Kinza Raza Khan (SE-17009) 9th Feb. 2021

Modelling & Simulation

SE-405

Final Examination

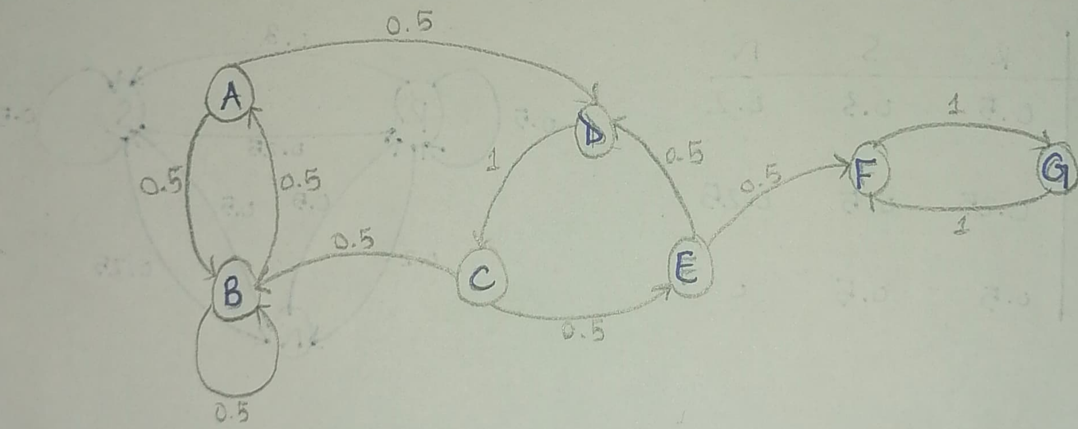
A1) There are 4 types of common markov models. They are:

① Markov Chain: modelling the system with respect to a random variable that changes with time.

② Hidden markov model: state can not be completely determined as system is partially observable.

③ Markov Decision Process: markov chain in which action vector is applied.

④ Partially Observable Markov Decision Process: Markov Decision Process in which state of the system is partially observable.



Class 1: {A, B}

class 2: {C, D, E}

class 3: {F, G}

- Accessible: All States / Classes
- Communicate: States A, B, F, G
- Recurrent: Class 1, 3
- Transient: Class 2
- Absorbing: Class 3

- Periodic/Aperiodic:

A: (A - B - A, A - D - C - B - A) → hops 2,4

B: (B - A - B, B - B) → hops 2,1

GCD(4, 2, 1) = 1 so aperiodic

C: (C - E - D - C, C - B - A - D - C) → hops 3,4

D: (D - C - E - D) → hops 3

E: (E - D - C - E) → hops 3

GCD(4, 3) = 1 so aperiodic

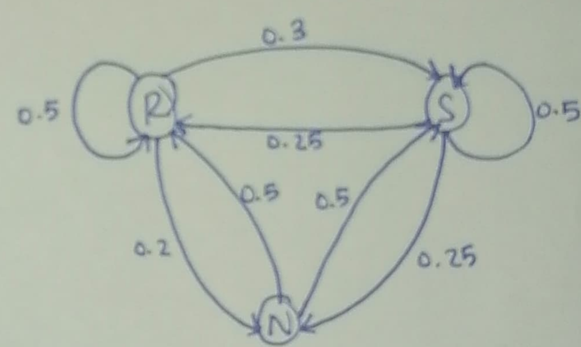
F: (F - G - F) → hops 2

G: (G - F - G) → hops 2

GCD(2, 2) = 2 so periodic

b)

	R	S	N
R	0.5	0.3	0.2
S	0.25	0.5	0.25
N	0.5	0.5	0



$$P^3 = \begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.25 & 0.5 & 0.25 \\ 0.5 & 0.5 & 0 \end{bmatrix} \begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.25 & 0.5 & 0.25 \\ 0.5 & 0.5 & 0 \end{bmatrix} \begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.25 & 0.5 & 0.25 \\ 0.5 & 0.5 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0.400 & 0.415 & 0.1875 \\ 0.3875 & 0.425 & 0.1875 \\ 0.400 & 0.425 & 0.175 \end{bmatrix}$$

The highest probability is that there will be ~~snowy~~ snowy days after a nice or snowy day. The probability of it being a nice day is the lowest therefore people should expect snow.

A2) a) validation is a fundamental process & part of model development. Through validation we can confirm that the model is the correct depiction of the system. We can validate by carrying out an iterative process of calibrating the model.

