

Zoom meeting ID

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Today < > March 2020

MON
16

GMT-04 AMMI Teaching

06:00

07:00 Lecture: Recap of neural ne
07:00, https://campuswire.c

08:00 Lecture: language modeling
08:00, https://campuswire.c

09:00

10:00 Lab: GCP and Jupyter Setup
09:30, https://campuswire.c

11:00 Lab: language modeling
11:00, https://campuswire.c

12:00

13:00 Lab: FF- PC Meet ICLR202
13:00, h 13:00 - 13:00, L

14:00

15:00

16:00

17:00

18:00 beer with keunwoo
18:00 - 20:15

19:00 Garage fMRI,

Lecture: language modeling
Monday, March 16 · 08:00 – 09:00

Join Zoom Meeting
<https://campuswire.com/c/GFADD516B/rooms/CF81B>

Kyunghyun Cho is inviting you to a scheduled Zoom meeting.

Join Zoom Meeting
<https://nyu.zoom.us/j/856462940>

Meeting ID: 856 462 940

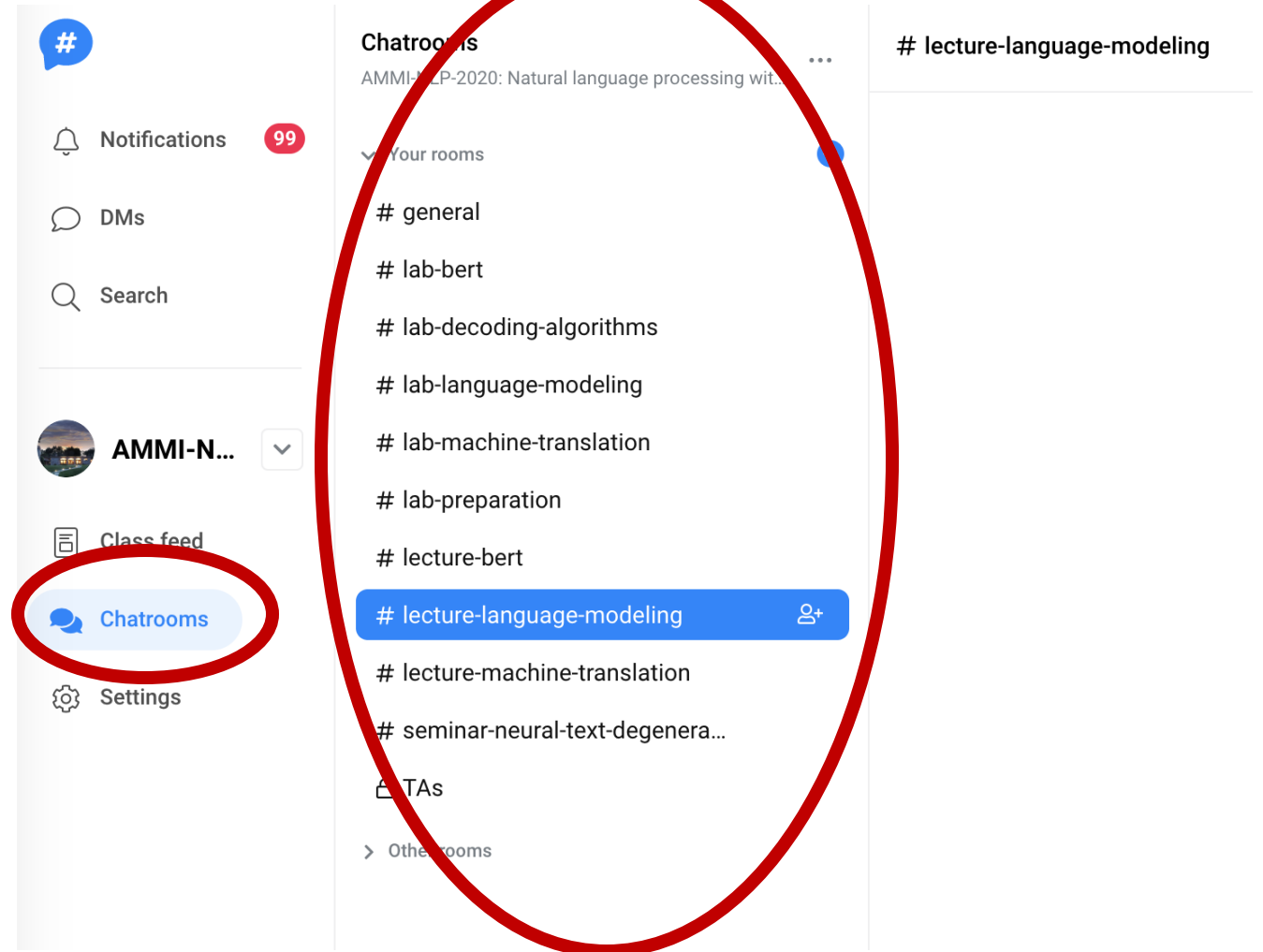
Join by SIP
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Join by H.323
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162.255.36.11 (US East)
221.122.88.195 (China)
115.114.131.7 (India Mumbai)
115.114.115.7 (India Hyderabad)
213.19.144.110 (EMEA)
103.122.166.55 (Australia)
209.9.211.110 (Hong Kong)
64.211.144.160 (Brazil)
69.174.57.160 (Canada)
207.226.132.110 (Japan)
Meeting ID: 856 462 940

