BAYES THEOREM

Date

* If one condition is given then apply conditional probability
* If more than one condition is given, Then apply bayes theorem.

Suppose $A_1, A_2 \ldots A_n$ represent n mutually exclusive & collectively exhaustive events with probabilities $P(A_1), P(A_2), \ldots, P(A_n)$. Let B be an arbitrary event with $P(B) \neq 0$ for which conditional probabilities $P(B/A_1), P(B/A_2), \ldots P(B/A_n)$ are also known. Given the information that outcome B has occurred, the revised probabilities $P(A_i/B)$ are determined with the helps of Bayes Theorem using the formula

$$P(A_i/B) = \frac{P(A_i \cap B)}{\sum_{i=1}^{n} P(A_i)\, P(B/A_i)}$$

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In a certain assembly plant three machines B_1, B_2, B_3 make 30%, 45%, 25% respectively of the products. It is known from past experience that 2%, 3%, 2% of prods made by each machine respectively are defective. Now suppose that a finished prod is randomly selected.

(a) What is the probability that it is defective?

(b) If a prod were chosen randomly & found to be defective, what is the probability that it was made by machine B_3?

Sol

Consider following events:

A = the product is defective.

B_1 = The prod is made by machine B_1

B_2 = " ——— B_2

B_3 = ——— B_3

$P(B_1) = 0.3$ $P(A/B_1) = 0.02$

$P(B_2) = 0.45$ $P(A/B_2) = 0.03$

$P(B_3) = 0.25$ $P(A/B_3) = 0.02$

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(a) $P(A) = P(A \cap B_1) + P(A \cap B_2) + P(A \cap B_3)$ —(1)

$P(A/B_1) = \frac{P(A \cap B_1)}{P(B_1)}$

$P(A \cap B_1) = P(A/B_1) \times P(B_1)$

$P(A \cap B_2) = P(A/B_2) \times P(B_2)$

$P(A \cap B_3) = P(A/B_3) \times P(B_3)$

$P(A \cap B_1) = 0.02 \times 0.3 = 3/500$

$P(A \cap B_2) = 0.03 \times 0.45 = 27/2000$

$P(A \cap B_3) = 0.02 \times 0.25 = 1/200$

Put in (1)

$P(A) = 0.0245$

(b) $P(B_3/A) = \frac{P(B_3 \cap A)}{P(A)}$

$P(B_3/A) = \frac{P(A/B_3)\, P(B_3)}{P(A)}$

$= \frac{0.25 \times 0.02}{0.0245}$

$= 0.204$

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Q2 Handout

Let

M = malfunction ~~was reported~~ by human error

A = Station A reported malfunction

B = " B

C = " C

$Q \Rightarrow P(C/M)$

$$P(C/M) = \frac{P(C \cap M)}{P(M)} \quad \text{A}$$

$$P(M) = P(M \cap A) + P(M \cap B) + P(M \cap C)$$

$$P(M/A) = \frac{P(M \cap A)}{P(A)}$$

$$P(M \cap A) = P(A) \times P(M/A)$$

$$P(M) = P(M/A)\,P(A) + P(M/B)\,P(B) + P(M/C)\,P(C)$$

$$P(M) = (7/48) \times (18/43) + (7/45)(15/43) + (5/45) \times (10/43)$$

$$\cancel{P(M) = 0.01627 + 0.1627 + 0.1627}$$

$$P(M) = \cancel{0.1435}\ \cancel{0.4887}\ 0.4419$$

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(A)-2)

$$P(C/M) = \frac{P(M/C)\,P(C)}{P(M)}$$

$$P(C/M) = \frac{5/40 \times 10/43}{0.4419} = 0.02907$$

$$P(C/M) = \frac{0.11627}{0.4419}$$

$$= 0.2632$$

(LAST Pages)

failure John pr depend karega $\Rightarrow P(E/J)$
John failure pe depend karega $\Rightarrow P(J/E)$

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Bayes

Q1.

J = John stamps 20%.
T = Tom stamps 60%.
F = Jeff stamps. 15%.
P = Pat stamps 5%.
E = Fail to stamp the expiration

$P(J) = 0.2$
$P(T) = 0.6$
$P(F) = 0.15$
$P(P) = 0.05$

$P(E/J) = \frac{1}{200} = 5\times10^{-3}$
$P(E/T) = \frac{1}{100} = 0.01$
$P(E/F) = \frac{1}{90} = 0.011$
$P(E/P) = \frac{1}{200} = 5\times10^{-3}$

$$P(J/E) = \frac{P(J\cap E)}{P(E)} = \frac{P(E/J)\,P(J)}{P(E)}$$

$$P(E) = P(E\cap J) + P(E\cap T) + P(E\cap F) + P(E\cap P)$$

$$P(E) = P(E/J)P(J) + P(E/T)P(T) + P(E/F)P(F) + P(E/P)P(P)$$

$$P(E) = 5\times10^{-3}\times0.2 + 0.01\times0.6 + 0.15\times0.011 + 0.05\times5\times10^{-3}$$

$$P(E) = 8.9\cancel{8}\times10^{-3}$$

$$P(J/E) = \frac{5\times10^{-3}\times0.2}{8.9\cancel{8}\times10^{-3}} = 0.11235$$

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Q 2.95

C = A person has cancer

D = A person is diagnosed with cancer.

$P(C) = 0.05$ $\quad P\left(\frac{D}{C}\right) = 0.78$

~~$P(B) = 0.78$~~

~~$P(C) = 0.06$~~ $\quad P\left(\frac{D}{C'}\right) = 0.06$

$P(C') = 1 - 0.05 = 0.95$

$P(D) = ?$

$P(D) = P(D \cap C) + P(D \cap C')$

$= P(D|C)\,P(C) + P(D|C')\,P(C')$

$= 0.096$.

Q 3.

$P(M_1) = 0.2$ $\quad P(D/M_1) = 0.01$

$P(M_2) = 0.3$ $\quad P(D/M_2) = 0.02$

$P(M_3) = 0.5$ $\quad P(D/M_3) = 0.03$

(a) $P\left(\frac{M_2}{D}\right) = ?$

$= \frac{P(M_2 \cap D)}{P(D)}$

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$P(D) = (D \cap M_1) + (D \cap M_2) + (D \cap M_3)$

$P(A \cap B) = P(A/B)\, P(B)$

$P(D) = 0.023.$

$P\left(\frac{M_2}{D}\right) = 0.26$ Ans

(b) $P\left(\frac{M_3}{D}\right) = 0.65$ Ans

Q4. $P(A) = 0.2$ $P(S/A) = 0.95$

$P(B) = 0.3$ $P(S/B) = 0.85$

$P(C) = 0.5$ $P(S/C) = 0.9$

(a) $P(S) = ?$ (b) $P\left(\frac{A}{S}\right) = ?$

(a) $P(S) = P(S \cap A) + P(S \cap B) + P(S \cap C)$

$= 0.895$

(b) $P\left(\frac{A}{S}\right) = \frac{P(A \cap S)}{P(S)} = 0.212$ Ans

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