lecture 10(b)

1) $xp + yq = z$

$P = x, \quad Q = y, \quad R = z$

$$\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z}$$

$$\frac{dx}{x} = \frac{dy}{y} \qquad \frac{dy}{y} = \frac{dz}{z}$$

$$\ln x = \ln y + \ln c_1 \qquad \ln y = \ln z + \ln c_2$$

$$\ln \frac{x}{y} = \ln c_1 \qquad \ln \frac{y}{z} = \ln c_2$$

$$\frac{x}{y} = c_1 \qquad \frac{y}{z} = c_2$$

$$\phi\left(\frac{x}{y}, \frac{y}{z}\right) = 0$$

Q3//

$$x^2 p + y^2 q = z^2$$

$P = x^2, \quad Q = y^2, \quad R = z^2$

$$\frac{dx}{x^2} = \frac{dy}{y^2} = \frac{dz}{z^2}$$

$$\frac{dx}{x^2} = \frac{dy}{y^2} \qquad \frac{dy}{y^2} = \frac{dz}{z^2}$$

$$\frac{-1}{x} = \frac{-1}{y} + c_1 \qquad \frac{-1}{y} = \frac{-1}{z} + c_2$$

$$\frac{1}{y} - \frac{1}{x} = c_1 \qquad \frac{1}{z} - \frac{1}{y} = c_2$$

$$\phi\left(\frac{1}{y} - \frac{1}{x}, \frac{1}{z} - \frac{1}{y}\right) = 0$$

lecture 10 (b)

Q5// $p+q = \sin x$

$P=1, \quad Q=1, \quad R=\sin x$

$$\frac{dx}{1} = \frac{dy}{1} = \frac{dz}{\sin x}$$

$$\frac{dx}{1} = \frac{dy}{1} \qquad | \qquad dx = \frac{dz}{\sin x}$$

$x = y + c_1$ | $\sin x\, dx = dz$

$-\cos x = z + c_2$

$x - y = c_1$ | $-z - \cos x = c_2$

$$\phi(x-y, -z-\cos x) = 0$$ Ans

Q7. $a(p+q) = z$

Now, $ap + aq = z$

$P=a, \quad Q=a, \quad R=z$

$$\frac{dx}{a} = \frac{dy}{a} = \frac{dz}{z}$$

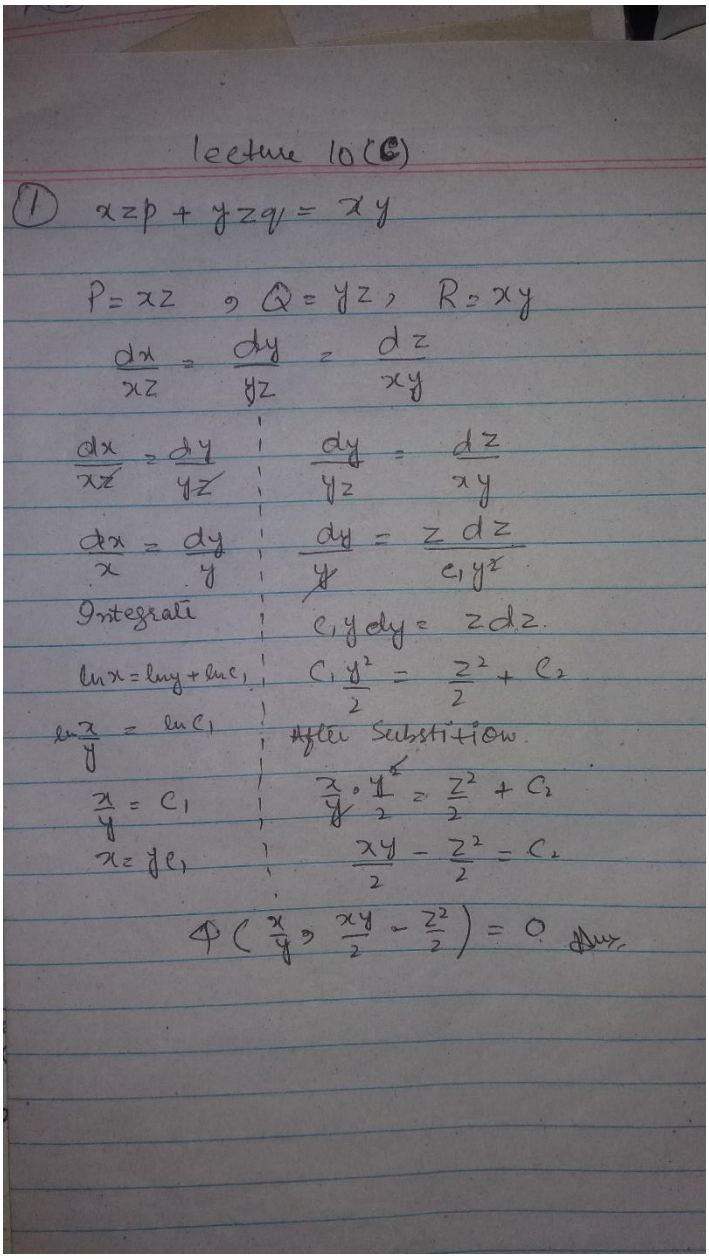
$$\frac{dx}{a} = \frac{dy}{a} \qquad | \qquad \frac{dy}{a} = \frac{dz}{z}$$

