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SE-036

MS (Assignment # 02)

Q#1: Patients arrive for a physical examination according to poisson process ----------, if 2 attenders are available.

Solution:

Given :-

$\lambda = 1/hr$

$\mu = \frac{1}{45_3} \times 60^4 = \frac{4}{3} = 1.33$ patients/hr.

$C = 2$

$L_q = ?$

(i) $\rho = \frac{\lambda}{C\mu} = \frac{1}{2(1.33)} = 0.3759$ $\boxed{= 37.6\%}$

(ii) $\rho_0 = \{[\sum_{n=0}^{c-1} \frac{(c\rho)^n}{n!}] + [(c\rho)^c(1/c!)(1/1-\rho)]\}^{-1}$

$\therefore c\rho = \frac{\lambda}{\mu} = \frac{1}{1.33} = 0.7518$

$\rho_0 = \{[\frac{(0.7518)^0}{0!} + \frac{(0.7518)^1}{1!}] + [(0.7518)^2(1/2!)(1/1-0.3759)]\}^{-1}$

$\rho_0 = \{1 + 0.7518 + [0.5653 \times 1/2 \times (1/0.6241)]\}^{-1}$

$\rho_0 = \{1 + 0.7518 + 0.4529\}^{-1}$

$\rho_0 = \{2.2047\}^{-1}$

$\boxed{\rho_0 = 0.4535\%}$ or

$\boxed{\rho_0 = 45.35\%}$

(iii) $L_s = C\rho + \frac{(C\rho)^{C+1} P_0}{C(C!)(1-\rho)^2}$

$L_s = 0.7518 + \frac{(0.7518)^3 (0.4535)}{2(2!)(1-0.3759)^2}$

$L_s = 0.7518 + \frac{0.4249(0.4535)}{4(0.3895)}$

$L_s = 0.7518 + \frac{0.1927}{1.5580}$

$L_s = 0.7518 + 0.1236$

$= 0.8754$

$\boxed{L_s \simeq 0 \text{ or } 1 \text{ patient}}$

(iv) $W_s = \frac{L_s}{\lambda} = \frac{0.8754}{1} = \boxed{0.8754 \text{ hrs}}$

(v) $W_q = W_s - \frac{1}{\mu}$

$= 0.8754 - \frac{1}{1.33}$

$\boxed{W_q = 0.1236 \text{ hrs}}$

(vi) $L_q = \lambda W_q$

$L_q = 1(0.1236)$

$= 0.1236$

$\boxed{L_q \simeq 0 \text{ or } 1 \text{ patient}}$

Ans!

Result :

Avg. no of delayed patient is 0 or 1

Q2: A tool crib with one attendant serves a group of 10 mechanics------ justify by comparing answers?

Solution:

Given:-

Mechanic's arrival time = 20 mins

Mechanic's arrival rate $(\lambda) = \frac{1}{20} = 0.05/min$

$= 0.05 \times 60 = 3/hr$

Service time = 3 mins

service rate $(\mu) = \frac{1}{3} = 0.33/mins$

$= 0.33 \times 60 = 20/hr$

Required:-

Whether 2 attendants are required or not = ?

WHEN THERE IS ONLY ONE ATTENDANT.

- $\rho = \frac{\lambda}{\mu}$

$= \frac{3}{20} = 0.15$

$\rho = 15\%$

- $\rho_0 = 1 - \rho$

$= 1 - 0.15 = 0.85$

$\rho_0 = 85\%$

- $L_s = \frac{\lambda}{\mu - \lambda}$

$= \frac{3}{20-3} = \frac{3}{17} = 0.176$

$L_s \cong 0$ or 1 mechanic

- $W_s = \frac{1}{\mu - \lambda}$

$= \frac{1}{20-3} = \frac{1}{17} = 0.058\ hrs$

- $W_q = \dfrac{\lambda}{\mu(\mu-\lambda)}$

$$= \frac{3}{20(20-3)} = \frac{3}{20\times17}$$

$$= 8.82\times10^{-3}$$

$$\boxed{W_q = 0.00882 \text{ hrs}}$$

- $L_q = \dfrac{\lambda^2}{\mu(\mu-\lambda)}$

$$= \frac{3^2}{20(20-3)} = \frac{9}{20\times17} = 0.026$$

$L_q \cong 0$ or 1 mechanic

WHEN THERE ARE 2 ATTENDANTS (c=2)

