Probability

Q2-53

N = 500

75 = 100$

150 = 25$

275 = 10$

P(X < 100$) = ?

Sol:

S = {$10, $25, $100}

$$P(\$10) = \frac{275}{500} = 0.55$$

$$P(\$25) = \frac{150}{500} = 0.3$$

$$P(\$100) = \frac{75}{500} = 0.15$$

Now

$$P(X < \$100) = P(\$25) + P(\$10)$$

$$= 0.55 + 0.3$$

$$P(X < \$100) = 0.85$$

Q2.54

$N = 500$

- $S = 210$ = smoke
- $D = 258$ = drink
- $E = 216$ = eat b/w meals

$S \cap D = 122$

$E \cap D = 83$

$S \cap E = 97$

$S \cap D \cap E = 52$

(a). $P(S \cap D') = ?$

(b). $P(E \cap D \cap S') = ?$

(c). $P(S' \cap E') = ?$

Sols

(a). $P(S \cap D') = P(S) - P(S \cap D)$

$= \frac{210}{500} - \frac{122}{500}$

$P(S \cap D') = \frac{88}{500} = 0.176$

(b). $P(E \cap D \cap S') = P(E \cap D) - P(E \cap D \cap S)$

$= \frac{83}{500} - \frac{52}{500}$

$P(E \cap D \cap S') = 0.062$

(c). $P(S' \cap E') = 1 - P(S \cup E)$

$= 1 - [P(S) + P(E) - P(S \cap E)]$

$= 1 - \left[\frac{210}{500} + \frac{216}{500} - \frac{97}{500}\right]$

$= 0.342$

professor)
tics Department)

Q. 2.55

$P(S) = 0.7$

$P(B) = 0.4$

$P(S \cup B) = 0.8$

(a). $P(S \cap B) = ?$

(b). $P(S' \cap B') = ?$

Sol.

(a) $P(S \cap B) = P(S) + P(B) - P(S \cup B)$

$= 0.7 + 0.4 - 0.8$

$= 0.3$

(b). $P(S' \cap B') = 1 - P(S \cup B)$

$= 1 - 0.8$

$P(S' \cap B') = 0.2$

Q. 2.56

$P(TF) = 0.6$

$P(MF) = 0.3$

$P(TF \cap MF) = 0.15$.

(a). $P(TF \cup MF) = ?$

(b). $P(TF' \cap MF') = ?$

Sol.

(a) $\Rightarrow$ $P(TF \cup MF) = P(TF) + P(MF) - P(TF \cap$

$= 0.6 + 0.3 - 0.15$

$= 0.75$

(b). $P(TF' \cap MF') = 1 - P(TF \cup MF)$

$= 1 - 0.75$

$= 0.25$ Ans

58

Q2.

$P(B) = 0.25$

$P(T) = 0.18$

$P(F) = 0.17$

$P(O) = 0.40$

(a). $P(B \cup F) = ?$

$P(B \cap F) = 0.15$

(b). $P(B' \cap F') = ?$

Sol:

(a). $P(B \cup F) = P(B) + P(F) - P(B \cap F)$

$= 0.25 + 0.17 - 0.15$

$P(B \cup F) = 0.27$

(b). $P(B' \cap F') = 1 - P(B \cup F)$

$= 1 - 0.27$

$P(B' \cap F') = 0.73$ Ans

Q2.57

(a). $\frac{5}{26} = 0.19$

(b). $\frac{9}{26} = 0.34$

(c). $\frac{19}{26} = 0.73$

Q2.59.

3 distinct letters — 4 distinct non-zero digits

$$\left(\frac{5}{26} \times \frac{25}{25} \times \frac{24}{24}\right) \times \left(\frac{8}{8} \times \frac{7}{7} \times \frac{6}{6} \times \frac{4}{9}\right)$$

(a, e, i, o, u) — (2, 4, 6, 8)

$$\Rightarrow \frac{5}{26} \times \frac{4}{9}$$

$$\Rightarrow 0.19 \times 0.44$$

$$\Rightarrow 0.084$$

Q2.60.

Two dices rolled.

