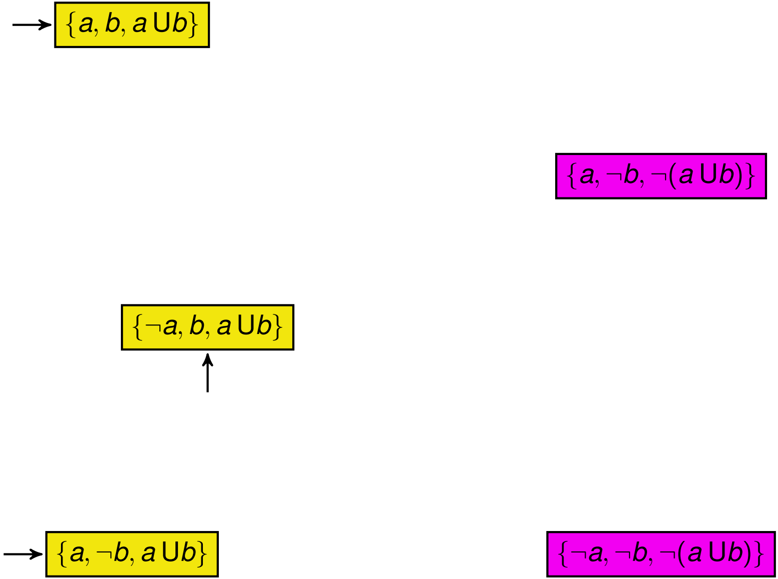
LTL to GNBA

- Let $\varphi = a \, \mathsf{U} b$.
- Subformulae of φ : $\{a, b, a \, \mathsf{U} b\}$. Let $B = \{a, \neg a, b, \neg b, a \, \mathsf{U} b, \neg(a \, \mathsf{U} b)\}$.
- Possibilities at each state : maximally consistent subsets of B
  - $\{a, \neg b, a \, \mathsf{U} b\}$
  - $\{\neg a, b, a \, \mathsf{U} b\}$
  - $\{a, b, a \, \mathsf{U} b\}$
  - $\{a, \neg b, \neg(a \, \mathsf{U} b)\}$
  - $\{\neg a, \neg b, \neg(a \, \mathsf{U} b)\}$
- The invariant to be fulfilled : all accepted words starting from a state B_i, satisfy formulae in B_i.
- All words accepted by the automaton must satisfy φ

LTL to GNBA

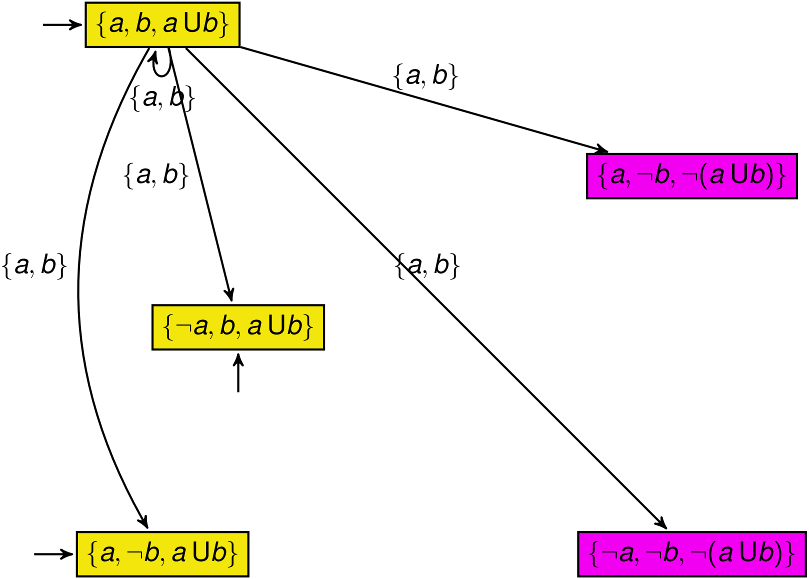
- Our initial state(s) must guarantee truth of $a \, \mathsf{U} b$. Thus, initial states: $\{a, b, a \, \mathsf{U} b\}$ and $\{\neg a, b, a \, \mathsf{U} b\}$ and $\{a, \neg b, a \, \mathsf{U} b\}$.
- All transitions outgoing from a state B are labeled with $B \cap AP$

