Answers of practice questions (week 1)

1.

$$y = A \sin^{-1}(x) \quad \text{— (i)}$$

$$y' = A \frac{1}{\sqrt{1-x^2}} \quad \text{— (ii)}$$

$$\sqrt{1-x^2}\, y' = A$$

Substitute in (i)

$$y = y' \sqrt{1-x^2}\, \sin^{-1}(x)$$

Ans.

Q2

$$y = ae^{x} + be^{2x}$$

$$y = e^{x}[a + be^{x}]$$

$$y \cdot e^{-x} = a + be^{x}$$

diff w. r to x.

$$y' \cdot e^{-x} + y \cdot (-e^{-x}) = be^{x}$$

$$y'e^{-x} - ye^{-x} = be^{x}$$

$$e^{-x}[y'e^{-x} - ye^{-x}] = b$$

$$y'e^{-2x} - ye^{-2x} = b$$

diff w. r to x

$$y'' \cdot e^{-2x} + y'(-2e^{-2x}) - [y'e^{-2x} + y(-2e^{-2x})] = 0$$

$$y''e^{-2x} - 2y'e^{-2x} - y'e^{-2x} + 2ye^{-2x} = 0$$

$$y''e^{-2x} - 3y'e^{-2x} + 2ye^{-2x} = 0$$

Ans

Q3 $y^2 = Ax^2 + Bx + c$ — (i)

diff w. r to x

$$2y \frac{dy}{dx} = 2Ax + B \quad \text{— (ii)}$$

diff again w. r to x

$$2[y'y' + yy''] = 2A \quad \text{— (iii)}$$

Again diff w. r to x

$$2[y''y' + y'y'' + y'y'' + yy'''] = 0$$

$)] = 0$

$$2[3y'y'' + yy'''] = 0$$

Ans.

Q4 $x^2 + y^2 + 2gx + 2fy + c = 0$

diff w. r to x

$$2x + 2y\frac{dy}{dx} + 2g + 2f\frac{dy}{dx} = 0$$

diff w. r to x

$$2 + 2[y'y' + yy''] + 2fy'' = 0$$

$$2 + 2(y')^2 + 2yy'' = -2fy''$$

$$\frac{2 + 2(y')^2 + 2yy''}{y''} = -2f$$

diff w. r to x

$$\frac{\left[4y'y''+2\{y'y''+yy'''\}\right]y''-\left[2+2(y')^2+2yy''\right]y'''}{(y'')^2}=0$$

$$\frac{4y'y''^2+2y'y''^2+yy''y'''-2y'''-2y'^2y'''-2yy''y'''}{(y'')^2}=0$$

$$\cancel{\frac{4y'y''^2}{y''^2}}+\cancel{\frac{2y'y''^2}{y''^2}}+\cancel{\frac{yy''y'''}{y''}}$$

$$4y'y''^2+2y'y''^2-yy''y'''-2y'''-2y'^2y'''=0$$

Ans

Q5 $(x-h)^2+(y-k)^2=0$ — (i)

diff w. r to x

$2(x-h)+2(y-k)y'=0$ — (ii)

diff w. r to x

$2(1)+2\left[y'y'+(y-k)y''\right]=0$

$2+2y'^2+(y-k)y''=0$

$y-k=\dfrac{-2-y'^2}{y''}$ ——— (iii)

put in Eq (ii)

$$2(x-h) + 2y'\left[\frac{-2-y'^2}{y''}\right] = 0$$

$$(x-h) = +\frac{\cancel{2}y'}{\cancel{2}}\left[\frac{2+y'^2}{y''}\right]$$

$$(x-h) = y'\left[\frac{2+y'^2}{y''}\right] \quad \text{— (iv)}$$

Put values of $(x-h)$ and $(y-k)$ in Eqn (1)

$$\left[y'\left\{\frac{2+y'^2}{y''}\right\}\right]^2 + \left[\frac{-2-y'^2}{y''}\right]^2 = 0$$

$$y'^2(2+y'^2)^2 + (2+y'^2)^2 = 0 \quad \text{Ans}$$

Remaining Part of Example 4

$$\int \frac{\sec\theta}{\tan\theta} \cdot \sec^2\theta \, d\theta$$

$$= \int \frac{\sec\theta}{\tan\theta}(1+\tan^2\theta)\, d\theta$$

$$= \int \frac{\sec\theta}{\tan\theta} + \frac{\sec\theta\tan^2\theta}{\tan\theta} \, d\theta$$

$$= \int \operatorname{cosec}\theta + \sec\theta\tan\theta \, d\theta$$

$$= \ln\left|\tan\frac{\theta}{2}\right| + \sec\theta + C \quad \text{Ans}$$