- $\rho = \dfrac{\lambda}{c\mu}$

$$\rho = \frac{3}{2\times20} = \frac{3}{40} = 0.075$$

$\rho = 7.5\%$

- $c\rho = \dfrac{\lambda}{\mu}$

$$= \frac{3}{20} = 0.15$$

$c\rho = 0.15$

- $\rho_0 = \left\{\left[\sum_{n=0}^{c-1}\frac{(c\rho)^n}{n!}\right] + \left[(c\rho)^c \times \frac{1}{c!}\times\left(\frac{1}{1-\rho}\right)\right]\right\}^{-1}$

$$\rho_0 = \left\{\left[\sum_{n=0}^{1}\frac{(0.15)^n}{n!}\right] + \left[(0.15)^2\times\frac{1}{2!}\times\frac{1}{(1-0.075)}\right]\right\}^{-1}$$

$$\rho_0 = \left\{\left[\frac{(0.15)^0}{0!} + \frac{(0.15)^1}{1!}\right] + \left[(0.0225)\times\frac{1}{2}\times\frac{1}{0.925}\right]\right\}^{-1}$$

$P_0 = \{[1+0.15]+[0.0225 \times 0.5 \times 1.08]\}^{-1}$

$P_0 = \{1.15+0.01215\}^{-1} = (1.162)^{-1} = 1/1.162$

$P_0 = 0.8604$

$P_0 = 86.04\%$

- $L_s = (C\rho) + \dfrac{(C\rho)^{C+1} P_0}{C \times C! \times (1-\rho)^2}$

$= 0.15 + \dfrac{(0.15)^3 \times 0.8604}{2 \times 2! \times (1-0.075)^2}$

$= 0.15 + \dfrac{(0.003375) \times 0.8604}{4 \times (0.855)}$

$L_s = 0.15 + 8.4 \times 10^{-4}$

$= 0.1508$

$L_s \cong 0$ or 1 mechanic

- $W_s = \dfrac{L_s}{\lambda}$

$= \dfrac{0.1508}{3}$

$W_s = 0.0502$ hrs

- $W_q = W_s - \dfrac{1}{\mu}$

$= 0.0502 - \dfrac{1}{20}$

$W_q = 0.0002$ hr

$L_q = \lambda \times W_q$

$= 3 \times 0.0002$

$= 0.0006$

$L_q = 0$ or 1 mechanic

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Results:-

The utilization of system decreases from 15% to 7.5%. when we have 2 attendants. However, the waiting time of mechanic is slightly decreased with two attendants but it does not have much impact. So, since, system utilization is more with only 1 attendant, it is adviseable to not have second attendant.

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Q3: The port of trop can services 3 ships at a time however .... operational characteristics

Solution:-

Given:

Arrival rate (λ) = 7/week

Service rate (μ) = 8/week

No. of servers (c) = 3

Waiting space in ship = 3

Required:

$\rho = ?$

$\rho_0 = ?$

$L_q = ?$

$L_s = ?$

$W_q = ?$

$W_s = ?$

1) $\rho = \frac{\lambda}{c\mu} = \frac{7}{3 \times 8} = \frac{7}{24} = 0.291$

$\boxed{\rho = 29.1\%}$

$c\rho = \frac{\lambda}{\mu}$

$= \frac{7}{8}$

$c\rho = 0.875$

2) $\rho_0 = \left\{\left[\sum_{n=0}^{c-1} \frac{(c\rho)^n}{n!}\right] + \left[(c\rho)^c \times \frac{1}{c!} \times \frac{1}{1-\rho}\right]\right\}^{-1}$

$= \left\{\left[\sum_{n=0}^{2} \frac{(0.875)^n}{n!}\right] + \left[(0.875)^3 \times \frac{1}{3!} \times \frac{1}{1-0.291}\right]\right\}^{-1}$

$= \left\{\left[\frac{0.875^0}{0!} + \frac{0.875}{1} + \frac{0.875^2}{2!}\right] + \left[0.669 \times 0.16 \times \frac{1}{0.709}\right]\right\}^{-1}$

