Linear Discriminant Analysis (LDA)

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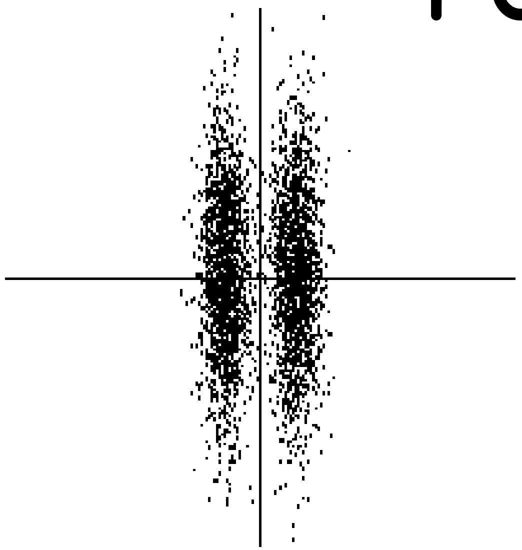
PCA

PCA

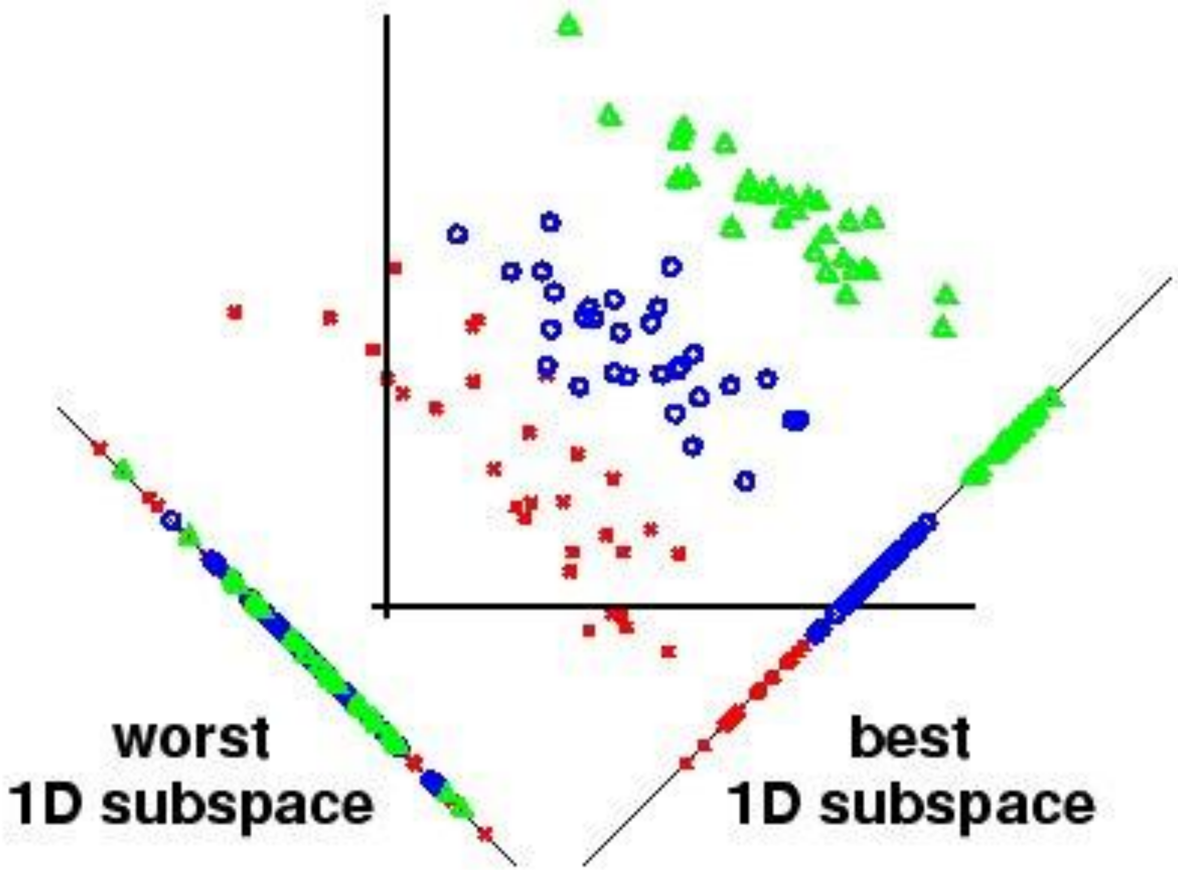


- The data is clearly divided into two clusters and each cluster has gaussian distribution
- PCA chooses the vertical axis as the principal component because of higher variance
  - Clustering information is lost
- Which projection would be more "interesting"?

