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WEEK-3.

LECTURES OF

STOCHASTIC PROCESSES.

SE-(410).

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Absorption Problems:

- A state in a Markov Chain that is not possible to leave is called an absorbing state.
- It is a recurrent state.
- $P_{KK} = 1$; Transition probability (P) of K to K.

EXAMPLE 1:

Consider the following Transition state diagram.

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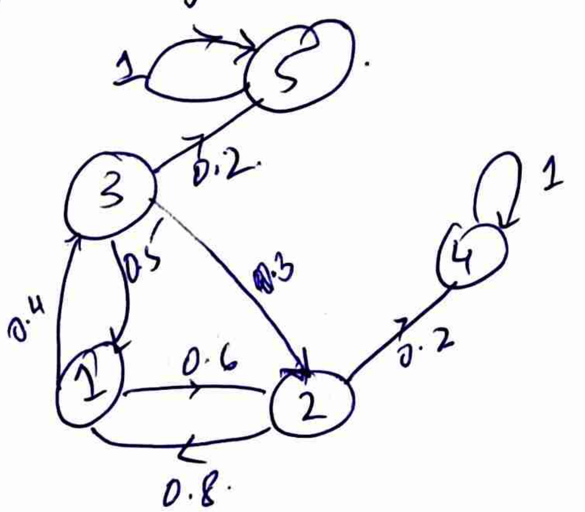

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State 4 and 5 are absorbing states.
If we were to find the probability as to what probability it is that we will end up in state 4 or state 5; such type of probability is called Absorption probability

probability that we will end up in state 4 is more if we started from state 2 & prob. of ending in state 5 is more if we started from state 3. With state 1, it is unclear.

→ So the absorption probability for any state heavily depends upon the initial state.

QUESTION 1: What is the probability a_i that the chain eventually settles in 4 given it started in i.

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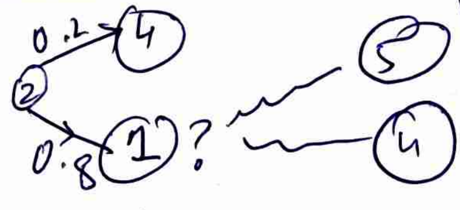
Now if $i = 4$, $a_i = 1$
$i = 5$, $a_i = 0$; There is no way that you go from state 5 to state 4.

But what if otherwise a_i? $i = 1, 2, 3$ then what?

If we start from state 2:



If you start from state 1, then you eventually get trapped in either state 4 or 5.

→ What are the probabilities of starting from 1 and ~~using the~~ ending in state 4 or 5?
Ans = we dont know.

→ The overall probability of interest which is $i = 2$; a_i; using the total probability theorem.

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$$a_2 = 0.2\,a_4 + 0.8\,a_1$$

Then,
$$a_1 = 0.6\,a_2 + 0.4\,a_3$$

$$a_3 = 0.3\,a_2 + 0.5\,a_1 + 0.2\,a_5$$

Now, solving these equations: 3/5

$a_2 = 0.2(1) + 0.8a_1$

$a_1 = 0.6a_2 + 0.4a_3$.

$a_3 = 0.3a_2 + 0.5a_1 + 0.2(0)$.

$\Rightarrow$ $a_2 = \frac{2}{10} + \frac{8a_1}{10} \longrightarrow \mathrm{I}$

$a_1 = \frac{6}{10}\left[\frac{2}{10} + \frac{8}{10}a_1\right] + 0.4a_3$ —— II

$a_3 = \frac{3}{10}\left[\frac{2}{10} + \frac{8}{10}a_1\right] + \frac{5}{10}a_1$ —— III.

$\Rightarrow a_3 = \frac{6}{100} + \frac{24}{100}a_1 + \frac{5}{10}a_1$

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$a_3 = \frac{6 + 24a_1 + 50a}{100}$

$a_3 = \frac{6 + 74a_1}{100}$ $\longrightarrow$ putting in II.

$\cancel{100a_3 = 6 + 74a_1}$.

$a_1 = \frac{6}{10}\left[\frac{2}{10} + \frac{8}{10}a_1\right] + \frac{4}{10}\left[\frac{6}{100} + \frac{74a_1}{100}\right]$

$74 \times 4 = 296$

$a_1 = \frac{12}{100} + \frac{48}{100}a_1 + \frac{24}{1000} + \frac{296a_1}{1000}$

$a_1 - \frac{48}{100}a_1 - \frac{296a}{1000} = \frac{12}{100} + \frac{24}{1000}$.

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$\frac{1000a_1 - 480a_1 - 296a_1}{\cancel{1000}} = \frac{120 + 24}{\cancel{1000}}$

$224a_1 = 144$

$a_1 = \cancel{\frac{144}{224}} = \cancel{\frac{72}{112}} = \cancel{\frac{36}{56}} = \cancel{\frac{18}{28}}\ \frac{9}{14}$

$\boxed{a_1 = \cancel{18/28}\ 9/14}$

Now,

$$a_2 = \frac{2}{10} + \frac{\cancel{84}^{2}}{\cancel{10}_{5}}\left(\frac{\cancel{18}^{9}}{\cancel{28}_{\cancel{14}\,7}}\right)$$

$$a_2 = \frac{2}{10\times7} + \frac{18}{35\times2}$$

$$a_2 = \frac{14+36}{70}$$

$$a_2 = \frac{\cancel{50}}{\cancel{70}} = \frac{5}{7}$$

$$\boxed{a_2 = \frac{5}{7}}$$

Now,

$$a_3 = \frac{6+74(9/14)}{100}$$

$$a_3 = \frac{6+\left(\frac{37\times9}{7}\right)}{100}$$

$$= \frac{6+\frac{333}{7}}{100}$$

$$= \frac{42+333}{7\times100}$$

$$a_3 = \frac{\cancel{375}}{\cancel{700}} = \frac{\cancel{75}}{140} = \frac{15}{28}$$

$$\boxed{a_3 = \frac{15}{28}}$$

Note that for 5 states you could have created a system ~~of~~ of equations with 5 unknown values. And you did create them But since values of a_4 & a_5 were 1 & 0 so the system became smaller.

Rough work

2	10, 35
5	5, 35
7	1, 7
	1, 1

$$\frac{\cancel{74}^{37}}{\cancel{14}_{7}}$$

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STION NO: 2.

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What is the probability b_i that the chain eventually settles in 5 given it started in i.

Ans= Repeat all procedure/steps for b_i with ending state as 5.

However; For any state i, given that you started in i, you will eventually with probability of 1 end up in either 4 or 5,

So $a_i + b_i = 1 \; \forall i$

$\forall$ = for all possible values of i

So once you have found out values for a_1, a_2, a_3, a_4 & a_5, you can find values of respective b_i through formula.

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To sum up, in general, the calculation of the probabilities to reach a given absorbing state (s), starting from any state (i) of Markov chain with (m) states, will be unique solution of m equations with m unknowns, with the additional conditions that

$a_s = 1$ & $a_s' = 0$; for the other absorbing state

Unique solution from $a_i = \sum_{j=1}^{m} P_{ij} a_j \; \forall i$

P_{ij} → probability of i to j

i = initial state

j = next state

X ——— X