LTL to GNBA



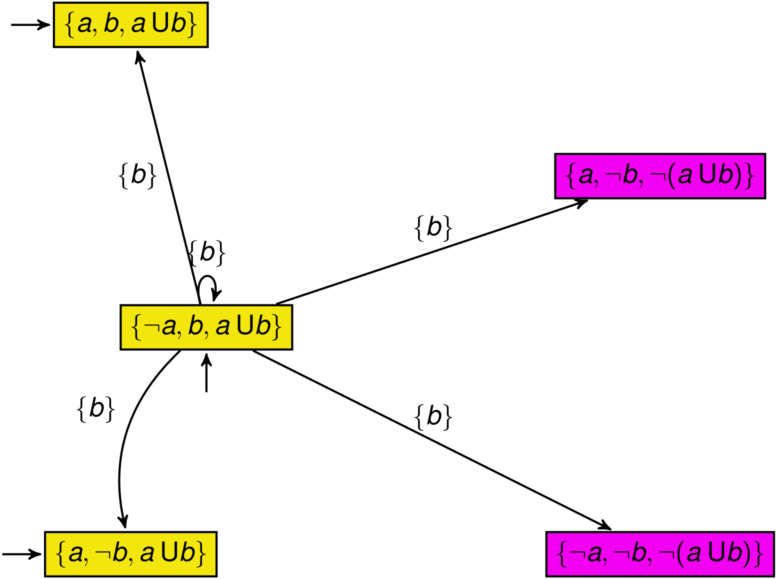
$\{a, b, a \mathsf{U} b\}$
$\{a, \neg b, \neg(a \mathsf{U} b)\}$
$\{\neg a, b, a \mathsf{U} b\}$
$\{a, \neg b, a \mathsf{U} b\}$
$\{\neg a, \neg b, \neg(a \mathsf{U} b)\}$

LTL to GNBA

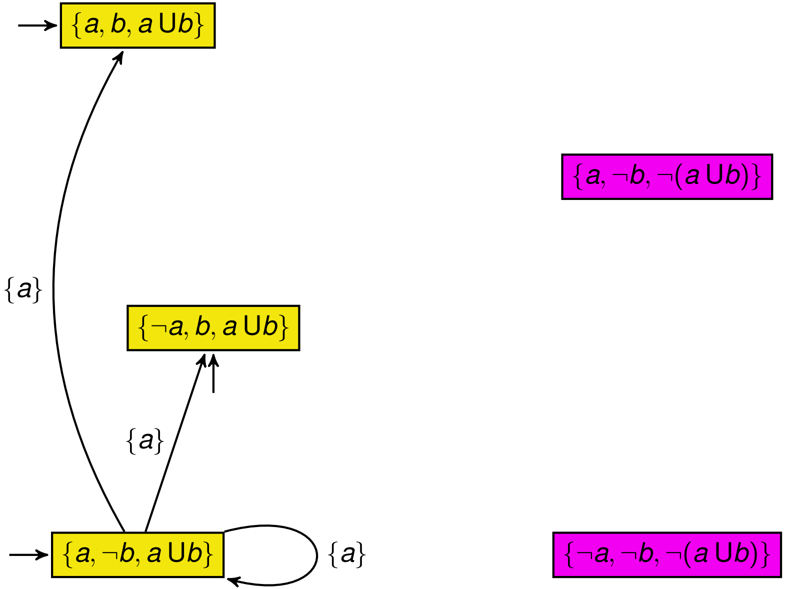


$\{a, b, a \,\mathsf{U}\, b\}$
$\{a, b\}$
$\{a, b\}$
$\{a, b\}$
$\{a, b\}$
$\{a, b\}$
$\{a, \neg b, \neg(a \,\mathsf{U}\, b)\}$
$\{\neg a, b, a \,\mathsf{U}\, b\}$
$\{a, \neg b, a \,\mathsf{U}\, b\}$
$\{\neg a, \neg b, \neg(a \,\mathsf{U}\, b)\}$

