CHAPMAN KOLMOGOROV DIFFERENTIAL EQUATION

FORWARD DIFFERENTIAL EQUATION

Using the CK equation

$$P_{ij}(m+n) = \sum_{k} P_{ik}(m) P_{kj}(n)$$

→ The sample space of states is S; such that i, j and $k \in S$.

→ Consider possibilities $k \neq j$ and $k = j$ as we know that there is direct transition possible from i to j.

So now we write the equation as

$$P_{ij}(m+n) = \sum_{k \neq j} P_{ik}(m) \cdot P_{kj}(n) + P_{ij}(m) \cdot P_{jj}(n) \rightarrow ①$$

Considering $P_{kj}(n)$ and $P_{jj}(n)$.

$$\left.\begin{aligned} P_{kj}(n) &= n\mu_{kj} \\ P_{jj}(n) &= 1 + n\mu_{jj} \end{aligned}\right\} \rightarrow \text{subs. these value in ①}$$

$$P_{ij}(m+n) = \sum_{k \neq j} P_{ik}(m) n\mu_{kj} + P_{ij}(m) \cdot (1 + n\mu_{jj})$$

$$P_{ij}(m+n) = \sum_{k \neq j} P_{ik}(m) n\mu_{kj} + P_{ij}(m) + P_{ij}(m) n\mu_{jj}$$

$$\frac{P_{ij}(m+n) - P_{ij}(m)}{n} = \sum_{k \neq j} \frac{P_{ik}(m) \cancel{n} \mu_{kj}}{\cancel{n}} + \frac{P_{ij}(m) n\mu_{jj}}{\cancel{n}}$$

Dividing by n.

① ②

$$\lim_{h \to 0} \frac{P_{ij}(m+h) - P_{ij}(m)}{h} = \sum_{k \neq j} P_{ik}(m)\, \mu_{kj} + P_{ij}(m)\, \mu_{jj} +$$

↳ partial differential Equation

Differential Equation can be of 2 types

1) Ordinary DE : dependant variable can be one or more than one but independent variable is one.

2) Partial DE : dependant variable single but independent variable is more than one

$$\frac{dx}{dt} + \frac{dy}{dt} = 0 \rightarrow ODE$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \rightarrow PDE$$

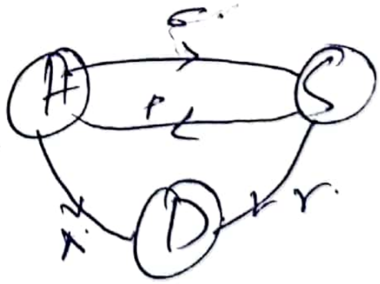
$$\boxed{\frac{d}{dt} P_{ij}(m) = \sum_{k} P_{ik}(m)\, \mu_{kj}}$$

(Now we have combined values for $k \neq j$ and $k = j$)

Forward Equation

Application # 1

HSD model



Now, we know.

$\mu_{HS} = \sigma$

$\mu_{SH} = \rho$

$\mu_{HD} = \lambda$

$\mu_{SD} = \gamma$

The generator matrix : (3)

$$\begin{array}{c} \\ H \\ S \\ D \end{array}\begin{array}{c} \begin{array}{ccc} H & S & D \end{array} \\ \begin{bmatrix} -(\sigma+\lambda) & \sigma & \lambda \\ \rho & -(\rho+\gamma) & \gamma \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

options

① H – H – H

② H – S – H.

③ H – D – H $\Rightarrow = 0$ No Transition

$$\frac{d}{dt} P_{HH}(m) = P_{HH}(m)\,\mu_{HH} + P_{SH}(m)\,\mu_{SH} + P_{HD}(m)\,\mu_{DH}^{=0}$$

$$\boxed{\frac{d}{dt} P_{HH}(m) = P_{HH}(m)\,-(\sigma+\lambda) + P_{HS}(m)\,\rho.}$$

If we want to find P_{HS}

$$\frac{d}{dt} P_{HS}(m) = P_{HS}(m)\,\mu_{SS} + P_{HH}(m)\,\mu_{HS} + P_{HD}(m)\,\mu_{DS} = 0$$

① H – S – S.

② H – H – S.

③ H – D – S = 0.

$$\boxed{\frac{d}{dt} P_{HS}(m) = P_{HS}(m)\,-(\rho+\gamma) + P_{HH}(m)\,\rho.}$$