LECTURE-5(C)

COURSE TITLE

LINEAR ALGEBRA

&

GEOMETRY

(MT-272)

DEPARTMENT OF TELECOMMUNICATION ENGINEERING

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Example -2

Example-2: Determining the dimension of a subspace of R^3

(a) $W = \{(d, c-d, c)$: c and d are real numbers$\}$

(b) $W = \{(2b, b, 0)$: b is a real number$\}$

Sol: (Hint: find a set of L.I. vectors that spans the subspace, i.e., find a basis for the subspace.)

(a) $(d, c-d, c) = c(0, 1, 1) + d(1, -1, 0)$

$\Rightarrow S = \{(0, 1, 1), (1, -1, 0)\}$ (S is L.I. and S spans W)

$\Rightarrow S$ is a basis for W

$\Rightarrow \dim(W) = \#(S) = 2$

(b) $\because (2b, b, 0) = b(2, 1, 0)$

$\Rightarrow S = \{(2, 1, 0)\}$ spans W and S is L.I.

$\Rightarrow S$ is a basis for W $\Rightarrow \dim(W) = \#(S) = 1$

Example -3

Example-3: Finding the dimension of a subspace of $M_{2\times2}$

Let W be the subspace of all symmetric matrices in $M_{2\times2}$. What is the dimension of W?

Sol:

$$W = \left\{ \begin{bmatrix} a & b \\ b & c \end{bmatrix} \middle| a, b, c \in R \right\}$$

$$\because \begin{bmatrix} a & b \\ b & c \end{bmatrix} = a\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + b\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + c\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow S = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$ spans W and S is L.I.

$\Rightarrow$ S is a basis for W $\Rightarrow$ dim(W) = #(S) = 3

Theorem

Methods to identify a basis in an *n*-dimensional space

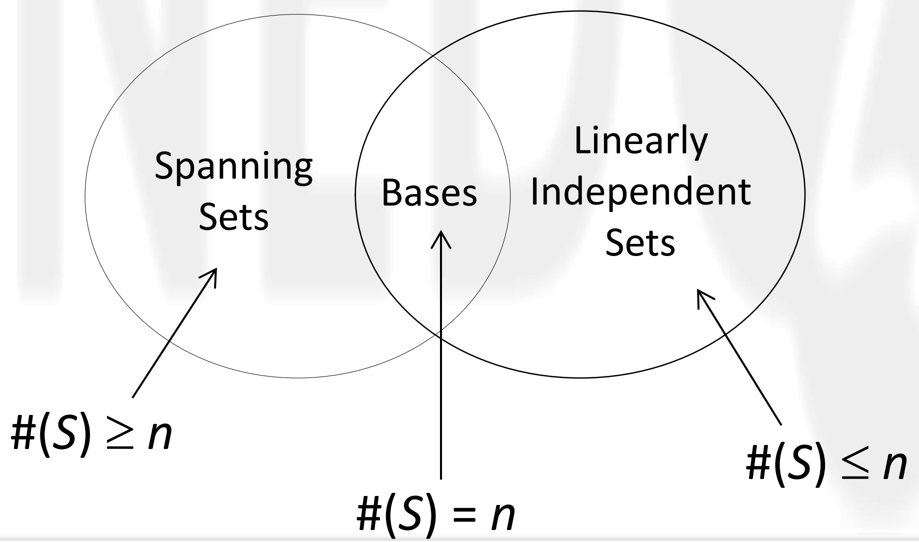
Let V be a vector space of dimension n

(1) If $S = \{\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_n\}$ is a linearly independent set of vectors in V, then S is a basis for V.

(2) If $S = \{\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_n\}$ spans V, then S is a basis for V

(Both results are due to the fact that $\#(S) = n$)

$\dim(V) = n$



Example -4

Example-4: Determine the dimension of and a basis for the solution space of the system

$$x_1 - 4x_2 + 3x_3 - x_4 = 0$$
$$2x_1 - 8x_2 + 6x_3 - 2x_4 = 0$$

Sol: The Augmented Matrix is

$$\begin{bmatrix} 1 & -4 & 3 & -1 & 0 \\ 2 & -8 & 6 & -2 & 0 \end{bmatrix} \xrightarrow{-2R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & -4 & 3 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Reduced echelon form

Retransform into linear equation

$x_1 - 4x_2 + 3x_3 - x_4 = 0$ or $x_1 = 4x_2 - 3x_3 + x_4$

Let $x_2 = s$ $x_3 = t$ & $x_4 = u$

$\therefore x_1 = 4s - 3t + u$

EXAMPLE-4

The solution in matrix form is

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4s - 3t + u \\ s \\ t \\ u \end{bmatrix} = \text{S} \begin{bmatrix} 4 \\ 1 \\ 0 \\ 0 \end{bmatrix} + \text{t} \begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \end{bmatrix} + \text{u} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Where s , t and u are scalars . Above equation represents the vector $(x_1, x_2 , x_3 , x_4)^T$ is a linear combination of vectors $\boldsymbol{v_1} = (4,1,0,0)^T$ $\boldsymbol{v_2} = (-3,0,1,0)^T$ and $\boldsymbol{v_3}$=(1,0,0,1)^T. Therefore $\{\boldsymbol{v_1}, \boldsymbol{v_2}, \boldsymbol{v_3}\}$ spans the solution space . Since neither of $\boldsymbol{v_1}, \boldsymbol{v_2}, \boldsymbol{v_3}$ expressed as a linear combination of other, therefore they are linearly independent vectors.

Hence $\{\boldsymbol{v_1}, \boldsymbol{v_2}, \boldsymbol{v_3}\}$ is a basis for solution space, therefore, the solution space has three dimension.

EXAMPLE-5

Example-5: Determine the dimension of a basis for the solution space of the system

$$\begin{aligned} x_1-3x_2+\ x_3 &= 0 \\ 2x_1-6x_2+2x_3 &= 0 \\ 3x_1-9x_2+3x_3 &= 0 \end{aligned}$$

Sol: The Augmented Matrix is

$$\begin{bmatrix} 1 & -3 & 1 & 0 \\ 2 & -6 & 2 & 0 \\ 3 & -9 & 3 & 0 \end{bmatrix} \xrightarrow[-3R_1+R_3]{-2R_1+R_2} \begin{bmatrix} 1 & -3 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Retransform into linear equation

$x_1 - 3x_2 + x_3 = 0$ or $x_1 = 3x_2 - x_3$

Let $x_2 = s$

$x_3 = t$

$x_1 = 3s - t$

EXAMPLE-5 Con'd

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3s - t \\ s \\ t \end{bmatrix} = \begin{bmatrix} 3s \\ s \\ 0 \end{bmatrix} + \begin{bmatrix} -t \\ 0 \\ t \end{bmatrix} = s\begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

Where s and t are scalars. Above equation represents the vector $(x_1, x_2, x_3)^T$ is a linear combination of vectors $\boldsymbol{v_1} = (3,1,0)^T$ & $\boldsymbol{v_2} = (-1,0,1)^{\mathrm{T}}$. Therefore $\{\boldsymbol{v_1}, \boldsymbol{v_2}\}$ spans the solution space . Since neither of v_1 & v_2 expressed as a linear combination of other, therefore they are linearly independent vectors.

Hence $\{\boldsymbol{v_1}, \boldsymbol{v_2}\}$ is a basis for solution space, therefore, the solution space has two dimension.