$x = y + c_1$ | $\frac{y}{a} = \ln z + c_2$

$x - y = c_1$ | $\frac{y}{a} - \ln z = c_2$

$$\phi\left(x-y, \frac{y}{a} - \ln z\right) = 0$$ Ans

Question 1



lecture 10 (C)

(1) $xzp + yzq = xy$

$P = xz$, $Q = yz$, $R = xy$

$$\frac{dx}{xz} = \frac{dy}{yz} = \frac{dz}{xy}$$

$$\frac{dx}{xz} = \frac{dy}{yz} \qquad \frac{dy}{yz} = \frac{dz}{xy}$$

$$\frac{dx}{x} = \frac{dy}{y} \qquad \frac{dy}{y} = \frac{z\,dz}{c_1 y^2}$$

Integrate $\qquad c_1 y\,dy = z\,dz$

$\ln x = \ln y + \ln c_1 \qquad \frac{c_1 y^2}{2} = \frac{z^2}{2} + c_2$

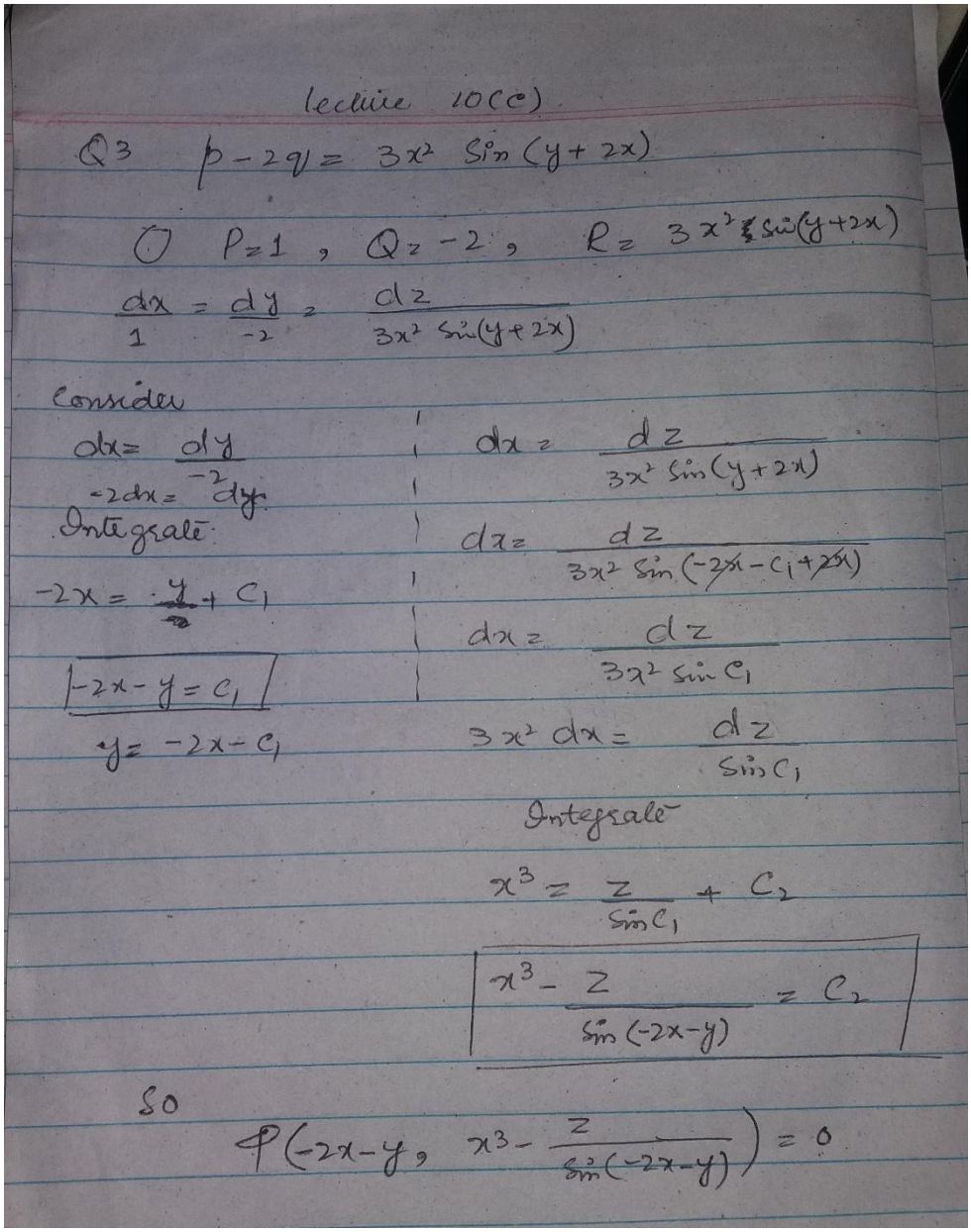
$\ln \frac{x}{y} = \ln c_1$ $\qquad$ After Substition