AMMI-NLP-2020
Created by: KyungHyun Cho

Course discussion/Q&A

- URL:
<https://campuswire.com/p/GFADD516B>
 - Code: 9109
 - Please join now!
- Q&A using chatrooms



Course Assistants

- **Sreyas Mohan**
 - PhD student at NYU Center for Data Science
- **Phu Mon Htut**
 - PhD student at NYU Center for Data Science
- **Sean Welleck**
 - PhD student at NYU Computer Science



Recap: Supervised Machine Learning

You all know already, but it's important enough to warrant repetition.

Supervised Learning – Overview

- Provided:
 1. a set of N input-output “training” examples
$$D = \{(x_1, y_1), \dots, (x_N, y_N)\}$$
 2. A per-example loss function
$$l(M(x), y) \geq 0$$
 3. Evaluation sets*: validation and test examples $D_{\text{val}}, D_{\text{test}}$
- What we must decide:
 1. Hypothesis sets $\mathcal{H}_1, \dots, \mathcal{H}_M$
 - Each set consists of all compatible models
 2. Optimization algorithm

* Often it is necessary to design a loss function.

* Often these sets are created by holding out subsets of training examples.

Supervised Learning – Overview

- Given:
 1. $D = \{(x_1, y_1), \dots, (x_N, y_N)\}$ and $D_{\text{val}}, D_{\text{test}}$
 2. $l(M(x), y) \geq 0$
 3. $\mathcal{H}_1, \dots, \mathcal{H}_M$
 4. Optimization algorithm
- Supervised learning finds an appropriate algorithm/model automatically
 1. For each hypothesis set \mathcal{H}_m , find the best model:

$$\hat{M}_m = \arg \min_{M \in \mathcal{H}_m} \sum_{n=1}^N l(M(x_n), y_n)$$

using the optimization algorithm.

Supervised Learning – Overview

- Given:
 1. $D = \{(x_1, y_1), \dots, (x_N, y_N)\}$ and $D_{\text{val}}, D_{\text{test}}$
 2. $l(M(x), y) \geq 0$
 3. $\mathcal{H}_1, \dots, \mathcal{H}_M$
 4. Optimization algorithm
- Supervised learning finds an appropriate algorithm/model automatically
 1. [Training] For each hypothesis set \mathcal{H}_m , find the best model:

$$\hat{M}_m = \arg \min_{M \in \mathcal{H}_m} \sum_{n=1}^N l(M(x_n), y_n)$$

using the optimization algorithm and the **training set**.

Supervised Learning – Overview

- Given:
 1. $D = \{(x_1, y_1), \dots, (x_N, y_N)\}$ and $D_{\text{val}}, D_{\text{test}}$
 2. $l(M(x), y) \geq 0$
 3. $\mathcal{H}_1, \dots, \mathcal{H}_M$
 4. Optimization algorithm
- Supervised learning finds an appropriate algorithm/model automatically
 2. [Model Selection]* Among the trained models, select the best one

$$\hat{M} = \arg \min_{M \in \{\mathcal{H}_1, \dots, \mathcal{H}_M\}} \sum_{(x, y) \in D_{\text{val}}} l(M(x), y)$$

using the **validation set** loss.

* If you're familiar with deep learning, “hyperparameter optimization” may be a more familiar term⁸ for you.

Supervised Learning – Overview

- Given:
 1. $D = \{(x_1, y_1), \dots, (x_N, y_N)\}$ and $D_{\text{val}}, D_{\text{test}}$
 2. $l(M(x), y) \geq 0$
 3. $\mathcal{H}_1, \dots, \mathcal{H}_M$
 4. Optimization algorithm
- Supervised learning finds an appropriate algorithm/model automatically
 3. [Reporting] Report how well the best model *would* work

$$R(\hat{M}) \approx \frac{1}{|D_{\text{test}}|} \sum_{(x,y) \in D_{\text{test}}} l(\hat{M}(x), y)$$

using the **test set** loss.

