# Stochastic Processes and Nonlinear Equilibrium System

Project: Fluctuations in Financial Markets

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# 1 Introduction

The precise shape of the return distribution (difference in the logarithm of stock prices at different times), which is characterized by a high probability of big volatility, has piqued interest. Stock price returns were described by models of volatility (standard deviation of returns) after it was discovered that substantial price movements tend to cluster together over time.

In this project we use an alternative strategy, inspired by the success of applying physics concepts to economic research. We look at price dynamics on a time scale $\Delta t$, which is long enough to average across the complexities of market micro-structure but short enough to clarify trading dynamics.

There is a quantitative relationship in physical systems between the strength of an observable's fluctuations and the pace at which energy is dissipated when the same observable is driven by an external force When studying the fluctuations of tiny particles in a fluid, Einstein was the first to uncover this relationship . The range of position fluctuations in this situation is related to temperature times particle mobility. The fluctuation-dissipation theorem is a generalization of this relationship that applies to a wide range of systems.

# 2 Methodology

## 2.1 Return

The observable in question for the project is the logarithmic change in the price of a share with in an interval $\Delta t$ given by

$$G_{\Delta t}(t) = lnS(t) - lnS(t - \Delta t) \tag{1}$$

where $S(t)$ is the price of a share a time $t$ [2].

We define a variable called the instantaneous return at time $t$ as $g(t) = \tau \frac{d}{dt} lnS(t)$, where $\tau$ is the average time between transactions. Therefore the observable returns on for time scale $\Delta t$ is related as

$$G(t)_{\Delta t} = \frac{1}{\tau} \int_{t-\Delta t}^{t} dt' g(t') \tag{2}$$

## 2.2 Fluctuation

The fluctuation to the system is caused by an applied force created by the difference between buyers and sellers of a given share. Analogous to instantaneous return, we define an instantaneous order/volume imbalance as $q(t) = \tau \frac{d}{dt} Q(t)$ [2]

## 2.3 Langevin Equation

The dynamics of the quantity $g(t)$ can be described by the stochastic equation

$$\tau \dot{g} = -rg(t) + \mu q(t) + f(t) \tag{3}$$

where $r$ is a constant and $f(t)$ is Gaussian white noise such that $\langle f(t) \rangle = 0$ and $\langle f(t) f(t') \rangle = \frac{2\tau}{r^2} \delta(t - t')$. The stochastic noise represents influences onto the price from news and other factors that are not accounted into the price or the order imbalance [2].

## 2.4 Response Function / Susceptibility

. The price impact function describes the relationship between the expectation value $\langle G_{\Delta t} \rangle_{\Delta Q}$ conditioned on the volume imbalance and the volume imbalance itself [2]. The susceptibility of $G$ with respect to the volume imbalance $\Delta Q$ [1]. The susceptibility $\chi$ is defined as as the slope of the price impact function close to zero order imbalance

$$\chi =_{\Delta Q \to 0}^{lim} \frac{\langle G_{\Delta t} \rangle_{\Delta Q}}{\Delta Q}. \tag{4}$$

### 2.5 Mean Squared Difference

The variance of price change for the unfluctuated system can be described by Einsteins diffusion law which states that

$$\langle (G(\Delta t))^2 \rangle_0 = 2D\Delta t \tag{5}$$

where $D$ is the diffusion coefficient[1]. In our case $D = \frac{1}{r^2\tau}$. which then gives the result

$$\langle (G(\Delta t))^2 \rangle_0 = \frac{2}{r^2\tau} t \tag{6}$$

## 3 Empirical Results

In order to attain the empirical results, I employed methods of data analysis to historical price data. The company that was under investigation was Naspers Limited. The company has the largest market cap in South Africa [3].

Historical trade data of the share was extracted from Yahoo Finance (ref:[4]). This data includes daily data of the Open price, Close price, High of the day, Low of the day and Volume traded of the share. And data for the past 10 years was used for the project.

This was all in order to realise the form of the response function of returns due do order imbalance $\chi$. The code used for the project can be found in the link ref:[5]

### 3.1 Response function

To in order to obtain results of the impact on return, from the historical data the price we will use is the Closing price. We compute the difference of logarithm price within the given daily time interval as mentioned in (1).

In order to determine whether the volume imbalance was a result of a buy or sell order, we assign the sign of the difference to the trade volume at the time $t - \Delta t$. A positive change in returns corresponds to the market having more buyers than sellers and hence a positive trade volume, a negative change in returns corresponds to the market having more sellers than buyers and hence a negative trade volume.