$\frac{x}{y} = c_1 \qquad \frac{x}{y} \cdot \frac{y^2}{2} = \frac{z^2}{2} + c_2$

$x = yc_1 \qquad \frac{xy}{2} - \frac{z^2}{2} = c_2$

$$\phi\left(\frac{x}{y}, \frac{xy}{2} - \frac{z^2}{2}\right) = 0$$

Question 3



lecture 10(c)

Q3 $p - 2q = 3x^2 \sin(y + 2x)$

① $P = 1$, $Q = -2$, $R = 3x^2 \sin(y+2x)$

$$\frac{dx}{1} = \frac{dy}{-2} = \frac{dz}{3x^2 \sin(y+2x)}$$

Consider

$$dx = \frac{dy}{-2}$$

$$-2dx = dy$$

Integrate

$$-2x = y + c_1$$

$$\boxed{-2x - y = c_1}$$

$$y = -2x - c_1$$

$$dx = \frac{dz}{3x^2 \sin(y+2x)}$$

$$dx = \frac{dz}{3x^2 \sin(-2x - c_1 + 2x)}$$

$$dx = \frac{dz}{3x^2 \sin c_1}$$

$$3x^2 dx = \frac{dz}{\sin c_1}$$

Integrate

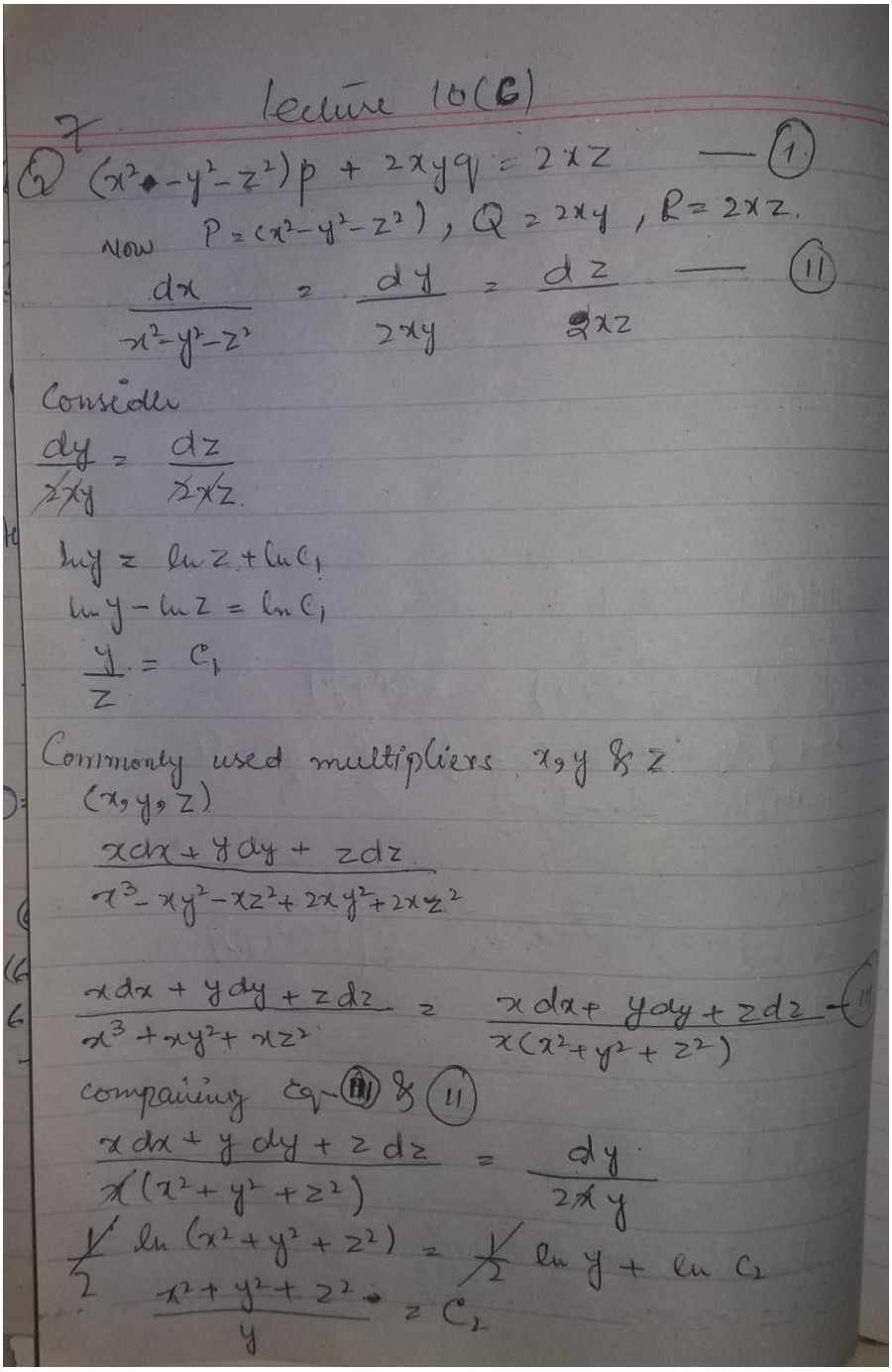
$$x^3 = \frac{z}{\sin c_1} + c_2$$

$$\boxed{x^3 - \frac{z}{\sin(-2x-y)} = c_2}$$

So

