13.8.25

0. Languages

$L \subseteq \Sigma^*$ $\quad \Sigma = \{0,1\}$

1. Grammars

$\langle \Sigma, V, S, P \rangle$

Generate-and-test

Complete — Sounders

Termination

$L = \{ w \in (0+1)^* \mid w \text{ is a prime} \}$

$L = \{ \langle w_1, w_2, w_3 \rangle \mid w_i \in (0+1)^* \text{ and } w_1, w_2, w_3 \text{ are the roots of } 7x^3 - 3y^2 3 + \dots = 0 \}$

2. Machines

~~FSA~~

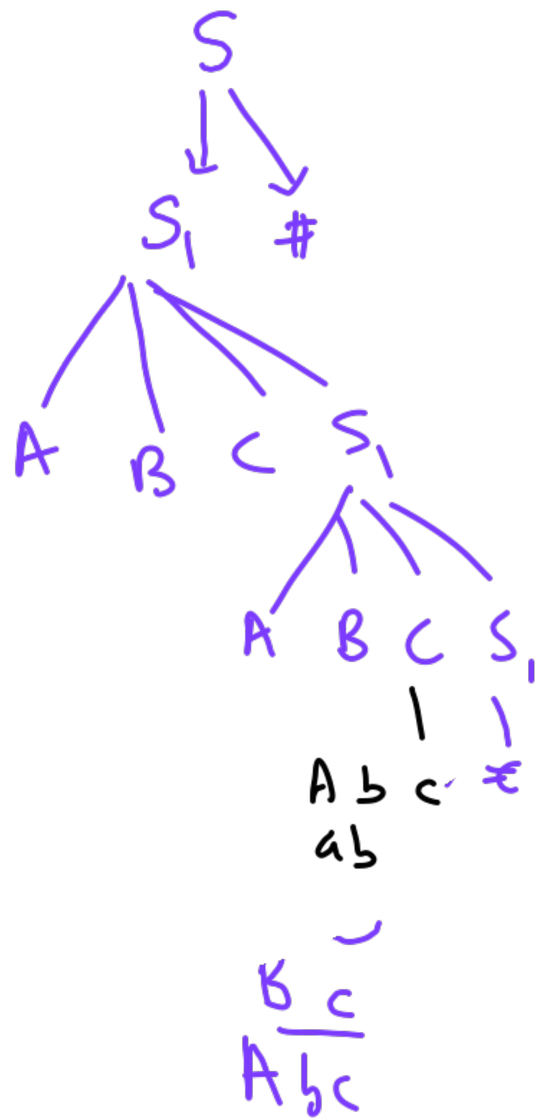
~~PDA~~ (Stack)