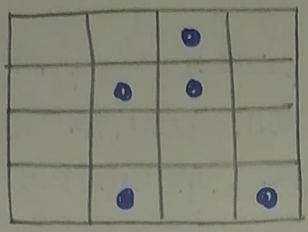
The techniques we can use to validate a model are as follows:

* building a model that has high face validity: we must ensure the model is as close to reality. Domain experts must be consulted before designing simulation model. The output must also be analysed timely by system experts.
* making the correct assumptions: there are two types of assumptions which include structural assumptions (how system operates) & data assumptions (collecting reliable data).
* compare simulation output with actual results
* using good programming logic & clean code.

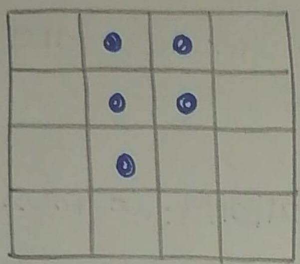
Different types of validity include:

- Subsystem validity
- Internal validity
- Face validity
- Sensitivity Analysis

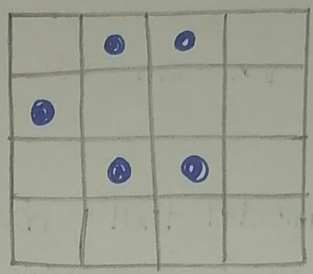
A2) b)



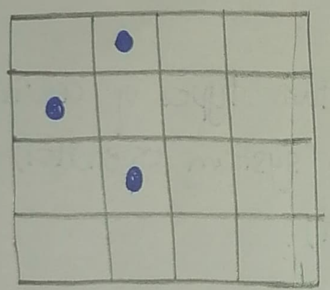
G-0



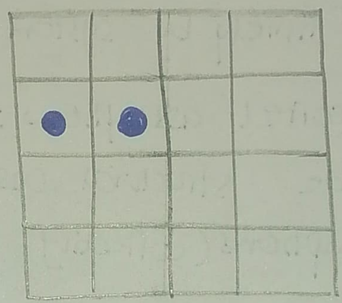
G-1



G-2



G-3



G-4

A3) a) System is a collection of ~~events~~ entities that act & interact together to accomplish a logical task.

(according to interconnection of events)

The types of systems are:

* Independent: none of the events depend on the other.

* Cascaded: the relationship between events is unilateral.

* Coupled: the relation between events is bilateral.

Advantages of Simulation:

- New rules and policies can be considered without disturbing the real-time operations of system.
- Time can be adjusted to investigate the simulation.
- Hypothesis can be easily tested.
- Interaction of variables can be analysed.
- Bottleneck analysis can be performed.
- "what if" questions can be answered.

Types of system according to uncertainties are:

* Deterministic/Stochastic: in stochastic input and output are both random whereas in deterministic output can be predicted on the basis of input.
* Static/Dynamic: Static represents system at particular point whereas in dynamic system changes with time.

A3) b) $\lambda = \frac{4}{9} = 0.44$ per hour

$\mu = \frac{1}{2} = 0.5$ per hour

i) $\rho = \frac{\lambda}{\mu} = \frac{0.44}{0.50} = 0.89 \approx 89\%$

ii) $L_s = \frac{\lambda}{\mu - \lambda} = \frac{0.44}{0.50 - 0.44} = 7.33 \approx 8$ machines

iii) $c = 3$

* $\rho = \frac{\lambda}{c\mu} = \frac{0.44}{3(0.50)} = 0.293 \approx 29.3\%$

* $P_0 = \left\{ \left[\sum_{n=0}^{2} \frac{(0.89)^n}{n!} \right] + \left[(0.89)^3 \left(\frac{1}{3!} \right) \left(\frac{1}{1 - 0.293} \right) \right] \right\}^{-1}$

$P_0 = (1 + 0.88 + 0.3872 + 0.160)^{-1}$

$P_0 = (2.4272)^{-1}$

$P_0 = 0.411 \approx 41.1\%$

* $L_s = c\rho + \frac{(c\rho)^{c+1} P_0}{c(c!)(1-\rho)^2}$

$L_s = 0.89 + \frac{(0.89)^4 (0.411)}{3(3!)(1 - 0.293)^2}$

$L_s = 0.89 + 0.027$

$L_s = 0.907 \approx 1$ machine

If three technicians are available the utilization factor decreases from 89% to approximately 29% and avg. no. of machines in service also decreases.

A4) a) The outputs generated by simulation models must be carefully analysed to understand how well the models are working. We can compare the performance through various methods.

Through estimation we can make an inference about the population based on the data received from the samples.

- Point Estimate: single value of population parameter statistics.
- Interval Estimate: two numbers between which the parameter is expected to lie.

