SUEUR correction ①

# Correction

$e_i = u_i - u$, $u_i$ solution sur le domaine $i$.

$$\begin{cases} -\dfrac{\partial^2 u_1^{2n+1}}{\partial x^2} + \dfrac{\partial^2 u}{\partial x^2} = f - f = 0 = -\dfrac{\partial^2 e_1^{2n+1}}{\partial x^2} \text{ sur } [0,\Gamma_1] \\ u_1^{2n+1}(0) - u(0) = e_1^{2n+1}(0) = \alpha - \alpha = 0 \\ e_1^{2n+1}(\Gamma_1) = e_2^{2n}(\Gamma_1) \end{cases} \longrightarrow e_1^{2n+1}(x) = ?$$

<u>Domaine 2</u> :
$$\begin{cases} -\dfrac{\partial^2 u_2^{2n+2}}{\partial x^2} + \dfrac{\partial^2 u}{\partial x^2} = f - f = 0 = -\dfrac{\partial^2 e_2^{2n+2}}{\partial x^2} \text{ sur } [\Gamma_2, 1] \\ u_2^{2n+2}(1) - u(1) = e_2^{2n+2}(1) = 0 \\ e_2^{2n+2}(\Gamma_2) = e_1^{2n+1}(\Gamma_2) \end{cases} \longrightarrow e_2^{2n+2}(x) = ?$$

<u>Polynôme caractéristique</u> :

$$-X^2 = 0 \Rightarrow X_1 = X_2 = 0$$

Donc, $e_1(x) = (a_1 x + b_1)\underbrace{e^{X_1 x}}_{1} = a_1 x + b_1$ $\quad e^{1}$

$$\text{Or} \begin{cases} e_1(0) = 0 \Rightarrow b_1 = 0 & (1) \\ e_1^{2n+1}(\Gamma_1) = e_2^{2n}(\Gamma_1) = a_1\Gamma_1 + b_1 = a_1\Gamma_1. & (2) \end{cases}$$

De la m. manière, $e_2^{2n+2}(x) = a_2 x + b_2$.

Méthode de Cramer

$$\begin{pmatrix} \Gamma_2 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} a_2 \\ b_2 \end{pmatrix} = \begin{pmatrix} e_1^{2n+1}(\Gamma_2) \\ 0 \end{pmatrix}$$

CL en 1 et $\Gamma_2$.

$$a_2 = \frac{\begin{vmatrix} e_1^{2n+1}(\Gamma_2) & 1 \\ 0 & 1 \end{vmatrix}}{\begin{vmatrix} \Gamma_2 & 1 \\ 1 & 1 \end{vmatrix}} = \frac{e^{2n+1}(\Gamma_2)}{\Gamma_2 - 1} \; ; \; b_2 = \frac{\begin{vmatrix} \Gamma_2 & e_1^{2n+1}(\Gamma_2) \\ 1 & 0 \end{vmatrix}}{\Gamma_2 - 1} = \frac{-e^{2n+1}(\Gamma_2)}{\Gamma_2 - 1}$$

$$e_1^{2n+1}(x) = \underbrace{\frac{e_2^{2n}(\Gamma_1)}{\Gamma_1}}_{a_1} x \quad \xrightarrow{\text{via (2)}}$$

$$e_2^{2n+2}(x) = e_1^{2n+1}(\Gamma_2) \frac{(x-1)}{\Gamma_2 - 1}$$

$\boxed{\text{Convergence ?}}$

$$e_1^{2n+1}(\Gamma_2) = \frac{e_2^{2n}(\Gamma_1)}{\Gamma_1} \Gamma_2$$

$$e_2^{2n}(\Gamma_1) = \frac{e_1^{2n-1}(\Gamma_2)}{\Gamma_2 - 1} \Gamma_1 - 1$$

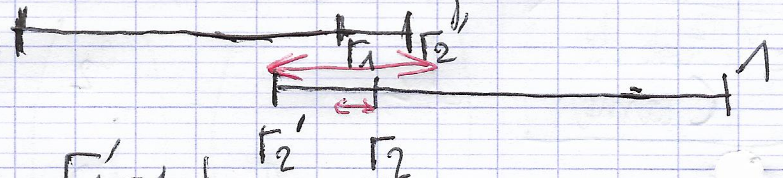
$$e_1^{2n+1}(\Gamma_1) = \frac{\Gamma_2}{\Gamma_1}\left(\frac{\Gamma_1 - 1}{\Gamma_2 - 2}\right) e_1^{2n-1}(\Gamma_2)$$

$\longrightarrow$ suite géométique de raison $r = \frac{\Gamma_2}{\Gamma_1}\left(\frac{\Gamma_1 - 1}{\Gamma_2 - 1}\right) < 1$

car $\Gamma_1 > \Gamma_2 \Rightarrow$ l'algo de Schwarz multiplicatif converge de manière purement linéaire ($r$ indpdt de $n$).

Remarques : 1) La convergence de Schwarz multiplicatif (additif) est d'autant plus rapide que le recouvrement $\Gamma_1 - \Gamma_2$ est grand.

$\Gamma_2' < \Gamma_2 < \Gamma_1 < \Gamma_1'$




$$\frac{\Gamma_2'}{\Gamma_1'} << \frac{\Gamma_2}{\Gamma_1} \qquad \frac{\Gamma_1 - 1}{\Gamma_2 - 1} >> \frac{\Gamma_1' - 1}{\Gamma_2' - 1} \quad \Bigg| \quad \gamma' << \gamma$$

SUEUR

correction 2

2) Schwarz multiplicatif cvg 2 fois + vite que Schwartz additif.

Exo

3) La cvg est purement linéaire (tx ne dépend pas de n)

ex $\gamma = 0,99999$ → converge en 2 itérations ? PRANK