~~TM~~

$$L = \{ a^n b^n c^n \mid n \geq 1 \}$$

$$L(G) = \{ w \mid w \in \Sigma^*, S \Rightarrow^* w \}$$

Derivation-tree



Generate template

$S \to S_1 \#$

$S_1 \to ABC S_1$

$S_1 \to \epsilon$

Swap rules

$CA \to AC$

$CB \to BC$

$C\# \to \epsilon$

$Cc \to cc$

$BA \to AB$

$Bc \to bc$

$Bb \to bb$

$Ab \to ab$

$Aa \to aa$

$Ca \to aC$

$Cb \to bC$

$Ba \to aB$

× loops $AB \to BA$, $BA \to AB$

$$L = \{ a^n b^n c^n \mid n \geq 1 \}$$

Base case 1. $S \rightarrow abc$

Longer 2. $S \rightarrow aAbc$

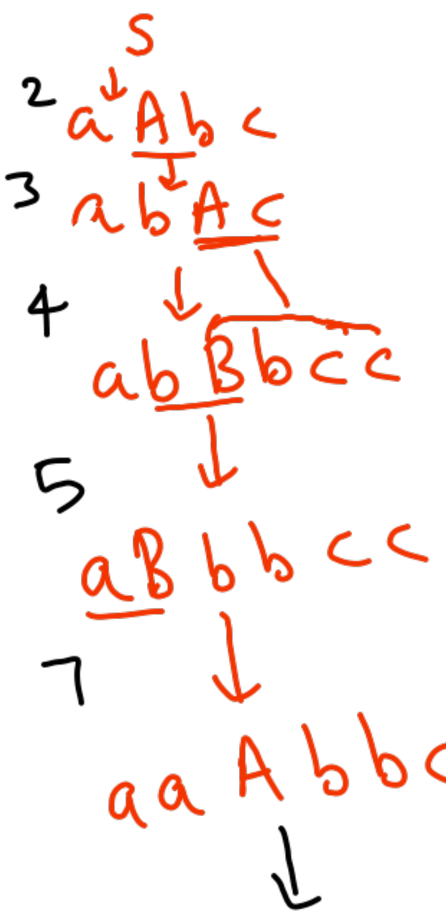
go find 3. $Ab \rightarrow bA$

4. $Ac \rightarrow Bbcc$

go back 5. $bB \rightarrow Bb$

6. $aB \rightarrow aa$

7. $aB \rightarrow aaA$



ambiguity

↓

2 different derivation trees.

Non-isomorphic

Type-0

CFG

$L = \{ w \in (a+b)^* \mid n_a(w) = n_b(w) \}$

Kolmogorov Complexity

1

$S \to \epsilon$

$S \to a\ S\ b$

$S \to b\ S\ a$

$S \to SS$

2

$S' \to \epsilon$

$S \to a\ B$

$S \to b\ A$

$B \to b\ S$

$B \to a\ B\ B$

$A \to a\ S$

$A \to b\ A\ A$

Closure Properties

$L = \{ a^n b^n c^n \}$

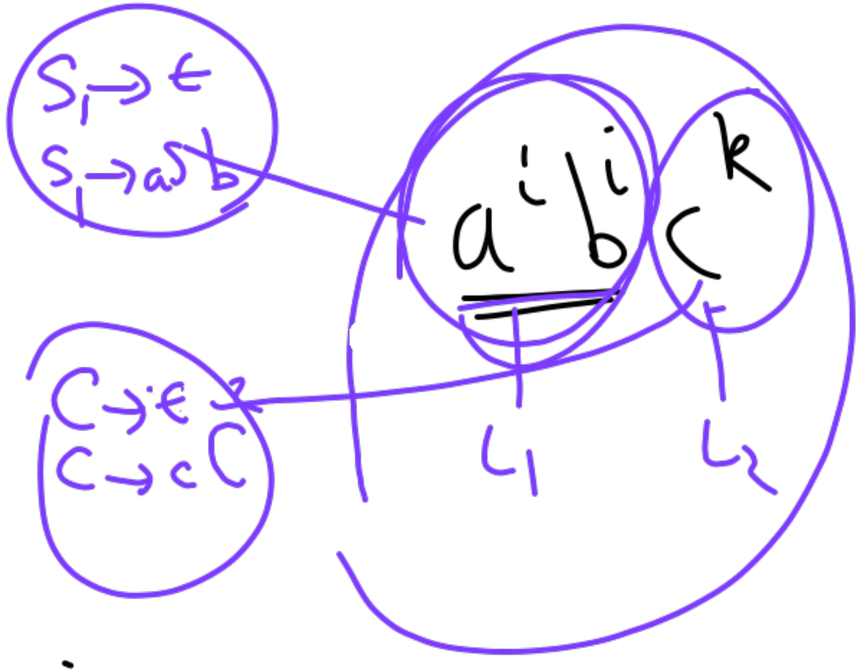
$L = \{ a^i b^j \mid i > j \}$

Concatenation

$S \to S_1 C$

$L = \{ a^i b^j c^k \mid i = j \text{ or } j = k \}$

$S \to S_1 \mid S_2$



$\cup$ $a^i b^j c^j$

$S \to S_1$

$S \to S_2$

Union

Left-linear
Right-linear — **Regular Grammars**

$L = \{ w \in (a+b)^* \mid n_a(w) \text{ is even AND } n_b(w) \text{ is odd} \}$

$S \to aA \quad C \to bA$
$C \to aB$
$S \to bB$
$A \to aS$
$A \to bC$
$B \to bS$
$B \to aC \quad B \to \epsilon$