LTL to GNBA

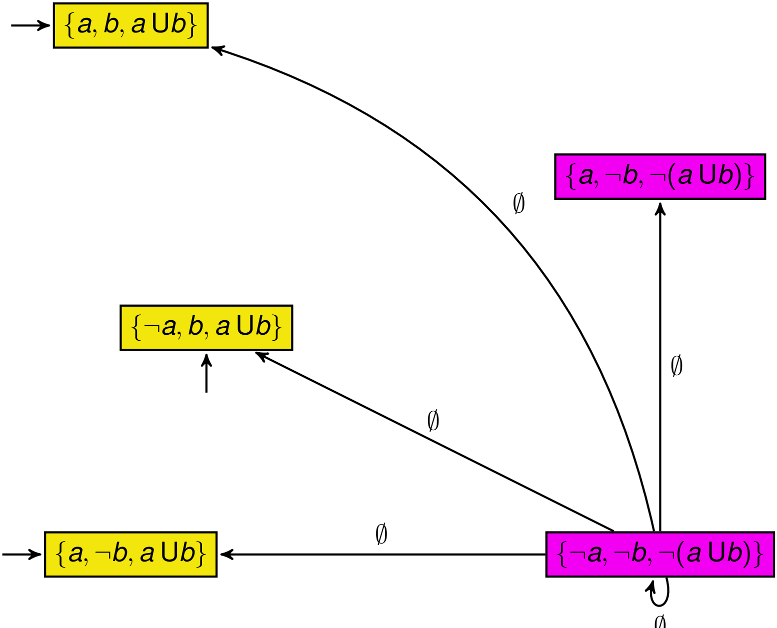

$\{a, b, a \,\mathsf{U}\, b\}$
$\{b\}$
$\{a, \neg b, \neg(a \,\mathsf{U}\, b)\}$
$\{b\}$
$\{b\}$
$\{\neg a, b, a \,\mathsf{U}\, b\}$
$\{b\}$
$\{b\}$
$\{a, \neg b, a \,\mathsf{U}\, b\}$
$\{\neg a, \neg b, \neg(a \,\mathsf{U}\, b)\}$

LTL to GNBA



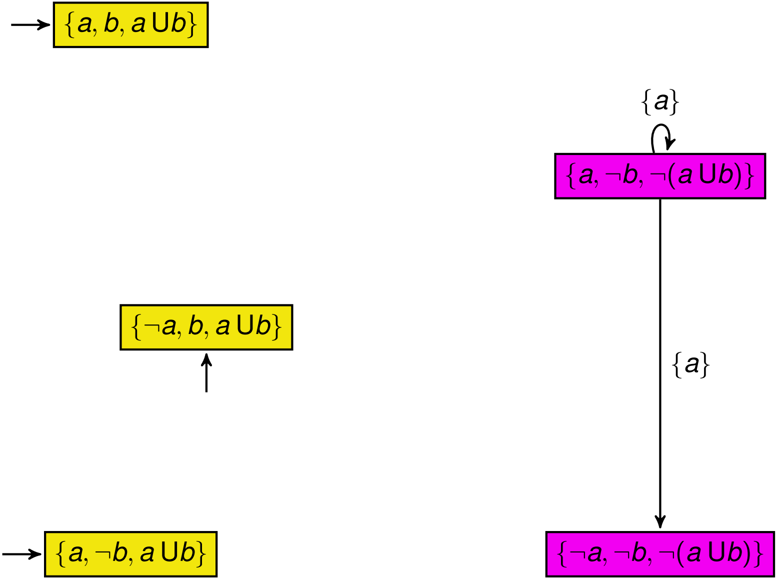
$\{a, b, a \,\mathsf{U}\, b\}$
$\{a, \neg b, \neg(a \,\mathsf{U}\, b)\}$
$\{a\}$
$\{\neg a, b, a \,\mathsf{U}\, b\}$
$\{a\}$
$\{a, \neg b, a \,\mathsf{U}\, b\}$
$\{a\}$
$\{\neg a, \neg b, \neg(a \,\mathsf{U}\, b)\}$

