Symbolic Math

Problem 1

Evaluate the following limits if they exist:

```
syms x
```

a. $\lim_{x \to 0} \frac{sinx}{x}$

```
limit(sin(x)/x, x, 0)
```

ans = 1

b. $\lim_{x \to \infty} \frac{x^3 + 3x^2 - 5}{2x^3 - 6x}$

```
limit((x^3 + 3*x^2 - 5) / (2*x^3 - 6*x), x, sym(inf))
```

ans =

$$\frac{1}{2}$$

c. $\lim_{x \to 0^+} \frac{1}{x}$

```
limit(1/x, x, 0, 'right')
```

ans = ∞

Problem 2

Find the derivatives of the following functions:

```
syms x
```

a. $y = 3x^3 + 7x^2 - 5$

```
diff(3*x^3 + 7*x^2 - 5)
```

ans = $9\,x^2 + 14\,x$

b. $y = \sqrt{1 + x^4}$

```
diff(sqrt(1 + x^4))
```

ans =

$$\frac{2\,x^3}{\sqrt{x^4 + 1}}$$

c. $y = x^{\ln x}$

```
diff(x^log(x))
```

```
ans =
```

$$x^{\log(x)-1}\log(x) + \frac{x^{\log(x)}\log(x)}{x}$$

Problem 3

Evaluate the following definite integrals

```
syms x
```

a. $\int_{x=0}^{1} (3x^3 + 2x^2 - 5)\,dx$

```
int(3*x^3 + 2*x^2 - 5, 0, 1)
```

```
ans =
```

$$-\frac{43}{12}$$

b. $\int_{x=0}^{1} \left(\frac{1}{\sqrt{x}}\right) dx$

```
int(1/sqrt(x), 0, 1)
```

`ans =` 2

c. $\int_{x=0}^{1} \left(e^{-x^2}\right) dx$

```
int(exp(-x^2), 0, 1)
```

```
ans =
```

