LECTURE-3(B)

COURSE TITLE

LINEAR ALGEBRA

&

GEOMETRY

(MT-272)

DEPARTMENT OF TELECOMMUNICATION ENGINEERING

COURSE TEACHER

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TYPES OF SOLUTION OF HOMOGENEOUS SYSTEM OF LINEAR EQUATIONS USING RANK

Consider a homogeneous system of linear equations with n-unknown. It can be written in matrix form as AX=0,

Where A = Matrix of coefficients, & X = Matrix of unknowns.

Let C = [A:0] is an Augmented matrix

The system AX=0 has

(1) trivial solution if Rank(A) = Rank(C) = n (no. of unknowns)

(2) non-trivial solution if Rank(A) = Rank(C) = r (any integer) such that $r < n$.

EXAMPLE-1

Example-1

Find the value of k such that the following system has non-trivial solution.

$$\begin{aligned} x + ky + 3z &= 0 \\ 4x + 3y + kz &= 0 \\ 2x + y + 2z &= 0 \end{aligned}$$

Solution: The Augmented matrix is

$$[A,0] = \begin{bmatrix} 1 & k & 3 & 0 \\ 4 & 3 & k & 0 \\ 2 & 1 & 2 & 0 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_3} \begin{bmatrix} 2 & 1 & 2 & 0 \\ 4 & 3 & k & 0 \\ 1 & k & 3 & 0 \end{bmatrix} \xrightarrow{-1/2R_1} \begin{bmatrix} 1 & 1/2 & 1 & 0 \\ 4 & 3 & k & 0 \\ 1 & k & 3 & 0 \end{bmatrix}$$

$$\xrightarrow[-R_1+R_3]{-4R_1+R_2} \begin{bmatrix} 1 & 1/2 & 1 & 0 \\ 0 & 1 & k-4 & 0 \\ 0 & k-1/2 & 2 & 0 \end{bmatrix} \xrightarrow{-(k-\frac{1}{2})R_2+R_3} \begin{bmatrix} 1 & 1/2 & 1 & 0 \\ 0 & 1 & k-4 & 0 \\ 0 & 0 & 2-(k-\frac{1}{2})(k-4) & 0 \end{bmatrix}$$

EXAMPLE-1 Con'd

In matrix-(a)

For non-trivial solution put $2-\left(k-\frac{1}{2}\right)(k-4)=0$

$\Rightarrow$ k=9/2 & k=0

$\Rightarrow$ Rank(A)=Rank(C)=2 < no. of unknowns

$\Rightarrow$ The system has non-trivial solution

EXAMPLE-2

Example-2

Determine ‘b’ such that the following system has non-trivial solution.

$$2x + y + 2z = 0$$
$$x + y + 3z = 0$$
$$4x + 3y + bz = 0$$

Solution: The Augmented matrix is

$$[A, 0] = \begin{bmatrix} 2 & 1 & 2 & 0 \\ 1 & 1 & 3 & 0 \\ 4 & 3 & b & 0 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & 1 & 3 & 0 \\ 2 & 1 & 2 & 0 \\ 4 & 3 & b & 0 \end{bmatrix} \xrightarrow[-4R_1+R_3]{-2R_1+R_2} \begin{bmatrix} 1 & 1 & 3 & 0 \\ 0 & -1 & -4 & 0 \\ 0 & -1 & b-12 & 0 \end{bmatrix}$$

$$\xrightarrow{-1R_2} \begin{bmatrix} 1 & 1 & 3 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & -1 & b-12 & 0 \end{bmatrix} \xrightarrow{-R_2+R_3} \begin{bmatrix} 1 & 1 & 3 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & b-8 & 0 \end{bmatrix} \quad \text{.....(a)}$$

In matrix (a)

If b=8 $\Rightarrow$ Rank(A)=Rank(C)=2< no. of unknowns

$\Rightarrow$ System has non-trivial solution.

VECTOR SPACES

VECTORS IN R^n

- An ordered n-tuple :

  a sequence of n real numbers $(x_1, x_2, \ldots, x_n)$

- R^n-space :

  the set of all ordered n-tuples

n = 1 R^1-space = set of all real numbers
(R^1-space can be represented geometrically by the x-axis)

n = 2 R^2-space = set of all ordered pair of real numbers (x_1, x_2)
(R^2-space can be represented geometrically by the xy-plane)

n = 3 R^3-space = set of all ordered triple of real numbers (x_1, x_2, x_3)
(R^3-space can be represented geometrically by the xyz-space)

n = 4 R^4-space = set of all ordered quadruple of real numbers (x_1, x_2, x_3, x_4)

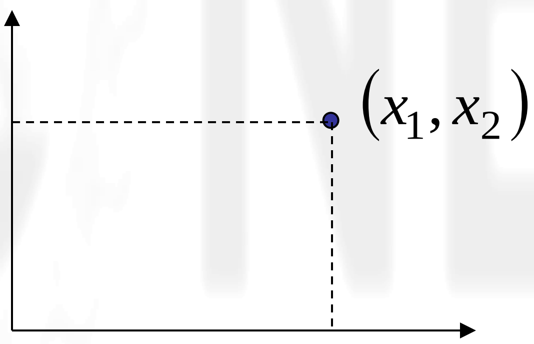
VECTORS IN R^n …Con'd

- Notes:

(1) An n-tuple $(x_1, x_2, \cdots, x_n)$ can be viewed as a point in R^n with the x_i's as its coordinates.