$= \left\{[1 + 0.875 + 0.382] + [0.669 \times 0.16 \times 1.41]\right\}^{-1}$

$= \{2.257 + 0.1509\}^{-1} = \{2.407\}^{-1}$

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$\rho_0 = 0.415$

$\rho_0 = 41.5\%$

(3) $L_s = (C\rho) + \dfrac{(C\rho)^{c+1}\rho_0}{C \times C! \times (1-\rho)^2}$

$= 0.875 + \dfrac{(0.875)^3 \times 0.415}{3 \times 3! \times (1-0.291)^2}$

$= 0.875 + \dfrac{0.5861 \times 0.415}{3 \times 6 \times 0.709^2}$

$= 0.875 + 0.026$

$= 0.901$

$\boxed{L_s \cong 0 \text{ or } 1 \text{ ship}}$

4) $W_s = \dfrac{L_s}{\lambda}$

$= \dfrac{0.901}{7}$

$\boxed{W_s = 0.128 \text{ week}}$

5) $W_q = W_s - 1/\mu$

$= 0.128 - \dfrac{1}{8} = 0.128 - 0.125$

$\boxed{W_q = 0.003 \text{ Week}}$

(6) $L_q = \lambda \times W_q$

$= 7 \times 0.003 = 0.021$

$\boxed{L_q \cong 0 \text{ or } 1 \text{ ship}}$

Results :

The utilization of system decreases from 15% to 7.5% when we have 2 attendants. However, the waiting time of mechanic is slightly decreased with the two attendants but its does not have much impact. So, since ~~re~~ system utilization is more with only 1 attendant, it is advisable to not have second attendant.

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Q4: Classic care has 3 workers...... to wash a car?

SOLUTION:

Given:-

$\mu = 1/36 \times 60 = 1.667$ cars/hr

$\lambda = 1/45 \times 60 = 1.333$ cars/hr

$C = 3$

Required:-

a) $L_s = ?$

b) $W_q = ?$

c) $W_s = ?$

a) $L_s = (C\rho) + \dfrac{(C\rho)^{c+1} \rho_0}{C(C!)(1-\rho)^2}$ ——— (i)

$\Rightarrow \rho = \dfrac{\lambda}{C\mu} = \dfrac{1.333}{3(1.667)} = 0.265 \times 100 = \boxed{26.5\%}$

$\rho_0 = \left\{\left[\sum_{n=0}^{c-1} \dfrac{(C\rho)^n}{n!}\right] + \left[(C\rho)^c \dfrac{1}{c!}\left(\dfrac{1}{1-\rho}\right)\right]\right\}^{-1}$

$\therefore C\rho = \dfrac{\lambda}{\mu} = \boxed{0.7996}$

$\rho_0 = \left\{\left[\sum_{n=0}^{3-1} \dfrac{(0.7996)^n}{n!}\right] + \left[0.7996^3 \times \dfrac{1}{3!} \times \dfrac{1}{(1-0.265)}\right]\right\}^{-1}$

$= \left\{\left[\sum_{n=0}^{2} \dfrac{(0.7996)^n}{n!}\right] + \left[\cancel{0.7}\, 0.5112 \times \dfrac{1}{6} \times 1.360\right]\right\}^{-1}$

$= \{[1 + 0.7996 + 0.3196] + [0.1158]\}^{-1}$

$= (2.235)^{-1}$

$= 0.447$

$\boxed{\rho_0 = 44.7\%}$

(i) $\rightarrow L_s = 0.7996 + \dfrac{0.7996^4(0.447)}{3(3!)(1-0.265)^2}$

$= 0.7996 + \dfrac{0.1827}{9.7240}$

$\boxed{L_s = 0.810 \simeq 0 \text{ or } 1 \text{ cars}}$

b) $W_s = \dfrac{L_s}{\lambda}$

$= \dfrac{0.818}{1.333}$

$\boxed{W_s = 0.613 \text{ hrs}}$

c) $W_q = W_s - \dfrac{1}{\mu}$

$= 0.613 - \dfrac{1}{1.667}$

$\boxed{W_q = 0.013 \text{ hrs}}$

Q5: The manager of a computer network ----- distribution?

