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SYMBOLIC MATH

BMES 678: Programming Assignment

Some problems are from the book Dynamical Systems with Matlab.

Setup & Definitions

```
from matplotlib import style
import sympy as sp

sp.init_printing(use_latex=True)
```

Question 1

Evaluate the following limits if they exist:

```
x = sp.symbols("x")
```

$$\lim_{x \to 0} \frac{\sin x}{x} \tag{a}$$

```
y = sp.sin(x) / x
sp.limit(y, x, 0)
```

1

$$\lim_{x \to \infty} \frac{x^3 + 3x^2 - 5}{2x^3 - 6x} \tag{b}$$

```
y = (x**3 + 3 * x**2 - 5) / (2 * x**3 - 6 * x)
sp.limit(y, x, sp.oo)
```

$\frac{1}{2}$

$$\lim_{x \to 0^+} \frac{1}{x} \tag{c}$$

```
y = 1 / x
sp.limit(y, x, 0, "+")
```

∞

Question 2

Find the derivatives of the following functions:

```
x = sp.symbols("x")
```

$$y = 3x^3 + 7x^2 - 6 \tag{a}$$

```
y = 3 * x**3 + 7 * x**2 - 6
sp.diff(y, x)
```

$9x^2 + 14x$

$$y = \sqrt{1 + x^4} \tag{b}$$

```
y = sp.sqrt(1 + x**4)
sp.diff(y, x)
```

$\frac{2x^3}{\sqrt{x^4 + 1}}$

$$y = x^{\ln x} \tag{c}$$

```
y = x ** sp.log(x)
sp.diff(y, x)
```

$\frac{2x^{\log(x)} \log(x)}{x}$

Question 3

Evaluate the following definite integrals

```
x = sp.symbols("x")
```

$$\int_0^1 3x^3 + 2x^2 - 8\,dx \tag{a}$$

```
y = 3 * x**3 + 2 * x**2 - 8
sp.integrate(y, (x, 0, 1))
```

$$-\frac{79}{12}$$

$$\int_0^1 \frac{1}{\sqrt{x}}\,dx \tag{b}$$

```
y = 1 / sp.sqrt(x)
sp.integrate(y, (x, 0, 1))
```

$$2$$

$$\int_0^1 e^{-x^2}\,dx \tag{c}$$

```
y = sp.exp(-(x**2))
sp.integrate(y, (x, 0, 1))
```

$$\frac{\sqrt{\pi}\,\mathrm{erf}\,(1)}{2}$$

Question 4

Graph the following:

```
x, y, t = sp.symbols("x y t")
```

$$y = e^{-x^2} \text{ for } -5 \le x \le 5 \tag{a}$$

```
expr = sp.exp(-(x**2))
sp.plot(expr, (x, -5, 5), size=(4, 3))
```

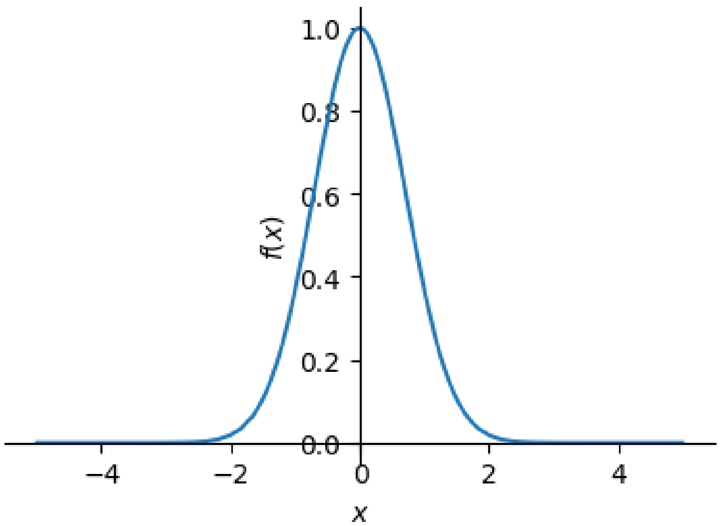

Figure 1: Plot of $y = e^{-x^2}$ for $-5 \leq x \leq 5$

$$x^2 - 7xy - y^2 = 2 \tag{b}$$

```
expr = sp.Eq(x**2 - 7 * x * y - y**2, 2)
sp.plot_implicit(expr, x, y, size=(4, 3))
```

