LECTURE-5(B)

COURSE TITLE

LINEAR ALGEBRA

&

GEOMETRY

(MT-272)

DEPARTMENT OF TELECOMMUNICATION ENGINEERING

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COORDINATES RELATIVE TO A BASIS

If S= $\{ \mathrm{v}_1, \mathrm{v}_2, \dots\dots, \mathrm{v}_\mathrm{n}\}$ is a basis for a vector space *V,* and

$$\mathbf{v} = c_1\mathbf{v_1} + c_2\mathbf{v_2}+\dots\dots+c_n\mathbf{v_n}$$

is the expression for a vector **v** in terms of the basis *S*, then the scalars $c_1, c_2, \dots, c_n$ are called the **coordinates** of **v** relative to the basis S.

The vector $(c_1, c_2, \dots, c_n)$ in R^n constructed from these coordinates vector of v relative to S; it is denoted by

$$(\mathbf{v})_S = (c_1, c_2, \dots, c_n)$$

EXAMPLE-1

Example-1: Let S = $(\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3})$ be the basis for R^3 where

$$\mathbf{v_1}=(1,2,1),\ \mathbf{v}_2=(2,9,0),\ \mathbf{v}_3=(3,3,4)$$

(a) Find the coordinate vector of **v** =(5,-1,9)with respect to *S*.

Sol: Let c_1,c_2, & c_3 be scalars, then vector equation can be written as

$$\mathbf{v}=c_1\mathbf{v}_1+c_2\mathbf{v}_2+c_3\mathbf{v}_3$$

In terms of components

$$(5,-1,9) = c_1(1,2,1) + c_2(2,9,0)+c_3(3,3,4)$$

On comparing both sides

$$\begin{aligned} c_1+2c_2+3c_3 &= 5 \\ 2c_1+9c_2+3c_3 &=-1 \quad \text{....(1)} \\ c_1 \qquad +4c_3 &= 9 \end{aligned}$$

EXAMPLE-1 ...Con'd

Solving the above system for $c_1, c_2, \& \ c_3$ the augmented matrix is

$$\begin{bmatrix} 1 & 2 & 3 & 5 \\ 2 & 9 & 3 & -1 \\ 1 & 0 & 4 & 9 \end{bmatrix} \xrightarrow{\text{G.J.E}} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\therefore c_1 = 1, c_2 = -1, c_3 = 2. \text{ Therefore,}$$

$$(\mathbf{v})_S = (1, -1\ , 2)$$

(b) Find the vector **v** in R^3 whose coordinate vector with respect to the basis S is $(\boldsymbol{v})_{\boldsymbol{s}} = (-1,3,2)$.

Sol: By using the definition of coordinate vector $(\boldsymbol{v})_{\boldsymbol{s}}$, we obtain

$\mathbf{v} = (-1)\ \mathbf{v}_1 + 3\mathbf{v}_2 + 2\mathbf{v}_3$

or v = (-1)(1,2,1) + (3)(2,9,0)+(2)(3,3,4)

EXAMPLE-1 ...Con'd

or $\mathbf{v} = (-1,-2,-1) + (6,27,0)+(6,6,8)$

or $\mathbf{v} = (-1+6+6,-2+27+6,-1+0+8)$

or $\mathbf{v} = (11,31,7)$

DIMENSION OF A VECTOR SPACE

The dimension of a vector space V is defined to be the number of vectors in a basis for V

V: a vector space $\qquad$ S: a basis for V

$\Rightarrow \dim(V) = \#(S)$ $\qquad$ (the number of vectors in a basis S)

Finite dimensional:

A vector space V is finite dimensional if it has a basis consisting of a finite number of elements

Infinite dimensional:

If a vector space V is not finite dimensional, then it is called infinite dimensional

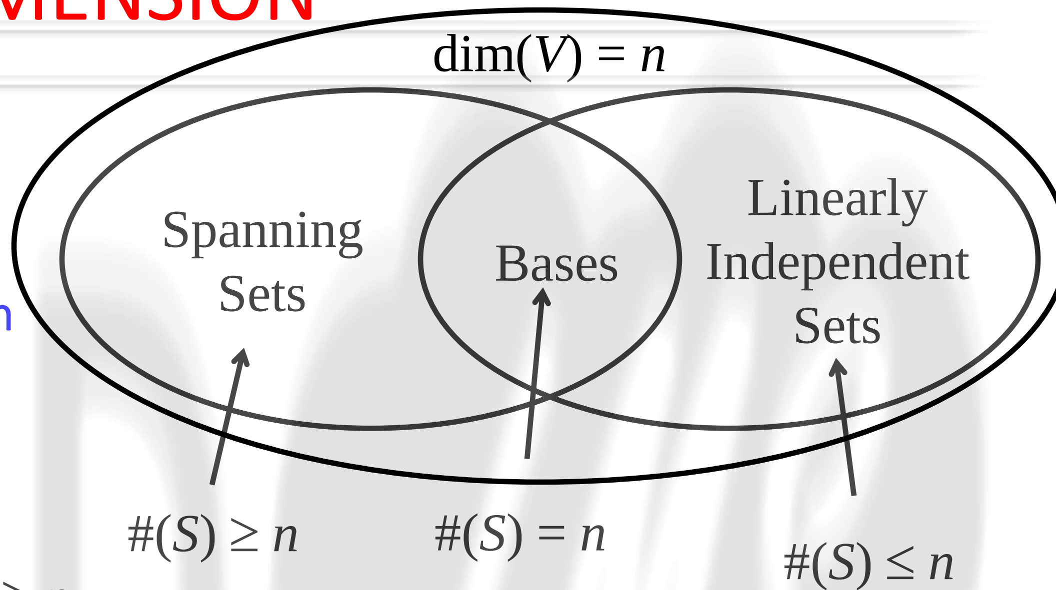
DIMENSION

- Notes:

(1) $\dim(\{\mathbf{0}\}) = 0$

(If V consists of the zero vector alone, the dimension of V is defined as zero)

(2) Given $\dim(V) = n$, for $S \subseteq V$



S: a spanning set $\Rightarrow \#(S) \geq n$

S: a L.I. set $\Rightarrow \#(S) \leq n$

S: a basis $\Rightarrow \#(S) = n$

(3) Given $\dim(V) = n$, if W is a subspace of $V \Rightarrow \dim(W) \leq n$

※ For example, if $V = R^3$, you can infer the dim(V) is 3, which is the number of vectors in the standard basis

Example -1

Example-1: Find the dimension of a vector space according to the standard basis

※ The simplest way to find the dimension of a vector space is to count the number of vectors in the standard basis for that vector space

(1) Vector space R^n $\Rightarrow$ standard basis $\{\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_n\}$

$$\Rightarrow \dim(R^n) = n$$

(2) Vector space $M_{m\times n}$ $\Rightarrow$ standard basis $\{E_{ij} \mid 1 \le i \le m, 1 \le j \le n\}$

$$\text{and in } E_{ij} \quad \begin{cases} a_{ij} = 1 \\ \text{other entries are zero} \end{cases}$$

$$\Rightarrow \dim(M_{m\times n}) = mn$$

(3) Vector space $P_n(x)$ $\Rightarrow$ standard basis $\{1, x, x^2, \ldots, x^n\}$

$$\Rightarrow \dim(P_n(x)) = n+1$$

(4) Vector space $P(x)$ $\Rightarrow$ standard basis $\{1, x, x^2, \ldots\}$