SOLUTION:

Given:-

$\alpha = 0.01$

1) H_0 = Service interruptions follows poisson distribution.

H_a = service interruptions donot follow poisson distribution.

2)

X_i	O_i	$P_i = \frac{e^{-\lambda}(\lambda)^x}{x!}$	$E_i = 500P_i$ (nP_i)	$X_o^2 = \frac{(O-E)^2}{E}$
0	160	$\frac{e^{-1.3}(1.3)^0}{0!} = 0.272$	136	$\frac{(160-136)^2}{136} = 4.235$
1	175	$\frac{e^{-1.3}(1.3)^1}{1!} = 0.354$	177	$\frac{(175-177)^2}{177} = 0.0225$
2	86	$\frac{e^{-1.3}(1.3)^2}{2!} = 0.230$	115	$\frac{(86-115)^2}{115} = 7.313$
3	41	$\frac{e^{-1.3}(1.3)^3}{3!} = 0.099$	49.5	$\frac{(41-49.5)^2}{49.5} = 1.459$
4	18	$\frac{e^{-1.3}(1.3)^4}{4!} = 0.032$	16	$\frac{(18-16)^2}{16} = 0.25$
5	12	$\frac{e^{-1.3}(1.3)^5}{5!} = 0.008$	4	$\frac{(12-4)^2}{4} = 16$
6	8	$\frac{e^{-1.3}(1.3)^6}{6!} = 0.001$	0.5	$\frac{(8-0.5)^2}{0.5} = 112.5$

∴ Finding λ:- $\lambda = \bar{X} = \frac{(0\times160)+(1\times175)+(2\times86)+(3\times41)+(4\times18)+(5\times12)+(6\times8)}{500}$

$$\lambda = \frac{0+175+172+123+72+60+48}{500}$$

$$\lambda = 650/500 = \boxed{1.3}$$

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3) Degree of freedom $= K - S - 1$

$= 7 - 1 - 1$

$= 5$

4) $\chi^2_{5,0.01} = 15.1$ (from table)

$\therefore \chi^2_0 < \chi^2_{5,0.01}$

$\boxed{\text{i.e } 83.837 \nless 15.1} \Rightarrow H_0$ is rejected

Hence; service interruptions don't follow poisson distribution.

Q6 : Suppose the Penn state........ 40% male?

SOLUTION:

Given;-

$\alpha = 0.05$

1) H_0 = Penn State student population is 60% female & 40% male.

H_a = Penn state student population isnot 60% female & 40% male.

2)

Categories	Observed(O)	Expected ($E = nP_i$)	$\chi^2_0 = \frac{(O-E)^2}{E}$
Male	47	$40\% \times 100 = 40$	$\frac{(47-40)^2}{40} = 1.225$
Female	53	$60\% \times 100 = 60$	$\frac{(53-60)^2}{60} = 0.816$
		Total	2.041

3) $DF = K - S - 1$

$= 2 - 0 - 1$

$= 1$

4) $\chi^2_{1,0.05} = 3.84$ (from table)

$\therefore \chi^2_0 < \chi^2_{1,0.05}$

i.e $2.041 < 3.84 \Rightarrow H_0$ is Accepted

Q7: The manager of commercial mortage...... distribution?

SOLUTION:

Given:-

$\alpha = 0.01, n = 104$

1) H_0 = The distribution is poisson distribution.

H_a = The distribution is not poisson distribution

Finding λ:-

$$\lambda = \bar{X} = \frac{(0\times13)+(1\times25)+(2\times32)+(3\times17)+(4\times9)+(5\times6)+(6\times1)+(7\times1)}{104}$$

$$= \frac{0+25+64+51+36+30+6+7}{104} = \frac{219}{104} = \boxed{2.11}$$

2)