$$\frac{\sqrt{\pi}\,\operatorname{erf}(1)}{2}$$

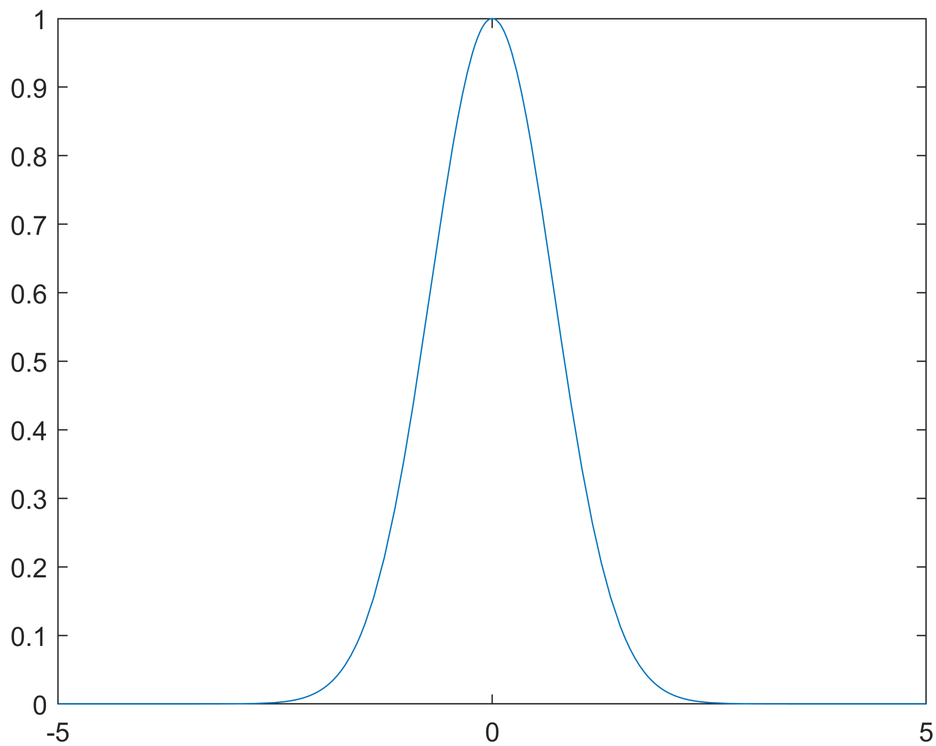
Problem 4

Graph the following:

```
syms x y t
```

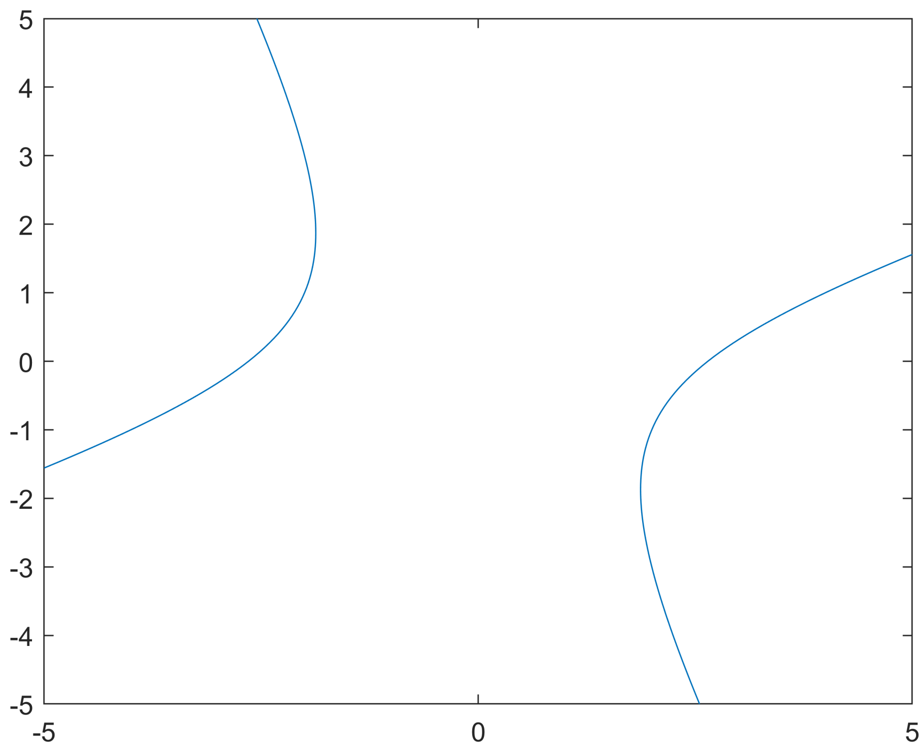
a. $y = e^{-x^2}\quad x \in [-5, 5]$

```
fplot(exp(-x^2), [-5,5])
```