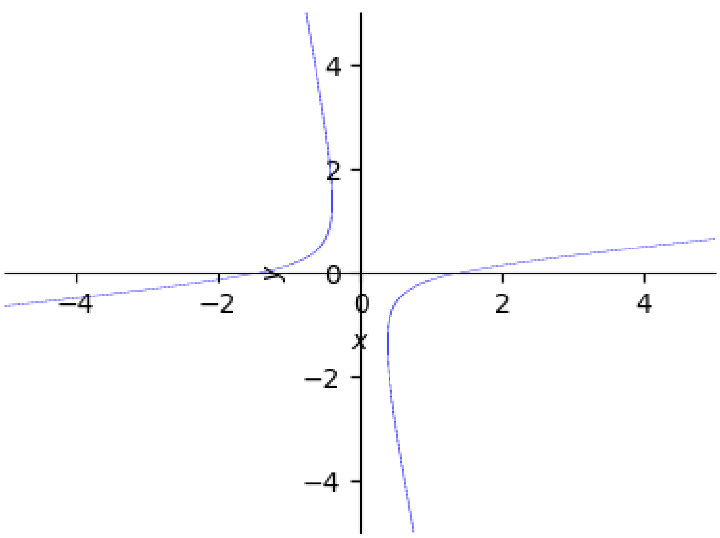

Figure 2: Plot of $x^2 - 7xy - y^2 = 2$

$$\begin{cases} x = t^2 - 3t \\ y = t^3 - 9t \end{cases} \quad \text{for } -4 \le t \le 4 \tag{c}$$

```
x = t**2 - 3*t
y = t**3 - 9*t
sp.plot_parametric(x, y, (t, -4, 4), size=(4, 3))
```

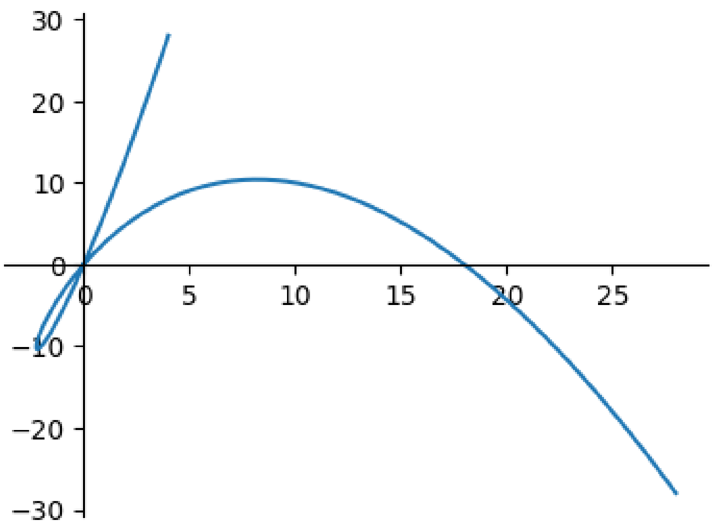

Figure 3: Plot of $x = t^2 - 3t$ and $y = t^3 - 9t$ for $-4 \le t \le 4$

Question 5

Show the folloiwing differential equations:

```
x = sp.symbols("x")
y = sp.Function('y')(x)
```

$$\frac{dy}{dx} = \frac{x}{2y}, \text{ given that } y(1) = 1. \text{ Plot } y \text{ for } 0 \le x \le 10 \tag{a}$$

```
diff = sp.Eq(sp.Derivative(y, x), x / (2*y))
soln = sp.dsolve(diff, y, ics={y.subs(x, 1): 1})
sp.plot(soln.rhs, (x, 0, 10), size=(4, 3))
```

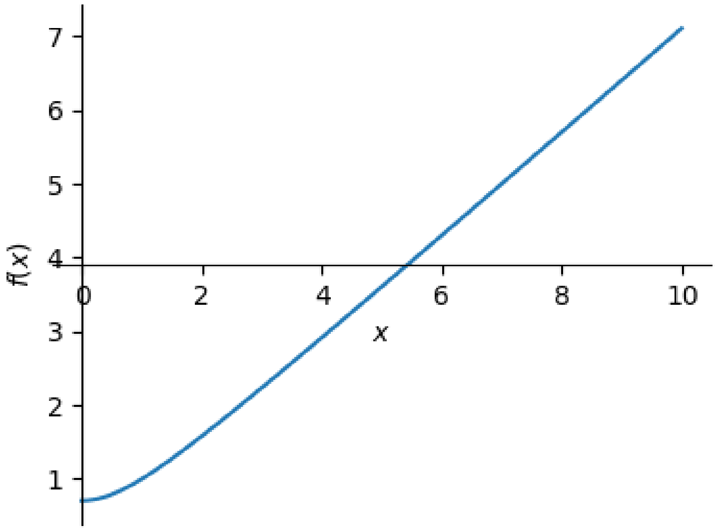

Figure 4: Plot of the solution to $\frac{dy}{dx} = \frac{x}{2y}$ given $y(1) = 1$

