BMES 678: Symbolic Math PA

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2024-04-04

Some problems are from the book Dynamical Systems with Matlab.

Question 1

Evaluate the following limits if they exist:

$$\lim_{x \to 0} \frac{\sin x}{x} \tag{a}$$

$$\lim_{x \to \infty} \frac{x^3 + 3x^2 - 5}{2x^3 - 6x} \tag{b}$$

$$\lim_{x \to \infty} \frac{1}{x} \tag{c}$$

Question 2

Find the derivatives of the following functions:

$$y = 3x^3 + 7x^2 - 6 \tag{a}$$

$$y = \sqrt{1 + x^4} \tag{b}$$

$$y = x^{\ln x} \tag{c}$$

Question 3

Evaluate the following definite integrals

$$\int_0^1 3x^3 + 2x^2 - 8\,dx \tag{a}$$

$$\int_0^1 \frac{1}{\sqrt{x}}\,dx \tag{b}$$

$$\int_0^1 e^{-x^2}\,dx \tag{c}$$

Question 4

Graph the following:

$$y = e^{-x^2} \text{ for } -5 \leq x \leq 5 \tag{a}$$

$$x^2 - 7xy - y^2 = 2 \tag{b}$$

$$\begin{cases} x = t^2 - 3t \\ y = t^3 - 9t \end{cases} \quad \text{for } -4 \leq t \leq 4 \tag{c}$$

Question 5

Show the folloiwing differential equations:

$$\frac{dy}{dx} = \frac{x}{2y}, \text{ given that } y(1) = 1. \text{ Plot } y \text{ for } 0 \leq x \leq 10 \tag{a}$$

$$\frac{dy}{dx} = -\frac{y}{x}, \text{ given that } y(2) = 3. \text{ Plot } y \text{ for } 0 \leq x \leq 10 \tag{b}$$

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0, \text{ given that } x(0) = 1 \text{ and } \dot{x}(0) = 0. \text{ Plot } x \text{ for } 0 \leq t \leq 10 \tag{c}$$

Question 6

[Courtesy of Ken Barbee] A cell has a total receptor concentration, R_T. When a ligand, with concentration L, is added, irreversible receptor-ligand bonds are formed according to the following reaction scheme:

$$R + L \xrightarrow{k} B$$

where R, L, and B are the concentrations of free (unbound) receptors, free ligands, and bound receptors on the surface of the cell, respectively. Bound receptors are also internalized (removed from the surface) at a rate proportional (internalization rate constant, k_{int}) to the surface concentration of bound receptors.

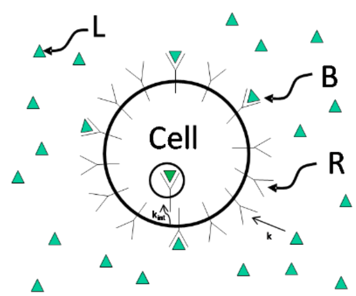

Figure 1: Receptor pic

Write the differential equation for the surface concentration of free and bound receptors in terms of the B, L, R, k and kint.

In many cases, ligand is present in concentrations much greater than the receptor concentration such that the concentration of ligand may be treated as a constant. Assuming a constant ligand concentration, L, solve for the bond concentration as a function of time with the initial condition that there are no bonds (B(0) = 0) and the initial surface receptor concentration is R0. Sketch a graph of the solution (B vs. time; use your own example values for the constants). You need to come up with your own values for the constants; these values have to be realistic (e.g., using 0 or negative concentrations would not be appropriate).

$B'(t) = R \cdot L \cdot K$

$R'(t) = -R \cdot L \cdot K$

L is constant -> either make L a numeric coefficient or set $L'(t) = 0$

rate of B being removed from surface -> $ -B k_{int}$

come up with your own numbers for ICs and then solve the equations, plot results. just have to be >0