CS 228 : Logic in Computer Science

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Recap

- Transition Systems as models of systems (read circuits, code, and so on)
- Traces of transition systems
- Properties as set of allowed traces
- These properties are certain languages over the alphabet 2^{AP}, and are called LT properties
- Writing properties in a language fashion
- Logic LTL to capture LT properties

Examples

- Whenever the traffic light is red, it cannot become green immediately:
  $\square(red \rightarrow \neg \bigcirc green)$
- Eventually the traffic light will become yellow
  $\Diamond yellow$
- Once the traffic light becomes yellow, it will eventually become green
  $\square(yellow \rightarrow \Diamond green)$
- Whenever the traffic light is red, it will eventually become green, but it must be yellow for sometime in between the red and the green
  $\square(red \rightarrow \bigcirc(red\ \mathrm{U}[yellow \wedge \bigcirc (yellow\ \mathrm{U} green)]))$

Semantics over Infinite Words

Given LTL formula φ over AP,

$$L(\varphi) = \{\sigma \in (2^{AP})^\omega \mid \sigma \models \varphi\}$$

Let $\sigma = A_0 A_1 A_2 \ldots$.

- $\sigma \models a$ iff $a \in A_0$
- $\sigma \models \varphi_1 \wedge \varphi_2$ iff $\sigma \models \varphi_1$ and $\sigma \models \varphi_2$
- $\sigma \models \neg\varphi$ iff $\sigma \not\models \varphi$
- $\sigma \models \bigcirc\varphi$ iff $A_1 A_2 \ldots \models \varphi$
- $\sigma \models \varphi \mathsf{U} \psi$ iff
  $\exists j \geqslant 0$ such that $A_j A_{j+1} \ldots \models \psi \wedge \forall 0 \leqslant i < j, A_i A_{i+1} \ldots \models \varphi$

Semantics over Infinite Words

Given LTL formula φ over AP,

$$L(\varphi) = \{\sigma \in (2^{AP})^\omega \mid \sigma \models \varphi\}$$

- $\sigma \models \Diamond\varphi$ iff $\exists j \geqslant 0$, $A_j A_{j+1} \ldots \models \varphi$
- $\sigma \models \Box\varphi$ iff $\forall j \geqslant 0$, $A_j A_{j+1} \ldots \models \varphi$
- $\sigma \models \Box\Diamond\varphi$ iff $\forall j \geqslant 0$, $\exists i \geqslant j$, $A_i A_{i+1} \ldots \models \varphi$
- $\sigma \models \Diamond\Box\varphi$ iff $\exists j \geqslant 0$, $\forall i \geqslant j$, $A_i A_{i+1} \ldots \models \varphi$

If $\sigma = A_0 A_1 A_2 \ldots$, $\sigma \models \varphi$ is also written as $\sigma, 0 \models \varphi$. This simply means $A_0 A_1 A_2 \ldots \models \varphi$. One can also define $\sigma, i \models \varphi$ to mean $A_i A_{i+1} A_{i+2} \ldots \models \varphi$ to talk about a suffix of the word σ satisfying a property.