There are a total of 54 reactions that involve Ca2+ binding to a state of the sensor and forming a different state.

$$Ca^{2+} + S_1 \rightarrow S_2$$

Let R_i be the event that in period of ΔT, the ith reaction takes place at least once.

$$r_i \Delta T = P(R_i) = k_i [Ca^{2+}][S_i]\Delta T$$

Let B_N be the event that exactly N reactions occur in ΔT.

Probability that no reactions occur in ΔT:

$$P(B_{N=0}) = lim_{K\rightarrow\infty}\big(1 - \sum_i r_i \epsilon\big)^K = e^{-\Delta T R_{tot}}$$

Let $r_{bind} = \sum r_i$, the sum of all the binding reaction rates.

Now,

$$P(R_i | B_{N=0}) = \frac{P(R_i \cap B_{N=0})}{P(B_{N=0})} = 0$$

Similarly we can calculate $P(B_{N=1})$:

$$\begin{aligned}
P(B_{N=1}) &= \sum_i P(R_i) lim_{K\rightarrow\infty}\big[1 - \sum_{j\neq i} r_j \epsilon\big]^K \\
&= \sum_i r_i \Delta T e^{-\Delta T(r_{bind} - r_i)} \\
P(R_l | B_{N=1}) &= \frac{P(R_l \cap B_{N=1})}{P(B_{N=1})} \\
&= \frac{r_l \Delta T e^{-\Delta T(R_{bind} - R_l)}}{\sum_i r_i \Delta T e^{-\Delta T(r_{bind} - r_i)}} \\
&= \frac{k_l [S_l]}{\sum_i k_i [S_i] e^{-\Delta T(r_l - r_i)}}
\end{aligned}$$

We can approximate the exponential to one.

This extends to $N = 2$ as follows:

$$
\begin{aligned}
P(B_{N=2}) &= \sum_{i,j} P(R_i)P(R_j) \lim_{K\to\infty} \big[1 - \sum_{k\neq i,j} r_k \epsilon\big]^K \\
&= \sum_{i,j} r_i r_j \Delta T e^{-\Delta T(r_{bind} - r_i - r_j)} \\
P(R_m, R_n | B_{N=2}) &= \frac{r_m r_n (\Delta T)^2 e^{-\Delta T(R_{bind} - R_m - R_n)}}{\sum_{i,j} r_i r_j (\Delta T)^2 e^{-\Delta T(R_{bind} - r_i - r_j)}} \\
&= \frac{k_m k_n [S_m][S_n]}{\sum_{i,j} k_i k_j [S_i][S_j] e^{-\Delta T((r_m + r_n) - (r_i + r_j))}} \\
&\approx \frac{k_m k_n [S_m][S_n]}{\sum_{i,j} k_i k_j [S_i][S_j]}
\end{aligned}
$$

We can similarly extend this to higher Ns. In extremely rare circumstances, N=5 can occur. We are assuming the independence of the R_is.