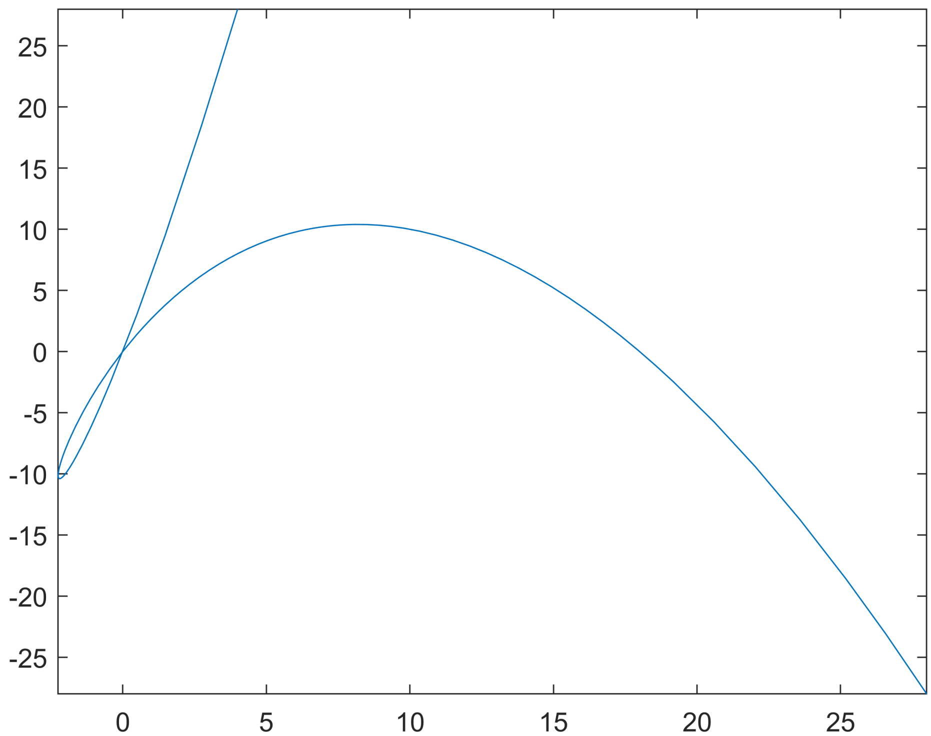
b. $x^2 - 2xy - y^2 = 7$

```
fimplicit(x^2 - 2*x*y - y^2 == 7)
```



c. $\begin{cases} x = t^2 - 3t \\ y = t^3 - 9t \end{cases} \quad t \in [-4, 4]$

```
x(t) = t^2 - 3*t;
y(t) = t^3 - 9*t;
fplot(x(t), y(t), [-4, 4])
```



Problem 5

Show the differential equations:

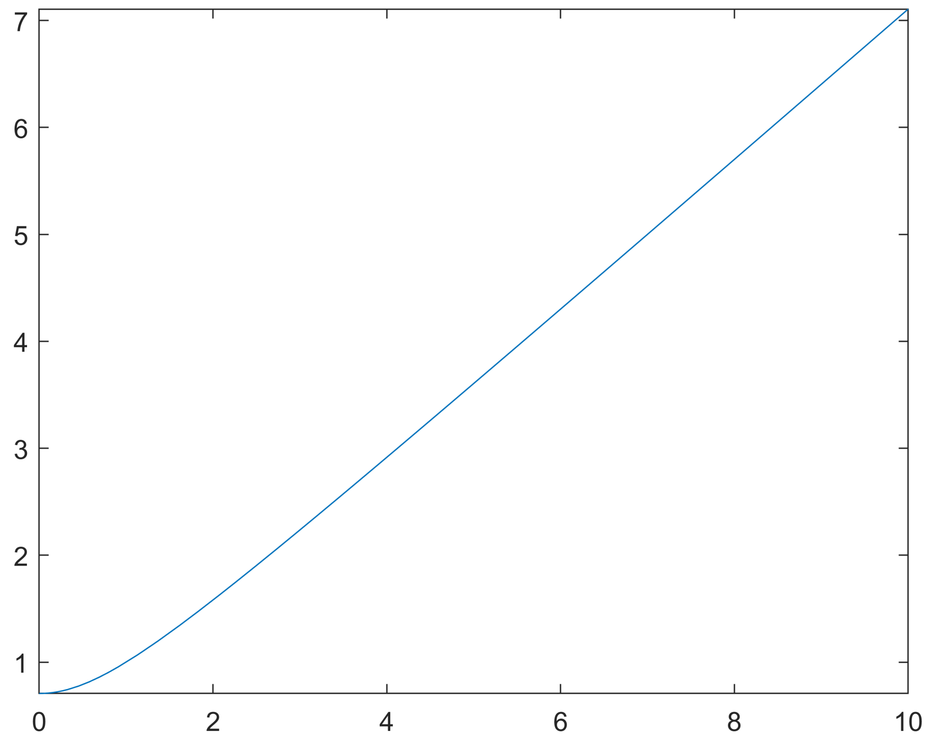
a. $\frac{dy}{dx} = \frac{x}{2y} \qquad y(1) = 1$

```
syms x y(x)
eqn = [diff(y, x) == x/(2*y), y(1) == 1];
y = dsolve(eqn)
```