Projection based on Interestingness (Projection Pursuit)

- Specify the projection index function $I(x,\alpha)$ that maximizes the interestingness of the projection
  - Measure of variation (PCA)
  - Departure from normality (negative entropy)
  - Class seperability (LDA, Bhattacharyya)
  - …
- Solve for α by maximizing $I(x,\alpha)$

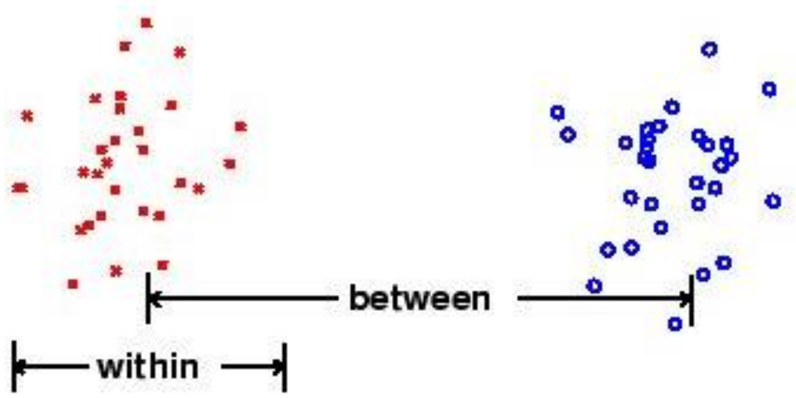
LDA



Within/between class scatter

- Within-class scatter (Sw)

$$S_w = \sum_j \mathrm{cov}_j$$
$$= \sum_j (X_j - \mu_j)(X_j - \mu_j)^T$$



  – where j is one of the red/blue classes.

- Between-class scatter (Sb)

$$S_b = \sum_j (\mu_j - \mu_*)(\mu_j - \mu_*)^T$$

LDA Criterion

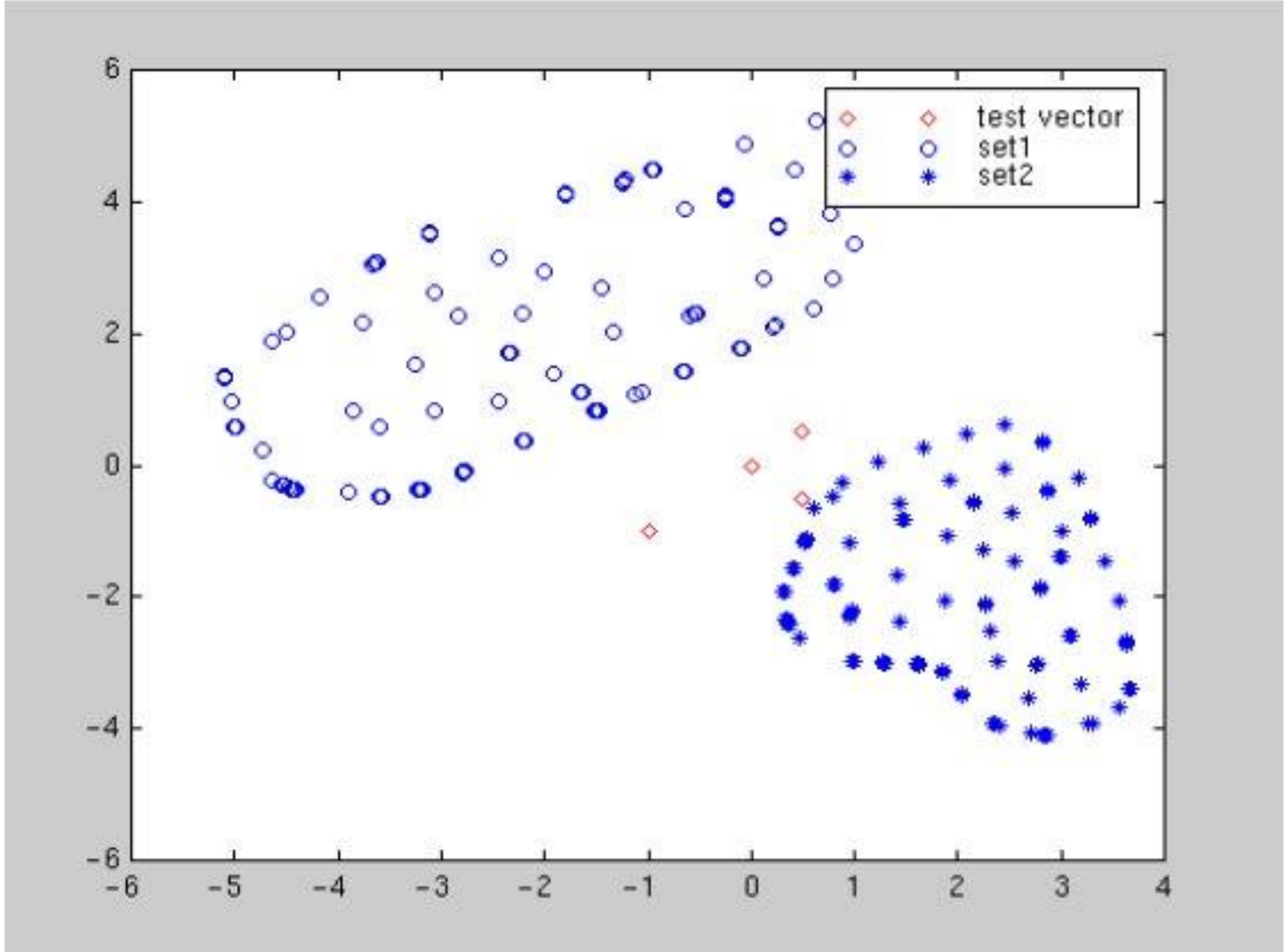
- Maximize ratio of overall variance to within class variance:

