Practice Problems of M/M/c

1. Alaska National Bank observed drive-in teller windows on a busy Saturday. CEO Ted Eschenbach estimates that customers arrive at a rate of about $\lambda = 18$ per hour, and that each teller can service about $\mu = 20$ customers per hour. If three windows are open, calculate operational characteristics.
2. Bill Youngdahl has been collecting data at the TU student grill. He has found that, between 5:00 P.M. and 7:00 P.M., students arrive at the grill at a rate of 25 per hour (Poisson distributed) and service time takes an average of 2 minutes (exponential distribution). There is only 1 server, who can work on only 1 order at a time. (service rate = 1 / 2 minutes) a) What is the average number of students in line? b) What is the average time a student is in the grill area? c) Suppose that a second server can be added to team up with the first (and, in effect, act as one faster server). This would reduce the average service time to 90 seconds. How would this affect the average time a student is in the grill area? (Service rate= 1 / 90 seconds) d) Suppose a second server is added and the 2 servers act independently, with each taking an average of 2 minutes. What would be the average time a student is in the system? (c=2)
3. Radovilsky's Department Store in Haywood, California, maintains a successful catalog sales department in which a clerk takes orders by telephone. If the clerk is occupied on one line, incoming phone calls to the catalog department are answered automatically by a recording machine and asked to wait. As soon as the clerk is free, the party who has waited the longest is transferred and serviced first. Calls come in at a rate of about 12 per hour. The clerk can take an order in an average of 4 minutes. Calls tend to follow a Poisson distribution, and service times tend to be exponential. a) What is the average time that catalog customers must wait before their calls are transferred to the order clerk? b) What is the average number of callers waiting to place an order? c) Radovilsky's is considering adding a second clerk to take calls. Should it hire another clerk? Explain your decision.
4. Eric Krassow's cabinet-making shop, in Memphis, has five tools that automate the drilling of holes for the installation of hinges. These machines need setting up for each order of cabinets. The orders appear to follow the Poisson distribution, averaging 3 per 8-hour day. There is a single technician for setting these machines. His service times are exponential, averaging 2 hours each. a) What is the service factor for this system? b) What is the average number of these machines in service? c) What impact on machines in service would there be if a 3 technicians are available?
5. At metropolis city hall, two workers "pull string" (make deal) every day. Strings arrived to be pulled on an average of one every 10 minutes throughout the day. It takes an average of 15 minutes to pull a string. Both times between arrivals and service times re exponentially distributed. What is the probability that there are no strings to be pulled in the system at a random point in time? What is the expected number of strings waiting to be pulled? What is the effect on performance if the third string puller, working at the same speed as the first two, is added to the system?
6. A computer consists of three processors. Their main task is to execute jobs from users. These jobs arrive according to a Poisson process with rate 15 jobs per minute. The execution time is exponentially distributed with mean 3 jobs per 10 seconds. When a processor completes a job and there are no other jobs waiting to be executed, the processor starts to execute maintenance jobs. These jobs are always available and they take an exponential time with mean 5 seconds. But as soon as a job from a user arrives, the processor interrupts the execution of the maintenance job and starts to execute the new job. The execution of the maintenance job will be resumed later (at the point where it was interrupted). i) What is the mean number of jobs from users are being requested?(Ls) ii) How many maintenance jobs are on average completed per minute? iii) What is the probability that a job from a user has to wait? (P) iv)Determine the mean waiting time of a job from a user.(Wq)

$\lambda = 18\,/hr$

$\mu = 20/hr.$

$c = 3.$

$$\rho = \frac{\lambda}{c\mu} = \frac{18}{3(20)} = \underline{0.3}$$

$$\therefore c\rho = \frac{\lambda}{\mu} = 0.9$$

$$P_0 = \left\{\left[\sum_{n=0}^{c-1} \frac{(c\rho)^n}{n!}\right] + \left[(c\rho)^c \left(\frac{1}{c!}\right) \frac{1}{1-\rho}\right]\right\}^{-1}$$

$$P_0 = \left\{\left[\sum_{n=0}^{3-1} \frac{(0.9)^n}{n!}\right] + \left[(0.9)^3 \left(\frac{1}{3!}\right)\left(\frac{1}{1-0.3}\right)\right]\right\}^{-1}$$

$$P_0 = \left\{\left[\frac{(0.9)^0}{0!} + \frac{(0.9)^1}{1!} + \frac{(0.9)^2}{2!}\right] + \left[0.729\left(\frac{1}{6}\right)\left(\frac{1}{0.7}\right)\right]\right\}^{-1}$$