y =

$$\frac{\sqrt{2}\ \sqrt{x^2+1}}{2}$$

```
fplot(y, [0 10])
```



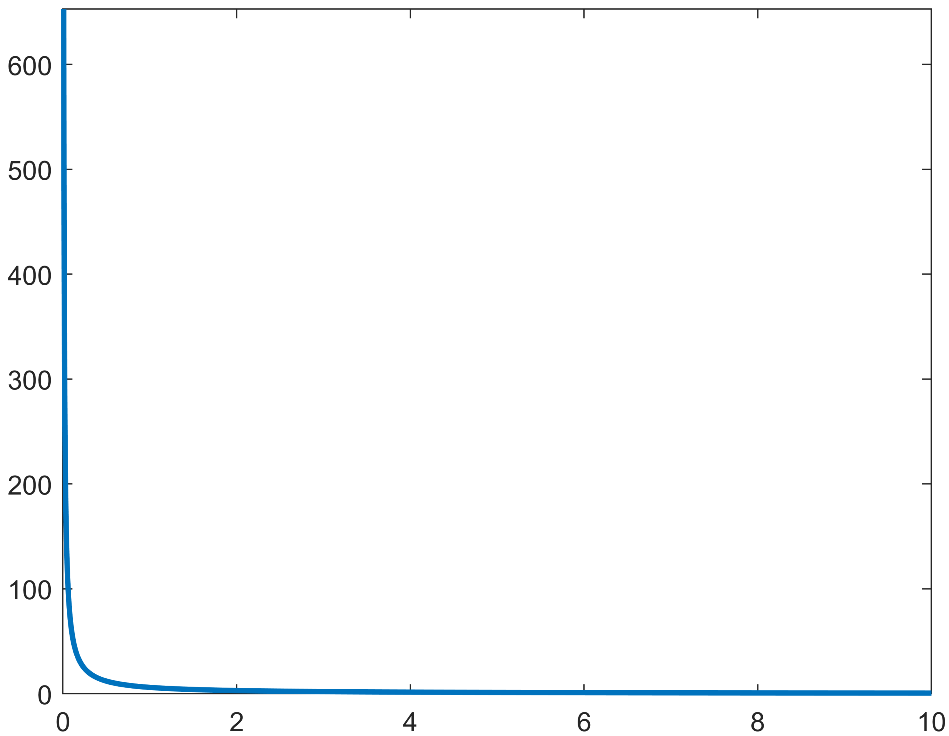
b. $\frac{dy}{dx} = \frac{-y}{x} \quad y(2) = 3$

```
syms x y(x)
eqn = [diff(y, x) == -y/x, y(2) == 3];
y = dsolve(eqn)
```

