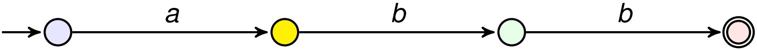
CS 228 : Logic in Computer Science

S. Krishna

Is it Regular? Is it FO-definable?

$\Sigma = \{a, b\}$. Consider the following languages $L \subseteq \Sigma^*$:

- Contains *abb*



$$\exists x \exists y \exists z (Q_a(x) \wedge Q_b(y) \wedge Q_b(z) \wedge S(x, y) \wedge S(y, z))$$

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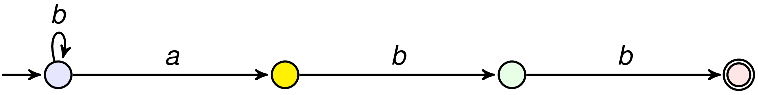

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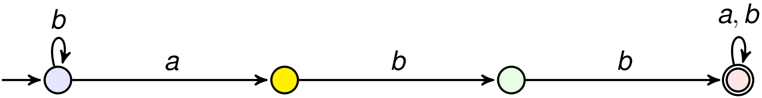

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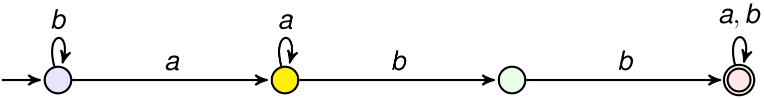

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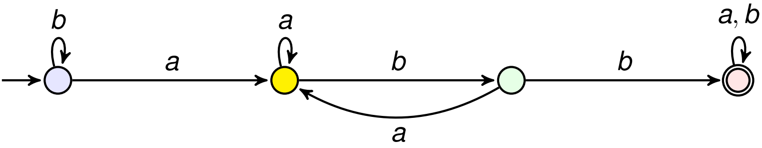

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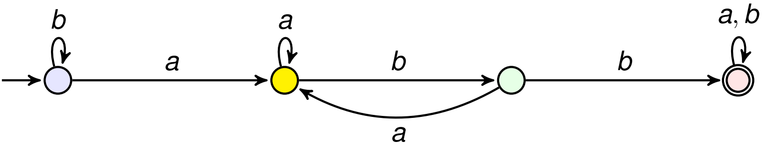

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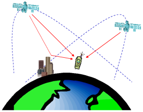
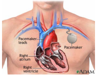
Verification through Model Checking





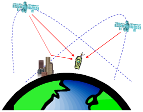
System

satisfy?

specification

good/bad properties

Verification through Model Checking



System

specification

satisfy?

System Model

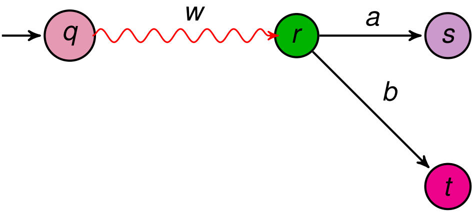
model-checking

$\models$?

Spec Model

Logic formula ϕ

DFA : Transition Function on Words



- $\hat{\delta}(q, wa) = s = \delta(\hat{\delta}(q, w), a) = \delta(r, a)$
- $\hat{\delta}(q, wb) = t = \delta(\hat{\delta}(q, w), b) = \delta(r, b)$

DFA Acceptance

- $w \in \Sigma^*$ is accepted iff $\hat{\delta}(q_0, w) \in F$
- $w \in \Sigma^*$ is rejected iff $\hat{\delta}(q_0, w) \notin F$

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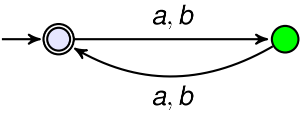
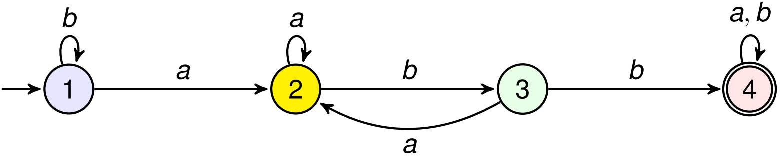
- Any string $w \in \Sigma^*$ is either accepted or rejected by a DFA A
- $L(A) = \{w \in \Sigma^* \mid \hat{\delta}(q_0, w) \in F\}$
- $\Sigma^* = L(A) \cup \overline{L(A)}$

Closer Look : DFA



- Blue state : ϵ, *ab*, *ba*, *bb*, *aa*, . . .
- Green state : *a*, *b*, *aaa*, *aba*, *baa*, *bbb*, *bba*, *bab*, . . .
- All words in Σ^* reach a unique state from the initial state
- Words reaching a final state are accepted; all others are rejected

Closer Look : DFA



- state 1 : b^*
- state 2: $b^*a, b^*aa^*, b^*aa^*(ba)^*$
- state 3 : $b^*ab, b^*aa^*b, b^*aa^*(ba)^*b$
- state 4 : $b^*abb\Sigma^*, b^*aa^*bb\Sigma^*, b^*aa^*(ba)^*bb\Sigma^*$
- All words in Σ^* reach a unique state from the initial state
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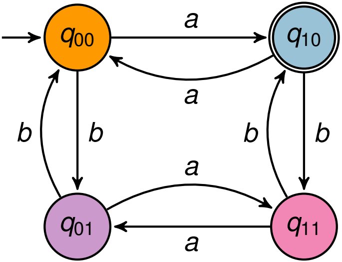
Closer Look : DFA