{ (1,1), (1,2), (1,3), (1,4), (1,5), (1,6),
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6) }

(a) $\frac{5}{36} = 0.138$

(b) $\frac{10}{36} = \frac{5}{18} = 0.27$

Q2.61

(a). $P(D) = \frac{({}^{1}C_{1} \times {}^{8}C_{2})}{{}^{9}C_{3}}$

$= \frac{28}{84}$

$P(D) = 0.33$

(b). $P(2N \cap 1P) = \frac{({}^{4}C_{2} \times {}^{3}C_{1})}{{}^{9}C_{3}}$

$= 0.35$

Q2.66.

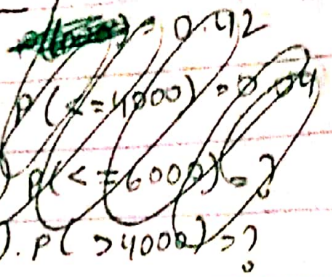
crusts — thin
thin with garlic & oregano
thin with cheese

sauces — standard
a new sauce with more garlic
a " " " fresh basil

(a). $3 \times 3 = 9$

(b) $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

Q2.66 P



$P(X > 6000) = 0.42$

$P(X < 4000) = 0.04$

(a). $P(X \leq 6000) = ?$

(b). $P(X >= 4000) = ?$

Sol:-

(a) → $P(X <= 6000) = 1 - P(X > 6000)$

$= 1 - 0.42$

$= 0.58$

(b) → $P(X >= 4000) = 1 - P(X < 4000)$

$= 1 - 0.04$

$= 0.96$

Ans

Q2.31.

$P(3) = 0.12$

$P(4) = 0.19$

$P(5) = 0.28$

$P(6) = 0.24$

$P(7) = 0.10$

$P(8) = 0.07$

$P(8 \text{ or more}) = 0.07$

$P(\text{atleast } 5 \text{ cars}) = ?$

Sol:

$P(\text{atleast } 5 \text{ cars}) = P(5) + P(6) + P(7) + P(8 \text{ or more})$

$= 0.28 + 0.24 + 0.10 + 0.07$

$= 0.69$

Q2.31

(a). P(no more than 4 cars) = P(3) + P(4)

" = 0.12 + 0.19

" = 0.31

(b). P(fewer than 8 cars) = P(3) + P(4) + P(5) + P(6) + P

" = 0.12 + 0.19 + 0.28 + 0.24

" = 0.93

(c). P(either 3 or 4 cars) = P(3) + P(4)

= 0.12 + 0.19

= 0.31

Q2.79.

Edu	Male		female		
Elementary	MNE	38	FNE	45	= 83
Secondary	MNS	28	FNS	50	= 78
College	MNC	22	FNC	17	= 39
		88		112	200

(a). $P(M/S) = \frac{P(M \cap S)}{P(S)} = \frac{\frac{28}{200}}{\frac{78}{200}} = 0.35$

(b). $P(C'/F) = \frac{P(C' \cap F)}{P(F)} = \frac{P(F) - P(C \cap F)}{P(F)}$

$= \frac{\frac{112}{200} - \frac{17}{200}}{\frac{112}{200}} = 0.84$

Q2.80

	Non	Moderate	Heavy
H	21	36	HNHS 30 → 87
NH	48 NSNH	26	19 → 93 NH
	69 NS	62	HS 49 → 180

$P(H/HS) = ?$

$P(NS/NH) = ?$

Q2.

(a) $P(H/HS) = \frac{P(H \cap HS)}{P(HS)} = \frac{\frac{30}{180}}{\frac{49}{180}} = 0.612$

(b). $P(NS/NH) = \frac{P(NS \cap NH)}{P(NH)} = \frac{\frac{48}{180}}{\frac{93}{180}} = 0.516$

Q2.51.