y =

$$\frac{6}{x}$$

```
fplot(y, [0 10], 'LineWidth', 2)
```

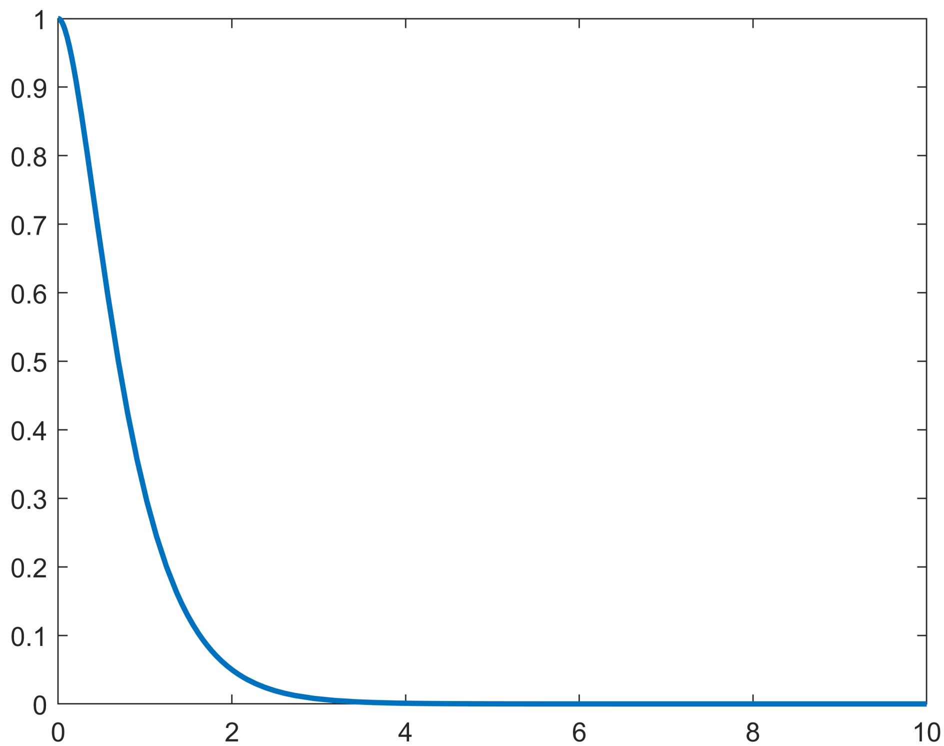


c. $\dfrac{d^2x}{dt^2} + 5\dfrac{dx}{dt} + 6x = 0 \qquad x(0) = 1 \;\; \dot{x}(0) = 0$

```
syms x(t) t
eqn = [diff(x,t,2) + 5*diff(x,t,1) + 6*x == 0; x(0) == 1; subs(diff(x,t,1), t, 0) == 0];
x = dsolve(eqn)
```