X_i	O_i	$P_i = [e^{-2.11}(2.11)^x]/x!$	$E_i = 104(P_i)$	$\chi_o^2 = \frac{(O-E)^2}{E}$
0	13	$\frac{e^{-2.11}(2.11)^0}{0!} = 0.121$	$104\times0.121 = 12.584$	$\frac{(13-12.584)^2}{12.584} = 0.0138$
1	25	$\frac{e^{-2.11}(2.11)^1}{1!} = 0.256$	26.624	$\frac{(25-26.624)^2}{26.624} = 0.0990$
2	32	$\frac{e^{-2.11}(2.11)^2}{2!} = 0.269$	27.976	$\frac{(32-27.976)^2}{27.976} = 0.5788$
3	17	$\frac{e^{-2.11}(2.11)^3}{3!} = 0.189$	19.656	$\frac{(17-19.656)^2}{19.656} = 0.3589$
4	9	$\frac{e^{-2.11}(2.11)^4}{4!} = 0.100$	10.4	$\frac{(9-10.4)^2}{10.4} = 0.1884$
5	6	$[e^{-2.11}(2.11)^5]/5! = 0.0423$	4.3992	
6	1 (8)	$[e^{-2.11}(2.11)^6]/6! = 0.015$	1.56 (6.4168)	$\frac{(8-4.6168)^2}{4.6168} = 0.3906$
7	1	$[e^{-2.11}(2.11)^7]/7! = 0.0044$	0.4576	
			Total	1.6295

3) $DF = K - S - 1 = 6 - 1 - 1 = 4$

4) $\chi^2_{DF,\alpha} = \chi^2_{4,0.01} = 13.277$ (from table)

$\therefore \chi_o^2 < \chi^2_{4,0.01}$

i.e: $\boxed{1.6295 < 13.277}$ So, H_0 is accepted.

Q9: One study of grand juries ----- proportion?

SOLUTION:

Given :-

$n = 66, \alpha = 0.05$

1) H_0 = For each age group, juniors proportion is consistent with country's proportion

H_a = For each age group, juniors proportion isn't consistent with country's proportion

2)

X_i	O_i	P_i	$E = nP_i$	$\chi = \frac{(O_i - E_i)^2}{E_i}$
21–40	5	42%	$66 \times \frac{42}{100} = 27.72$	$\frac{(5-27.72)^2}{27.72} = 18.62$
41–50	9	23%	$66 \times \frac{23}{100} = 15.18$	$\frac{(9-15.18)^2}{15.18} = 2.52$
51–60	19	16%	$66 \times \frac{16}{100} = 10.56$	$\frac{(19-10.56)^2}{10.56} = 6.75$
Over 60	33	19%	$66 \times \frac{19}{100} = 12.54$	$\frac{(33-12.54)^2}{12.54} = 33.38$
Total	66	100%	66	$\chi_0^2 = 61.27$

3) $DF = K - S - 1$

$= 4 - 0 - 1$

$= 3$

4) $\chi^2_{3, 0.05} = 7.81$ (from table)

$\therefore \chi_0^2 < \chi^2$

$\boxed{\text{i.e } 61.27 \nless 7.81}$

So, H_0 is rejected.

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Q9: The time required for 50 different ...... 1/6' are used.

SOLUTION:

Given:-

$n = 50, \alpha = 0.05, P = 1/6$

Since $P_i = 1/6 = 0.1666$ for all the classes, we don't need to calculate λ.

1) H_0 = Service times are exponentially distributed.
H_a = services times are not exponentially distributed.

2)

Classes (x_i)	Observed freq. (O_i)	Expected freq. ($E_i = nP_i$)	$\chi_0^2 = (O-E)^2/E$
[0, 0.220)	8	50×0.1666 = 8.33	$(8-8.33)^2/8.33 = 0.013$
[0.220, 0.489)	11	8.33	$(11-8.33)^2/8.33 = 0.855$
[0.489, 0.836)	9	8.33	$(9-8.33)^2/8.33 = 0.053$
[0.836, 1.325)	5	8.33	$(5-8.33)^2/8.33 = 1.331$
[1.325, 2.161)	10	8.33	$(10-8.33)^2/8.33 = 0.334$
[2.161, ∞)	7	8.33	$(7-8.33)^2/8.33 = 0.212$
Total:	50	49.98	$\chi_0^2 = 2.7983$

3) $DF = K - S - 1$
$= 6 - 1 - 1$
$= 4$

4) $\chi^{2*}_{4,0.05} = 9.49$ (from table)