$$= \{[1 + 0.9 + 0.405] + 0.1735\}$$

$$\{2.305 + 0.1735\}^{-1}$$

$$= \{2.4785\}^{-1} \Rightarrow \boxed{0.4034}$$

$$L_s = c\rho + \frac{(c\rho)^{c+1} P_0}{c(c!)(1-\rho)^2}$$

$$= 0.9 + \frac{(0.9)^{3+1}(0.4034)}{3(3!)(1-0.3)^2}$$

$$= 0.9 + \frac{(0.9)^4(0.4034)}{3(6)(0.49)}$$

$$= 0.9 + \frac{0.2646}{8.82}$$

$$= 0.9 + 0.030008$$

$$\boxed{L_s = 0.930}$$

$$W_s = \frac{L_s}{\lambda}$$

$$= \frac{0.930}{18}$$

$$\boxed{W_s = 0.0516}$$

$$W_q = W_s - \frac{1}{\mu}$$

$$= 0.0516 - \frac{1}{20}$$

$$\boxed{W_q = 0.0016}$$

$$L_q = \lambda W_q$$

$$= 18(0.0016)$$

$$\boxed{L_q = 0.0288}$$

(2)

$\lambda = 25/hr$

$\mu = 1/2min \Rightarrow 30/hr$

a: $L_q = \dfrac{\lambda^2}{\mu(\mu-\lambda)}$

$= \dfrac{(25)^2}{30(30-25)} = \dfrac{625}{30(5)}$

$\boxed{L_q = 4.16}$

b: $W_s = \dfrac{1}{\mu-\lambda}$

$= \dfrac{1}{30-25} = \dfrac{1}{5}$

$\boxed{W_s = 0.2}$

c: $\lambda = 25/hr$

$\mu = 1/90sec \Rightarrow 1 \div \dfrac{90}{60}$

$= 1 \div \dfrac{3}{2} = \dfrac{2}{3} \times 60$

$= 40/hr.$

$W_s = \dfrac{1}{\mu-\lambda}$

$= \dfrac{1}{40-25} = \dfrac{1}{15}$

$\boxed{W_s = 0.067}$

d:- $\lambda = 25/h$

$\mu = 30/hr$

$c = 2.$

$W_s = ?$

3

$\rho = \dfrac{25}{2(30)} = \underline{0.41}$

$c\rho = \dfrac{25}{30} = \underline{0.83}.$

$P_0 = \left\{\left[\sum_{n=0}^{2-1} \dfrac{(0.83)^n}{n!}\right] + \left[(0.83)^2\left(\dfrac{1}{2!}\right)\left(\dfrac{1}{1-0.41}\right)\right]\right\}^{-1}$

$P_0 = \left\{\left[\dfrac{(0.83)^0}{0!} + \dfrac{(0.83)^1}{1!}\right] + \left[0.68 \times \dfrac{1}{2} \times \dfrac{1}{0.59}\right]\right\}^{-1}$

$= \{[1 + 0.83] + [0.57]\}^{-1}$

$= \{1.83 + 0.57\}^{-1} = \{2.40\}^{-1}$

$\boxed{P_0 = 0.41}$

$L_s = 0.83 + \dfrac{(0.83)^3(0.41)}{2(2!)(0.59)^2}$

$= 0.83 + \dfrac{(0.68)(0.41)}{4(0.34)}$

$= 0.83 + \dfrac{0.278}{1.36}$

$= 0.83 + 0.204$

$\boxed{L_s = 1.03}$

$W_s = \dfrac{L_s}{\lambda} = \dfrac{1.03}{25}$

$\boxed{W_s = \cancel{0.23}}$ 0.0412.

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3

$\lambda = 12/hr$

$\mu = 1/4min \Rightarrow 15/hr.$

a: $W_q = ?$

$$W_q = \frac{\lambda}{\mu(\mu-\lambda)}$$

$$= \frac{12}{15(15-12)} = \frac{12}{15(3)}$$

$$\boxed{W_q = 0.266}$$

b: $L_q = ?$

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} \Rightarrow \frac{12^2}{15(15-12)}$$

$$= \frac{144}{15(3)} = \boxed{3.2}$$

c: $\lambda = 12/hr$, $\mu = 15/hr$

$c = 2$

$$\rho = \frac{\lambda}{c\mu} = \frac{12}{2(15)}$$

$$\boxed{\rho = 0.4} = 40\%$$

$$c\rho = \frac{12}{15} = 0.8$$

$$P_0 = \left\{\left[\sum_{n=0}^{c-1} \frac{(c\rho)^n}{n!}\right] + \left[(c\rho)^c \left(\frac{1}{c!}\right)\left(\frac{1}{1-\rho}\right)\right]\right\}^{-1}$$

