

INDIAN INSTITUTE OF TECHNOLOGY BOMBAY

Department of Mathematics

SI 427 (Probability I)

Tutorial Sheet-V

All random objects are in (Ω, \mathcal{F}, P) unless told otherwise.

1. Let X be a random variable with distribution function F . Show that

$$P\{X = x\} = F(x) - F(x-), \quad x \in \mathbb{R}.$$

2. An unbiased coin is tossed 3 times and the trials are independent. Let X denote the number of H 's. Find the distribution function of X .
3. Give an example of a random variable which is neither discrete nor continuous.
4. An X -coin is tossed twice independently, where X is a random variable taking values $\frac{1}{2}$ and $\frac{1}{3}$ with equal probability. Let Y denote the number of H 's. Find the distribution function of Y .
5. Let X be an exponential(1) random variable and denote the distribution function of X by F . Find the distribution of $F \circ X$.

6. Define $F : \mathbb{R} \rightarrow [0, 1]$ by

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{x^2}{4} & \text{if } 0 \leq x \leq 2 \\ 1 & \text{if } x > 2. \end{cases}$$

Show that F is a distribution function. Does pdf exists?

7. Define $f : \mathbb{R} \rightarrow \mathbb{R}$ by

$$f(x) = \frac{1}{2}e^{-|x|}, \quad x \in \mathbb{R}.$$

Show that f is a pdf of some random variable say X . Also find $P(1 \leq |X| \leq 4)$.

8. Let X be exponentially distributed with parameter λ . Find the pdf of cX , $c > 0$.