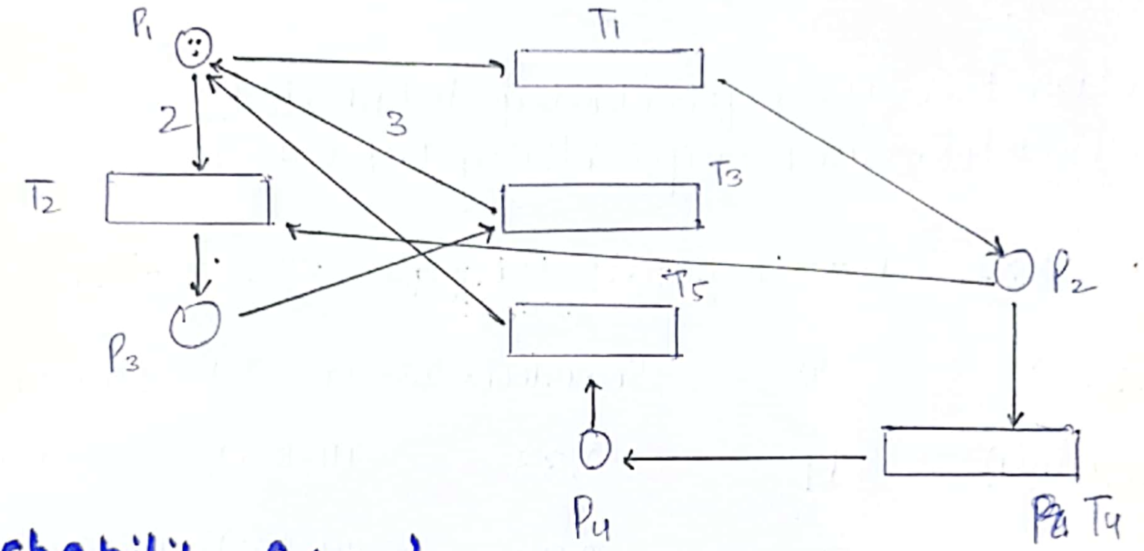
$\therefore \chi_0^2 < \chi^2_{4,0.05}$

$\boxed{\text{ie } 2.798 < 9.49}$

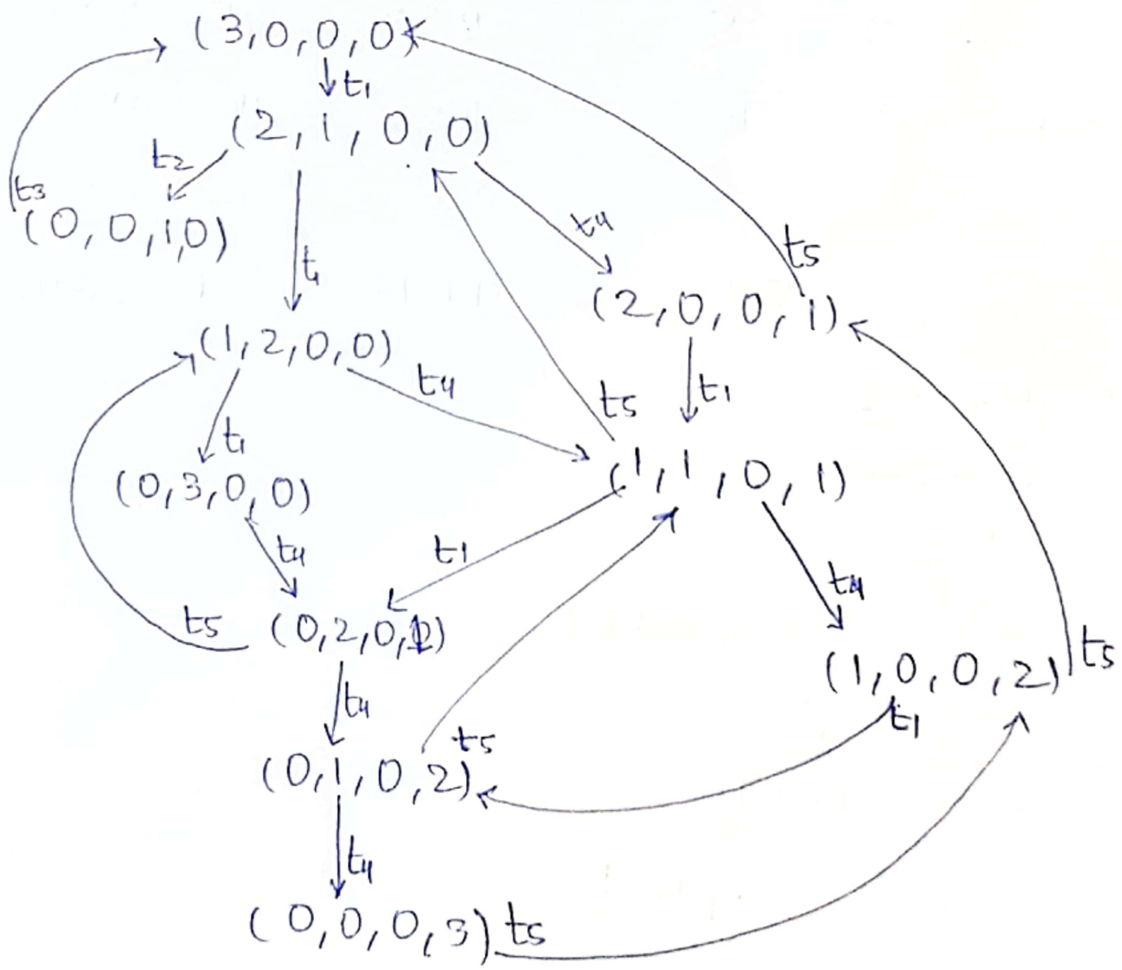
H_0 is accepted.

