(Fall 22) (Batch 19) Solution Assignment # 2

Q1.

$\lambda = 1/hr = 24/day$

$\mu = 1/45 mins = 32/day$

$L_q = ?$

$c = 2.$

$\because L_q = \lambda W_q$

$\therefore cp = \frac{\lambda}{\mu} = \frac{24}{32} = 0.75$

$p = \frac{\lambda}{c\mu} = \frac{24}{2(32)} = 0.375$

$$P_0 = \left\{\left[\sum_{n=0}^{c-1}\left(\frac{cp^n}{n!}\right)\right] + \left[(cp)^c\left(\frac{1}{c!}\right)\frac{1}{1-p}\right]\right\}^{-1} \Rightarrow \left\{\left[\sum_{n=0}^{2-1}\left(\frac{cp^n}{n!}\right)\right] + \left[(cp)^c\frac{1}{c!}\frac{1}{1-p}\right]\right\}^{-1}$$

$$P_0 = \left\{\left[\frac{0.75^0}{0!} + \frac{(0.75)^1}{1!}\right] + \left[(0.75)^2\left(\frac{1}{2!}\right)\frac{1}{1-0.375}\right]\right\}^{-1}$$

$$= \left\{\left[1 + 0.75\right] + \left[0.562\left(\frac{1}{2}\right)\left(\frac{1}{0.625}\right)\right]\right\}^{-1}$$

$$= \left\{1.75 + 0.449\right\}^{-1}$$

$$= \left\{2.199\right\}^{-1}$$

$$\boxed{P_0 = 0.454}$$

$$L_s = cp + \frac{(cp)^{c+1} P_0}{c(c!)(1-p)^2}$$

$$= 0.75 + \frac{(0.75)^{2+1}(0.454)}{2(2!)(1-0.375)^2}$$

$$= 0.75 + \frac{(0.4218)(0.454)}{4(0.3906)}$$

$$= 0.75 + \frac{0.191}{1.562} \Rightarrow 0.75 + 0.122$$

$$\boxed{L_s = 0.872}$$

$$W_s = \frac{L_s}{\lambda}$$

$$W_s = \frac{0.872}{24}$$

$$\boxed{W_s = 0.036}$$

$$W_q = W_s - \frac{1}{\mu} = 0.036 - \frac{1}{32}$$

$$W_q = 0.036 - 0.0312$$

$$\boxed{W_q = 0.0051}$$

$$L_q = \lambda W_q \Rightarrow 24(0.0051).$$

$$\boxed{L_q = 0.123}$$

Scanned with CamScanner

Q2:

$\lambda = \frac{1}{20} \times 60 = 3/hr$

$\mu = \frac{1}{3} \times 60 = 20/hr.$

if $c = 1$.

$\rho = \frac{\lambda}{\mu} = \frac{3}{20} = 0.15 = 15\%$

$P_0 = 1-\rho = 1-0.15$

$P_0 = 0.85 \Rightarrow \boxed{85\%}$

$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)}$

$= \frac{3^2}{20(20-3)} = \frac{9}{20(17)}$

$L_q = \frac{9}{340} = 0.026$

if $c = 2$

$\rho = \frac{\lambda}{c\mu} = \frac{3}{2(20)} = \frac{3}{40}$

$\rho = 0.075 \Rightarrow \boxed{7.5\%}$

$P_0 = \left\{\left[\sum_{n=0}^{c-1} \frac{(c\rho)^n}{n!}\right] + \left[(c\rho)^c\left(\frac{1}{c!}\right)\left(\frac{1}{1-\rho}\right)\right]\right\}^{-1}$

$\therefore c\rho = \frac{3}{20} = 0.15$

$= \left\{\left[\sum_{n=0}^{2-1} \frac{(0.15)^n}{n!}\right] + \left[(0.15)^2\left(\frac{1}{2!}\right)\left(\frac{1}{1-0.075}\right)\right]\right\}^{-1}$

$= \left\{\left[1 + \frac{0.15}{1!}\right] + \left[0.0225 \times \frac{1}{2} \times \frac{1}{0.925}\right]\right\}^{-1}$

$= \left\{1.15 + \left[0.0225 \times \frac{1}{1.85}\right]\right\}^{-1}$

$= \{1.15 + 0.0121\}^{-1}$

$= \{1.16\}^{-1}$

$\boxed{P_0 = 0.86} \Rightarrow 86\%$

$L_s = 0.15 + \frac{(0.15)^{2+1}(0.86)}{2(2!)(1-0.075)^2}$

$= 0.15 + \frac{(0.00337)(0.86)}{2(2)(0.855)}$

$= 0.15 + \frac{0.00289}{3.42} \Rightarrow 0.15 + 0.00084$

$\boxed{L_s = 0.150}$

$W_s = \frac{0.15}{3} = \boxed{0.05}$

$W_q = 0.05 - \frac{1}{20}$

$= 0.05 - 0.05$

$\boxed{W_q = 0}$

$L_q = \lambda W_q$

$= 3(0)$

$\boxed{L_q = 0}$

Q3

Assignment # 2.