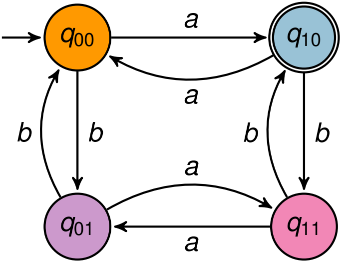
- Each state is a bucket holding infinitely many words
- Thus we have good and bad buckets
- The buckets partition Σ^*
- Good buckets determine the language accepted by the DFA
- Words that land in bad buckets are not accepted by the DFA

Language Acceptance : Proof



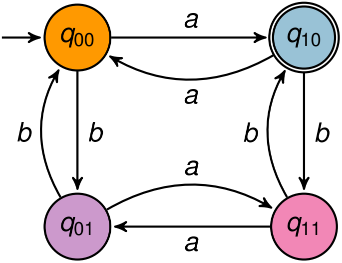
- $L = \{w \in \{a, b\}^* \mid |w|_a \text{ is odd and } |w|_b \text{ is even}\}$

Language Acceptance : Proof



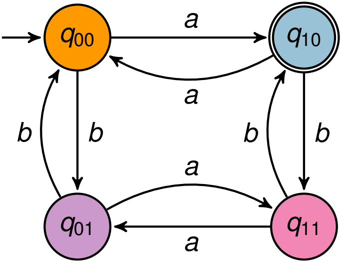
- $L = \{w \in \{a,b\}^* \mid |w|_a$ is odd and $|w|_b$ is even$\}$
- Show that for any $w \in \Sigma^*$,
  - $\hat{\delta}(q_{00}, w) = q_{ij}$ with $i, j \in \{0, 1\}$, parity of i same as $|w|_a$ and parity of j same as $|w|_b$

Language Acceptance : Proof



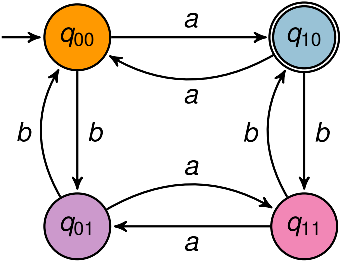
- Prove by induction on $|w|$

Language Acceptance : Proof



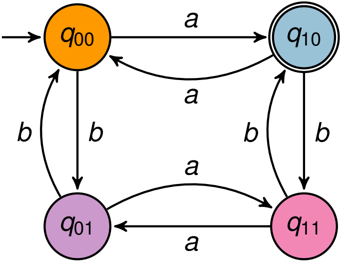
- Prove by induction on $|w|$
- Base case : For $|w| = \epsilon$, $\hat{\hat{\delta}}(q_{00}, \epsilon) = q_{00}$

Language Acceptance : Proof



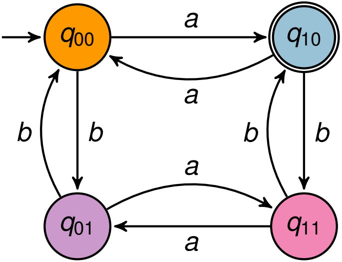
- Prove by induction on $|w|$
- Base case : For $|w| = \epsilon$, $\hat{\hat{\delta}}(q_{00}, \epsilon) = q_{00}$
- Assume the claim for $x \in \Sigma^*$, and show it for $xc, c \in \{a, b\}$.

Language Acceptance : Proof



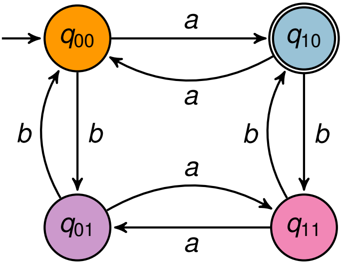
- $\hat{\delta}(q_{00}, xc) = \delta(\hat{\delta}(q_{00}, x), c)$

Language Acceptance : Proof



- $\hat{\delta}(q_{00}, xc) = \delta(\hat{\delta}(q_{00}, x), c)$
- By induction hypothesis, $\hat{\delta}(q_{00}, x) = q_{ij}$ iff
  - parity of i and $|x|_a$ are the same
  - parity of j and $|x|_b$ are the same

Language Acceptance : Proof



- Case Analysis : If $|x|_a$ odd and $|x|_b$ even, then $i = 1, j = 0$
  - $\delta(q_{10}, a) = q_{00}, \delta(q_{10}, b) = q_{11}$
  - $|xa|_a$ is even and $|xa|_b$ is even
  - $|xb|_a$ is odd and $|xb|_b$ is odd
- Other Cases : Similar
- $\hat{\delta}(q_{00}, x) = q_{10}$ iff $|x|_a$ odd and $|x|_b$ even

Recall : Bucket Analogy for DFA

- Finite states, infinite number of words
- Each state is a bucket holding (potentially) infinitely many words
- Thus we have good and bad buckets
- The buckets partition Σ^*
- Good buckets determine the language accepted by the DFA
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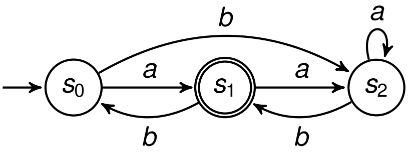
Closure under Complementation



Closure under Complementation