

**COUNTING SAMPLE POINT**

- 2.21 Registrants at a large convention are offered 6 sightseeing tours on each of 3 days. In how many ways can a person arrange to go on a sightseeing tour planned by this convention?
- 2.22 In a medical study patients are classified in 8 ways according to whether they have blood type  $AB+$ ,  $AB-$ ,  $A+$ ,  $A-$ ,  $B+$ ,  $B-$ ,  $O+$ , or  $O-$ , and also according to whether their blood pressure is low, normal, or high. Find the number of ways in which a patient can be classified.
- 2.23 If an experiment consists of throwing a die and then drawing a letter at random from the English alphabet, how many points are there in the sample space?
- 2.24 Students at a private liberal arts college are classified as being freshmen, sophomores, juniors, or seniors, and also according to whether they are male or female. Find the total number of possible classifications for the students of that college.
- 2.25 A certain shoe conies in 5 different styles with each style available in 4 distinct colors. If the store wishes to display pairs of these shoes showing all of its various styles and colors, how many different pairs would the store have on display?
- 2.27 A developer of a new subdivision offers a prospective home buyer a choice of 4 designs, 3 different heating systems, a garage or carport, and a patio or screened porch. How many different plans are available to this buyer?
- 2.28 A drug for the relief of asthma can be purchased from 5 different manufacturers in liquid, tablet, or capsule form, all of which come in regular and extra strength. How many different ways can a doctor prescribe the drug for a patient suffering from asthma?
- 2.29 In a fuel economy study, each of 3 race cars is tested using 5 different brands of gasoline at 7 test sites located in different regions of the country. If 2 drivers are used in the study, and test runs are made once under each distinct set of conditions, how many test runs are needed?
- 2.30 In how many different ways can a true-false test consisting of 9 questions be answered?
- 2.31 If a multiple-choice test consists of 5 questions each with 4 possible answers of which only 1 is correct,  
(a) In how many different ways can a student check off one answer to each question?  
(b) In how many ways can a student, check off one answer to each question and get all the answers wrong?
- 2.32 (a) How many distinct permutations can be made from the letters of the word *columns*?  
(b) How many of these permutations start with the letter m?
- 2.33** A witness to a hit-and-run accident told the police that the license number contained the letters RLH followed by 3 digits, the first of which is a 5. If the witness cannot recall the last 2 digits, but is certain that all 3 digits are different, find the maximum number of automobile registrations that the police may have to check.
- 2.34** (a) In how many ways can 6 people be lined up to get on a bus?  
(b) If 3 specific persons, among 6, insist on following each other, how many ways are possible?  
(c) If 2 specific persons, among 6, refuse to follow each other, how many ways are possible?
- 2.35** A contractor wishes to build 9 houses, each different in design. In how many ways can he place these houses on a street if 6 lots are on one side of the street and 3 lots are on the opposite side?
- 2.36** (a) How many three-digit numbers can be formed from the digits 0, 1, 2, 3, 4, 5, and 6, if each digit can be used only once?  
(b) How many of these are odd numbers?  
(c) How many are greater than 330?
- 2.37** In how many ways can 4 boys and 5 girls sit in a row if the boys and girls must alternate?
- 2.40 In how many ways can 5 starting positions on a basketball team be filled with 8 men who can play any of the positions?

## Handout #1

## PROBABILITY & STATISTICS (MT-331)

- 2.41** Find the number of ways that 6 teachers can be assigned to 4 sections of an introductory psychology course if no teacher is assigned to more than one section.
- 2.43** In how many ways can 5 different trees be planted in a circle?
- 2.44** In how many ways can a caravan of 8 covered wagons from Arizona be arranged in a circle?
- 2.45** How many distinct permutations can be made from the letters of the word *infinity*?
- 2.46** In how many ways can 3 oaks, 4 pines, and 2 maples be arranged along a property line if one does not distinguish among trees of the same kind?
- 2.47** A college plays 12 football games during a season. In how many ways can the team end the season with 7 wins, 3 losses, and 2 ties?
- 2.48 Nine people are going on a skiing trip in 3 cars that hold 2, 4, and 5 passengers, respectively. In how many ways is it possible to transport the 9 people to the ski lodge, using all cars?