$$criterion = \frac{S_b}{S_w}$$

Two LDA approaches

- Class-independent transformation
  - All data is transformed using the same transformation, regardless of their class
  - Maximize ratio of overall between class variance to overall within class variance $criterion = \frac{S_b}{S_w}$
- Class-dependent transformation
  - Transform data from each class separately
  - Maximize ratio of overall between class variance to within class variance of a specific j'th class.
    - $criterion_j = \frac{S_b}{cov_j}$

LDA



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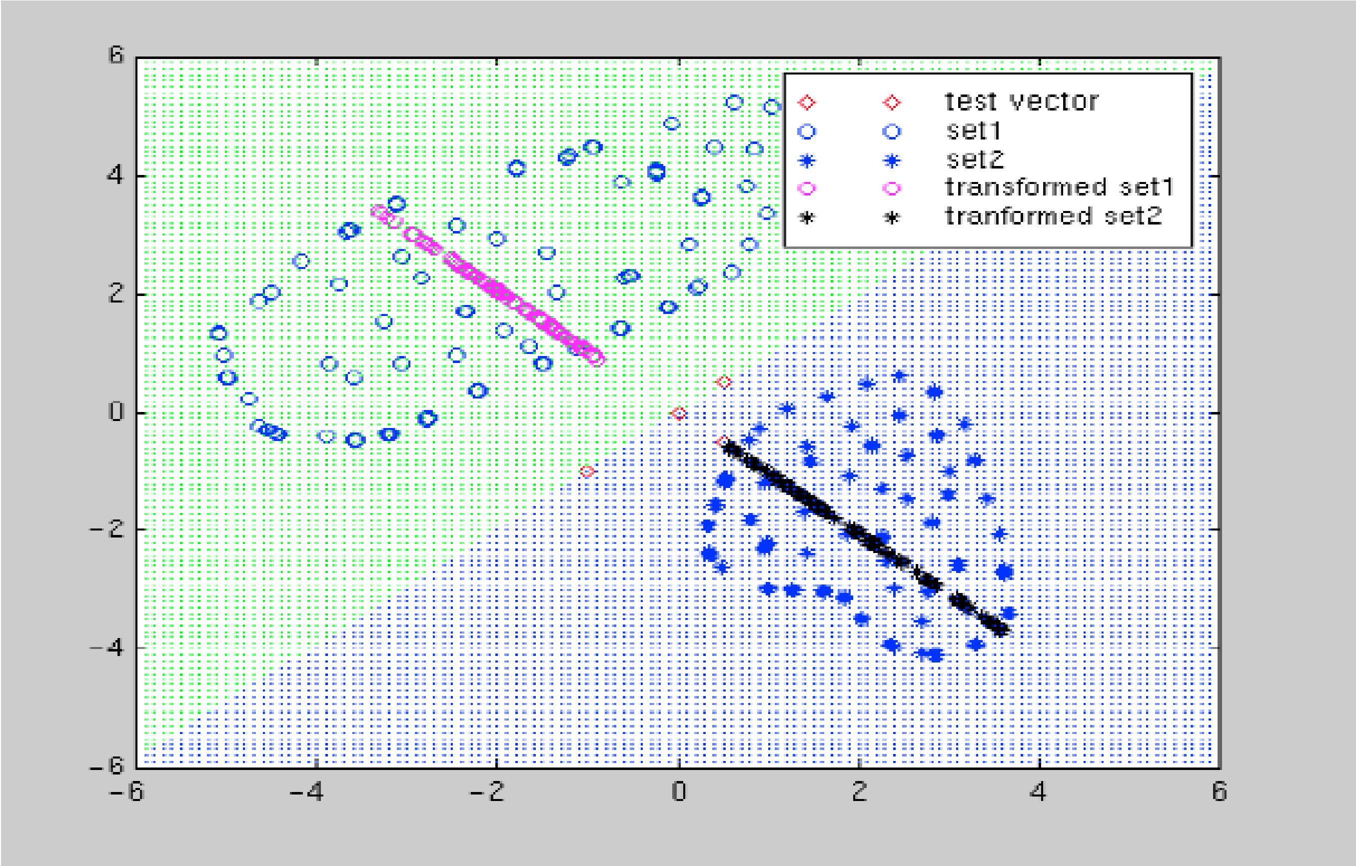
Class-Independent LDA