$N = 100$

$N = 42$

$P = 68$

$H = 54$

$N \cap H = 22$

$N \cap P = 25$

$H \cap (N' \cap P') = 7$

$H \cap N \cap P = 10$

$H' \cap N' \cap P' = 8$

(a). $P(H \cap N/P) = \frac{P(H \cap N \cap P)}{P(P)} = \frac{\frac{10}{100}}{\frac{68}{100}} = 0.147$

(b). $P\left(H \cap \overline{N}/P'\right) = \frac{P(H \cap N \cap P')}{P(P')} = \frac{P(H \cap N) - P(H \cap N \cap P)}{1 - P(P)}$

$= \frac{\frac{22}{100} - \frac{10}{100}}{1 - \frac{68}{100}}$

$= \frac{\frac{12}{100}}{\frac{32}{100}} = 0.375$

Q2.82

B_1' = 0.10

B_2' = 0.08

B_3' = 0.12
(independent)

(a). $P(B_1 \cap B_2') = P(B_1) \cdot P(B_2')$

$= (1 - P(B_1')) \times (P(B_2'))$

$= (1 - 0.10)(0.08)$

$= (0.9)(0.08)$

$= 0.072$

(b). $P(B_1 \cap B_2 \cap B_3') = P(B_1) \cdot P(B_2) \cdot P(B_3')$

$= (1 - P(B_1'))\ (1 - P(B_2'))\ P(B_3')$

$= (1 - 0.10)(1 - 0.08)(0.12)$

$= 0.099$

Q2.84.

P(Oil damage) = 0.25

P(OF) = 0.40

P(OC ∩ OF) = 0.14

(a). $P(OF/OC) = \frac{P(OF \cap OC)}{P(OC)} = \frac{0.14}{0.25} = 0.56$

(b). $P(OC/OF) = \frac{P(OC \cap OF)}{P(OF)} = \frac{0.14}{0.40} = 0.35$

Q2.92. $P(T) = 1/4 = 0.25$ (every fourth disk is tested)
(independent)

$P(T_1') = 0.01$
$P(T_2') = 0.03$
$P(T_3') = 0.02$
$P(T_4') = 0.01$
} failure rates

$P(T_1) = 1 - P(T_1') = 1 - 0.01 = 0.99$
$P(T_2) = 1 - P(T_2') = 1 - 0.03 = 0.97$
$P(T_3) = 1 - P(T_3') = 1 - 0.02 = 0.98$
$P(T_4) = 1 - P(T_4') = 1 - 0.01 = 0.99$
} success rate

$P(\text{all programs are successful}) = P(T_1) \cdot P(T_2) \cdot P(T_3) \cdot P(T_4)$
$= 0.99 \times 0.97 \times 0.98 \times 0.99$
$= 0.931$

defective CD
$P(D) = 1 - P(\text{all programs are successful})$
if any program fails

$P(D) = 1 - 0.931$
$P(D) = 0.068$

(a). $P(T \cap D) = P(T) \cdot P(D)$
$= 0.25 \times 0.068$
$= 0.017$

(b). $P(\text{failed test 2 or 3}) = P(T_2) \cdot P(T_3) \cdot (1 - P(T_2) \cdot P($

$= (0.99)(0.99)(1 - 0.99 \cdot 0.98)$

$= (0.9801)(0.0494)$

$= 0.0484$

(c). $P(\text{CD's rejected in a sample of } 100) = 100 \cdot P(D)$

$= 100 \cdot 0.068$

$= 6.8 \approx 7$

(d). $P(T/D) = \frac{P(T \cap D)}{P(D)} = \frac{P(T) \cdot P(D)}{P(D)}$

$= \frac{0.25 \times 0.068}{0.068}$

$= 0.25$

$P(T_2' \cap T_3' / T) = \frac{P(T_2' \cap T_3' \cap T)}{P(T)} = \frac{P(T_2') \cdot P(T_3') \cdot P}{P(T)}$

Q2.93.

(independent)

$P(S-E) = 0.96$

(a).

Q2.94.

$P(T) = 0.7$

$P(N) = 0.9$

$P(T' \cap N') = ?$

$\Rightarrow P(T' \cap N') = P(T') \times P(N')$

$= (1 - P(T)) \times (1 - P(N))$

$= (1 - 0.7) \times (1 - 0.9)$

$= (0.3)(0.1)$

$P(T' \cap N') = 0.03$

Ans