$$P_0 = \left\{\left[\sum_{n=0}^{2-1} \frac{(0.8)^n}{n!}\right] + \left[(0.8)^2\left(\frac{1}{2!}\right)\left(\frac{1}{1-0.4}\right)\right]\right\}^{-1}$$

$$= \left\{[1 + 0.8] + \left[0.64 \times \frac{1}{2} \times \frac{1}{0.6}\right]\right\}^{-1}$$

$$= \{1.8 + 0.53\}^{-1} = \{2.33\}^{-1}$$

$$\boxed{P_0 = 0.42}$$

$$L_s = c\rho + \frac{(c\rho)^{c+1} P_0}{c(c!)(1-\rho)^2}$$

$$= 0.8 + \frac{(0.8)^3(0.42)}{2(2!)(1-0.4)^2}$$

$$= 0.8 + \frac{(0.512)(0.42)}{4(0.36)}$$

$$= 0.8 + \frac{0.215}{1.44}$$

$$= 0.8 + 0.149$$

$$\boxed{L_s = 0.949}$$

$$W_s = \frac{L_s}{\lambda} = \frac{0.949}{12} = \boxed{0.079}$$

$$W_q = W_s - \frac{1}{\mu} = 0.079 - \frac{1}{15}$$

$$\boxed{W_q = 0.013}$$

$$L_q = \lambda W_q = \frac{\cancel{15}}{12}(0.013)$$

$$\boxed{L_q = \cancel{0.195}} \quad 0.156$$

∴ No: of call waiting (L_q) is reduced so, company should hire 2nd clerk

$\lambda = 3$ per 8-hr day.

$\mu = 1/2hr \Rightarrow 4$ per 8-hr day

a: $\rho = \frac{\lambda}{\mu}$

$\rho = \frac{3}{4} = 0.75$

$\boxed{\rho = 75\%}$

b: $L_s = \frac{\lambda}{\mu - \lambda}$

$= \frac{3}{4-3}$

$\boxed{L_s = 3}$

c: $\lambda = 3/8$ hr day

$\mu = 4/8$ hr day

$C = 3$

$\rho = \frac{\lambda}{c\mu} = \frac{\cancel{3}}{\cancel{3}(4)}$

$\boxed{\rho = 0.25}$

$cp = \frac{\lambda}{\mu} = \frac{3}{4} = \underline{0.75}$

$= \left\{\left[\sum_{n=0}^{3-1} \frac{(0.75)^n}{n!}\right] + \left[(0.75)^3\left(\frac{1}{3!}\right)\left(\frac{1}{1-0.25}\right)\right]\right\}^{-1}$

(1) $P_0 = \left\{\left[\frac{(0.75)^0}{0!} + \frac{(0.75)^1}{1!} + \frac{(0.75)^2}{2!}\right] + \left[0.42 \times \frac{1}{6} \times \frac{1}{0.75}\right]\right\}^{-1}$

$= \left\{\left[1 + 0.75 + \overset{0.14}{0.56}\right] + \left[0.42 \times \frac{1}{4.5}\right]\right\}^{-1}$

$= \{2.31 + 0.93\}^{-1}$

$= \{3.24\}^{-1} \Rightarrow \boxed{0.30}$

$L_s = cp + \frac{(cp)^{c+1} P_0}{c(c!)(1-\rho)^2}$

$= 0.75 + \frac{(0.75)^{3+1}(0.30)}{3(3!)(1-0.25)^2}$

$= 0.75 + \frac{(0.316)(0.30)}{18(0.93)}$

$= 0.75 + \frac{0.094}{16.87}$

$= 0.75 + 0.0056$

$\boxed{L_s = 0.75}$ 0.76

$W_s = \frac{L_s}{\lambda} = \frac{0.75}{3}$

$\boxed{W_s = 0.25}$

$W_q = W_s - \frac{1}{\mu} \Rightarrow 0.25 - \frac{1}{4}$

$\boxed{W_q = 0}$

$L_q = \lambda W_q \Rightarrow 3(0)$

$\boxed{L_q = 0}$

$\lambda = 1/10\text{min} \Rightarrow 6/\text{hr}.$

$\mu = \frac{1}{15\text{min}} \Rightarrow 4/\text{hr}.$

$c = 2.$

$$\rho = \frac{\lambda}{c\mu} = \frac{6}{2(4)} = \frac{6}{8}$$

$\boxed{\rho = 0.75}$, $c\rho = \frac{6}{4}$ $\boxed{c\rho = 1.5}$

$$P_0 = \left\{\left[\sum_{n=0}^{2-1} \frac{(1.5)^n}{n!}\right] + \left[(1.5)^2\left(\frac{1}{2!}\right)\left(\frac{1}{1-0.75}\right)\right]\right\}^{-1}$$

