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Lec # 3

Number System and Binary Arithmetic

- When we type some letters or words, the computer translates them in numbers as computers can understand only numbers. A computer can understand the positional number system where there are only a few symbols called digits and these symbols represent different values depending on the position they occupy in the number.
- The value of each digit in a number can be determined using −the digit
- The position of the digit in the number

The base of the number system (where the base is defined as the total number of digits available in the number system

Decimal Number System

- The number system that we use in our day-to-day life is the decimal number system.
- Decimal number system has base 10 as it uses 10 digits from 0 to 9.
- In decimal number system, the successive positions to the left of the decimal point represent units, tens, hundreds, thousands, and so on.
- Each position represents a specific power of the base (10). For example, the decimal number 1234 consists of the digit 4 in the units position, 3 in the tens position, 2 in the hundreds position, and 1 in the thousands position. Its value can be written as
- **(1 x 1000)+ (2 x 100)+ (3 x 10)+ (4 x l)**
- **(1 x 10^3)+ (2 x 10^2)+ (3 x 10^1)+ (4 x l00)**
- **1000 + 200 + 30 + 4**
- **1234**

Types of Number System

As a computer programmer or an IT professional, you should understand the following number systems which are frequently used in computers.

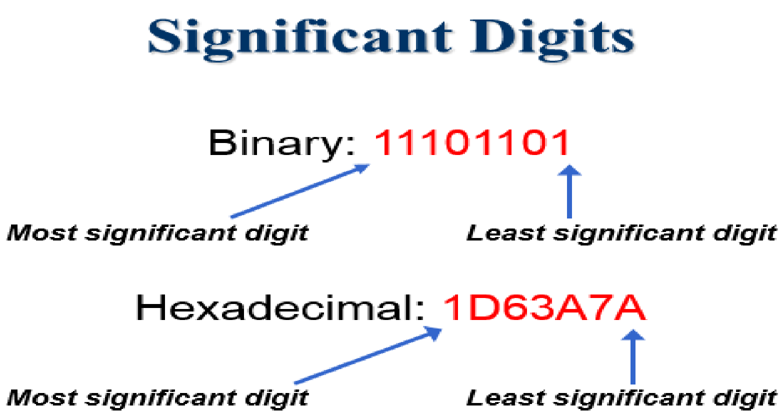
S.No.	Number System and Description
1	**Binary Number System** Base 2. Digits used : 0, 1
2	**Octal Number System** Base 8. Digits used : 0 to 7
3	**Hexa Decimal Number System** Base 16. Digits used: 0 to 9, Letters used : A- F

- ✓ ***We are all familiar with the decimal number system (Base 10). Some other number systems that we will work with are***:
  - **Binary → Base 2**
  - **Octal → Base 8**
  - **Hexadecimal → Base 16**

Characteristics of Numbering Systems

Important Key points:

- The digits are consecutive.
- The number of digits is equal to the size of the base.
- Zero is always the first digit.
- The base number is never a digit.
- When 1 is added to the largest digit, a sum of zero and a carry of one results.
- Numeric values are determined by the implicit positional values of the digits



1).Binary Number System

- Also called the **"Base 2 system"**
- The binary number system is used to model the series of electrical signals computers use to represent information
- 0 represents the no voltage or an **off state**
- 1 represents the presence of voltage or an **on state**

Binary Numbering Scale

Base 2 Number	Base 10 Equivalent	Power	Positional Value
000	0	2^0	1
001	1	2^1	2
010	2	2^2	4
011	3	2^3	8
100	4	2^4	16
101	5	2^5	32
110	6	2^6	64
111	7	2^7	128

Decimal to Binary Conversion

- The easiest way to convert a decimal number to its binary equivalent is to use the ***Division Algorithm***
- This method repeatedly divides a decimal number by 2 and records the quotient and remainder
  - ***The remainder digits (a sequence of zeros and ones) form the binary equivalent in least significant to most significant digit sequence***

Division Algorithm

Convert 67 to its binary equivalent:

$67_{10} = x_2$

Step 1: 67 / 2 = 33 R 1 — *Divide 67 by 2. Record quotient in next row*

Step 2: 33 / 2 = 16 R 1 — *Again divide by 2; record quotient in next row*

Step 3: 16 / 2 = 8 R 0 — *Repeat again*

Step 4: 8 / 2 = 4 R 0 — *Repeat again*

Step 5: 4 / 2 = 2 R 0 — *Repeat again*

Step 6: 2 / 2 = 1 R 0 — *Repeat again*

Step 7: 1 / 2 = 0 R 1 — *STOP when quotient equals 0*

$\boxed{1\ 0\ 0\ 0\ 0\ 1\ 1_2}$

Example 4: Convert the decimal number 17 to binary.
Solution:

2	17		
2	8 – 1	⟶	L.S.B.
2	4 – 0		
2	2 – 0		
2	1 – 0		
	0 – 1	⟶	M.S.B.