Supervised Learning – Overview

- Given:
 1. $D = \{(x_1, y_1), \dots, (x_N, y_N)\}$ and $D_{\text{val}}, D_{\text{test}}$
 2. $l(M(x), y) \geq 0$
 3. $\mathcal{H}_1, \dots, \mathcal{H}_M$
 4. Optimization algorithm
- Supervised learning finds an appropriate algorithm/model automatically
- It results in an algorithm \hat{M} with an expected performance of $R(\hat{M})$.

Supervised Learning

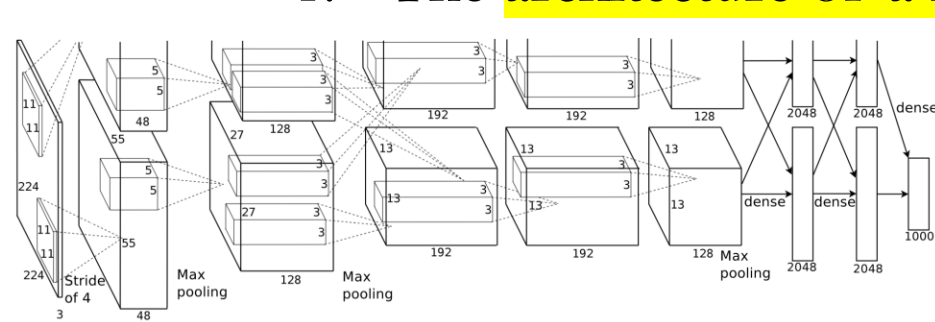
- Three points to consider both in research and in practice
 1. How do we decide/design a **hypothesis set**?
 2. How do we decide a **loss function**?
 3. How do we **optimize** the loss function?

Hypothesis set – Neural Networks

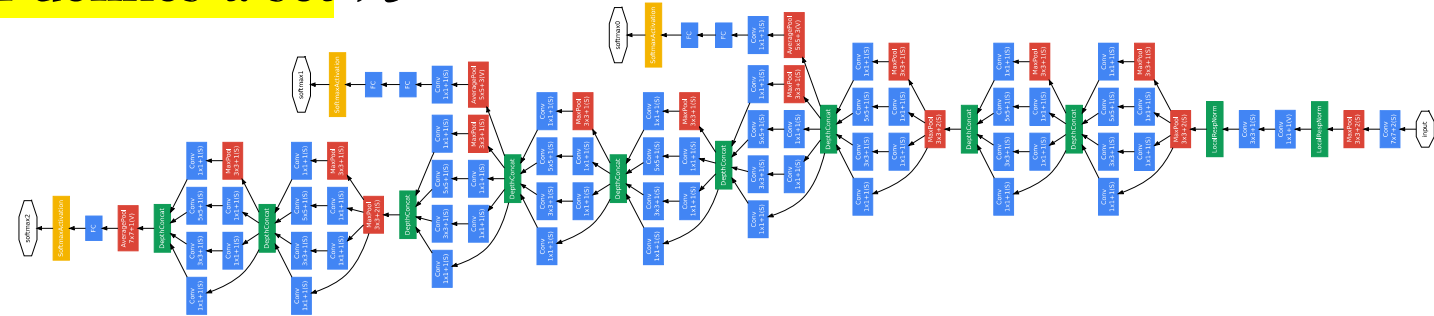
- What kind of machine learning approach will we consider?
 - Classification:
 - Support vector machines, Naïve Bayes classifier, logistic regression, ...?
 - Regression:
 - Support vector regression, Linear regression, Gaussian process, ...?
- How are the hyperparameters sets?
 - Support vector machines: regularization coefficient C
 - Gaussian process: kernel function $k(\cdot, \cdot)$

Hypothesis set – Neural Networks

- In the case of deep learning/artificial neural networks,
 1. The architecture of a network defines a set \mathcal{H}