LTL to GNBA



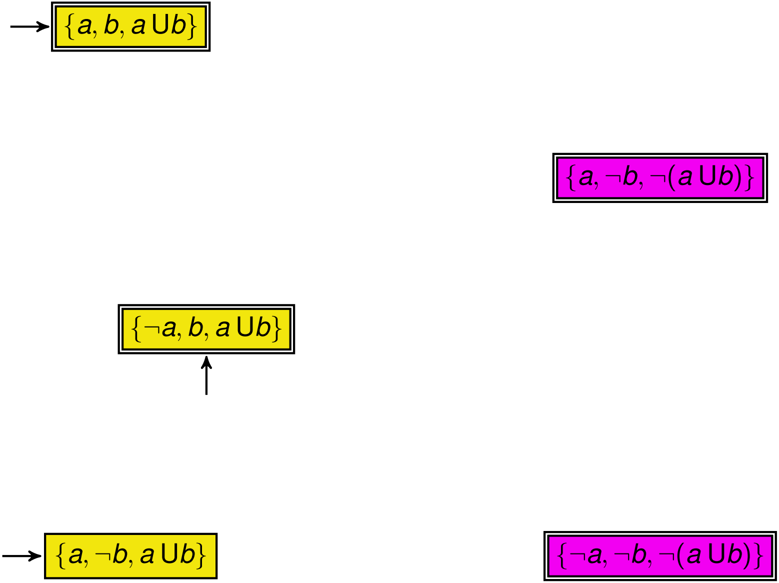
$\{a, b, a \,\mathsf{U}\, b\}$
$\{a, \neg b, \neg(a \,\mathsf{U}\, b)\}$
$\emptyset$
$\{\neg a, b, a \,\mathsf{U}\, b\}$
$\emptyset$
$\emptyset$
$\emptyset$
$\{a, \neg b, a \,\mathsf{U}\, b\}$
$\{\neg a, \neg b, \neg(a \,\mathsf{U}\, b)\}$
$\emptyset$

LTL to GNBA



$\{a, b, a \,\mathsf{U}\, b\}$
$\{a\}$
$\{a, \neg b, \neg(a \,\mathsf{U}\, b)\}$
$\{\neg a, b, a \,\mathsf{U}\, b\}$
$\{a\}$
$\{a, \neg b, a \,\mathsf{U}\, b\}$
$\{\neg a, \neg b, \neg(a \,\mathsf{U}\, b)\}$

LTL to GNBA : Accepting States



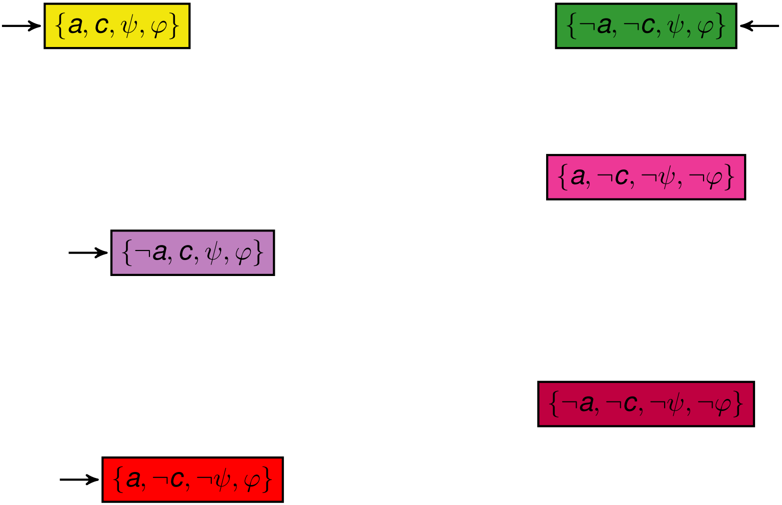
LTL to GNBA

Construct GNBA for $\neg(a \, \mathsf{U} b)$.