$$P_0 = \left\{\left[\frac{(1.5)^0}{0!} + \frac{(1.5)^1}{1!}\right] + \left[2.25 \times \frac{1}{2} \times \frac{1}{0.25}\right]\right\}^{-1}$$

$$P_0 = \{[1 + 1.5] + 4.5\}^{-1}$$

$$P_0 = \{2.5 + 4.5\}^{-1} = \{7\}^{-1}$$

$\boxed{P_0 = 0.142}$

$$L_s = 1.5 + \frac{(1.5)^{2+1}(0.142)}{2(2!)(1-0.75)^2}$$

$$= 1.5 + \frac{3.375 \times 0.142}{4(0.25)^2}$$

$$= 1.5 + \frac{0.479}{0.25}$$

$$= 1.5 + 1.916$$

$\boxed{L_s = 3.416}$

(5) $W_s = \frac{L_s}{\lambda}$

$= \frac{3.416}{6}$

$\boxed{W_s = 0.569}$

$W_q = W_s - \frac{1}{\mu}$

$= 0.569 - \frac{1}{4}$

$= 0.569 - 0.25$

$\boxed{W_q = 0.319}$

$L_q = \lambda W_q$

$= 6(0.319)$

$\boxed{L_q = 1.916}$

$c = 3$, then.

$$\rho = \frac{6}{3(4)} = \frac{6}{12}$$

$$\boxed{\rho = 0.5}$$

$$c\rho = \frac{6}{4} = 1.5$$

$$P_0 = \left\{\left[\sum_{n=0}^{3-1} \frac{(1.5)^n}{n!}\right] + \left[(1.5)^3\left(\frac{1}{3!}\right)\frac{1}{1-0.5}\right]\right\}^{-1}$$

$$= \left\{\frac{(1.5)^0}{0!} + \frac{(1.5)^1}{1!} + \frac{(1.5)^2}{2!} + \left[3.375 \times \frac{1}{6} \times \frac{1}{0.5}\right]\right\}^{-1}$$

$$= \left\{[1 + 1.5 + 1.125] + 1.125\right\}^{-1}$$

$$= \{3.628 + 1.125\}^{-1}$$

$$= \{4.753\}^{-1}$$

$$\boxed{P_0 = 0.210}$$

$$L_s = 1.5 + \frac{(1.5)^{3+1}(0.210)}{3(3!)(1-0.5)^2}$$

$$= 1.5 + \frac{(5.062)(0.21)}{18(0.5)^2}$$

$$= 1.5 + \frac{1.063}{4.5}$$

$$= 1.5 + 0.236$$

$$\boxed{L_s = 1.736}$$

$$W_s = \frac{L_s}{\lambda}$$

$$= \frac{1.736}{6}$$

$$\boxed{W_s = 0.289}$$

$$W_q = W_s - \frac{1}{\mu}$$

$$= 0.289 - 0.25$$

$$\boxed{W_q = 0.039}$$

$$L_q = \lambda W_q$$

$$= 6(0.039)$$

$$\boxed{L_q = 0.236}$$

$\lambda = 15/min$

$\mu = 3/10sec = \frac{3}{10} \times 60 = 18/min$

$c = 3$

a. $L_s = ?$

b. —

c: p

d. W_q

$\therefore p = \frac{\lambda}{c\mu} = \frac{15}{3 \times 18} = \frac{15}{54} = 0.277.$

$cp = \frac{\lambda}{\mu} = \frac{15}{18} = 0.833.$

$\therefore$ maintenac jobs:
1 in 5 sec.
$\frac{60}{5}$ for minutes
12 jobs/min

$P_0 = \{[\frac{(0.833)^0}{0!} + \frac{(0.833)^1}{1!} + \frac{(0.833)^2}{2!}] + [(0.833)^3(\frac{1}{3!})(\frac{1}{1-0.277})$

$= \{[1 + 0.833 + 0.347] + [0.5780(\frac{1}{6})(\frac{1}{0.723})]\}^{-1}$

$= (2.18 + 0.1332)^{-1} = (2.313)^{-1}$

$\boxed{P_0 = 0.432}$

$L_s = cp + \frac{(cp)^{c+1} P_0}{c(c!)(1-p)^2}$

$= 0.833 + \frac{(0.833)^4(0.432)}{3(3!)(0.723)^2}$

$= 0.833 + \frac{(0.481)(0.432)}{9.409}$

$= 0.833 + 0.0220$

$\boxed{L_s = 0.855}$

$\therefore W_s = \frac{L_s}{\lambda}$

$= \frac{0.855}{15}$

$W_s = 0.0570$

$W_q = W_s - \frac{1}{\mu}$

$= 0.0570 - \frac{1}{18}$

$= 0.0570 - 0.0555$

$\boxed{W_q = 0.002}$

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