Computational Project
due Monday May 4th 2020 at 10:00 am

Two-nucleon scattering by a central potential

Consider two nucleons each of mass $m = 938$ MeV$/c^2$ interacting via a central potential given by

$$V(r) = \begin{cases} -V_0 & \text{for } r \leq R \\ 0 & \text{otherwise} \end{cases}$$

where V_0 is positive and $R = 1.45$ fm.

1. Set $V_0 = 20$ MeV and determine the S, P, and D-wave phase shifts as function of the energy E in the range of $0 - 200$ MeV, and plot them.

2. Plot the radial wave functions, $u_\ell(r)$, for few representative energies, *e.g.*, $E = 1$, 10, 100, and 200 MeV.

3. Evaluate the total cross sections at these energies.

4. Plot the total and partial cross sections as function of the energy.

5. The S-wave phase shift can be calculated analytically. Use the analytical solution of the phase shift and the cross section in S-wave to validate your code.

6. Repeat the steps above with $V_0 = 60$ MeV. Note that there is an ambiguity in the definition of the phase shifts. This will make your phase shift in S-wave discontinuous when approaching $\pi/2$. Make your phase shift continuous by adding the appropriate phase.

7. Write a report describing your code and its validation against the analytical solutions, and your findings. If you have time, you may take this opportunity to practicing with Latex. If you are working in pairs and writing the report together you may find overleaf useful.

If you are working with a colleague please try to each contribute to the final product. If this is the first time you are coding, take this opportunity to get exposed to and play with coding, and contribute to the project by, *e.g.*, working out the analytical solutions, writing the project report, working on the figures and so on. In practice, work *together* (each at their respective homes).

Implementation

We seek the solution, $u(r) = r\,R(r)$, of the radial equation

$$-\frac{\hbar^2}{2\,\mu} u''(r) + V(r)\,u(r) + \frac{\ell(\ell+1)}{r^2} = E\,u(r)\,, \tag{1}$$

Figure 1: Your figure here.

where

$$E = \frac{\hbar^2\, k^2}{2\,\mu}\,, \quad \text{and} \qquad \mu = \frac{m}{2}\,. \tag{2}$$

We define

$$v(r) = \frac{2\,\mu}{\hbar^2}\, V(r)\,, \tag{3}$$

and rewrite the equation above as

$$u''(r) + \left[k^2 - v(r)\, u(r) - \frac{\ell(\ell+1)}{r^2}\right]\, u(r) = 0\,,$$

or

$$u''(r) + K(r) u(r) = 0\,, \tag{4}$$

with

$$K(r) = \frac{2\,\mu}{\hbar^2}\,[E - V(r)] - \frac{\ell(\ell+1)}{r^2}\,. \tag{5}$$

As $r \to 0$, $u(r) \to r^{(\ell+1)}$. We use two grid points close to zero (*e.g.*, $r_1 = h$ and $r_2 = 2 \times h$, where h is the step in r), and the solution calculated in these points, that is $u_1 = u(r_1) = h^{\ell+1}$ and $u_2 = u(r_2) = (2 \times h)^{\ell+1}$, to start building the solution outwards using the Numerov's method.

The subsequent u_i's calculated at r_i's are obtained using the following algorithm

$$u_{i+1}\left(1 + \frac{h^2}{12} K_{i+1}\right) - u_i\left(2 - \frac{5\,h^2}{6} K_i\right) + u_{i-1}\left(1 + \frac{h^2}{12} K_{i-1}\right) + O(h^6) = 0\,.$$

TO BE CONTINUED...

You will need the physical constant $\hbar\, c = 197.326980$ MeV fm.