$$\sum_{d|N} \varphi(d)\tau(n/d) = \sigma(n)$$

$\sum_{d|p^k} \varphi(d)\tau(p^k/d) = \varphi(1)\tau(p^k) + \varphi(p)\tau(p^{k-1}) + \ldots + \varphi(p^k)\tau(1) = \varphi(1)(k+1) + \varphi(p)k + \ldots + \varphi(p^k)(k+1) = (k+1) + (p-1)k + \ldots + (p^k - p^{k-1}) = k + k(p-1) + \ldots + k(p^k - p^{k-1}) + (k+1) = k(1 + (p-1) + \ldots + (p^k - p^{k-1})) + (k+1) = k(p^k - 1) + (k+1) = kp^k - k + k + 1 = p^{k+1} - 1 = \sigma(p^k$