CS 228 : Logic in Computer Science

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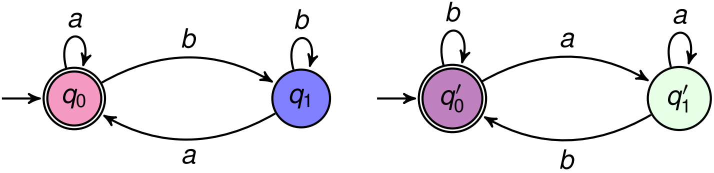
So Far

- ω-automata with Büchi acceptance, also called Büchi automata
- Non-determinism versus determinism

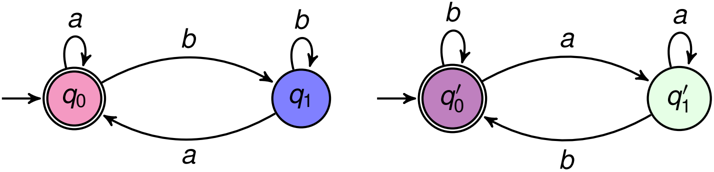
Büchi Acceptance

For Büchi Acceptance, *Acc* is specified as a set of states, $G \subseteq Q$. The ω-word α is accepted if there is a run ρ of α such that $\mathit{Inf}(\rho) \cap G \neq \emptyset$.

Union and Intersection of NBA



Union and Intersection of NBA



- States as $Q_1 \times Q_2 \times \{1, 2\}$, start state $(q_0, q'_0, 1)$

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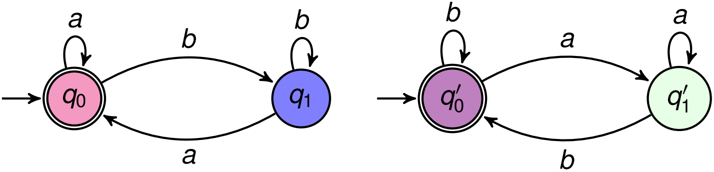

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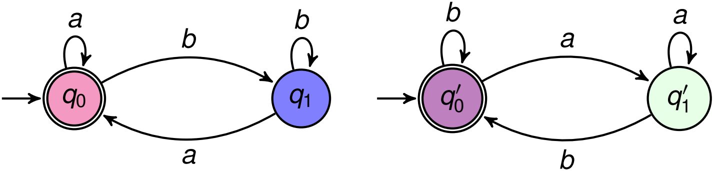

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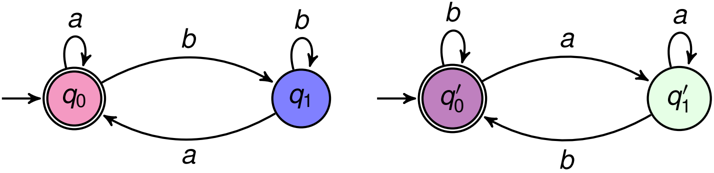

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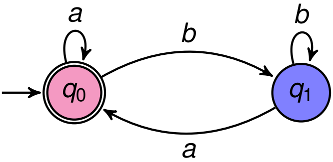
- Good states=$Q_1 \times G_2 \times \{2\}$ or $G_1 \times Q_2 \times \{1\}$

Emptiness

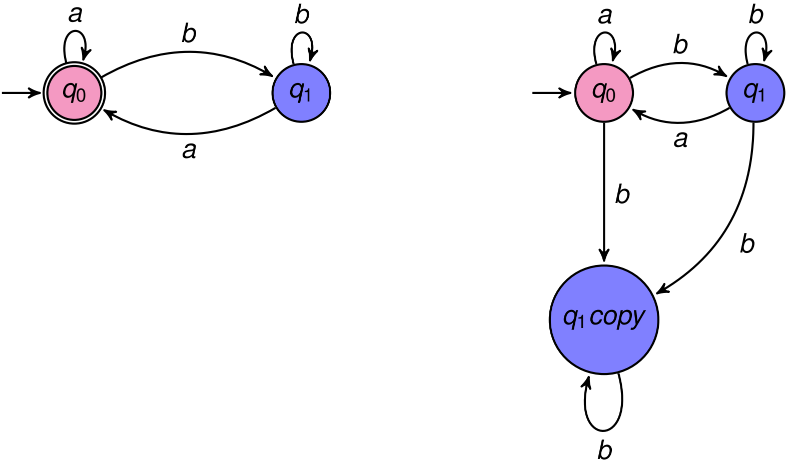
Given an NBA/DBA $\mathcal{A}$, how do you check if $L(\mathcal{A}) = \emptyset$?

- Enumerate SCCs
- Check if there is an SCC containing a good state

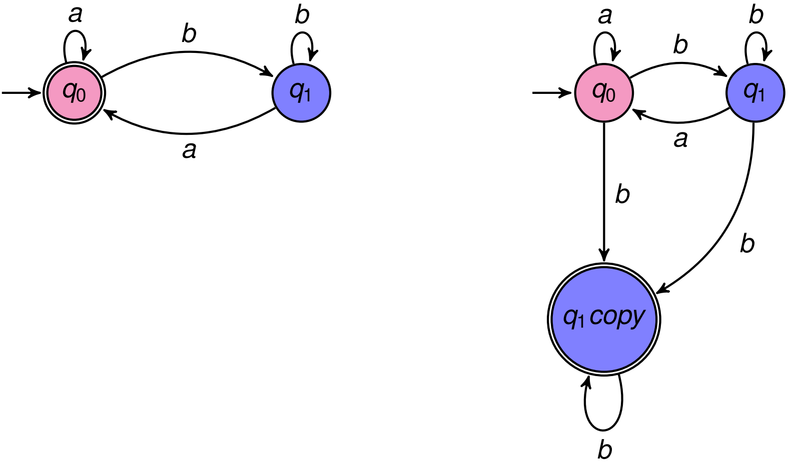
Complementation of DBA



Complementation of DBA



Complementation of DBA



Complementation of DBA

- Given $\mathcal{A}$ is a DBA, and $w \notin L(\mathcal{A})$, then after some finite prefix, the unique run of w settles in bad states.
- Idea for complement: "copy" states of $Q - G$, once you enter this block, you stay there.
- View this as the set of good states, any word w that was rejected by $\mathcal{A}$ has two possible runs in this automaton: the original run, and one another, that will settle in the $Q - G$ copy, and will be accepted.
- What we get now is an NBA for $\overline{L(\mathcal{A})}$, not a DBA.

Complementing NBA non-trivial, can be done.

GNBA

- Generalized NBA, a variant of NBA
- Only difference is in acceptance condition
- Acceptance condition in GNBA is a set $\mathcal{F} = \{F_1, \ldots, F_k\}$, each $F_i \subseteq Q$
- An infinite run ρ is accepting in a GNBA iff

$$\forall F_i \in \mathcal{F}, \mathit{Inf}(\rho) \cap F_i \neq \emptyset$$

- Note that when $\mathcal{F} = \emptyset$, all infinite runs are accepting
- GNBA and NBA are equivalent in expressive power.