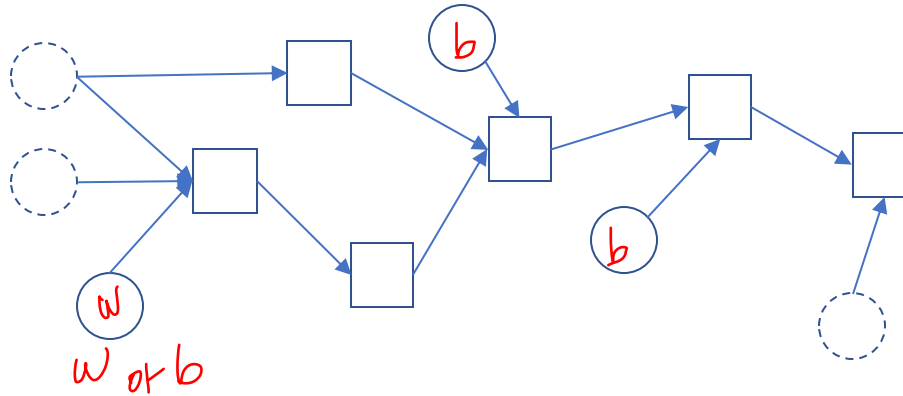
vs.



2. Each model in the set $M \in \mathcal{H}$ is characterized by its parameters θ
 - Weights and bias vectors define one model in the hypothesis set.
- There are infinitely many models in a hypothesis set.
- We use optimization to find “a” good model from the hypothesis set.

Network Architectures

- What is a neural network? – An (arbitrary) directed acyclic graph (DAG)



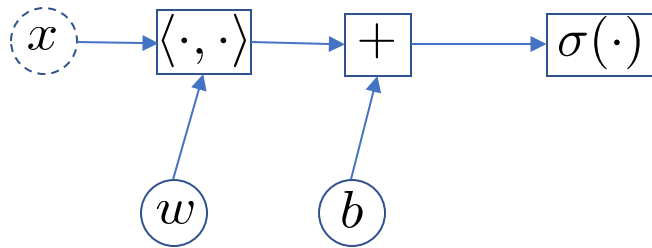
1. Solid Circles \bigcirc : parameters (to be estimated or found)
2. Dashed Circles \bigcirc : vector inputs/outputs (given as a training example)
3. Squares \square : compute nodes (functions, often continuous/differentiable)

Network Architectures

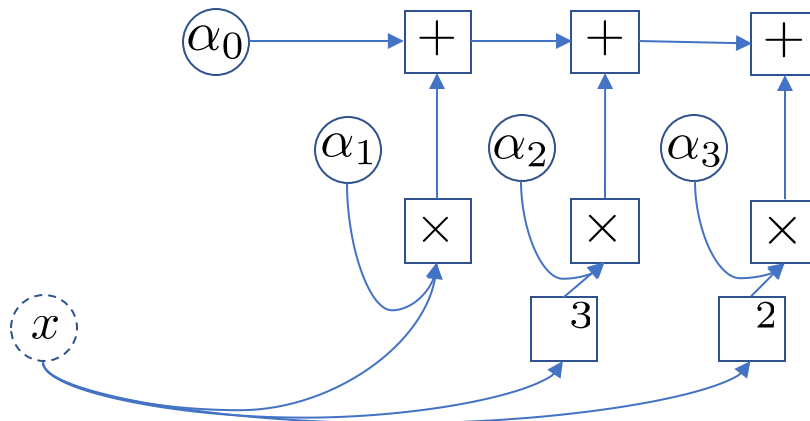
- What is a neural network? – An (arbitrary) directed acyclic graph (DAG)

1. Logistic regression

$$p_{\theta}(y = 1|x) = \sigma(w^{\top}x + b) = \frac{1}{1 + \exp(-w^{\top}x - b)}$$



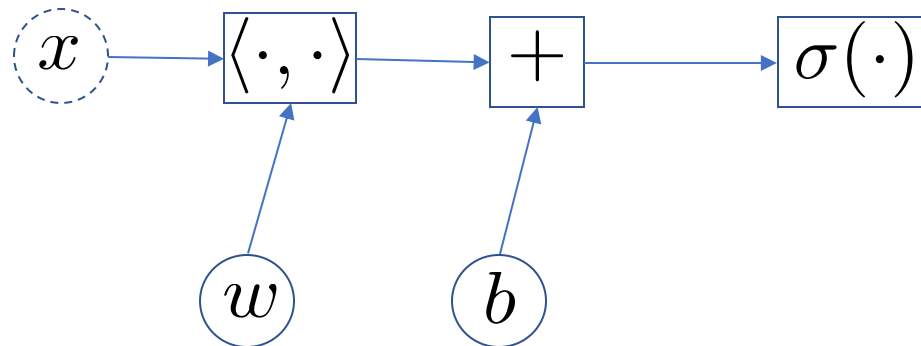
2. 3rd-order polynomial function $y = \alpha_0 + \alpha_1x + \alpha_2x^2 + \alpha_3x^3$