$$\phi\left(-2x - y,\ x^3 - \frac{z}{\sin(-2x-y)}\right) = 0$$

Question 5



Lecture 10(C)

7

(2) $(x^2-y^2-z^2)p + 2xyq = 2xz$ — (1)

Now $P = (x^2-y^2-z^2)$, $Q = 2xy$, $R = 2xz$.

$$\frac{dx}{x^2-y^2-z^2} = \frac{dy}{2xy} = \frac{dz}{2xz} \quad \text{— (II)}$$

Consider

$$\frac{dy}{2xy} = \frac{dz}{2xz}$$

$$\ln y = \ln z + \ln c_1$$

$$\ln y - \ln z = \ln c_1$$

$$\frac{y}{z} = c_1$$

Commonly used multipliers x, y & z.

(x, y, z)

$$\frac{x\,dx + y\,dy + z\,dz}{x^3 - xy^2 - xz^2 + 2xy^2 + 2xz^2}$$

$$\frac{x\,dx + y\,dy + z\,dz}{x^3 + xy^2 + xz^2} = \frac{x\,dx + y\,dy + z\,dz}{x(x^2+y^2+z^2)} \quad \text{— (III)}$$

comparing eq- (III) & (II)

$$\frac{x\,dx + y\,dy + z\,dz}{x(x^2+y^2+z^2)} = \frac{dy}{2xy}$$

$$\frac{1}{2}\ln(x^2+y^2+z^2) = \frac{1}{2}\ln y + \ln c_2$$

$$\frac{x^2+y^2+z^2}{y} = c_2$$