### PROBABILITY

- 2.53 A box contains 500 envelopes of which 75 contain \$100 in cash, 150 contain \$25, and 275 contain \$10. An envelope may be purchased for \$25. What is the sample space for the different amounts of money? Assign probabilities to the sample points and then find the probability that the first envelope purchased contains less than \$100.
- 2.54 Suppose that in a senior college class of 500 students it is found that 210 smoke, 258 drink alcoholic beverages, 216 eat between meals, 122 smoke and drink alcoholic beverages, 83 eat between meals and drink alcoholic beverages, 97 smoke and eat between meals, and 52 engage in all three of these bad health practices. If a member of this senior class is selected at random, find the probability that the student
- (a) smokes but does not drink alcoholic beverages;
  - (b) eats between meals and drinks alcoholic beverages but does not smoke;
  - (c) neither smokes nor eats between meals.
- 2.55** The probability that an American industry will locate in Shanghai, China is 0.7, the probability that it will locate in Beijing, China is 0.4, and the probability that it will locate in either Shanghai or Beijing or both is 0.8. What is the probability that the industry will locate
- (a) in both cities?
  - (b) in neither city?
- 2.56** From past experiences a stockbroker believes that under present economic conditions a customer will invest in tax-free bonds with a probability of 0.6, will invest in mutual funds with a probability of 0.3, and will invest in both tax-free bonds and mutual funds with a probability of 0.15. At this time, find the probability that a customer will invest
- (a) in either tax-free bonds or mutual funds:
  - (b) in neither tax-free bonds nor mutual funds.
- 2.57 2.57** If a letter is chosen at random from the English alphabet, find the probability that the letter (a) is a vowel exclusive of y;
- (b) is listed somewhere ahead of the letter *j*
  - (c) is listed somewhere after the letter *g*.
- 2.58** An automobile manufacturer is concerned about a possible recall of its best-selling four-door sedan. If there were a recall, there is 0.25 probability that a defect is in the brake system, 0.18 in the transmission, 0.17 in the fuel system, and 0.40 in some other area.
- (a) What is the probability that the defect is the brakes or the fueling system if the probability of defects in both systems simultaneously is 0.15?
  - (b) What is the probability that there are no defects in either the brakes or the fueling system?
- 2.59** If each coded item in a catalog begins with 3 distinct letters followed by 4 distinct nonzero digits, find the probability of randomly selecting one of these coded items with the first letter a vowel and the last digit even.
- 2.60 A pair of fair dice is tossed. Find the probability of getting
- (a) a total of 8
  - (b) at most a total of 5.

- 2.62 If 3 books are picked at random from a shelf containing 5 novels, 3 books of poems, and a dictionary, what is the probability that
- the dictionary is selected?
  - 2 novels and 1 book of poems are selected?

2.66 Dom's Pizza Company uses taste testing and statistical analysis of the data prior to marketing any new product. Consider a study involving three types of crusts (thin, thin with garlic and oregano, and thin with bits of cheese). Dom's is also studying three sauces, (standard, a new sauce with more garlic, and a new sauce with fresh basil).

- How many combinations of crust and sauce are involved?
- What is the probability that a judge will get a plain thin crust with a standard sauce for his first taste test?

2.68 Interest centers around the life of an electronic component. Suppose it is known that the probability that the component survives for more than 6000 hours is 0.42. Suppose also that the probability that the component survives *no longer than* 4000 hours is 0.04.

- What is the probability that the life of the component is less than or equal to 6000 hours?
- What is the probability that the life is greater than 4000 hours?

Q2.31 If the probabilities that an automobile mechanic will service 3, 4, 5, 6, 7, or 8 or more cars on any given workday are, respectively, 0.12, 0.19, 0.28, 0.24, 0.10, and 0.07, what is the probability that he will service at least 5 cars on his next day at Work?

2.71 Consider the situation of Example 2.31 on page 54.

- What is the probability that no more than 4 cars will be serviced by the mechanic?
- What is the probability that he will service fewer than 8 cars?
- What is the probability that he will service either 3 or 4 cars?