LTL to GNBA

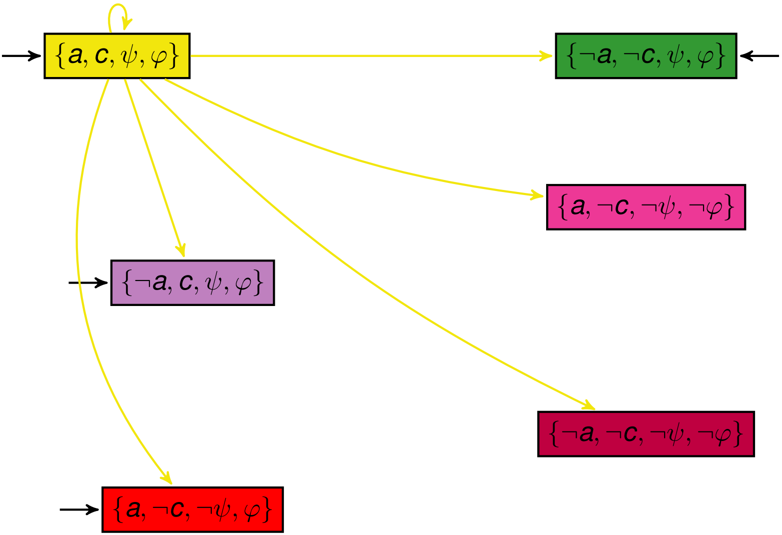
- Let $\varphi = a \,\mathsf{U} (\neg a \,\mathsf{U} c)$. Let $\psi = \neg a \,\mathsf{U} c$
- Subformulae of φ : $\{a, \neg a, c, \psi, \varphi\}$. Let $B = \{a, \neg a, c, \neg c, \psi, \neg\psi, \varphi, \neg\varphi\}$.
- Possibilities at each state : some consistent subset of B holds
  - $\{a, c, \psi, \varphi\}$
  - $\{\neg a, c, \psi, \varphi\}$
  - $\{a, \neg c, \neg\psi, \varphi\}$
  - $\{a, \neg c, \neg\psi, \neg\varphi\}$
  - $\{\neg a, \neg c, \psi, \varphi\}$
  - $\{\neg a, \neg c, \neg\psi, \neg\varphi\}$

LTL to GNBA

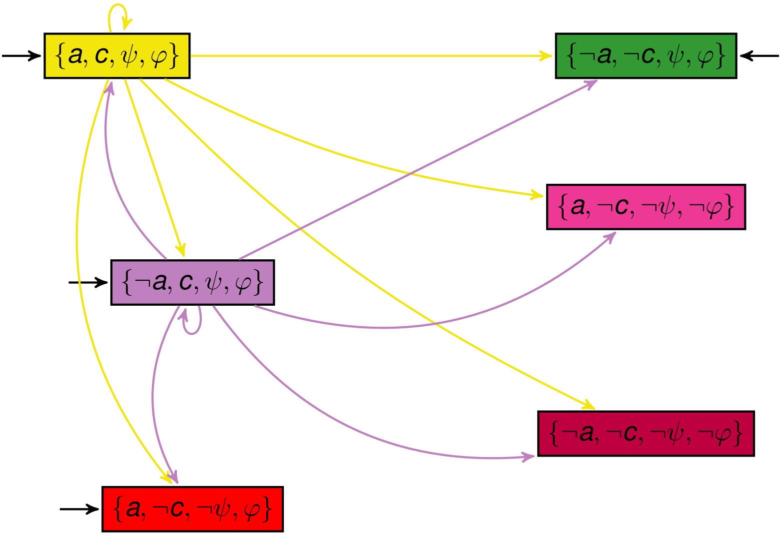


$\{a, c, \psi, \varphi\}$
$\{\neg a, \neg c, \psi, \varphi\}$
$\{a, \neg c, \neg\psi, \neg\varphi\}$
$\{\neg a, c, \psi, \varphi\}$
$\{\neg a, \neg c, \neg\psi, \neg\varphi\}$
$\{a, \neg c, \neg\psi, \varphi\}$