Therefore, $17 = 10001$

Example 5: Convert the decimal number 0.625 to binary.
Solution:

$0.625 \times 2 = 1.250$	1 (M S B)
$0.250 \times 2 = 0.500$	0
$0.25 \times 2 = 1.00$	1 (L S B)

Therefore, $0.625 = 0.101$

Note: Any further multiplication by 2 in example 5 will equal to 0; therefore the multiplication can be terminated. However, this, is not so. Often it will be necessary to terminate the multiplication when an acceptable degree of accuracy is obtained. The binary number obtained will then be an approximation

Example: Convert decimal 0.6 in to binary number system

$0.6 \times 2 = 1.2$	1 (M S B)
$0.2 \times 2 = 0.4$	0
$0.4 \times 2 = 0.8$	0
$0.8 \times 2 = 1.6$	1
$0.6 \times 2 = 1.2$	1 (LSB)

Therefore, $0.6 = 0.10011$

1b).Binary to Decimal Conversion

- The easiest method for converting a binary number to its decimal equivalent is to use the ***Multiplication Algorithm***
- Multiply the binary digits by increasing powers of two, starting from the right
- Then, to find the decimal number equivalent, sum those products

Multiplication Algorithm

Convert $(10101101)_2$ to its decimal equivalent:

Binary ⟶	1	0	1	0	1	1	0	1
	×	×	×	×	×	×	×	×
Positional Values ⟶	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

Products ⟶ $128 + 32 + 8 + 4 + 1$

$\boxed{173_{10}}$

Example 1: Convert the following binary number to their decimal equivalent. (a) 1101 (b) 1001

Solution:

(a) $1101 = (1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0)$

$= 8 + 4 + 0 + 1 = 13$

(b) $1001 = (1 \times 2^3) + (0 \times 2^2) + (0 \times 2^1)(1 \times 2^0)$

$= 8 + 0 + 0 + 1 = 9$

Example 2: Convert the following binary numbers to their decimal equivalent. (a) 0.011 (b) 0.111

Solution:

(a) $0.011 = (0 \times 2^{-1}) + (1 \times 2^{-2}) + (1 \times 2^{-3})$

$= 0 + \frac{1}{4} + \frac{1}{8}$

$= 0.25 + 0.125 = 0.375$

(b) $0.111 = (1 \times 2^{-1}) + (1 \times 2^{-2}) + (1 \times 2^{-3})$

$= \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$

$= 0.5 + 0.25 + 0.125 = 0.875$

Example 3: Convert the binary number 110.011 to its decimal equivalent.

Solution:

$110.011 = (1 \times 2^2) + (1 \times 2^1) + (0 \times 2^0) + (0 \times 2^{-1})$
$+ (1 \times 2^{-2}) + (1 \times 2^{-3})$

$= 4 + 2 + 0 + 0 + \frac{1}{4} + \frac{1}{8}$

$= 4 + 2 + 0.25 + 0.125 = 6.375$

2).Octal Number System

- Also known as the Base 8 System
- Uses digits 0 - 7
- Readily converts to binary
- Groups of three (binary) digits can be used to represent each octal digit
- Also uses multiplication and division algorithms for conversion to and from base 10

2b).Octal to Decimal & Decimal to Octal:

Example 8: Convert the following octal numbers to their decimal, equivalent.

Solution: (a) 35_8 (b) 100_8 (c) 0.24_8

(a) $35_8 = (3 \times 8^1) + (5 \times 8^0)$
$= 24 + 5 = 29$

(a) $100_8 = (1 \times 8^2) + (0 \times 8^1) + (0 \times 8^0)$
$= 64 + 0 + 0 = 64_{10}$

(b) $0.24_8 = (2 \times 8^{-1}) + (4 \times 8^{-2})$
$= \frac{2}{8} + \frac{4}{64}$
$= 0.3125_{10}$

10.10 Decimal-to-Octal Conversion:

To convert decimal numbers to their octal equivalent, the following procedures are employed:

- Whole-number conversion: Repeated division-by-8.
- Fractional number conversion: Repeated multipli-cation-by-8.