$$\frac{dy}{dx} = -\frac{y}{x}, \text{ given that } y(2) = 3. \text{ Plot } y \text{ for } 0 \leq x \leq 10 \tag{b}$$

```
diff = sp.Eq(sp.Derivative(y, x), -y / x)
soln = sp.dsolve(diff, y, ics={y.subs(x, 2): 3})
sp.plot(soln.rhs, (x, 0, 10), size=(4, 3))
```

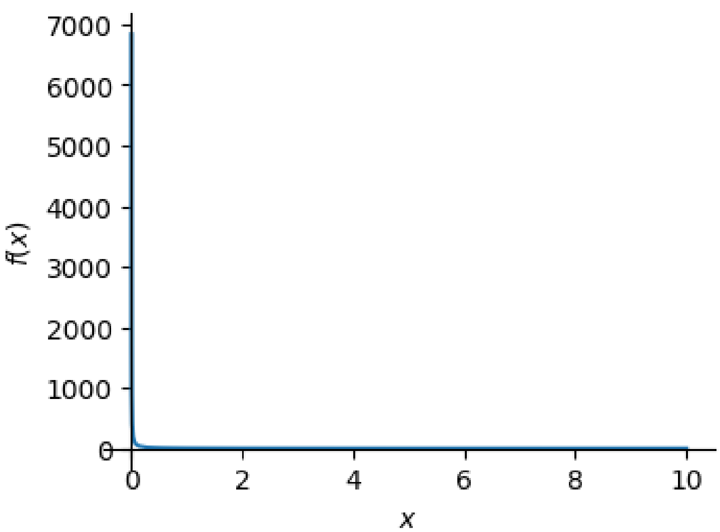

Figure 5: Plot of the solution to $\frac{dy}{dx} = \frac{-y}{x}$ given $y(2) = 3$

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0, \text{ given that } x(0) = 1 \text{ and } \dot{x}(0) = 0. \text{ Plot } x \text{ for } 0 \leq t \leq 10 \tag{c}$$

```python
t = sp.symbols("t")
x = sp.Function("x")(t)

diff = sp.Eq(sp.Derivative(x, t, t) + 5*sp.Derivative(x, t) + 6*x, 0)
soln = sp.dsolve(diff, x, ics={
    x.subs(t, 0): 1,
    sp.Derivative(x, t).subs(t, 0): 0
})
sp.plot(soln.rhs, (t, 0, 10), size=(4, 3))
```

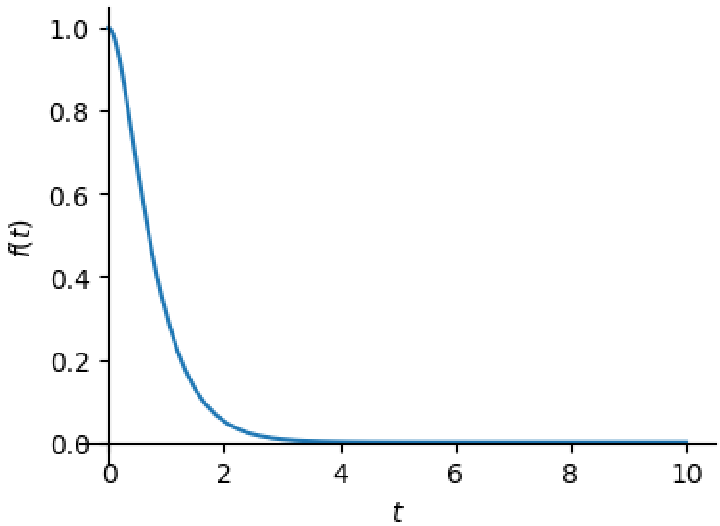

Figure 6: Plot of the solution to $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0$ given $x(0) = 1$ and $\dot{x}(0) = 0$

Question 6

[Courtesy of Ken Barbee] A cell has a total receptor concentration, R_T. When a ligand, with concentration L, is added, irreversible receptor-ligand bonds are formed according to the following reaction scheme:

$$R + L \xrightarrow{k} B$$

where R, L, and B are the concentrations of free (unbound) receptors, free ligands, and bound receptors on the surface of the cell, respectively. Bound receptors are also internalized (removed from the surface) at a rate proportional (internalization rate constant, k_{int}) to the surface concentration of bound receptors.