2.70 Factory workers are constantly encouraged to practice zero tolerance when it comes to accidents in factories. Accidents can occur because the working environment or conditions themselves are unsafe. On the other hand, accidents can occur due to carelessness or so-called human error. In addition, the worker's shift 7:00 A.M.-3:00 P.M. (day shift), 3:00 P.M.-11:00 P.M. (evening shift), and 11:00 P.M.-7:00 A.M. (graveyard shift) may be a factor. During the last year, 300 accidents have occurred. The percentages of the accidents for the condition combinations are as follows:

Shift	Unsafe Conditions	Human Error
Day	5%	32%
Evening	6%	25%
Graveyard	2%	30%

If an accident report is selected randomly from the 300 reports,

- What is the probability that the accident occurred on the graveyard shift?
- What is the probability that the accident occurred due to human error?
- What is the probability that the accident occurred due to unsafe conditions?
- What is the probability that the accident occurred on either the evening or graveyard shift?

### CONDITIONAL PROBABILITY

2.79 A random sample of 200 adults are classified below by sex and their level of education attained.

Education	Male	Female
Elementary	38	45
Secondary	28	50
College	22	17

If a person is picked at random from this group, find the probability that

- The person is a male, given that the person has a secondary education
- The person does not have a college degree, given that the person is a female.

# Handout #1

# PROBABILITY & STATISTICS (MT-331)

**2.80** In an experiment to study the relationship of hypertension and smoking habits, the following data are collected for 180 individuals:

	<b>Nonsmokers</b>	<b>Moderate Smokers</b>	<b>Heavy Smokers</b>
<i>H</i>	21	36	30
<i>NH</i>	48	26	19

Where *H* and *NH* in the table stand for *Hypertension* and *No hypertension*, respectively. If one of these individuals is selected at random, find the probability that the person is

- Experiencing hypertension, given that the person is a heavy smoker;
- a nonsmoker, given that the person is experiencing no hypertension.

**2.81** In the senior year of a high school graduating class of 100 students, 42 studied mathematics, 68 studied psychology, 54 studied history, 22 studied both mathematics and history, 25 studied both mathematics and psychology, 7 studied history but neither mathematics nor psychology, 10 studied all three subjects, and 8 did not take any of the three. If a student is selected at random, find the probability that

- a person enrolled in psychology takes all three subjects
- a person not taking psychology is taking both history and mathematics.

**2.82** A manufacturer of a flu vaccine is concerned about the quality of its flu serum. Batches of serum are processed by three different departments having rejection rates of 0.10, 0.08, and 0.12, respectively. The inspections by the three departments are sequential and independent.

- What is the probability that a batch of serum survives the first departmental inspection but is rejected by the second department?
- What is the probability that a batch of serum is rejected by the third department?

**2.84** The probability that an automobile being filled with gasoline will also need an oil change is 0.25; the probability that it needs a new oil filter is 0.40; and the probability that both the oil and filter need changing is 0.14.

- If the oil had to be changed, what is the probability that a new oil filter is needed?
- If a new oil filter is needed, what is the probability that the oil has to be changed?

**2.92** Before the distribution of certain statistical software every fourth compact disk (CD) is tested for accuracy. The testing process consists of running four independent programs and checking the results. The failure rates for the 4 testing programs are, respectively, 0.01, 0.03, 0.02, and 0.01.

- What is the probability that a CD was tested and failed any test?
- Given that a CD was tested, what is the probability that it failed program 2 or 3?
- In a sample of 100, how many CDs would you expect to be rejected?
- Given a CD was defective, what is the probability that it. Was tested?

**2.93** A town has 2 fire engines operating independently. The probability that a specific engine is available when needed is 0.96.

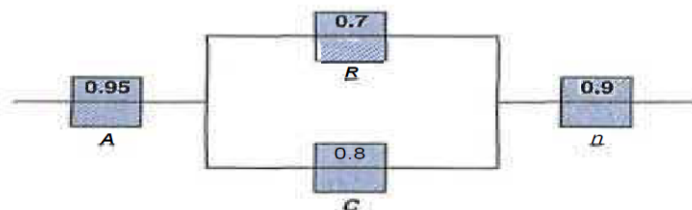
- What is the probability that neither is available when needed?
- What is the probability that a fire: engine is available when needed?