The change in returns is then sorted according to the assigned volume imbalance allowing us to take the average of returns with respect to each volume imbalance. This results in the following figure

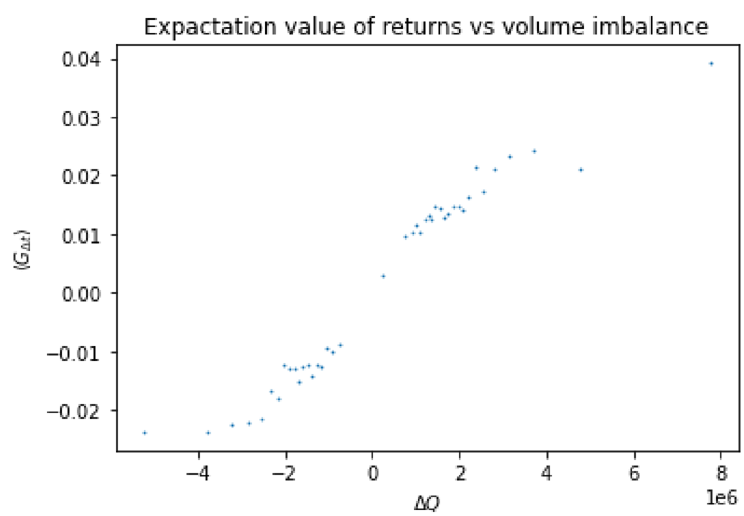


Figure 1: This is the plot of the Price impact, which is the expected return due to an order imbalance, plotted against the order imbalance in the markets for NPN.JO. Returns are measured in percent, the order imbalance in number of stocks.

## 3.2 Relation of susceptibility to standard deviation

We then sort the quarterly data according to the fluctuations (corresponding to $\frac{1}{r}$ which is the liquidity of the market) per trade and calculate the price impact function for each fluctuation strength. This is illustrated in the following figure

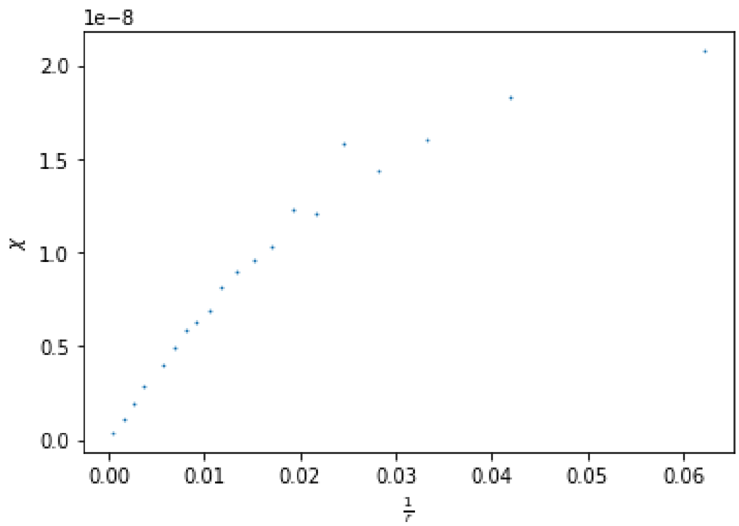


Figure 2: Susceptibility for stock price changes (slope of the price impact function for small order imbalances) plotted against standard deviation $1/r$ of returns per trade.

# 4 Discussion

From figure 1 return response function $\chi$ can be visualised by plotting the average price change for a given order imbalance against the order imbalance $\Delta Q$. It's characteristic form of with linear part for small $\Delta Q$ and less steep part for larger $\Delta Q$ can be described by a hyperbolic tangent.

From figure 2 we get linear relationship of between $\chi$ and $1/r$. From 6 we can get the relation

$$\frac{1}{r} \sim \sqrt{\langle (G(\Delta t))^2 \rangle_0} \tag{7}$$

This along with the result from figure 2 gives the result that

$$\chi \sim \sqrt{\langle (G(\Delta t))^2 \rangle_0} \tag{8}$$

and their proportionality constant is the square root of trades executed within an interval, which is analogous to temperature in physical systems[2].

Thus the empirical analysis support the linear relationship between fluctuation strength and susceptibility.

# 5 Conclusion

The relationship between the strength of stock price swings and the friction constant, which links order imbalance and price variations, has been explored both theoretically and practically. We used a coarse-grained time scale to describe stock price dynamics and discovered a similarity to the fluctuation-dissipation theorem in physics, which is validated by our empirical analysis.

# References


[1] ROHWER, C. (2021). Stochastic Processes and Non-Equilibrium Systems 2021. Lecture Notes.

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[3] Value.today. 2021. South Africa Top Companies by Market Cap as on 2020. [online] Available at: ¡https://www.value.today/headquarters/south-africa¿ [Accessed 20 July 2021].

[4] Yahoo is now a part of Verizon Media. (2000). Yahoo Finance. https://finance.yahoo.com/quote/NPN.JO/

[5] https://github.com/Kabelo741/Fluctuations-in-Financial-Markets-Project.git