x = $e^{-3\,t}\,(3\,e^{t} - 2)$

```
fplot(x, [0 10], 'LineWidth', 2)
```



Problem 6

Compute the fifth derivative of $cos(x^2)$

```
syms x
diff(cos(x^2), x, 5)
```

ans = $120\,x\sin(x^2) + 160\,x^3\cos(x^2) - 32\,x^5\sin(x^2)$

Problem 7

Compute an expanded form of the expression $(x^2 - y)^5$

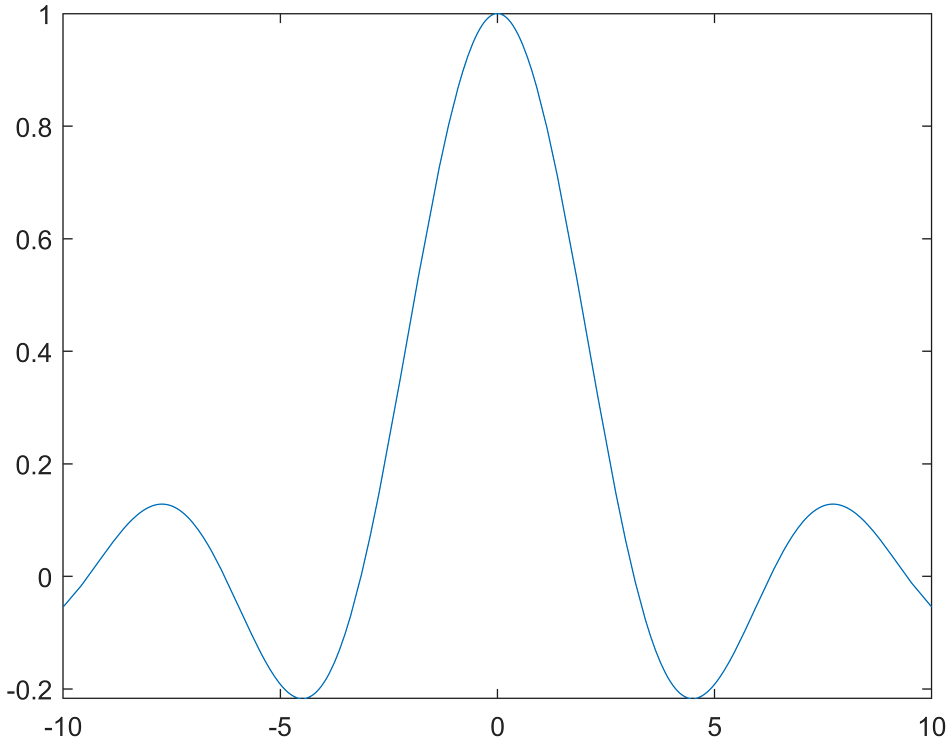
```
syms x y
expand((x^2 - y)^5)
```

ans = $x^{10} - 5\,x^8\,y + 10\,x^6\,y^2 - 10\,x^4\,y^3 + 5\,x^2\,y^4 - y^5$

Problem 8

Generate a plot of the function $\frac{sinx}{x}$ $x \in [-10, 10]$

```
syms x
fplot(sin(x)/x, [-10 10])
```



Problem 9

Find a simplified expression for the following sum $\sum_{k=1}^{n}(k^2+2k+1)$

```
syms k n
simplify(symsum(k^2 + 2*k + 1, 1, n))
```

```
ans =
```

$$\frac{n\ (2\,n^2+9\,n+13)}{6}$$

Problem 10

Compute $(2A-8B)$, AB, and $(A-B)^{-1}$ for the following matrices:

$$A=\begin{pmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 7 & 8 & 9\end{pmatrix}\quad B=\begin{pmatrix}1 & 1 & 0\\ 1 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}$$

```
syms A B
A = sym([1 2 3; 4 5 6; 7 8 9]);
B = sym([1 1 0; 1 0 1; 0 1 0]);
2*A - 8*B
```

```
ans =
```

$$\begin{pmatrix} -6 & -4 & 6 \\ 0 & 10 & 4 \\ 14 & 8 & 18 \end{pmatrix}$$

```
A*B
```

```
ans =
```

$$\begin{pmatrix} 3 & 4 & 2 \\ 9 & 10 & 5 \\ 15 & 16 & 8 \end{pmatrix}$$

```
(A - B) ^ -1
```

```
ans =
```

$$\begin{pmatrix} -\frac{5}{17} & -\frac{6}{17} & \frac{5}{17} \\ -\frac{4}{17} & \frac{21}{34} & -\frac{9}{34} \\ \frac{7}{17} & -\frac{7}{34} & \frac{3}{34} \end{pmatrix}$$

Problem 11

Find all solutions to the following system of linear equations

$$3x - 2y + 3z = 10$$
$$x - y = -4$$
$$-x + y - z = 20$$

```
syms x y z

% I'll use the matrix approach to solve this system:
% Av=b
%  v=(A^-1)b
A = sym([3 -2 3; 1 -1 0; -1 1 -1]);
v = [x; y; z];
b = sym([10; -4; 20]);
isempty(null(A))
```

```
ans = logical

   1
```

Since the Null space of A is empty, the solutions to this system are uniqe

```
% Evaluate solution
[x; y; z] == (A^-1)*b
```

```
ans =
```

$$\begin{pmatrix} x = 66 \\ y = 70 \\ z = -16 \end{pmatrix}$$

Problem 12

Compute the following interval:

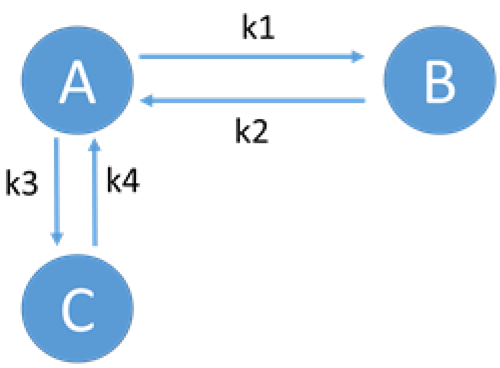
$$\int_{-\pi}^{\pi} (3 + 2\sin x + 3\cos x)(1 + 2\sin x)\, dx$$

```
syms x
int((3 + 2*sin(x) + 3*cos(x))*(1 + 2*sin(x)), [-pi pi])
```

ans = $10\,\pi$

Problem 13

A certain metabolite A in a cell is converted to B and C at rates of $k_1 = 0.5min^{-1}$ and $k_3 = 0.3min^{-1}$, respectively. The reverse reactions occur at rates of $k_2 = 0.2min^{-1}$ and $k_4 = 0.1min^{-1}$, respectively. Initially, the concentrations of A, B and C are $700\,nM$, $500\,nM$, and $0\,nM$ respectively.




```
syms k_1 k_2 k_3 k_4 A(t) B(t) C(t) t
sys = [diff(A(t), t) == k_2*B(t) + k_4*C(t) - (k_1 + k_3)*A(t);
         diff(B(t), t) == k_1*A(t) - k_2*B(t);
             diff(C(t), t) == k_3*A(t) - k_4*C(t);
             A(0) == 700; B(0) == 500; C(0) == 0];
ode = subs(sys, {k_1, k_2, k_3, k_4}, cellfun(@vpa, {0.5, 0.2, 0.3, 0.1}))
```

ode =

$$\begin{pmatrix} \frac{\partial}{\partial t}\ A(t) = 0.2\ B(t) - 0.8\ A(t) + 0.1\ C(t) \\ \frac{\partial}{\partial t}\ B(t) = 0.5\ A(t) - 0.2\ B(t) \\ \frac{\partial}{\partial t}\ C(t) = 0.3\ A(t) - 0.1\ C(t) \\ A(0) = 700 \\ B(0) = 500 \\ C(0) = 0 \end{pmatrix}$$

a. Solve the system of ODEs for the concentrations of these metabolites. Plot their concentrations over time.

```
soln = dsolve(ode);
A(t) = simplify(soln.A)
```

A(t) = $45.16497474473821865461668896808 6\,e^{-0.13466880685409625737078686275462\,t} + 470.2196406398771659607679$