$\lambda = 7/\text{week}$

$\mu = 8/\text{week}$

$c = 3$

i: $\rho = ?$

ii: $P_0 = ?$

iii: $W_s = ?$

iv: $W_q = ?$

v. $L_s = ?$

vi. $L_q = ?$

Sol:-

$$\rho = \frac{\lambda}{c\mu} = \frac{7}{8(3)} = \frac{7}{24} = 0.29$$

$$\boxed{\rho = 29\%}$$

$$\therefore c\rho = \frac{7}{8} = 0.87$$

$$P_0 = \left\{\left[\sum_{n=0}^{c-1} \frac{(c\rho)^n}{n!}\right] + \left[(c\rho)^c \left(\frac{1}{c!}\right)\left(\frac{1}{1-\rho}\right)\right]\right\}^{-1}$$

$$= \left\{\left[\sum_{n=0}^{3-1} \frac{(0.87)^n}{n!}\right] + \left[(0.87)^3\left(\frac{1}{3!}\right)\left(\frac{1}{1-0.29}\right)\right]\right\}^{-1}$$

$$\left\{\left[\frac{(0.87)^0}{0!} + \frac{(0.87)^1}{1!} + \frac{(0.87)^2}{2!}\right] + \left[(0.65)\left(\frac{1}{6}\right)\left(\frac{1}{0.71}\right)\right]\right\}^{-1}$$

$$\left\{\left[1 + 0.87 + 0.37\right] + \left[\frac{0.65}{4.26}\right]\right\}^{-1}$$

$$\{2.24 + 0.15\}^{-1}$$

$$\{2.39\}^{-1}$$

$$\boxed{P_0 = 0.41 = 41\%}$$

iii: $L_s = c\rho + \dfrac{(c\rho)^{c+1} P_0}{c(c!)(1-\rho)^2}$

$$= 0.87 + \frac{(0.87)^{3+1}(0.41)}{3(3!)(1-0.29)^2}$$

$$= 0.87 + \frac{(0.57)(0.41)}{3(6)(0.50)}$$

$$= 0.87 + \frac{0.23}{9}$$

$$= 0.87 + 0.025$$

$$\boxed{L_s = 0.89}$$

iv. $W_s = \dfrac{L_s}{\lambda}$

$$W_s = \frac{0.89}{7}$$

$$\boxed{W_s = 0.127}$$

v. $W_q = W_s - \dfrac{1}{\mu}$

$$= 0.127 - \frac{1}{8}$$

$$= 0.127 - 0.125$$

$$\boxed{W_q = 0.002}$$

vi. $L_q = \lambda W_q$

$$= 7(0.0021)$$

$$\boxed{L_q = 0.014}$$

Scanned with CamScanner

4 $\lambda = 1/45\text{ min} = 1.33/\text{hr}$

$\mu = 1/9\text{ min} \times 4 = \frac{1}{36\text{ min}} \quad 1.66/\text{hr}$

$c = 3$

i. $W_q = ?$

ii. $L_s = ?$

iii. Average time to wash car?

$P = \frac{\lambda}{c\mu} = \frac{1.33}{3(1.66)} = \frac{1.33}{4.98}$

$\boxed{P = 0.267}$

$cp = \frac{\lambda}{\mu} = \frac{1.33}{1.66}$

$\boxed{cp = 0.801}$

$P_0 = \left\{\left[\frac{(0.801)^0}{0!} + \frac{(0.801)^1}{1!} + \frac{(0.801)^2}{2!}\right] + \left[(0.801)^3\left(\frac{1}{3!}\right)\left(\frac{1}{1-0.267}\right)\right]\right\}^{-1}$