**2.94** The probability that Tom will be alive in 20 years is 0.7, and the probability that Nancy will be alive in 20 years is 0.9. If we assume independence for both, what is the probability that neither will be alive in 20 years?

**2.95** One overnight case contains 2 bottles of aspirin and 3 bottles of thyroid tablets. A second tote bag contains 3 bottles of aspirin, 2 bottles of thyroid tablets, and 1 bottle of laxative tablets. If 1 bottle of tablets is taken at random from each piece of luggage, find the probability that

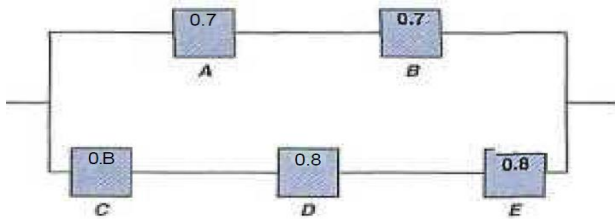
- Both bottles contain thyroid tablets
- Neither bottle contains thyroid tablets
- The 2 bottles contain different tablets.

**2.98** Suppose the diagram of an electrical system is given in Figure 2.10. What is the probability that the system works? Assume the components fail independently.



**2.99** A circuit system is given in Figure 2.11. Assume the components fail independently.

- What is the probability that the entire system works?
- Given that the system works, what is the probability that the component A is not working?



**2.100** In the situation of Exercise 2.99, it is known that the system does not work. What is the probability that the component A also does not work?

**Q1-**Disks of polycarbonate plastic from a supplier are analyzed for scratch and shock resistance. The results from 100 disks are summarized as follows:

		shock resistance	
		high	low
scratch resistance	high	70	9
	low	16	5

Let  $A$  denote the event that a disk has high shock resistance, and let  $B$  denote the event that a disk has high scratch resistance. If a disk is selected at random, determine the following probabilities:

- (a)  $P(A)$       (b)  $P(B)$   
 (c)  $P(A')$     (d)  $P(A \cap B)$   
 (e)  $P(A \cup B)$    (f)  $P(A' \cup B)$

**Q2-**Disks of polycarbonate plastic from a supplier are analyzed for scratch and shock resistance. The results from 100 disks are summarized as follows:

		shock resistance	
		high	low
scratch resistance	high	70	9
	low	16	5

- (a) If a disk is selected at random, what is the probability that its scratch resistance is high and its shock resistance is high?  
 (b) If a disk is selected at random, what is the probability that its scratch resistance is high or its shock resistance is high?  
 (c) Consider the event that a disk has high scratch resistance and the event that a disk has high shock resistance. Are these two events mutually exclusive?

**Q3-**The analysis of shafts for a compressor is summarized by conformance to specifications:

		roundness conforms	
		yes	no
surface finish conforms	yes	345	5
	no	12	8

- (a) If we know that a shaft conforms to roundness requirements, what is the probability that it conforms to surface finish requirements?  
 (b) If we know that a shaft does not conform to roundness requirements, what is the probability that it conforms to surface finish requirements?

**Q4.** The following table summarizes the analysis of samples of galvanized steel for coating weight and surface roughness:

		coating weight	
		high	low
surface roughness	high	12	16
	low	88	34

- (a) If the coating weight of a sample is high, what is the probability that the surface roughness is high?  
 (b) If the surface roughness of a sample is high, what is the probability that the coating weight is high?  
 (c) If the surface roughness of a sample is low, what is the probability that the coating weight is low?

## Handout #1

## PROBABILITY & STATISTICS (MT-331)

**Q5-** A batch of 500 containers for frozen orange juice contains 5 that are defective. Two are selected, at random, without replacement from the batch.

- What is the probability that the second one selected is defective given that the first one was defective?
- What is the probability that both are defective?
- What is the probability that both are acceptable?