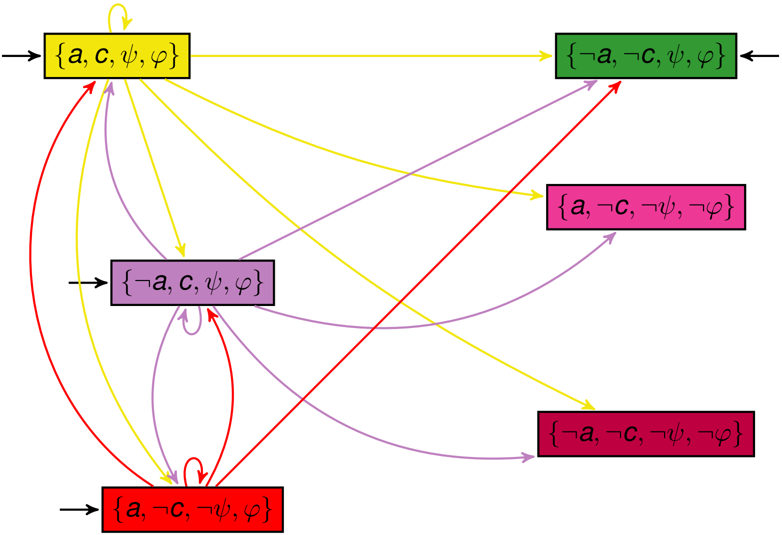
LTL to GNBA


$\{a, c, \psi, \varphi\}$
$\{\neg a, \neg c, \psi, \varphi\}$
$\{a, \neg c, \neg\psi, \neg\varphi\}$
$\{\neg a, c, \psi, \varphi\}$
$\{\neg a, \neg c, \neg\psi, \neg\varphi\}$
$\{a, \neg c, \neg\psi, \varphi\}$

LTL to GNBA

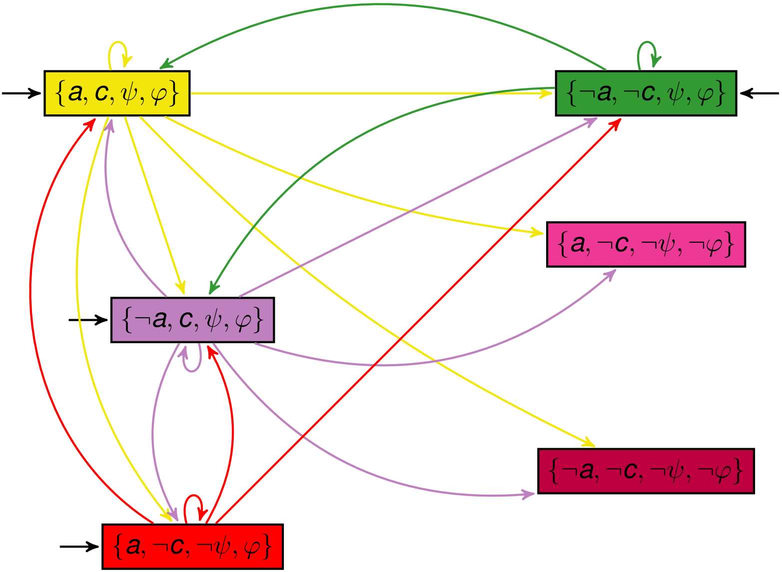

$\{a, c, \psi, \varphi\}$
$\{\neg a, \neg c, \psi, \varphi\}$
$\{a, \neg c, \neg\psi, \neg\varphi\}$
$\{\neg a, c, \psi, \varphi\}$
$\{\neg a, \neg c, \neg\psi, \neg\varphi\}$
$\{a, \neg c, \neg\psi, \varphi\}$