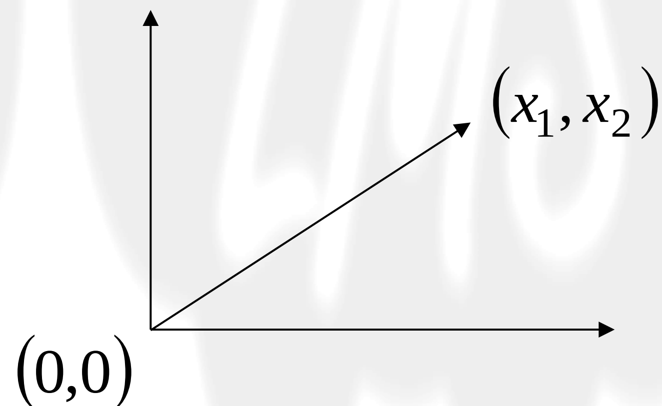
(2) An n-tuple $(x_1, x_2, \cdots, x_n)$ also can be viewed as a vector $\mathbf{x} = (x_1, x_2, \cdots, x_n)$ in R^n with the x_i's as its components.

- Ex:1



a point

or



a vector

※ A vector on the plane is expressed geometrically by a directed line segment whose initial point is the origin and whose terminal point is the point (x_1, x_2)

VECTORS IN R^n ...Con'd

$$\mathbf{u} = (u_1, u_2, \cdots, u_n), \quad \mathbf{v} = (v_1, v_2, \cdots, v_n) \qquad \text{(two vectors in } R^n\text{)}$$

- Equality:

  $\mathbf{u} = \mathbf{v}$ if and only if $u_1 = v_1$, $u_2 = v_2$,……, $u_n = v_n$

- Vector addition (the sum of **u** and **v**):

  $$\mathbf{u} + \mathbf{v} = (u_1 + v_1, u_2 + v_2, \cdots, u_n + v_n)$$

- Scalar multiplication (the scalar multiple of **u** by *c*):

  $$c\mathbf{u} = (cu_1, cu_2, \cdots, cu_n)$$

- Notes:

  The sum of two vectors and the scalar multiple of a vector in R^n are called the standard operations in R^n

VECTORS IN R^n …Con'd

- Difference between **u** and **v**:

$$\mathbf{u}-\mathbf{v} \equiv \mathbf{u}+(-1)\mathbf{v}=(u_1-v_1,\ u_2-v_2,\ u_3-v_3,\ldots,\ u_n-v_n)$$

- Zero vector :

$$\mathbf{0}=(0,0,\ldots,0)$$

VECTORS IN R^n ...Con'd

- Notes:

A vector $\mathbf{u} = (u_1, u_2, \ldots, u_n)$ in R^n can be viewed as:

Use comma to separate components

a $1 \times n$ row matrix (row vector): $\mathbf{u} = [u_1 \; u_2 \; \cdots \; u_n]$

Use blank space to separate entries

or

a $n \times 1$ column matrix (column vector): $\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}$

VECTORS IN R^n ...Con'd

Vector addition

$$\mathbf{u}+\mathbf{v}=(u_1,u_2,\cdots,u_n)+(v_1,v_2,\cdots,v_n)$$
$$=(u_1+v_1,u_2+v_2,\cdots,u_n+v_n)$$

Scalar multiplication

$$c\mathbf{u}=c(u_1,\ u_2,\cdots,\ u_n)$$
$$=(cu_1,cu_2,\cdots,\ cu_n)$$

Regarded as $1\times n$ row matrix

$$\mathbf{u}+\mathbf{v}=[u_1\ u_2\ \ \cdots\ \ u_n]+[v_1\ v_2\ \ \cdots\ \ v_n]$$
$$=[u_1+v_1\ \ u_2+v_2\ \ \cdots\ \ u_n+v_n]$$

$$c\mathbf{u}=c[u_1\ u_2\cdots u_n]$$
$$=[cu_1\ \ cu_2\ \ \cdots\ \ cu_n]$$

Regarded as $n\times 1$ column matrix

$$\mathbf{u}+\mathbf{v}=\begin{bmatrix}u_1\\u_2\\\vdots\\u_n\end{bmatrix}+\begin{bmatrix}v_1\\v_2\\\vdots\\v_n\end{bmatrix}=\begin{bmatrix}u_1+v_1\\u_2+v_2\\\vdots\\u_n+v_n\end{bmatrix}$$

$$c\mathbf{u}=c\begin{bmatrix}u_1\\u_2\\\vdots\\u_n\end{bmatrix}=\begin{bmatrix}cu_1\\cu_2\\\vdots\\cu_n\end{bmatrix}$$