LECTURE-3(A)

COURSE TITLE

LINEAR ALGEBRA

&

GEOMETRY

(MT-272)

DEPARTMENT OF TELECOMMUNICATION ENGINEERING

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RANK OF A MATRIX

- A rank of a matrix A is defined as the no. of non-zero rows in echelon form of matrix A.

Example-1

Determine the rank of following matrices.

(a) $A=\begin{bmatrix}1 & 2 & 3\\ 2 & 4 & 7\\ 3 & 6 & 10\end{bmatrix}$

Solution:

$$\begin{bmatrix}1 & 2 & 3\\ 2 & 4 & 7\\ 3 & 6 & 10\end{bmatrix} \xrightarrow{-2R_1+R_2} \begin{bmatrix}1 & 2 & 3\\ 0 & 0 & 1\\ 3 & 6 & 10\end{bmatrix}$$

EXAMPLE-1 Con'd

$$\xrightarrow{-3R_1+R_3} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{-R_2+R_3} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Echelon Form

In the above echelon form matrix, the non-zero rows are 2. Hence the Rank(A) = 2

EXAMPLE-1 Con'd

(b) $$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 2 \\ 2 & 2 & 3 \end{bmatrix}$$

Solution:

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 2 \\ 2 & 2 & 3 \end{bmatrix} \xrightarrow{-R_1+R_2} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 2 & 2 & 3 \end{bmatrix} \xrightarrow{-2R_1+R_3} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\xrightarrow{-R_3} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Echelon Form

In the above echelon form matrix, the non-zero rows are 3. Hence the Rank(A) = 3

CONSISTENCY OF NON-HOMOGENEOUS SYSTEM OF LINEAR EQUATIONS

Consider a non-homogeneous system of linear equations with n-unknown. It can be written in matrix form as AX=b,

Where A = Matrix of coefficients, X = Matrix of unknowns, & b = Matrix of constants

Let C = [A:b] is an Augmented matrix

The system AX=b has

(1) Solution or consistent if Rank(A) = Rank(C)

(2) unique solution if Rank(A) = Rank(C) = n (no. of unknowns)

(3) infinitely many solution if Rank(A) = Rank(C) = r (any integer) such that $r < n$.

(4) no Solution or inconsistent if Rank(A) $\neq$ Rank(C)

EXAMPLE-1

Example-1

For what value of λ the system
$$x+2y=1$$
$$5x+\lambda y=5$$

has (i) unique solution (ii) infinitely many solution.

Solution: The Augmented matrix is

$$[A,b]=\begin{bmatrix}1 & 2 & 1\\ 5 & \lambda & 5\end{bmatrix}\xrightarrow{-5R_1+R_2}\begin{bmatrix}1 & 2 & 1\\ 0 & \lambda-10 & 0\end{bmatrix}\text{.....(1)}$$

In matrix-(1)

If $\lambda \neq 10$ $\Rightarrow$ Rank(A)=Rank(C)=2=no. of unknowns

Therefore, given system has unique solution.

If $\lambda = 10$ $\Rightarrow$ Rank(A)=Rank(C)=1< no. of unknowns

Therefore, given system has infinitely many solution.

EXAMPLE-2

Example-2

Determine for what values of λ & μ the following the system of linear equations has

(i) No solution (ii) unique solution (iii) infinitely many solution.

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = \mu$$

Solution: The Augmented matrix is

$$[A, b] = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{bmatrix} \xrightarrow[\,]{\substack{-R_1+R_2 \\ -R_1+R_3}} \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda-1 & \mu-6 \end{bmatrix}$$

$$\xrightarrow{-R_2+R_3} \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{bmatrix} \quad \text{.....(a)}$$

In matrix-(a)

If $\lambda = 3$ & $\mu \neq 10$ $\Rightarrow$ Rank(A)$\neq$Rank(C) $\Rightarrow$ The system is inconsistent or has no solution.

EXAMPLE-2 Con'd

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{bmatrix} \quad .....(a)$$

In matrix-(a)

If $\lambda \neq 3$, & μ may have any value $\Rightarrow$ Rank(A)=Rank(C)=3=no. of unknowns

$\Rightarrow$ The system has unique solution

If $\lambda = 3$ & $\mu = 10$ $\Rightarrow$ Rank(A)=Rank(C)=2 < no. of unknowns

$\Rightarrow$ The system has infinitely many solution