⚙ 1. SYSTEM OVERVIEW

A current-mode controlled buck converter has two feedback loops:

(a) Outer loop → Voltage control

- Maintains the desired output voltage.
- Generates a reference current (i_{ref}).

(b) Inner loop → Current control

- Forces the inductor current (i_L) to follow (i_{ref}) on every switching cycle.

🧩 2. POWER STAGE MODEL

Let's start from the averaged buck converter equations (continuous conduction mode, CCM):

$$L\frac{di_L}{dt} = DV_{in} - V_o$$

$$C\frac{dV_o}{dt} = i_L - \frac{V_o}{R}$$

Where:

- (D) = duty ratio
- (i_L) = inductor current
- (V_o) = output voltage

⚡ 3. LINEARIZATION (SMALL-SIGNAL MODEL)

We write each variable as:

$$x = X + \hat{x}$$

where (X) is steady-state value, ($\hat{x}$) is a small perturbation.

So:

$$L\frac{d\hat{i}_L}{dt} = D\hat{v}_{in} + V_{in}\hat{d} - \hat{v}_o$$

$$C\frac{d\hat{v}_o}{dt} = \hat{i}_L - \frac{\hat{v}_o}{R}$$

🔹 4. VOLTAGE-MODE CONTROL TRANSFER FUNCTION (for reference)

In voltage-mode control, the control variable is (d) directly, so the small-signal control-to-output transfer is:

$$G_{vd}(s) = \frac{\hat{v}_o(s)}{\hat{d}(s)} = \frac{V_{in}}{1 + s\frac{R_c C}{1+sL/R_c}}; (\text{approx.})$$

This is second-order double pole at ($\omega_0 = 1/\sqrt{LC}$).

🔹 5. CURRENT-MODE CONTROL INNER LOOP BEHAVIOR

In CMC, duty cycle is not controlled directly by voltage error.

Instead:

1. The outer voltage loop outputs a current command (i_{ref}).
2. The inner loop ensures (i_L) follows (i_{ref}) every cycle.

So, we can write the inner current loop dynamics:

$$\hat{i_L}(s) = \frac{K_c}{1 + s/\omega_p}; \hat{i_{ref}}(s)$$

- (K_c) = DC gain of current loop (≈ 1 if loop is tight)
- (ω_p) = crossover of current loop (usually near half switching frequency)
- For frequencies below the switching frequency, ($i_L \approx i_{ref}$).

Hence, inductor current dynamics are suppressed and the inductor behaves almost instantaneously controlled.

🧮 6. EFFECTIVE SMALL-SIGNAL MODEL

We can substitute ($i_L \approx i_{ref}$) into the capacitor equation:

$$C\frac{d\hat{v}_o}{dt} = \hat{i_{ref}} - \frac{\hat{v}_o}{R}$$

Taking Laplace transform:

$$\hat{v}_o(s) = \frac{R}{1 + sRC}; \hat{i_{ref}}(s)$$

So the control-to-output transfer function becomes:

$$G_{vi}(s) = \frac{\hat{v}_o(s)}{\hat{i_{ref}}(s)} = \frac{R}{1 + sRC}$$

✅ This is a first-order system!

🔹 7. COMBINED TRANSFER FUNCTION

Now combine both loops:

- Outer voltage loop controls (i_{ref}) based on voltage error.
- Inner loop ensures ($i_L = i_{ref}$).
- The plant (power stage) seen by the voltage loop is first-order instead of second-order**.

Thus:

$$\hat{v_o}(s) = \underbrace{\frac{R}{1 + sRC}}_{\text{effective plant}} \cdot \hat{i_{ref}}(s)$$

and since (i_{ref}) is generated by voltage error amplifier:

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$$i_{ref}(s) = G_c(s) \cdot (V_{ref}(s) - \hat{v_o}(s))$$

Therefore, the overall closed-loop transfer function is:

$$\frac{\hat{v_o}(s)}{V_{ref}(s)} = \frac{G_c(s) \cdot \frac{R}{1+sRC}}{1 + G_c(s) \cdot \frac{R}{1+sRC}}$$

🧠 8. INTERPRETATION

Concept	Voltage-Mode	Current-Mode
Effective plant order	2nd order (LC double pole)	1st order (C only)
Dominant pole	LC double pole	Output capacitor pole only
Needed compensator	Type III (for 180° phase lag)	Type II (for 90° lag only)
Phase margin	Harder to stabilize	Easier (simpler compensation)
Inner dynamics	Inductor current uncontrolled	Inductor current directly controlled

So, current-mode control linearizes the inductor, making it appear as a current source to the outer loop.

🧮 9. INCLUDING EFFECT OF CURRENT LOOP GAIN

In a more realistic model, the inner loop is not perfectly ideal.

Let the inner loop have gain ($H_i(s) = \frac{Ki}{1+s/\omega\pi}$).

Then:

$$G_{vd}(s) = \frac{V_{in} H_i(s)}{(1 + sRC)(1 + sL/R + s^2 LC(1 - H_i(s)))}$$

If ($H_i(s) \to \infty$), the ($s^2 LC(1 - H_i)$) term cancels the inductor pole — leaving a first-order system.

📈 10. FREQUENCY RESPONSE COMPARISON

Property	Voltage-Mode	Current-Mode
DC gain	V_{in}	R
Number of poles	2	1
ESR zero	Adds phase boost	Still adds boost if ESR present
Phase margin (uncompensated)	~ 0°–90°	~ 90°–135°
Bode shape	Resonant dip around (f_{LC})	Smooth roll-off

🧩 11. PHYSICAL INTERPRETATION

- The inner current loop makes the inductor look like a resistor instead of a reactive element.
- From the outer loop's view:

$V_o = I_{ref}. R_{eq}$

where ($R_{eq} \approx R_{Load} \,||\, 1/(sC)$)

- Hence, voltage control becomes as simple as controlling the current through an RC network.

🔬 12. PRACTICAL INSIGHT

Scenario	Behavior
Line transient (V_{in} step)	Inner loop reacts almost instantly — fast correction
Load transient	Outer voltage loop detects Vout deviation, adjusts Iref; inner loop enforces new current quickly
Stability tuning t	Easier — Type II compensator sufficien
Subharmonic oscillation	May appear for duty > 0.5 → requires slope compensation

⚙ 13. FINAL SUMMARY

Parameter	Voltage-Mode Control	Current-Mode Control
Effective order	2nd (LC)	1st (C only)
Transfer function	$G_{vd}(s) = \frac{V_{in}(1+sR_cC)}{LCs^2 + s(R_cC + \frac{L}{R}) + 1}$	$G_{vi}(s) = \frac{R(1+sR_cC)}{1+sRC}$
Inner variable	Duty ratio	Inductor current
Outer variable	Output voltage	Output voltage
Compensation	Type III	Type II
Line/load response	Slower	Faster
Stability	Needs ramp comp. only if D > 0.5	Needs slope comp. (for inner loop)