CS 171 - Lab 3

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Content by Professor Mark Boady

Detailed instructions to the lab assignment are found in the following pages.

• Complete all the exercises and type your answers in the space provided.

What to submit:

Lab sheet in PDF

Submission must be done via Gradescope

- Please make sure you have tagged all questions and your partner.
- We only accept submissions via Gradescope

Students' Names: Tony Kabilan Okeke (tko35)

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Possible Points: 87

Your score out of 87:

Lab Grade on 100% scale:

Graded By (TA Signature):

Question 1: 11 points

Binary Numbers are represented as powers of two. For this lab, we will work with **8-bit** numbers. Let us go through the powers of two.

For each power of two, write what each power of 2 evaluates to.

(a) (1 point)
$$2^0$$

 $2^0 = 1$

(b) (1 point)
$$2^1$$

 $2^1 = 2$

(c) (1 point)
$$2^2$$

 $2^2 = 4$

(d) (1 point)
$$2^3$$

 $2^3 = 8$

(e)
$$(1 \text{ point}) 2^4$$

 $2^4 = 16$

(f) (1 point)
$$2^5$$

 $2^5 = 32$

(g) (1 point)
$$2^6$$

 $2^6 = 64$

(h) (1 point)
$$2^7$$
 $2^7 = 128$

(i) (1 point)
$$2^8$$

 $2^8 = 256$

(j) (1 point)
$$2^9$$

 $2^9 = 512$

(k) (1 point)
$$2^{10}$$
 $2^{10} = 1024$

Question 2: 10 points

Each bit in a Binary Number relates to a power of 2.

Power of Two	27	26	25	24	23	22	21	20
Binary 83	0	1	0	1	0	0	1	1

If a bit is set to 1, that means we add that number.

$$83 = 2^6 + 2^4 + 2^1 + 2^0$$

Write what integer value each Binary Value represents.

Note: I like to put spaces every 4 bits for readability.

- (a) (1 point) 0000 1000 = 8
- (b) (1 point) 0001 1000 = **24**
- (c) (1 point) 0010 1110 = 46
- (d) (1 point) 0011 0001 = 49
- (e) (1 point) 0011 1011 = **59**
- (f) (1 point) 0101 0110 = 86
- (g) (1 point) 0111 1110 = 126
- (h) (1 point) 1101 0110 = **214**
- (i) (1 point) 1101 1101 = **221**
- (j) (1 point) 1111 0100 = **244**

Question 3: 11 points

The Process to convert a number from an integer to binary is to repeatedly compute the quotient and remainder. We want to convert 139 to binary.

$$139 \% 2 = 1$$
 $139 // 2 = 69$
 $69 \% 2 = 1$
 $69 // 2 = 34$
 $34 \% 2 = 0$
 $34 // 2 = 17$
 $17 \% 2 = 1$
 $17 // 2 = 8$
 $8 \% 2 = 0$
 $8 // 2 = 4$
 $4 \% 2 = 0$
 $4 // 2 = 2$
 $2 \% 2 = 0$
 $2 // 2 = 1$
 $1 \% 2 = 1$
 $1 // 2 = 0$

The binary value is 1000 1011. Notice that the result of each remainder calculation produces one bit.

(a) (2 points) The first remainder is 139 % 2 = 1.
Is this 1 the first (highest) or last (lowest) bit in the binary number?
It is the last (lowest) bit

Compute the following remainders to find the binary value of 78.

(b)
$$(1 \text{ point}) 78 \% 2 = 0$$

(c)
$$(1 \text{ point}) 39 \% 2 = 1$$

(d)
$$(1 \text{ point}) 19 \% 2 = 1$$

(e)
$$(1 \text{ point}) 9 \% 2 = 1$$

(f)
$$(1 \text{ point}) 4 \% 2 = 0$$

(g)
$$(1 \text{ point}) 2 \% 2 = 0$$

(h)
$$(1 \text{ point}) 1 \% 2 = 1$$

(i) (2 points) Write the binary value of 78. Make sure to give all 8 Bits. 0100 1110

Question 4: 20 points

Convert each of the following numbers to binary. Write all 8 bits for each number.