**Q6.** A maintenance firm has gathered the following information regarding the failure mechanisms for air conditioning systems:

		evidence of gas leaks	
		yes	no
evidence of electrical failure	yes	55	17
	no	32	3

The units without evidence of gas leaks or electrical failure showed other types of failure. If this is a representative sample of AC failure, find the probability

- That failure involves a gas leak
- That there is evidence of electrical failure given that there was a gas leak
- That there is evidence of a gas leak given that there is evidence of electrical failure

(Ans key)

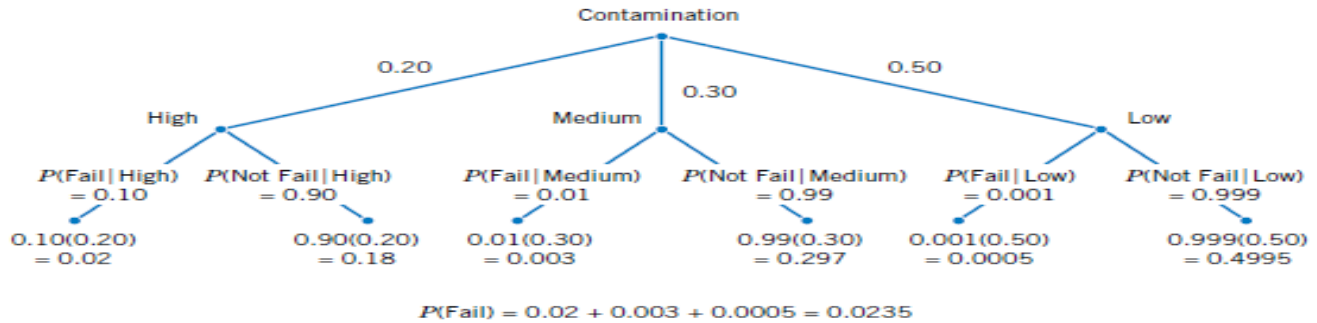
- 2-65. (a)  $4/499 = 0.0080$   
 (b)  $(5/500)(4/499) = 0.000080$   
 (c)  $(495/500)(494/499) = 0.98$   
 2-67. (a) 0.813 (b) 0.632  
 (c) 0.764

**2-78.** Computer keyboard failures are due to faulty electrical connects (12%) or mechanical defects (88%). Mechanical defects are related to loose keys (27%) or improper assembly (73%). Electrical connect defects are caused by defective wires (35%), improper connections (13%), or poorly welded wires (52%).

- Find the probability that a failure is due to loose keys.
- Find the probability that a failure is due to improperly connected or poorly welded wires.

Continuing with the semiconductor manufacturing example, assume the following probabilities for product failure subject to levels of contamination in manufacturing:

Probability of Failure	Level of Contamination
0.10	High
0.01	Medium
0.001	Low



In a particular production run, 20% of the chips are subjected to high levels of contamination, 30% to medium levels of contamination, and 50% to low levels of contamination. What is the probability that a product using one of these chips fails? Let

$H$  denote the event that a chip is exposed to high levels of contamination

$M$  denote the event that a chip is exposed to medium levels of contamination

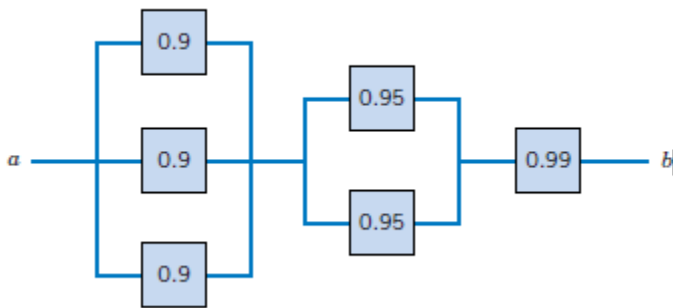
$L$  denote the event that a chip is exposed to low levels of contamination

Then,

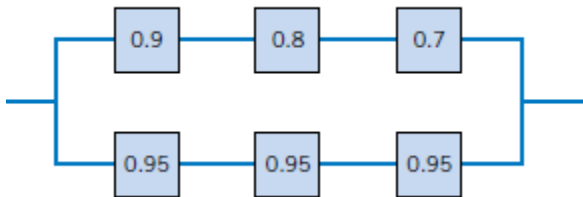
$$\begin{aligned}
 P(F) &= P(F|H)P(H) + P(F|M)P(M) + P(F|L)P(L) \\
 &= 0.10(0.20) + 0.01(0.30) + 0.001(0.50) = 0.0235
 \end{aligned}$$

This problem is also conveniently solved using the tree diagram in Fig. 2-16.