10.11 Repeated Division-by-8 Method:

The repeated-division by 8 method of converting decimal to octal applies only to whole numbers. The procedure is illustrated in the following example.

Example 8: Convert the following decimal numbers to their octal equivalent:

(a) 245 (b) 175

Solution:

8	245	
8	30 – 5	(LSD)
8	3 – 6	
	0 – 6 (MSD)	

8	175
8	21 – 7
8	2 – 5
	0 – 2

Therefore, $245_{10} = 365_8$, therefore , $175_{10} = 257_8$

10.12 Repeated Multiplication-by-8 Method:

To convert decimal fractions to their Octal equivalent requires repeated Multiplication by 8, as shown in the following example.

Example 9: Convert the decimal fraction 0.432 to octal equivalent .

Solution:

		Carry
0.432×8	$= 3.456$	3(MSD)
0.456×8	$= 3.648$	3
0.648×8	5.184	5
0.184×8	$= 1.472$	1 (LSD)

The first carry is nearest the octal point, therefore,

$0.432_{10} = 0.3351_8$

The conversion to octal is not precise, since there is a remainder. If greater accuracy is required, we simply continue multiplying by 8 to obtained more octal digits.

2c).Octal to Binary & Binary to Octal Conversion:

Example 10: Convert the following octal numbers to their binary equivalent.

(a) 247_8 (b) 501_8

Solution:

(a)	2	4	7	Octal
	010	100	111	binary

Thus, $247_8 = 010100111_2$

(b)	5	0	1	octal
	101	000	001	binary

Thus, $501_8 = 101000001_2$

10.15 Binary-to-Octal Conversion:

In printing out octal numbers, the modern electronic digital computer performs a binary-to-octal conversion. This is a simple procedure. The binary number is divided into groups to three bits, counting to the right and to the left from the binary point and then each group Of three is interpreted as an octal digit; as shown in above table.

Example 11: Convert 11010101 . 01101 to an octal-number.

Solution:	011	010	101	.	011	10
	011	010	101		011	010
	3	2	5		3	2

Therefore $11010101.0110_2 = 325.32_8$

3).

Hexadecimal Number System

- Base 16 system
- Uses digits 0-9 & letters A,B,C,D,E,F
- Groups of four bits represent each base 16 digit

Decimal	Hexadecimal
0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
10	A
11	B
12	C
13	D
14	E
15	F

Decimal to Hexadecimal Conversion

Convert 830_{10} to its hexadecimal equivalent:

830 / 16 = 51 R14 ← = E in Hex
51 / 16 = 3 R3
3 / 16 = 0 R3

$\boxed{33E_{16}}$

Hexadecimal to Decimal Conversion

Convert 3B4F16 to its decimal equivalent:

Hex Digits →	3	B	4	F
	×	×	×	×
Positional Values →	16^3	16^2	16^1	16^0
Products →	12288 +	2816 +	64 +	15

$\boxed{15{,}183_{10}}$

Binary to Hexadecimal Conversion

- The easiest method for converting binary to hexadecimal is to use a substitution code
- Each hex number converts to 4 binary digits

Substitution Code

0000 = 0	**0100 = 4**	**1000 = 8**	**1100 = C**
0001 = 1	**0101 = 5**	**1001 = 9**	**1101 = D**
0010 = 2	**0110 = 6**	**1010 = A**	**1110 = E**
0011 = 3	**0111 = 7**	**1011 = B**	**1111 = F**

10.16 Binary Arithmetic:

Binary arithmetic includes the basic arithmetic operations of addition, subtraction, multiplication and division. The following sections present the rules that apply to these operations when they are performed on binary numbers.

Binary Addition:

Binary addition is performed in the same way as addition in the decimal-system and is, in fact, much easier to master. Binary addition obeys the following four basic rules:

$$\begin{array}{r}0\\+0\\\hline 0\end{array}\quad\begin{array}{r}0\\+1\\\hline 1\end{array}\quad\begin{array}{r}1\\+0\\\hline 1\end{array}\quad\begin{array}{r}1\\+1\\\hline 10\end{array}$$

The results of the last rule may seem some what strange, remember that these are binary numbers. Put into words, the last rule states that

binary one + binary one = binary two = binary "one zero"

When adding more than single-digit binary number, carry into, higher order columns as is done when adding decimal numbers. For example 11 and 10 are added as follows:

$$\begin{array}{r}11\\+10\\\hline 101\end{array}$$

In the first column (L S C or 2^0) '1 plus 0 equal 1. In the second column (2^1) 1 plus 1 equals 0 with a carry of 1 into the third column (2^2).