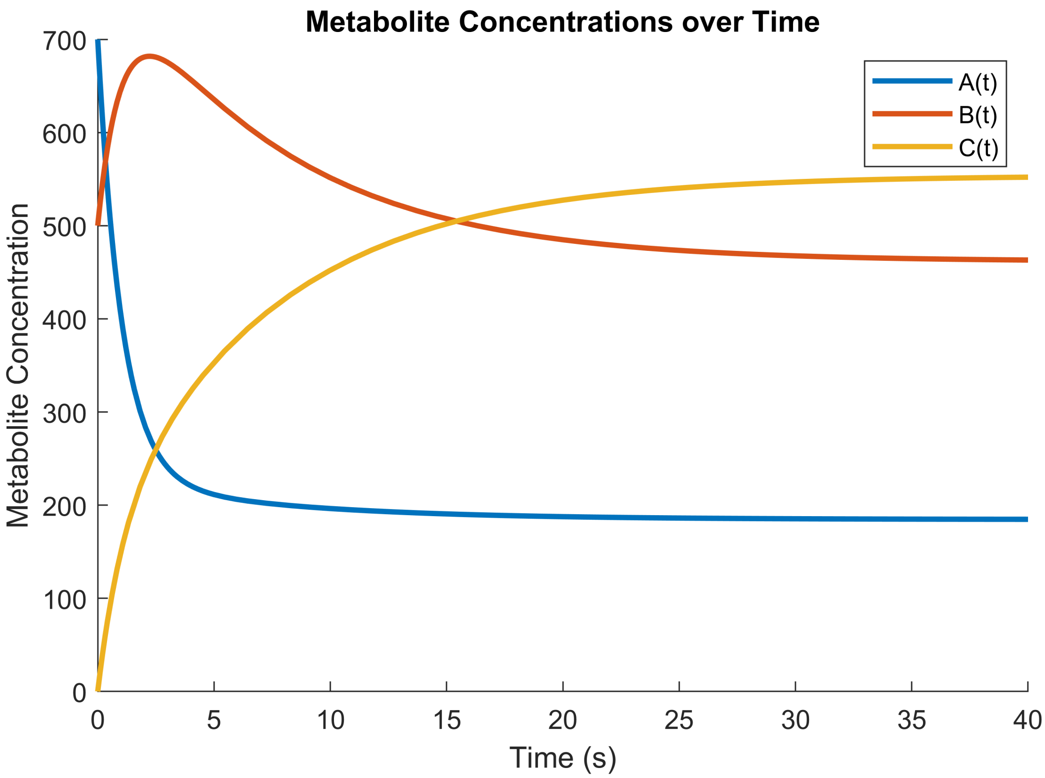
```
B(t) = simplify(soln.B)
```

B(t) = $345.66164009795110206643841890523\,e^{-0.13466880685409625737078686275462\,t} - 307.2001016364126405279768$

```
C(t) = simplify(soln.C)
```

C(t) = $553.84615384615384615384615384615 - 163.01953900346452543279104597284\,e^{-0.96533119314590374262 92}$

```
% Plot concentration
figure('Position', [0 0 600 400]);
hold on;
fplot(A(t), [0 40], 'LineWidth', 2);
fplot(B(t), [0 40], 'LineWidth', 2);
fplot(C(t), [0 40], 'LineWidth', 2);
xlabel('Time (s)');
ylabel('Metabolite Concentration');
title('Metabolite Concentrations over Time');
hold off;
legend({'A(t)', 'B(t)', 'C(t)'})
```



b. At what time are the concentrations of A and B equal?

```
t_equal = solve(A(t) == B(t), t, 'Real', true)
```

t_equal = 0.33193208047118125966764239874726

c. What are the steady-state concentrations of the metabolites?

```
% Steady state concentration of A
limit(A(t), t, inf)
```

ans = 184.61538461538461538461538461538

```
% Steady state concentration of B
limit(B(t), t, inf)
```

ans = 461.53846153846153846153846153846

```
% Steady state concentration of C
limit(C(t), t, inf)
```

ans = 553.84615384615384615384615384615

Problem 14

The differential equation $\frac{1}{A^2}\frac{d^2x}{dt^2} + x = \sin(Bt)$ is the governing equation for a "forced harmonic oscillator." It describes the behavior of an enerfy conserving system that vibrates freely at a frequency $A\left(\frac{radians}{sec}\right)$, which is excited by an external force at frequency B. Use symbolic math to solve the equation with initial conditions $x = 0$ and $\frac{dx}{dt} = 0$ (*simplify the solution, and use* `IgnoreSpecialCases`). Plot the solution for a time interval of $60\,sec$, with $A = 1$ and $B = 1.2$.

```
syms A B x(t) t
ode = [(1/(A^2) * diff(x(t), t, 2)) + x == sin(B*t); x(0) == 0; subs(diff(x(t), t) == 0, t, 0)
ode = subs(ode, {A, B}, {1, 1.2});
% Solve ode
soln = simplify(dsolve(ode))
```

soln =

$$\frac{30\sin(t)}{11} - \frac{25\sin\left(\frac{6t}{5}\right)}{11}$$

```
% plot solution
figure;
fplot(soln, [0 60])
xlabel('t'); ylabel('x');
```