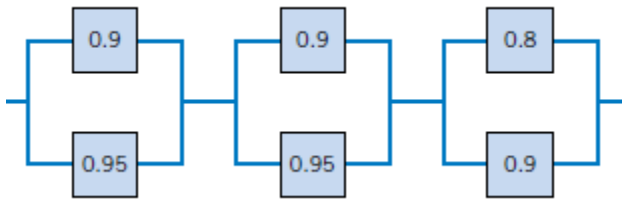
Q1-The following circuit operates only if there is a path of functional devices from left to right. The probability that each device functions is shown on the graph. Assume that devices fail independently. What is the probability that the circuit operates?



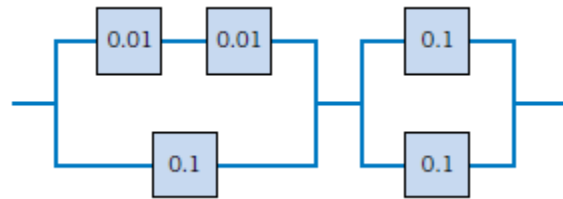
Q2-The following circuit operates if and only if there is a path of functional devices from left to right. The probability that each device functions is as shown. Assume that the probability that a device is functional does not depend on whether or not other devices are functional. What is the probability that the circuit operates?



Q3-The following circuit operates if and only if there is a path of functional devices from left to right. The probability each device functions is as shown. Assume that the probability that a device functions does not depend on whether or not



Q4- The following circuit operates if and only if there is a path of functional devices from left to right. Assume devices fail independently and that the probability of failure of each device is as shown. What is the probability that the circuit operates?



Q5-. In the layout of a printed circuit board for an electronic product, there are 12 different locations that can accommodate chips. If five different types of chips are to be placed on the board, how many different layouts are possible? If the five chips that are placed on the board are of the same type, how many different layouts are possible?

#### BAYES THEOREM:

Q1- Suppose that the four inspectors at a film factory are supposed to stamp the expiration date on each package of film at the end of the assembly line. John, who stamps 20% of the packages, fails to stamp the expiration date once in every 200 packages; Tom, who stamps 60% of the packages, fails to stamp the expiration date once in every 100 packages; Jeff, who stamps 15% of the packages, fails to stamp the expiration date once in every 90 packages; and Pat, who stamps 5% of the packages, fails to stamp the expiration date once in every 200 packages. If a customer complains that her package of film does not show the expiration date, what is the probability that it was inspected by John?

Q2- A regional telephone company operates three identical relay stations at different locations. During a one-year period, the number of malfunctions reported by each station and the causes are shown below.

Station	A	B	C
Problems with electricity supplied	2	1	1
Computer malfunction	4	3	2
Malfunctioning electrical equipment	5	4	2
Caused by other human errors	7	7	5

Suppose that a malfunction was reported and it was found to be caused by other human errors. What is the probability that it came from station C?

2.95 In a certain region of the country it is known from past experience that the probability of selecting an adult over 40 years of age with cancer is 0.05. If the probability of a doctor correctly diagnosing a person with cancer as having the disease is 0.78 and the probability of incorrectly diagnosing a person without cancer as having the disease is 0.06, what is the probability that an adult over 40 years of age is diagnosed as having cancer?



Q3- Three different machines M1, M2, and M3 are used to produce similar electronic components. Machines M1, M2, and M3 produce 20%, 30% and 50% of the components respectively. It is known that the probabilities that the machines produce defective components are 1% for M1, 2% for M2, and 3% for M3. If a component is selected randomly from a large batch, and that component is defective, find the probability that it was produced:

(a) by M2, and (b) by M3.

Q4. It is known that of the articles produced by a factory, 20% come from Machine A, 30% from Machine B, and 50% from Machine C. The percentages of satisfactory articles among those produced are 95% for A, 85% for B and 90% for C. An article is chosen at random.

f) What is the probability that it is satisfactory?

g) Assuming that the article is satisfactory, what is the probability that it was produced by Machine A?