- (a) (2 points) $19 = 0001 \ 0011$
- (b) (2 points) 28 = 0001 1100
- (c) (2 points) 57 = 0011 1001
- (d) (2 points) 60 = 0011 1100
- (e) (2 points) 112 = 0111 0000
- (f) (f) (2 points) $148 = 1001 \ 0100$
- (g) (2 points) 156 = 1001 1100
- (h) (2 points) 171 = **1010 1011**
- (i) (2 points) 178 = **1011 0010**
- (j) (2 points) 232 = 1110 1000

Question 5: 10 points

Binary Addition is based on four simple formulas.

$$0 + 0 = 00$$

$$1 + 0 = 01$$

$$0 + 1 = 01$$

$$1 + 1 = 10$$

Notice that the final value 1 + 1 = 10 requires two places. This addition creates a carry to the next column.

(a) (5 points) In Binary, show the addition of 19 + 28. Show both the result and carry bits.

(b) (5 points) In Binary, show the addition of 57 + 60. Show both the result and carry bits.

Question 6: 15 points

Negative numbers in Binary are represented using Two's Complement.

To convert a positive number to negative in Two's Complement.

- 1) Flip All Bits (1 to 0, 0 to 1)
- 2) Add 1 to the Number

For example, we can make 17 into -17 using these steps.

Value	128	64	32	16	8	4	2	1
17	0	0	0	1	0	0	0	1
Flip Bits	1	1	1	0	1	1	1	0
-17 (add 1)	1	1	1	0	1	1	1	1

(a) (1 point) Convert 1110 1111 to a positive integer

Value	128	64	32	16	8	4	2	1
-17	1	1	1	0	1	1	1	1
Subtract 1	1	1	1	0	1	1	1	0
-17 (Flip Bits)	0	0	0	1	0	0	0	1

 $1110\ 1111 = -17$

(b) (1 point) What is result of 239 + 17?

 $239 + 17 = 256 = 0000\ 0000\ (On\ an\ 8-bit\ processor)$

- (c) (1 point) How many bits does it take to store 256 in binary? It would take 9 bits to store 256 in binary (1 0000 0000).
- (d) (2 points) When a processor computes an addition that has more result bits than the computer supports, it ignores the extra bits. For example, 255 + 5 = 4 on an 8-bit processor. This is called **overflow**. Explain how overflow makes Two's Complement numbers act negative.

The difference between 2 integers, say 91-32, can be expressed as a sum of a positive and negative integer, 91+(-32). In Binary, this would be the sum of 0101 1011 and 1110 0000 (the Two's complement of 32). The sum of these results in an overflow bit in the 9-bit position. Once this is discarded, the remaining bits provide the difference between the two integers. This is how overflow allows Two's complement numbers to act negative.

Convert each of the following Binary Numbers to Two's Complement

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(e) (2 points) 0000 1001 = 1111 0110 + 0000 0001 = 1111 0111
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(f)
$$(2 \text{ points})$$
 0000 1010 = 1111 0101 + 0000 0001 = **1111 0110**

(h) (2 points)
$$0011 \ 1111 = 1100 \ 0000 + 0000 \ 0001 = 1100 \ 0001$$

(i)
$$(2 \text{ points})$$
 0100 0010 = 1011 1101 + 0000 0001 = **1011 1110**

Question 7: 10 points

Complete each of the below subtractions.

First, convert both numbers to Binary. Use Two's Complement for negative numbers. Then show the addition with result and carry bits. Remember to discard the overflow bit.

(a) (5 points) In Binary, show the computation of 91 - 32. Show both the result and carry bits.

$$91 = 0101 \ 1011$$

 $32 = 0010 \ 0000$

Two's Complement: $-32 = 1101\ 1111 + 0000\ 0001 = 1110\ 0000$

(1 - represents the discarded overflow bit)

$$\rightarrow$$
 91 - 32 = 0101 1011 + 1110 0000 = **0011 1011 = 59**

(b) (5 points) In Binary, show the computation of 100 - 47. Show both the result and carry bits.

$$100 = 0110\ 0100$$

 $47 = 0010\ 1111$

Two's Complement: $-47 = 1101\ 0000 + 0000\ 0001 = 1101\ 0001$

(1 - represents the discarded overflow bit)

$$\rightarrow$$
 100 - 47 = 0110 0100 + 1101 0001 = **0011 0101** = **53**