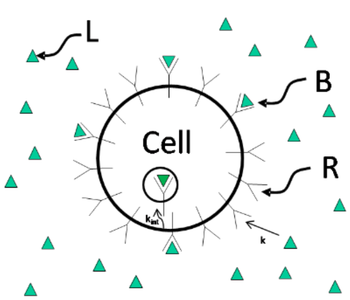

Figure 7: Receptor pic

Write the differential equation for the surface concentration of free and bound receptors in terms of the B, L, R, k and kint.

In many cases, ligand is present in concentrations much greater than the receptor concentration such that the concentration of ligand may be treated as a constant. Assuming a constant ligand concentration, L, solve for the bond concentration as a function of time with the initial condition that there are no bonds (B(0) = 0) and the initial surface receptor concentration is R0. Sketch a graph of the solution (B vs. time; use your own example values for the constants). You need to come up with your own values for the constants; these values have to be realistic (e.g., using 0 or negative concentrations would not be appropriate).

ODEs for receptor ligand interaction:

Rate of ligands binding to receptors: $B'(t) = kRL - k_{int}B$

Rate of decrease of free receptors: $R'(t) = -kRL$

given: $B(0) = 0, \;\; L$ is constant

since $R_T = R + B \implies R(0) = 100$

```
# define ODE parameters
k = 1 * 10**-3
L = 10**3
k_int = 0.1
R0 = 100
B0 = 0

# declare symbols
t = sp.symbols("t")
B, R = sp.symbols("B R", cls=sp.Function)

# define ODEs
odes = [
    sp.Eq(B(t).diff(t), k*R(t)*L - k_int*B(t)),
    sp.Eq(R(t).diff(t), -k*R(t)*L),
]

# solve
soln = sp.dsolve(odes, [B(t), R(t)], ics={
    B(t).subs(t, 0): B0,
    R(t).subs(t, 0): R0,
})
print("solution:")
soln
```

```
solution:
```

$$\left[B(t) = -111.111111111111e^{-1.0t} + 111.111111111111e^{-0.1t}, \;\; R(t) = 100.0e^{-1.0t}\right]$$

```
sp.plot(
    soln[0].rhs,
    (t, 0, 80),
```

```
    size=(6, 3),
    xlabel="time, $t$",
    ylabel="Bound receptor conc, $B(t)$",
)
```

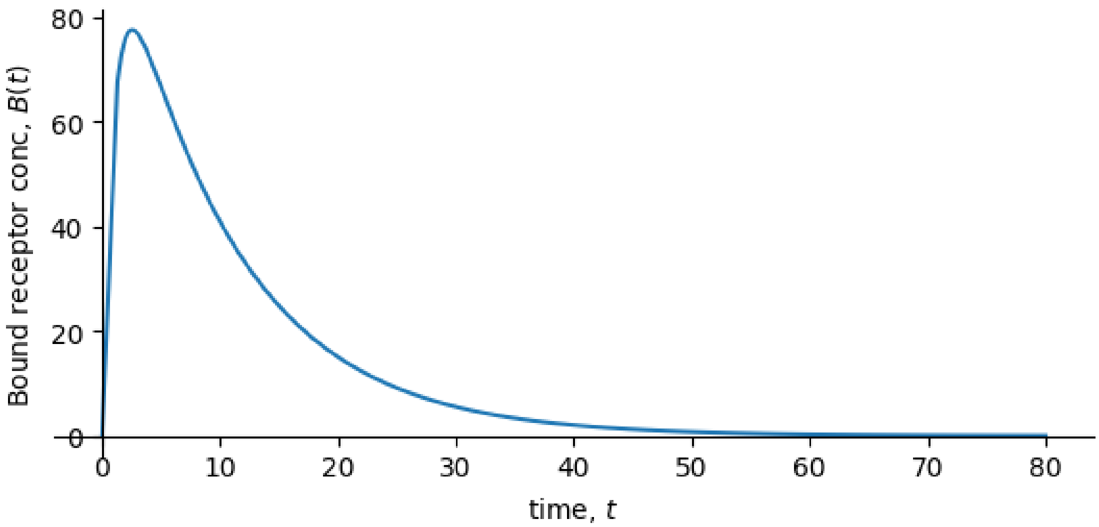

Figure 8: Plot of the solution to the ODEs for the bound receptor concentration as a function of time.