```

```
    if ((r2 - r1) % g ≠ 0)
        return false;

    T c1, c2;
    auto nrem = gcdext(m1 / g, m2 / g, c1, c2);
    assert(nrem == 1);
    assert(c1 * (m1 / g) + c2 * (m2 / g) == 1);
    T a = c1;
    a *= (r2 - r1) / g;
    a %= (m2 / g);
    m = m1 / g * m2;
    r = a * m1 + r1;
    r = r % m;
    if (r < 0)
        r += m;

    assert(r % m1 == r1 && r % m2 == r2);
    return true;
}
```

7.2 Integer points under a rational line

```
// integer (x, y) : 0 ⩽ x < n, 0 < y ⩽ (kx + b) / d
// (real division)
// In other words, sum_{x=0}^{n-1} [(kx+b)/d]
ll trapezoid (ll n, ll k, ll b, ll d)
{
    if (k == 0)
        return (b / d) * n;
    if (k ⩾ d || b ⩾ d)
        return (k / d) * n * (n - 1) / 2 + (b / d) * n + trapezoid(
            ↪ n, k % d, b % d, d);
    return trapezoid((k * n + b) / d, d, (k * n + b) % d, k);
}
```