Confidence Interval is also a good measure of estimation as it expresses the degree of uncertainty.

		C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
b)	IAT	2	1	5	4	6	4	3	5
A4	ST	5	2	4	5	2	3	5	2
		C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8

Clock	LQ(t)	LS(t)	Checkout	FEL	S	N_D	F
0	0	1	$(C_1, 0)$	$(D, 5, C_1)$ $(A, 2, C_2)$	0	0	0
2	1	1	$(C_1, 0)(C_2, 2)$	$(D, 5, C_1)$ $(A, 3, C_3)$	0	0	0
3	2	1	$(C_1, 0)(C_2, 2)$ $(C_3, 3)$	$(D, 5, C_1)$ $(A, 8, C_4)$	0	0	0
5	1	1	$(C_2, 2)$ $(C_3, 3)$	$(D, 7, C_2)$ $(A, 8, C_4)$	5	1	1
7	0	1	$(C_3, 3)$	$(D, 11, C_3)$ $(A, 8, C_4)$	10	2	2
8	1	1	$(C_3, 3)(C_4, 8)$	$(D, 11, C_3)$ $(A, 12, C_5)$	10	2	2
11	0	1	$(C_4, 8)$	$(D, 16, C_4)$ $(A, 12, C_5)$	18	3	3
12	1	1	$(C_4, 8)(C_5, 12)$	$(D, 16, C_4)$ $(A, 18, C_6)$	18	3	3
16	0	1	$(C_5, 12)$	$(D, 21, C_5)$ $(A, 18, C_6)$	26	4	4
18	0	1	$(C_6, 18)$	$(D, 21, C_5)$ $(A, 22, C_7)$	32	5	5
21	0	0	—	$(A, 22, C_7)$	35	6	6
22	0	1	$(C_7, 22)$	$(D, 27, C_7)$ $(A, 25, C_8)$	35	6	6
25	1	1	$(C_7, 22)$ $(C_8, 25)$	$(D, 27, C_7)$ $(A, 30, C_9)$	35	6	6
27	0	1	$(C_8, 25)$	$(D, 29, C_3)$ $(A, 30, C_9)$	40	7	7
29	0	0	—	$(A, 30, C_9)$	44	8	8
30	0	1	$(C_9, 30)$	$(D, -, C_9)$	44	8	8

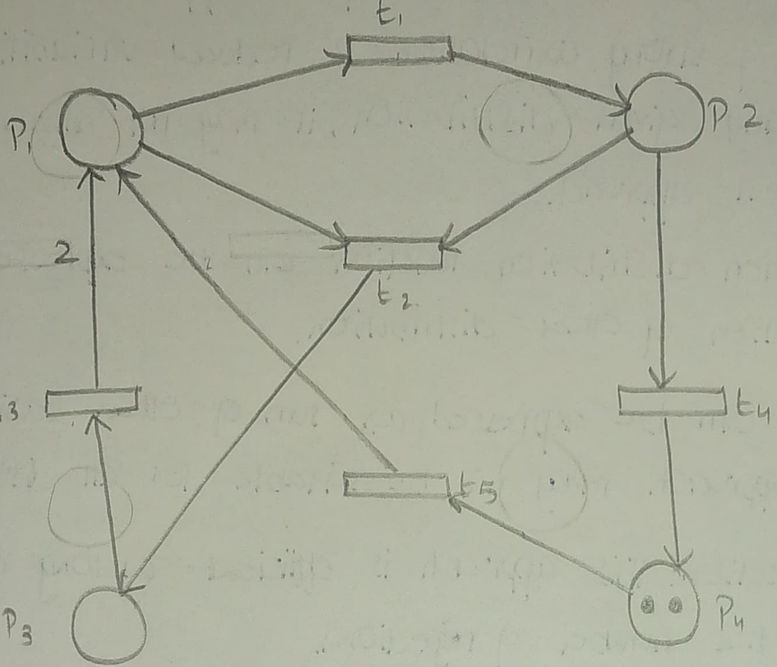
A5) a) A random variate is a particular outcome of random variable.