LTL to GNBA



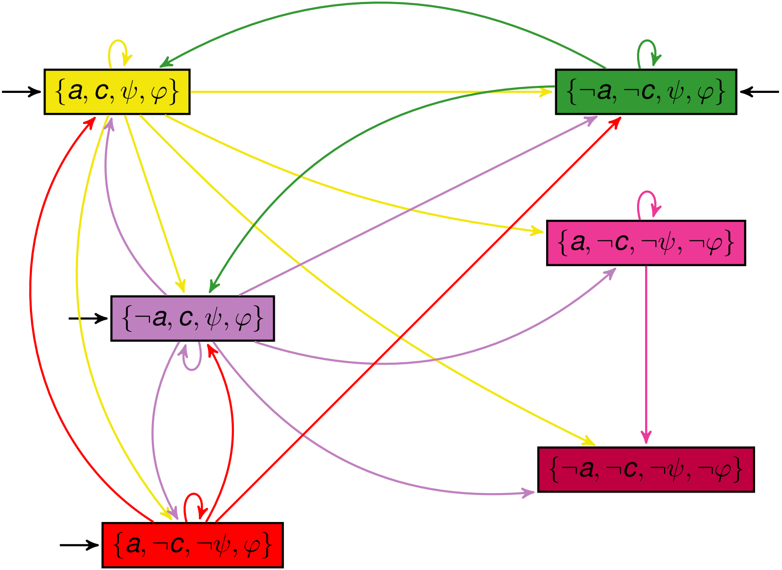
$\{a, c, \psi, \varphi\}$
$\{\neg a, \neg c, \psi, \varphi\}$
$\{a, \neg c, \neg\psi, \neg\varphi\}$
$\{\neg a, c, \psi, \varphi\}$
$\{\neg a, \neg c, \neg\psi, \neg\varphi\}$
$\{a, \neg c, \neg\psi, \varphi\}$

LTL to GNBA



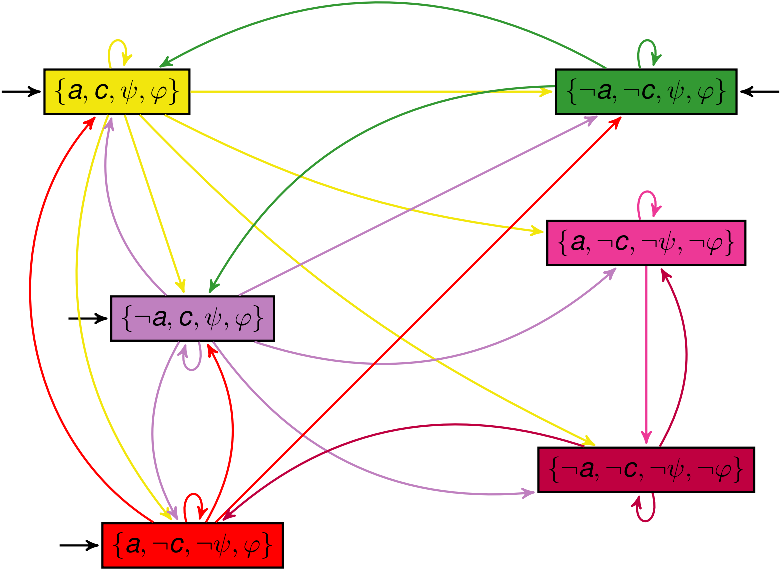
$\{a, c, \psi, \varphi\}$
$\{\neg a, \neg c, \psi, \varphi\}$
$\{a, \neg c, \neg\psi, \neg\varphi\}$
$\{\neg a, c, \psi, \varphi\}$
$\{\neg a, \neg c, \neg\psi, \neg\varphi\}$
$\{a, \neg c, \neg\psi, \varphi\}$

LTL to GNBA



$\{a, c, \psi, \varphi\}$
$\{\neg a, \neg c, \psi, \varphi\}$
$\{a, \neg c, \neg\psi, \neg\varphi\}$
$\{\neg a, c, \psi, \varphi\}$
$\{\neg a, \neg c, \neg\psi, \neg\varphi\}$
$\{a, \neg c, \neg\psi, \varphi\}$

LTL to GNBA


$\{a, c, \psi, \varphi\}$
$\{\neg a, \neg c, \psi, \varphi\}$
$\{a, \neg c, \neg\psi, \neg\varphi\}$
$\{\neg a, c, \psi, \varphi\}$
$\{\neg a, \neg c, \neg\psi, \neg\varphi\}$
$\{a, \neg c, \neg\psi, \varphi\}$

GNBA Acceptance Condition

- $\psi = \neg a \, \mathsf{U} c$
- $\varphi = a \, \mathsf{U} \psi$
- $F_1 = \{B \mid \psi \in B \to c \in B\}$
- $F_2 = \{B \mid \varphi \in B \to \psi \in B\}$
- $\mathcal{F} = \{F_1, F_2\}$

