# 1

## a)

$$f = \operatorname{arctg}\frac{u}{v}$$
$$df = d(\arctan\frac{u}{v}) = \frac{1}{1+\frac{u}{v}^2}\cdot\frac{du\cdot v - u\cdot dv}{v^2}$$

## b)

$$f = \frac{1}{u^2+v^2}$$
$$df = d(\frac{1}{\sqrt{u^2+v^2}}) = -\frac{d(v^2+u^2)}{2(u^2+v^2)\sqrt{u^2+v^2}}$$

# 2

## a)

$$\cos{(ax+b)}^{(n)} = a^n\cos{(ax+b+\frac{\pi}{2}n)}$$

$n=1$: $-a\sin(ax+b) = \cos(ax+b+\frac{\pi}{2})$
$\sqsupset$ $P_n-$ верно, тогда верно и $P_{n+1}$
$P_{n+1}:(\cos{ax+b})^{n+1} = \Big(\cos{(ax+b)^{(n)}}\Big)' = \big(a^n\cos{(ax+b+\frac{\pi}{2}n)}\big)' = -a^{n+1}\sin\big(ax+b+\frac{\pi}{2}n\big) = a^{n+1}\cos\big(ax+b+\frac{\pi}{2}(n+1)\big)$

## b)

$$(\ln(ax+b))^{(n)} = \frac{(-1)^{n+1}(n-1)!a^n}{(ax+b)^n}$$

$n=1:(\ln(ax+b))' = \frac{a}{ax+b}$
$\sqsupset$ $P_n$верно, тогда верно и $P_{n+1}$ $(\ln(ax+b))^{n+1} = ((\ln{ax+b})^n)' = (\frac{(-1)^{n-1}(n-1)!a^n}{(ax+b)^n})' = \frac{-(-1)^{n-1}(n-1)!a^n\cdot a\cdot n\cdot(ax+b)^n-1}{(ax+b)^2n} = \frac{(-1)^n\cdot n!\cdot a^{n+1}}{(ax+b)^{n+1}}$

# 3

## a)

$$\frac{x-13}{x^2-x-6} = \frac{3}{x+2}+\frac{-2}{x-3}$$

$(\frac{3}{x+2}+\frac{-2}{x-3})^{(n)} = (\frac{3}{x+2})^{(n)}+(\frac{-2}{x-3})^{(n)} = \frac{3\cdot(-1)^n\cdot n!}{(x+2)^n} - \frac{2\cdot(-1)^n\cdot n!\cdot 2}{(x-3)^n}$

## b)

$$f(x) = (x^2+x+1)e^{-3x}$$

### c)

$$\cos^2(5x) = \frac{1+\cos 10x}{2}$$

$$\sin(3x)\cdot(\frac{1+\cos 10x}{2}) = \frac{\sin 3x + \cos(10x)\cdot\sin(3x)}{2} = \frac{\sin 3x + \sin(13x)/2 - \sin(7x)/2}{2}$$

$$f^{(n)} = \frac{3^n\cdot\sin\left(3x+\frac{\pi n}{2}\right)}{2} + \frac{13^n\cdot\sin\left(13x+\frac{\pi n}{2}\right)}{4} - \frac{7^n\cdot\sin\left(7x+\frac{\pi n}{2}\right)}{4}$$

## 4

### a)

$$\lim_{x\to 0}\frac{\arcsin 2x - 2\arcsin(x)}{x^3} = \frac{\frac{2\sqrt{1-x^2}-2\sqrt{1-4x^2}}{3x^2\sqrt{1-5x^2+4x^4}}}{}$$

=

$$\lim_{x\to 0}\frac{-2x\sqrt{1-4x^2}+8\sqrt{1-x^2}}{\sqrt{(1-x^2)(1-4x^2)\sqrt{4x^4-5x^2+1}}\cdot(48x^5\cdot 45x^4+6)} = 1$$