$= \{[1 + 0.801 + 0.320] + [0.513(0.166)(1.364)]\}^{-1}$

$\{2.121 + 0.116\}^{-1} = \{2.237\}^{-1}$

$\boxed{P_0 = 0.447}$

$L_s = 0.801 + \frac{(0.801)^{3+1}\,0.447}{3(3!)(1-0.267)^2}$

$= 0.801 + \frac{0.411(0.447)}{3(6)(0.537)}$

$= 0.801 + \frac{0.183}{9.671}$

$= 0.801 + 0.0189$

$\boxed{L_s = 0.819}$

$W_s = \frac{L_s}{\lambda} = \frac{0.819}{1.33}$

$\boxed{W_s = 0.616}$

$W_q = W_s - \frac{1}{\mu}$

$= 0.616 - \frac{1}{1.66}$

$= 0.646 - 0.602$

$\boxed{W_q = 0.014}$

iii

Average time to wash car:

1 service for 9 mins

4 " " = 9 x 4

= 36 mins

Sol.

1: H_0 = Service interruption follows poisson dist.

H_1 = Service interruptions do not follow poisson dist.

2: finding λ.

$$\lambda = \bar{X} = \sum_{n=0}^{n} \frac{x_i}{n} = \frac{0(160)+1(175)+2(86)+3(41)+4(18)+5(12)+6(8)}{500}$$

$$= \frac{650}{500} \Rightarrow \boxed{\lambda = 1.3}$$

X_i	O_i	$P_i = \frac{e^{-1.3}(1.3)^x}{x!}$	$E_i = 500(p_i)$	$(O-E)^2/E$	
0	160	$= \frac{e^{-1.3}(1.3)^0}{0!} = 0.272$	136	$\frac{(160-136)^2}{136} = 4.23$	
1	175	0.354	177	0.022	
2	86	0.230	115	7.313	
3	41	0.099	49.5	1.459	
4	18	0.032	16	0.25	
5	12	0.008	4	16.03	
6	8	0.0018	0.9	56.011	$X_0^2 = 85.28$

$\therefore$ DF = 7-1-1 = 5.

$CV = X_{0.01,} = 15.18$

$$X_0^2 > X^2_{0.01,5}$$

$$85.28 > 15.18$$

Ho is rejected.

H_0 = Assumed prob. of ...
H_1 = " " "

∴ Prob of female : 60% = 0.60
Prob of male : 40% = 0.40

Categories	observed	Expected	$\frac{(O-E)^2}{E}$
Female	53	60	$\frac{(53-60)^2}{60} = 0.816$
male	47	40	$\frac{(47-40)^2}{40} = 1.225$
		Total	= 2.041

Hence,

$$X_0^2 = 2.041$$

F = 2 - 0 - 1
= 1

V = $X^2_{0.05,1} = 3.84$

$$\therefore X_0^2 > X^2_{0.05,1} \Rightarrow 2.041 \not> 3.84$$

H_0 is accepted

① H_0 = Commercial mortgage approved/week ...
H_1 = " " " do not "

2. finding λ:

$$\lambda = \bar{X} = \sum_{n=0}^{n} \frac{X_i}{n} = \frac{0(13)+1(25)+2(32)+3(17)+4(9)+5(6)+6(1)+7(1)}{104}$$

$$= \frac{219}{104} \Rightarrow \lambda = 2.1$$

n	O_i	$P_i = e^{-2.1}(2.1)^n/n!$	$E_i = 104(P_i)$	$(O-E)^2/E$
0	13	$\frac{e^{-2.1}(2.1)^0}{0!} = 0.122$	12.688	0.0076
1	25	0.257	26.728	0.1117
2	32	0.270	28.08	0.5472
3	17	0.189	19.656	0.3588
4	9	0.099	10.296	0.1631
5	6	0.041	4.264	
6	1 8	0.014	1.456 6.167	0.5448
7	1	0.0043	0.4472	

Total $X_0^2 = 1.7332$

DF = 6 − 1 − 1 = 4

CV = $X_{0.01,4}$ = 13.28

$\therefore X_0^2 > X^2_{0.01,4}$

$\therefore 1.7332 \not> 13.28$

H_0 is accepted

Scanned with CamScanner

: 8:

H_0 = The proportion of jurors is consistent with the county proportion.

H_1 = " " " is not "

Groups.	Observed	count wide %	$E = n(P_i) = 66(P_i)$	$\frac{(O-E)^2}{E}$
21-40	5	42% = 0.42	27.72	18.621
41-50	9	23% = 0.23	15.18	2.515
51-60	19	16% = 0.16	10.56	6.745
over 60	33	19% = 0.19	12.54	33.382

Total = X_0^2 = 61.263

DF = 4-0-1 = 3

CV = $X_{0.05,3}$ = 7.81

$X_0^2 > X^2_{0.05,3}$

61.263 > 7.81

H_0 is rejected.