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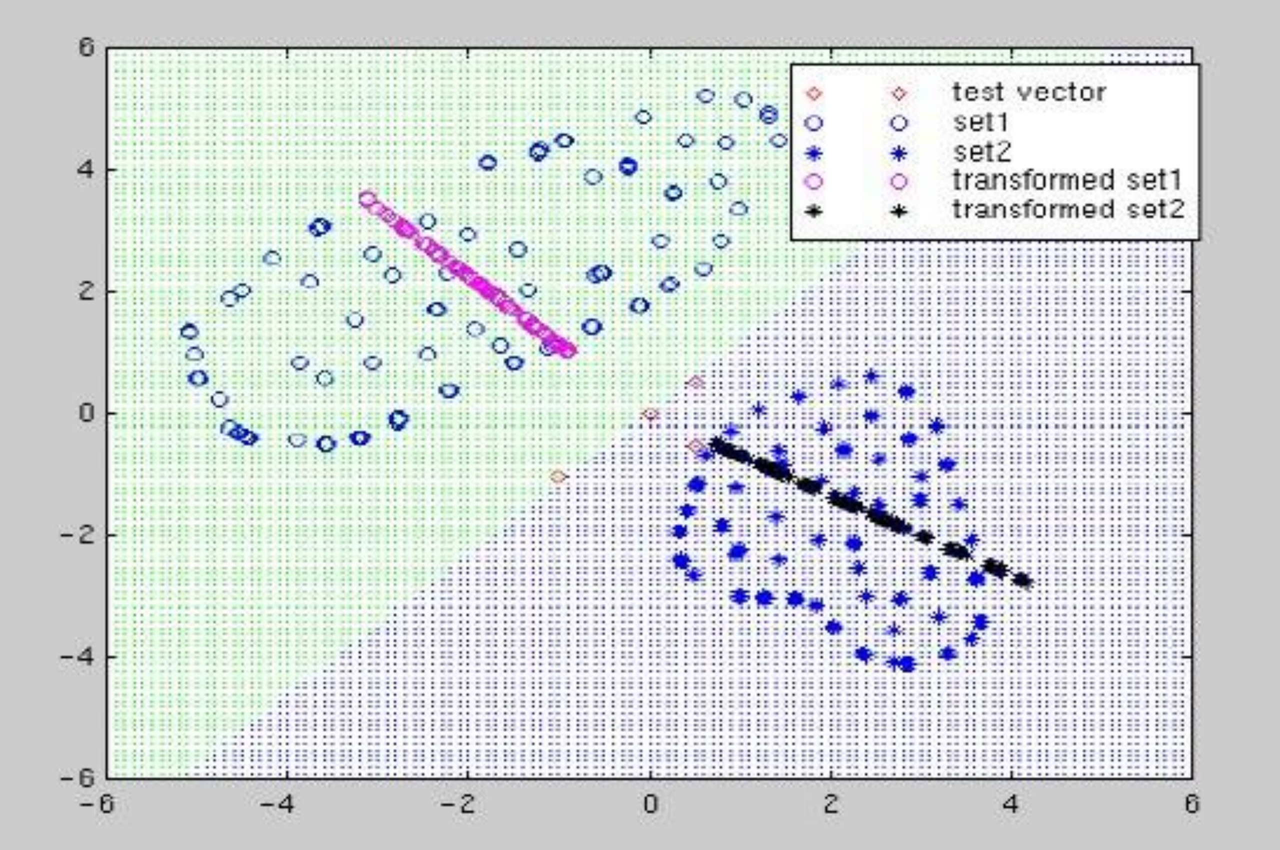
Class-Dependent LDA



Thoughts on LDA

- The goal in LDA is typically classification. We typically don't show the projections.
- LDA does not work well for:
  - complex problems with nonlinear patterns.
  - high dimensional data
- One solution to high dimensional data:
  - Perform PCA first, reduce to a few dimensions.
  - Then perform LDA