8.8.2025

Agenda

1. Languages
2. Enumeration/ Encoding
3. Define (How to?)
4. Grammars
5. Properties < Decision / closure

Alphabet

Σ (symbols)

0, 1, 2
a, b, c

Strings
Words

Concatenation ·

Σ^k

Σ^3 → 3 letters

a · b · c

$\Sigma^* = \bigcup_{k=0}^{\infty} \Sigma^k$ → Sentence

all possible strings

length

0

ϵ λ

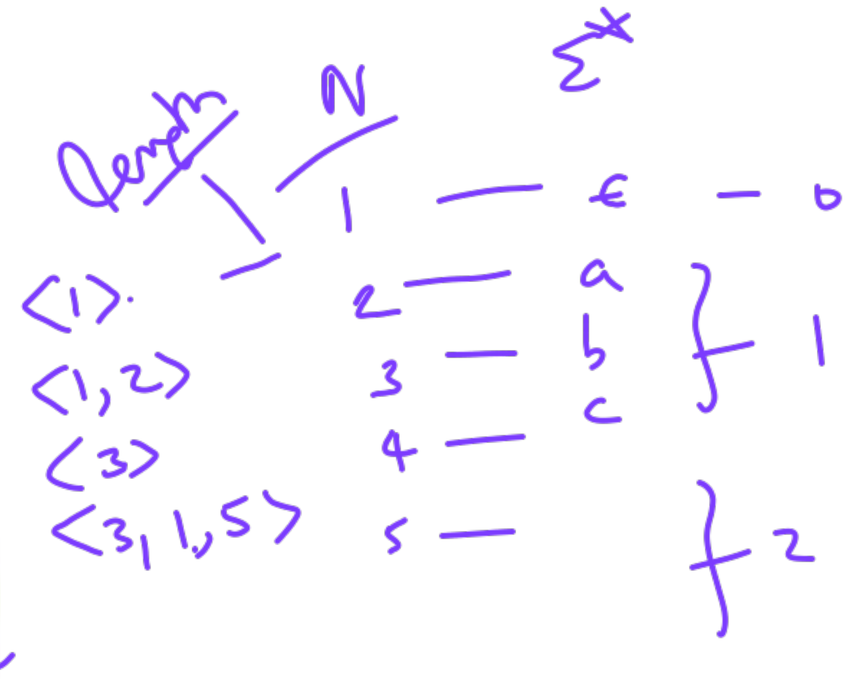
- Moodle post
- Assign 1

LLM

Satan-Cantor

· Enumeration

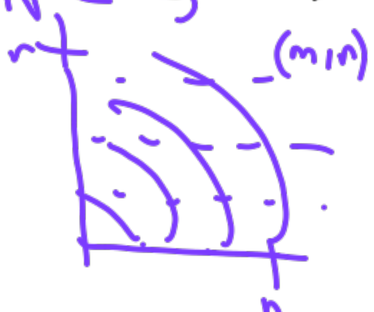
$\Sigma = \{a, b, c\}$



Pairs

Bijection

$N \times N \leftrightarrow N$



$N \times N \times N \hookrightarrow N$

$N \leftrightarrow \Sigma^*$

Dovetailing

$N \leftrightarrow N^k$ ← unknown (finite)

Σ Σ^*

Define Language $L \subseteq \Sigma^*$

Can we enumerate all Languages?

$\Sigma = \{a\}$

$N = \{1, 2, 3, \ldots\}$

$S_1 = \{a, a^2, a^4, \ldots\}$ 2^N

	a	a^2	a^3	.....	a^{100}	...
1 – S_1	0	1	1	0	...	
2 – S_2	1	0	1	0	0	-
3 – S_3	1	1	1			
4 – S_4	1	0	1	0		
.	.					

diagonal

$S_d = \{ a^i \mid a^i \notin S_i \}$

Terminals

Non Terminals

Grammars (to define languages)

$G = \langle \Sigma, V, S, P \rangle$

Σ — alphabet

V — Variables

S — Starting Variable

P — Set of Productions / Rules

Productions

$lhs \longrightarrow rhs$

$\alpha \longrightarrow \beta$

$\alpha, \beta \in (\Sigma \cup V)^*$

$+ \; |\alpha| \geq |\beta|$

1) Type 0 (Unrestricted)

2) CSL — Context Sensitive Language

3)

$\Sigma = \{a, b, c, 0, 1\}$

$V = \{S, A, B, P, R, L\}$

$S \rightarrow aA$

$A \rightarrow aBcA$

$\quad \mid cBBAd$

$\quad \mid AAcBa$

aA

AA

BB

$$\alpha \longrightarrow \beta$$

① Type-0 (Unrestricted)

$$\alpha, \beta \in (\Sigma \cup V)^*$$

② Context-Sensitive (CSG)

lhs at least as long as rhs

$|\alpha| \geq |\beta|$

③ Context-Free Grammar (CFG)

$lhs \in V$ (single variable)

④ Reg. Type

Right linear

Left linear

$\alpha \in V$, $\beta \rightarrow$ has at most one variable.

$S \rightarrow aBc$ $S \rightarrow Acc$ $S \rightarrow ccBa$ $S \rightarrow aaa$

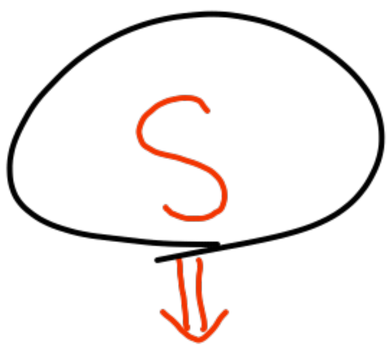
Left linear

right linear

$S \rightarrow abC$

Derivations

$Xb \to bbY$



$L(G) = \{ w \in \Sigma^* \mid S \Rightarrow^* w \}$

$bbYa \to aa$

"1-step replacement"

$abcXbaYD$

$abcbbYaYD$

Equal # of a's and b's

$L = \{ w \in \Sigma^* \mid n_a(w) = n_b(w) \}$

$\Sigma = \{a, b\}$ $L_1 = \{ a^n b^n \mid n \geq 1 \}$

$a^n b^{2n}$ $\quad a^{10} b^{20}$

Write a CFG G $\quad L(G) = L_1$

Technique

Induction or Recursion

Base

Build-up

$S \to ab$

$S \to aSb$

1. $S \to abb$
2. $S \to aSbb$
3. $S \to bb$

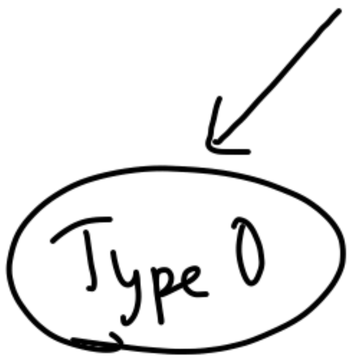
All Good strings

$aSbb$

$aaSbbbb$

No BAD String

$$L = \{ a^n b^n c^n \mid n \geq 1 \}$$



$$L = \{ a^{i^2} \mid i \geq 1 \}$$