Q10: Create a petrinet-----

$$E^+ = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \\ P_4 \end{array} \begin{array}{c} \begin{array}{ccccc} T_1 & T_2 & T_3 & T_4 & T_5 \end{array} \\ \begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \quad E^- = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \\ P_4 \end{array} \begin{array}{c} \begin{array}{ccccc} T_1 & T_2 & T_3 & T_4 & T_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 3 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \quad M_0 = \begin{bmatrix} 3 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$



Reachability Graph:



Boundness:

P_1, P_2, P_3 are 3-bounded

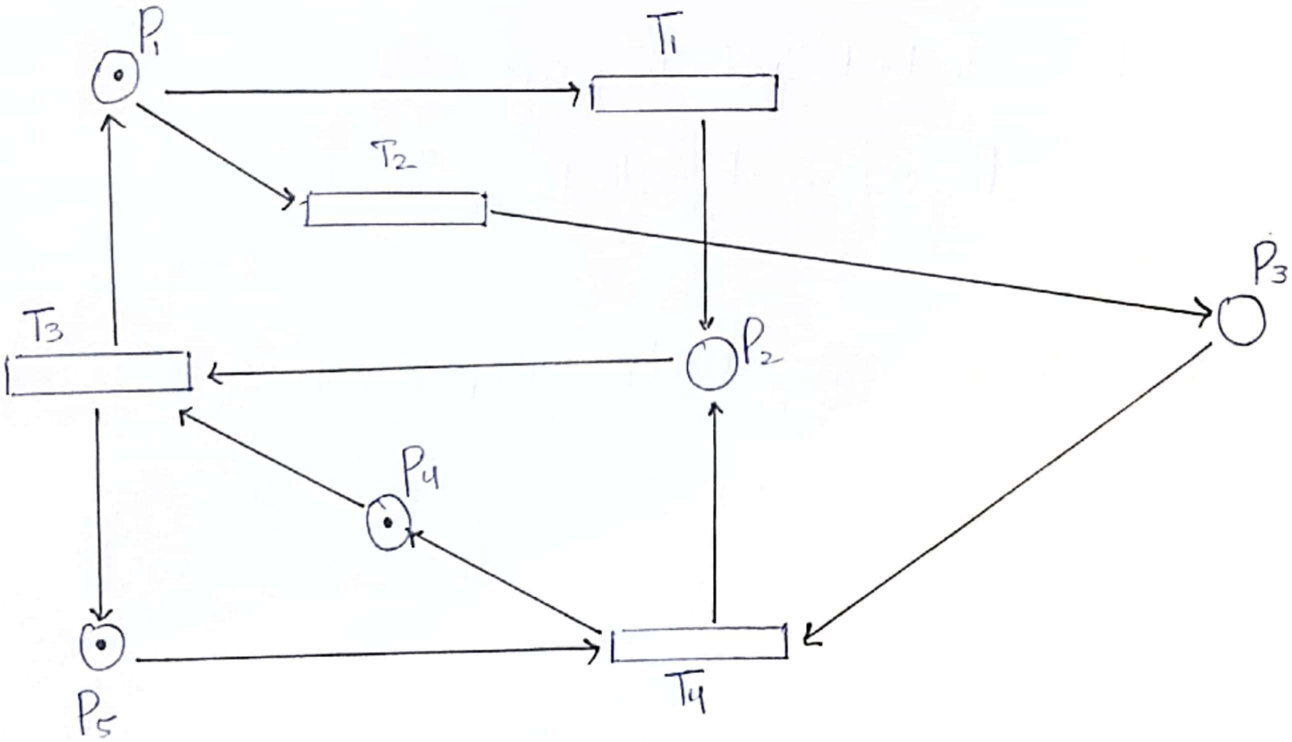
P_3 is 1-bounded → safe.