Techniques of generating random variates:

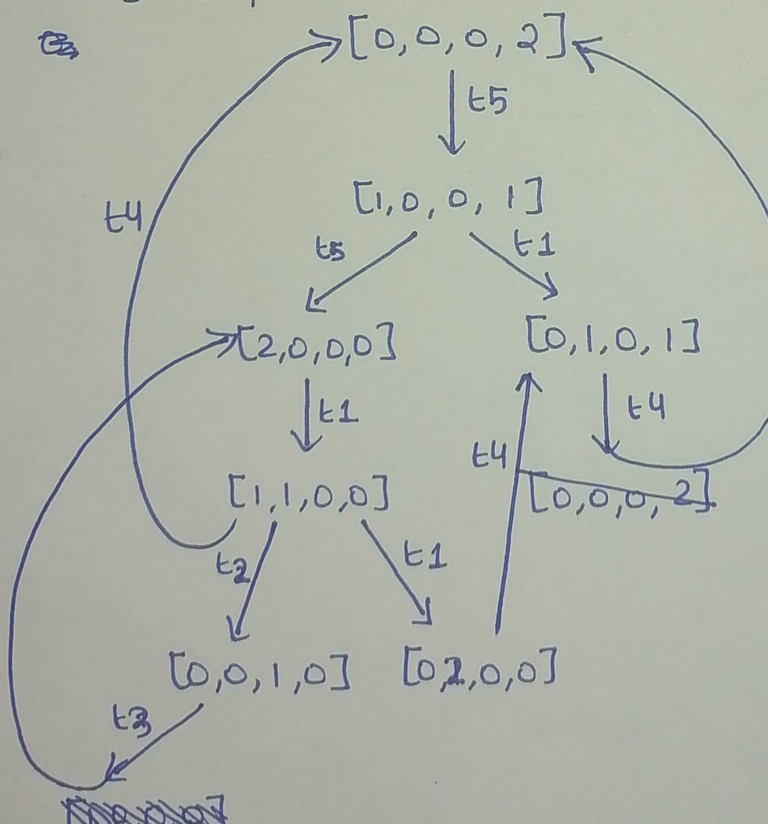
- Inverse Transform: ~~The~~ It is the simplest approach & provides the advantage of strong correlation & reduced variability. However for any given distribution, it may not always be the fastest or simplest approach.
- composition: When distribution function can be expressed as convex combination of other distribution.
- Convolution: X can be expressed as sum of other variables. However this approach may not be suitable for all distributions.
- Acceptance - Rejection: This approach is efficient as long as we can minimize the number of rejections.

A5) b)

$$E^{+} = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \\ P_4 \end{array} \begin{array}{c} \begin{array}{ccccc} t_1 & t_2 & t_3 & t_4 & t_5 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \quad E^{-} = \begin{bmatrix} 0 & 0 & 2 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad m_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 2 \end{bmatrix}$$



Reachability Graph:



Boundedness:

- P1, P2, P4 are 2-bounded (Petri Net is not bounded)
- P3 is safe.

Liveness:

- All ~~transactions~~ transitions are live.
- Petri Net is live

A6) a) i) ~~Geom~~ Geometric Distribution: The random variable X is the number of trials required to achieve the first success.

It is represented by:

$$P(X) = \begin{cases} q^{x-1} p & x = 1, 2, 3 \ldots \\ 0 & \text{otherwise} \end{cases}$$

ii) List Processing: keeping track of events. Entities must be known and recorded as well as Event Notice which refers to tracking the occurrence of events.

In the list, the following common operations can be performed such as removing record from any location (index) or adding record to any location or to the bottom/top.

iii) Next Event Time Advance Mechanism: clock is initialized from zero. All the times of event occurrence are recorded. The clock is then advanced to the time of occurrence of the first event. Therefore the sequence of events are updated & all future events recorded.

~~H's distribution of commercial mortgages approved~~ r

b) A6

H_0: ~~distribution of~~ commercial mortgages approved per week follows Poisson distribution

H_1: commercial mortgages approved per week does not follow Poisson distribution.

categories	observed (O_i)	$P_i = \frac{e^{-\bar{x}}(\bar{x})^x}{x!}$	Expected E_i	$X_0^2 = \frac{(O_i - E_i)^2}{E_i}$
0	12	0.112	11.424	0.029
1	26	0.246	25.092	0.033
2	30	0.268	27.336	0.259
3	16	0.196	19.992	0.393
4	8	0.107	10.914	0.778
5	6	0.0467	4.763	
6	1	0.017	1.734	
7	3	0.0053	0.543	
			5–7: 7.040	1.244

$\boxed{X_0^2 = 3.137}$

$$\bar{x} = \frac{(0\times12)+(1\times26)+(2\times30)+(3\times16)+(4\times8)+(5\times6)+(6\times1)+(7\times3)}{102}$$

$\bar{x} = 2.186$

DF = k − s − 1
= 6 − 1 − 1
= 4

CV = $X^2_{0.01,4}$
= 13.28

Since $X_0^2 < X^2_{0.01,4}$

$3.137 < 13.28$

therefore H_0 is accepted & commercial mortgages approved per week follows Poisson distribution.