1

$$J_k = \int \frac{1}{(y^2+\alpha^2)^k} = \begin{bmatrix} g(y) = \frac{1}{(y^2+\alpha^2)^k} \\ f(y) = 1 \\ F(y) = y \\ g^{'}(y) = \frac{-2ky}{(y^2+\alpha^2)^{k+1}} \end{bmatrix} = \frac{y}{(y^2+\alpha^2)^k} + 2k \cdot \int \frac{y^2 \pm a^2}{(y^2+\alpha^2)^{k+1}} =$$

$$= \frac{y}{(y^2+\alpha^2)^k} + 2k \cdot J_k - 2k\alpha^2 J_{k+1}$$

$$J_{k+1} = \frac{1}{2k\alpha^2}\left(\frac{y}{(y^2+\alpha^2)^k} + (2n-1)\cdot J_n\right)$$

2)

a)

$$\int \frac{x^4}{(x+1)^3} = \int x - 3 + \frac{6}{x+1} + \frac{-4}{(x+1)^2} + \frac{1}{(x+1)^3} = \frac{x^2}{2} - 3x + 6J_1 - 4J_2 + J_3$$

$$J_3 = \frac{1}{4}\cdot\frac{x}{(x+1)^2} + \frac{3}{4}\cdot J_2$$

$$J_2 = \frac{1}{2}\cdot\frac{x}{x+1} + \frac{1}{2}J_1$$

$$J_1 = \ln\|x+1\|$$

$$\int \frac{x^4}{(x+1)^3} = -\frac{x^4}{2(x+1)^2} + x^2 - 4x + 6\ln\|x+1\| + \frac{2}{x+1} + C$$

b)

$$\int \frac{1}{x^4+1}$$

$$x^4+1 = (x-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i)(x-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i)(x+\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i)(x+\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i) = (x^2+\sqrt{2}x+1)(x^2-\sqrt{2}x+1)$$

$$\frac{Ax+C}{(x^2+\sqrt{2}x+1)} + \frac{Bx+D}{(x^2-\sqrt{2}x+1)} = \frac{1}{(x^2+\sqrt{2}x+1)(x^2-\sqrt{2}x+1)}$$

$$\begin{cases} 1 = D - C \\ 0 = -A - \sqrt{2}C + B - \sqrt{2}B \\ 0 = -\sqrt{2}A - C + \sqrt{2}B - D \\ 0 = B - A \end{cases} \rightarrow \begin{cases} A = \frac{1}{2\sqrt{2}} \\ C = -\frac{1}{2} \\ B = \frac{1}{2\sqrt{2}} \\ D = \frac{1}{2} \end{cases}$$

$$\int \frac{1}{x^4+1} = \int -\frac{\frac{x}{2\sqrt{2}} - \frac{1}{2}}{x^2-\sqrt{2}x+1} + \frac{\frac{x}{2\sqrt{2}} + \frac{1}{2}}{x^2+\sqrt{2}x+1} =$$

$$\frac{1}{4\sqrt{2}}\left(-\ln(x^2-\sqrt{2}x+1) + \ln(x^2+\sqrt{2}x+1) - 2\,\mathrm{arctg}(1-\sqrt{2}x) + \mathrm{arctg}(\sqrt{2}x+1)\right) + C$$

d)

$$\int \frac{}{3\sin x + 4\cos x + 10} = \begin{bmatrix} t = \operatorname{tg}\frac{x}{2} \\ \sin x = \frac{2t}{1+t^2} \\ \cos x = \frac{1-t^2}{1+t^2} \\ = \frac{2}{1+t^2} \end{bmatrix} = \int \frac{}{\frac{6t}{1+t^2} + \frac{4-4t^2}{1+t^2} + 10} \frac{2}{1+t^2} =$$

$$\int \frac{1}{3t^2+3t+7} = \frac{1}{\sqrt{19/4}} \cdot \operatorname{arctg}\frac{t+3/2}{\sqrt{19/4}} + C$$

e)

$$\int \frac{1-\sqrt{x^2+x+1}}{x\sqrt{x^2+x+1}} \begin{bmatrix} \sqrt{x^2+x+1} = xt+1 \\ = \frac{2t^2-2t+2}{(1-t^2)^2} \\ x = \frac{2t-1}{1-t^2} \end{bmatrix} = -2\int \frac{t}{2t-1} \cdot \frac{t^2-t+1}{1-t^2} = 2\int \frac{t(t^2-t+1)}{(2t-1)(t^2-1)^2} =$$

$$\int \frac{t}{(2t-1)(t^2-1)} = \int -\frac{1}{6t+6} - \frac{2}{3(t-1)} + \frac{1}{2(t+1)} = -\frac{1}{6}\ln(|t+1|) - \frac{2}{3}\ln|t-1| + \frac{1}{2}\ln|t+1| + C$$