Final States

$\{a, c, \psi, \varphi\} \in F_1, F_2$

$\{\neg a, \neg c, \psi, \varphi\} \in F_2$

$\{a, \neg c, \neg\psi, \neg\varphi\} \in F_1, F_2$

$\{\neg a, c, \psi, \varphi\} \in F_1, F_2$

$\{\neg a, \neg c, \neg\psi, \neg\varphi\} \in F_1, F_2$

$\{a, \neg c, \neg\psi, \varphi\} \in F_1$

Putting Together

- Given φ, build $Cl(\varphi)$, the set of all subformulae of φ and their negations
- Consider those $B \subseteq Cl(\varphi)$ which are consistent
  - $\varphi_1 \wedge \varphi_2 \in B \leftrightarrow \varphi_1 \in B$ and $\varphi_2 \in B$
  - $\psi \in B \rightarrow \neg\psi \notin B$ and $\psi \notin B \rightarrow \neg\psi \in B$
  - Whenever $\psi_1 \, \mathrm{U} \psi_2 \in Cl(\varphi)$,
    - $\psi_2 \in B \rightarrow \psi_1 \, \mathrm{U} \psi_2 \in B$
    - $\psi_1 \, \mathrm{U} \psi_2 \in B$ and $\psi_2 \notin B \rightarrow \psi_1 \in B$

Putting Together

Given φ over AP, construct $A_\varphi = (Q, 2^{AP}, \delta, Q_0, \mathcal{F})$,

- $Q = \{B \mid B \subseteq Cl(\varphi) \text{ is consistent }\}$
- $Q_0 = \{B \mid \varphi \in B\}$
- $\delta : Q \times 2^{AP} \to 2^Q$ is such that
  - For $C = B \cap AP$, $\delta(B, C)$ is enabled and is defined as :
  - If $\bigcirc\psi \in Cl(\varphi)$, $\bigcirc\psi \in B$ iff $\psi \in \delta(B, C)$
  - If $\varphi_1 \,\mathsf{U} \varphi_2 \in Cl(\varphi)$,
    $\varphi_1 \,\mathsf{U} \varphi_2 \in B$ iff $(\varphi_2 \in B \vee (\varphi_1 \in B \wedge \varphi_1 \,\mathsf{U} \varphi_2 \in \delta(B, C)))$
- $\mathcal{F} = \{F_{\varphi_1 \mathsf{U} \varphi_2} \mid \varphi_1 \,\mathsf{U} \varphi_2 \in Cl(\varphi)\}$, with
  $F_{\varphi_1 \mathsf{U} \varphi_2} = \{B \in Q \mid \varphi_1 \,\mathsf{U} \varphi_2 \in B \to \varphi_2 \in B\}$
- Prove that $L(\varphi) = L(A_\varphi)$

GNBA Size

- States of A_φ are subsets of $Cl(\varphi)$

GNBA Size

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- Maximum number of states $\leqslant 2^{|\varphi|}$

GNBA Size

- States of A_{φ} are subsets of $Cl(\varphi)$
- Maximum number of states $\leqslant 2^{|\varphi|}$
- Number of sets in $\mathcal{F} = |\varphi|$

GNBA Size

- States of A_φ are subsets of $Cl(\varphi)$
- Maximum number of states $\leqslant 2^{|\varphi|}$
- Number of sets in $\mathcal{F} = |\varphi|$
- LTL $\varphi \rightsquigarrow$ NBA A_φ : Number of states in $A_\varphi \leqslant |\varphi|.2^{|\varphi|}$
- Lower Bound : Find a family of LTL formulae φ_n such that the state space of $A_{\varphi_n} \geqslant |\varphi|.2^{|\varphi|}$

GNBA Size

- States of A_φ are subsets of $Cl(\varphi)$
- Maximum number of states $\leqslant 2^{|\varphi|}$
- Number of sets in $\mathcal{F} = |\varphi|$
- LTL $\varphi \rightsquigarrow$ NBA A_φ : Number of states in $A_\varphi \leqslant |\varphi|.2^{|\varphi|}$
- Lower Bound : Find a family of LTL formulae φ_n such that the state space of $A_{\varphi_n} \geqslant |\varphi|.2^{|\varphi|}$
- $\varphi_n = \Diamond[a \wedge \bigcirc^n \Box \phi]$ over $AP = \{a\}$.