When we add 1 + 1.+ 1 (carry) produces 11, recorded as 1 with a carry to the next column.

Example 12: Add (a) 111 and 101 (b) 1010, 1001 and 1101.

Solution:

(a) $\begin{array}{r}{\scriptstyle (1)(1)}\\1\,1\,1\\1\,0\,1\\\hline 1\,1\,0\,0\end{array}$ (b) $\begin{array}{r}{\scriptstyle (2)(1)(1)(1)}\\1\,0\,1\,0\\1\,0\,0\,1\\1\,1\,0\,1\\\hline 1\,0\,0\,0\,0\end{array}$

- Consider a 4 bit binary number

Decimal	Binary	Decimal	Binary
0	0000	4	0100
1	0001	5	0101
2	0010	6	0110
3	0011	7	0111

- Examples of binary arithmetic

$3 + 2 = 5$

$$\begin{array}{ccccc} & & 1 & & \\ & 0 & 0 & 1 & 1\\ + & 0 & 0 & 1 & 0\\\hline & & 1 & 0 & 1\end{array}$$

$3 + 3 = 6$

$$\begin{array}{ccccc} & & 1 & 1 & \\ & 0 & 0 & 1 & 1\\ + & 0 & 0 & 1 & 1\\\hline & & 1 & 1 & 0\end{array}$$

Binary Subtraction:

Binary subtraction is just as simple as addition subtraction of one bit from another obey the following four basic rules

$0 - 0 = 0$

$1 - 1 = 0$

$1 - 0 = 1$

$10 - 1 = 1$ with a transfer (borrow) of 1.

When doing subtracting, it is sometimes necessary to borrow from the next higher-order column. The only it will be necessary to borrow is when we try to subtract a 1 from a 0. In this case a 1 is borrowed from the next higher-order column, which leaves a 0 in that column and creates a

10 i.e., 2 in the column being subtracted. The following examples illustrate binary subtraction.

Example 13: Perform the following subtractions.

(a) 11 - 01 , (b) 11-10 (c) 100 - 011

Solution:

(a) $\begin{array}{r} 11 \\ -01 \\ \hline 10 \end{array}$ (b) $\begin{array}{r} 11 \\ -10 \\ \hline 01 \end{array}$ (c) $\begin{array}{r} 100 \\ -011 \\ \hline 001 \end{array}$

Part (c) involves to borrows, which handled as follows. Since a 1 is to be subtracted from a 0 in the first column, a borrow is required from the next higher-order column. However, it also contains a 0; therefore, the second column must borrow the 1 in the third column. This leaves a 0 in the third column and place a 10 in the second column. Borrowing a 1 from 10 leaves a 1 in the second column and places a 10 i.e, 2 in the first column:

Octal Addition & Subtraction

Example – Addition

$456_8 + 123_8 = 601_8$

1 1	carry	
4 5 6	$= 302_{10}$	
+ 1 2 3	$= 83_{10}$	
6 0 1	$= 385_{10}$	

Octal Subtraction

The subtraction of octal numbers follows the same rules as the subtraction of numbers in any other number system. The only variation is in borrowed number. In the decimal system, you borrow a group of 10_{10}. In the binary system, you borrow a group of 2_{10}. In the octal system you borrow a group of 8_{10}.

Example – Subtraction

Example:

$456_8 - 173_8 = 333_8$

8	borrow
^{3}4 5 6	$= 302_{10}$
- 1 7 3	$= 123_{10}$
2 6 3	$= 179_{10}$

Hexadecimal Addition & Subtraction:

Example – Addition

$4A6_{16} + 1B3_{16} = 659_{16}$

	1	carry
	4 A 6	$= 1190_{10}$
+	1 B 3	$= 435_{10}$
	6 5 9	$= 1625_{10}$

Hexadecimal Subtraction

The subtraction of hexadecimal numbers follow the same rules as the subtraction of numbers in any other number system. The only variation is in borrowed number. In the decimal system, you borrow a group of 10_{10}. In the binary system, you borrow a group of 2_{10}. In the hexadecimal system you borrow a group of 16_{10}.

Example - Subtraction

$4A6_{16} - 1B3_{16} = 2F3_{16}$

	16	borrow
	^{3}4 A 6	$= 1190_{10}$
-	1 B 3	$= 435_{10}$
	2 F 3	$= 755_{10}$