Q9-

H_0 = Service times are exponentially distributed
H_1 = Service times are not exponentially dist:

Intervals	O_i	$E_i = 50\left(\frac{1}{6}\right)$	$(O-E)^2/E$
1	8	8.33	0.013
2	11	8.33	0.855
3	9	8.33	0.053
4	5	8.33	1.331
5	10	8.33	0.334
6	7	8.33	0.212

$X_0^2 = 2.798$

DF = 6-1-1 = 4

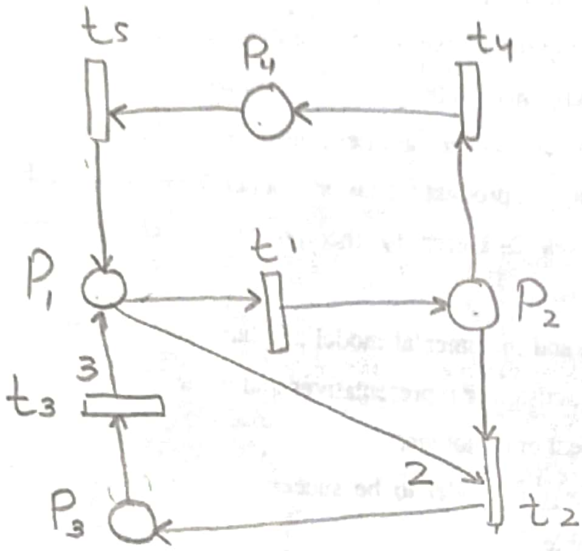
CV- $X^2_{0.05,4} = 9.49$

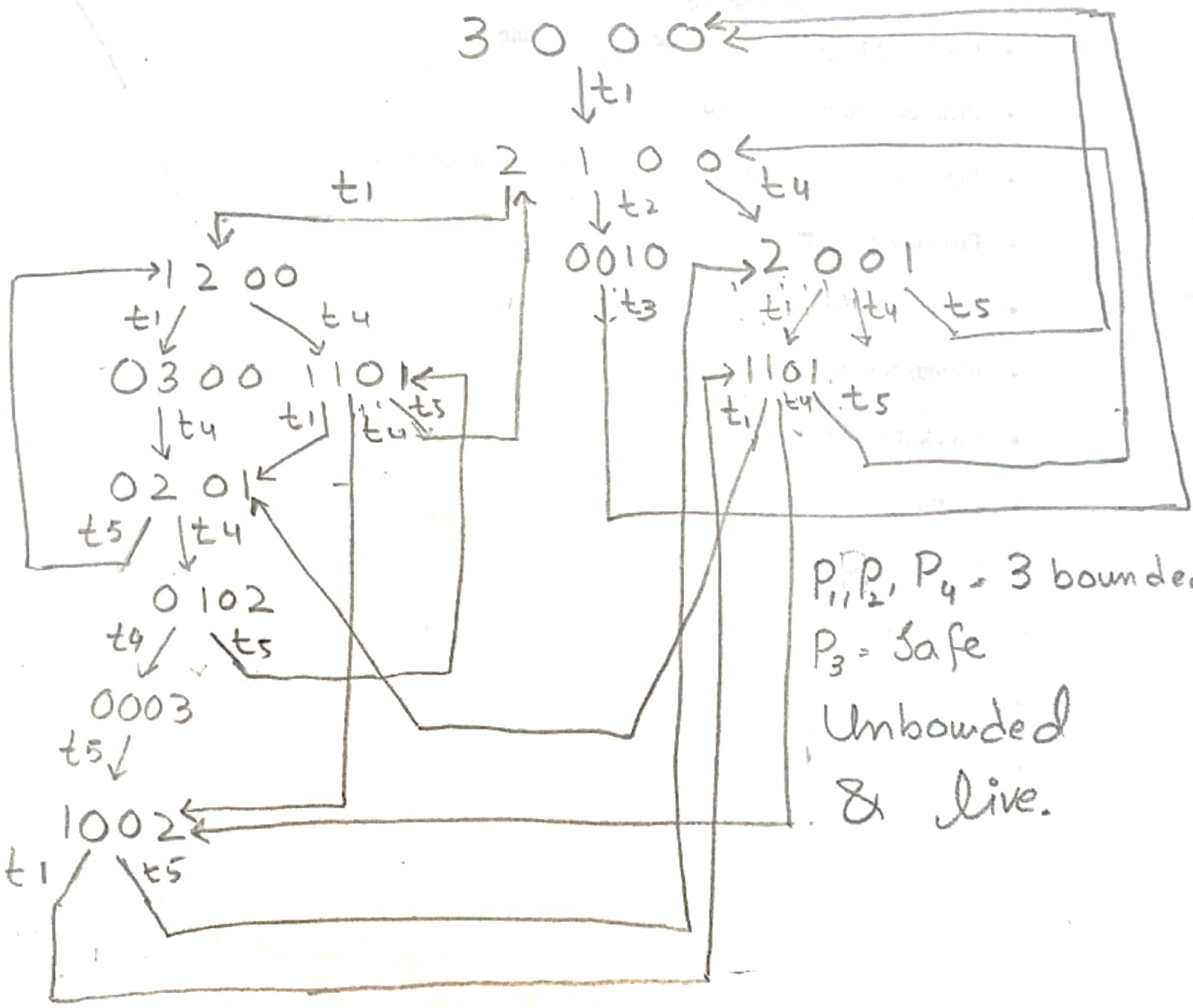
$\therefore \quad X_0^2 < X^2_{0.05,4}$

$2.798 < 9.49$

Ho is accepted.

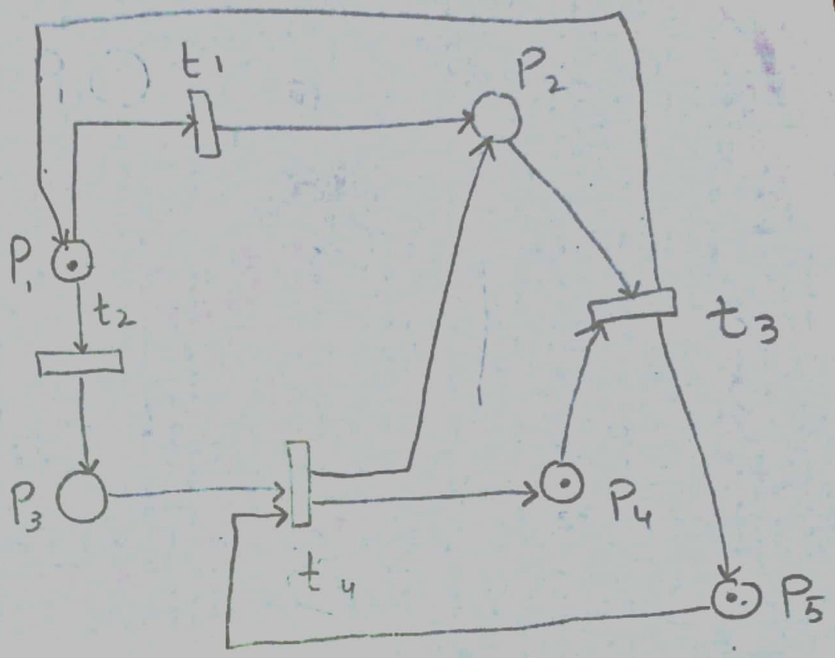
10- $E^+ = \begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ $E^- = \begin{bmatrix} 0 & 0 & 3 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$ $M_0 = \begin{bmatrix} 3 \\ 0 \\ 0 \\ 0 \end{bmatrix}$





P_1, P_2, P_4 - 3 bounded
P_3 - Safe
Unbounded
& live.

$$E^+ = \begin{array}{c} \begin{array}{cccc} t_1 & t_2 & t_3 & t_4 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \quad E^- = \begin{array}{c} \begin{array}{cccc} t_1 & t_2 & t_3 & t_4 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \quad M_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$



1 0 0 1 1

$\xrightarrow{t_1}$ (0 1 0 1 1) $\xrightarrow{t_3}$ (1, 0, 0, 0, 2)

$\xrightarrow{t_2}$ (0, 0, 1, 1, 1) $\xrightarrow{t_4}$ (0, 1, 0, 2, 0) $\xrightarrow{t_3}$ 1 0 0 1 1

(1, 0, 0, 0, 2) $\xrightarrow{t_1}$ (0, 1, 0, 0, 2)

(1, 0, 0, 0, 2) $\xrightarrow{t_2}$ (0, 0, 1, 0, 2) $\xrightarrow{t_4}$ (0, 1, 0, 1, 1) $\xrightarrow{t_3}$ (1, 0, 0, 0, 2)

P_1, P_2, P_3 = safe (1 bounded)

P_4, P_5 = 2 bounded.

Petrinet is unbounded & live.