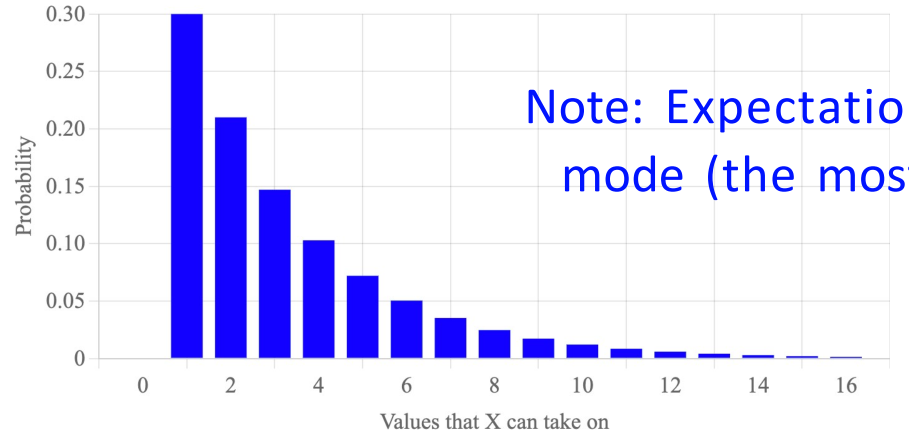
Expected Value of The Geometric

If $X \sim \mathrm{Geo}(p)$, then $E[X] = \frac{1}{p}$

This definition has intuition built in:

- If a coin has probability ½ of a head, then on average, it will take him two tosses to get a head. E[X] = $(1/2)^{-1}$ = 2.



Note: Expectation is often **not** the mode (the most likely outcome)

Expected Value of The Geometric

$$E[Y] = \sum_{n=1}^{\infty} n \cdot \underbrace{(1-p)^{n-1} \cdot p} = \frac{1}{p}$$

$$= 1 \cdot p + 2(1-p)p + 3(1-p)^2 p + 4(1-p)^3 p$$

$$= p\left(1 + 2(1-p) + 3(1-p)^2 + \cdots\right) = Sp$$

$$= p\left(\underbrace{(1-p) + 2(1-p)^2 + 3(1-p)^3 + \cdots}\right) =$$

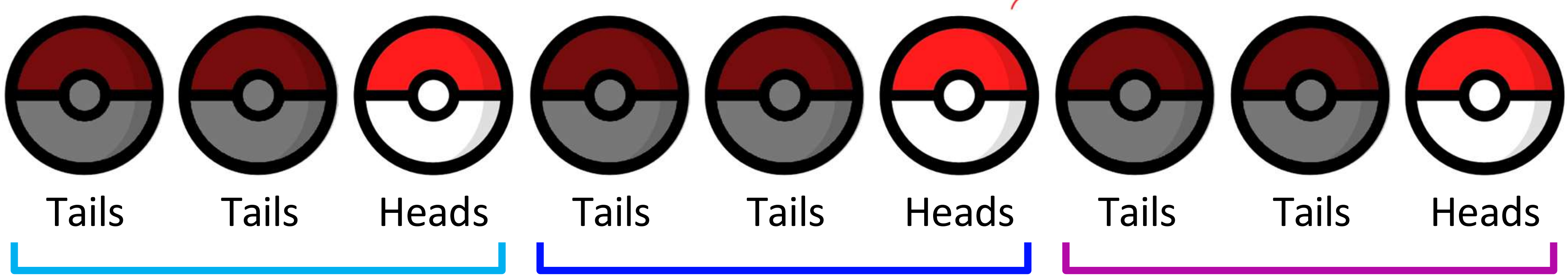
$$=$$

Recall JEE math - -

Expected Value of The ~~Negative Binomial~~

We can derive using the **sum of expectations** property, similar to binomials.

The Negative Binomial ...is a sum of Geometric random variables



Expected Value of The Negative Binomial

We can derive using the **sum of expectations** property, similar to binomials.

Let $X_i \sim \text{Geo}(p)$, for each i from 1 to r.

$$E[X_i] = \frac{1}{p}$$

Let $Y \sim \text{NegBin}(r, p)$.

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St. Petersburg Paradox

The Game:

- We have a fair coin (lands on heads with p = 0.5)
- Let n = number of coin flips (tails) to get the first heads
- You will win: $\$2^n$

How much would you pay to play?

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How much would you pay to play?

Let X be your winnings. $g(x) = 2^x$

$E(g(x))$

$$E[\cancel{X}] = \underbrace{\left(\frac{1}{2}\right)^1 2^1 + \left(\frac{1}{2}\right)^2 2^2 + \left(\frac{1}{2}\right)^3 2^3 + \ldots = \sum_{i=0}^{\infty} 1 = \infty}_{g(x)}$$

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What if you could play this game for only \$1000...but just once?

Expectations of Classic Random Variables

$X \in \{1, 2, \ldots \infty\}$

$X \sim \text{Geo}(p)$

$P(X = n) = (1-p)^{n-1} p$

$$E[X] = \frac{1}{p}$$

$X \in \{0, 1\}$

$X \sim \text{Bern}(p)$

$P(X = x) = p^x (1-p)^{1-x}$

$$E[X] = p$$

$Y \in \{r, r+1, \ldots \infty\}$

$Y \sim \text{NegBin}(r, p)$

$P(Y = n) = \binom{n-1}{r-1} (1-p)^{n-r} p^r$

$$E[Y] = \frac{r}{p}$$

$Y = \sum_{i=1}^{r} X_i \quad X_i \sim \text{Geo}(p)$

$Y \sim \text{Bin}(n, p)$ $\quad Y \in \{0, 1, \ldots n\}$

$P(Y = k) = \binom{n}{k} p^k (1-p)^{n-k}$

$$E[Y] = n \cdot p$$

$Y = \sum_{i=1}^{n} X_i \quad X_i \sim \text{Bern}(p)$

Variance of Classic Random Variables

$X \sim \text{Geo}(p)$

Homework

$$Var(X) = \frac{1-p}{p^2}$$

$X \sim \text{Bern}(p)$

$$Var(X) = p(1-p)$$

$Y \sim \text{NegBin}(r, p)$

$$Var(X) = \frac{r \cdot (1-p)}{p^2}$$

$Y \sim \text{Bin}(n, p)$

$$Var(Y) = n \cdot p(1-p)$$