Liveness:-

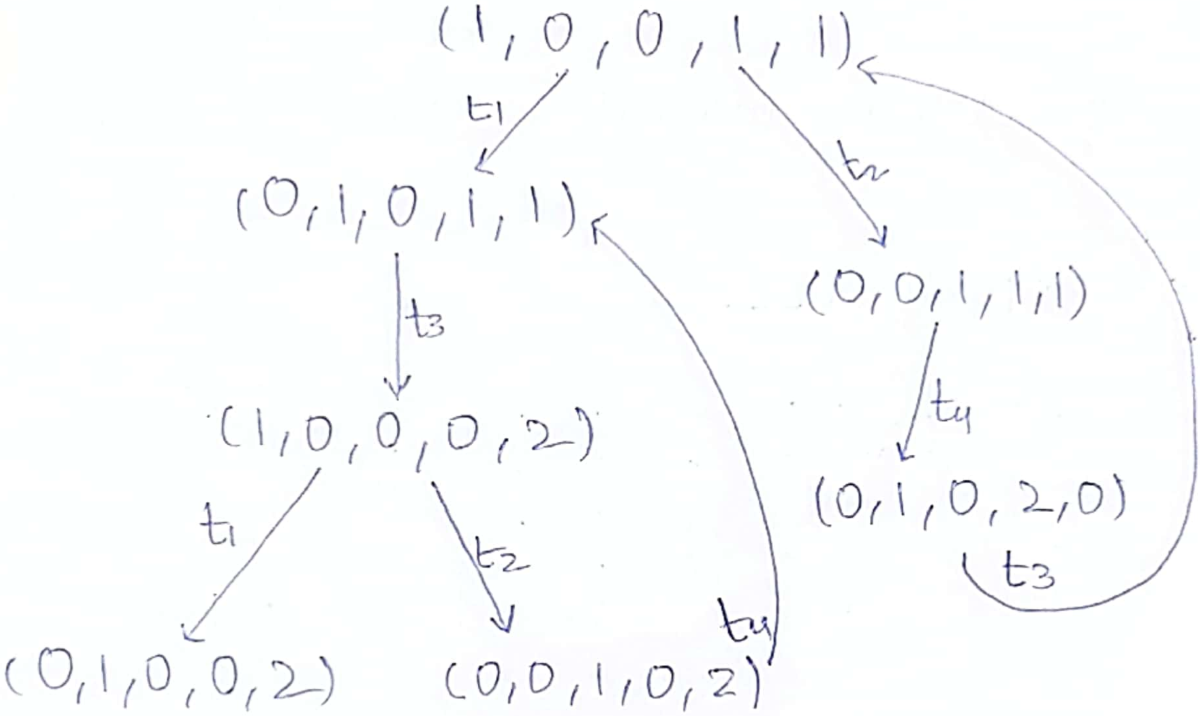
Petrinet is live because all transitions are fired.

Q11: Create a petrinet......

$$E^+ = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad E^- = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad M_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$



Reachability Graph:



Boundness

P_1, P_2, P_3 are 1-bounded => safe

P_4 and P_5 are 2-bounded

Liveness

Petrinet is live as all transitions are fired.