Inference – Forward Computation

- What is a neural network? – An (arbitrary) directed acyclic graph (DAG)
- Forward computation: how you “use” a trained neural network.
 - Topological sweep (breadth-first)
 - Logistic regression

$$p_{\theta}(y = 1|x) = \sigma(w^{\top}x + b) = \frac{1}{1 + \exp(-w^{\top}x - b)}$$



DAG \leftrightarrow Hypothesis Set

- What is a neural network? – An (arbitrary) directed acyclic graph (DAG)
- Implication in practice
 - Naturally supports high-level abstraction
 - Object-oriented paradigm fits well.*
 - Base classes: variable (input/output) node, operation node
 - Define the internal various types of variables and operations by inheritance
 - Maximal code reusability
 - See the success of PyTorch, TensorFlow, DyNet, ...
- You define a hypothesis set by designing a directed acyclic graph.
- The hypothesis space is then a set of all possible parameter settings.

Supervised Learning

- Three points to consider both in research and in practice
 1. How do we decide/design a **hypothesis set**?
 2. How do we decide a **loss function**?
 3. How do we **optimize** the loss function?

Loss Functions

- Per-example loss function
 - Computes how good a model is doing on a given example:
$$l(M(x), y) \geq 0$$
- So many loss functions...
 - Classification: hinge loss, log-loss, ...
 - Regression: mean squared error, mean absolute error, robust loss, ...
- In this lecture, we stick to **distribution-based loss functions.**

A Neural network

computes a conditional distribution

- Supervised learning: what is y given x ?

$$f_{\theta}(x) = ?$$

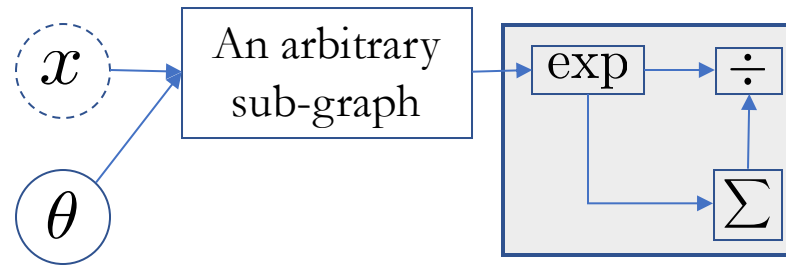
- In other words, how probable is a certain value y' of y given x ?

$$p(y = y' | x) = ?$$

- What kind of distributions?
 - Binary classification: Bernoulli distribution
 - Multiclass classification: Categorical distribution
 - Linear regression: Gaussian distribution
 - Multimodal linear regression: Mixture of Gaussians

Important distributions – Categorical

- How probable is a certain value y' of y given x ? $p(y = y'|x) = ?$
- Multi-class classification: Categorical distribution $\mathcal{C}(\{\mu_1, \mu_2, \dots, \mu_C\})$
 - Probability: $p(y = v|x) = \mu_v$, where $\sum_{v=1} \mu_v = 1$
 - Fully characterized by $\{\mu_1, \mu_2, \dots, \mu_C\}$.
 - A neural network then should turn the input x into a vector $\mu =$



$$\begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_C \end{bmatrix}$$

using a **softmax** function: $\text{softmax}(a) = \frac{1}{\sum_{v=1}^C \exp(a_v)} \exp(a) .$

Important distributions – Gaussian

- How probable is a certain value y' of y given x ? $p(y = y'|x) = ?$
- Regression: Gaussian distribution $\mathcal{N}(\mu, \mathbb{I})$ with an identity covariance
 - Probability: $p(y|x) = \frac{1}{Z} \exp(-\frac{1}{2}(y - \mu)^\top (y - \mu))$
 - Fully characterized by $\mu \in \mathbb{R}^q$.
 - A neural network then should turn the input x into a vector μ .
 - Can be done trivially by affine transformation.

Loss Function – negative log-probability

- Once a neural network outputs a conditional distribution $p_\theta(y|x)$, a natural way to define a loss function arises.
- Make sure training data is maximally likely:
 - Equiv. to making sure each and every training example is maximally likely.

$$\arg \max_{\theta} \log p_{\theta}(D) = \arg \max_{\theta} \sum_{n=1}^N \log p_{\theta}(y_n|x_n)$$

- Why *log*? – many reasons... but out of the lecture's scope.
- Equivalently, we want to minimize the *negative* log-probability.
 - A loss function is the sum of negative log-probabilities of correct answers.

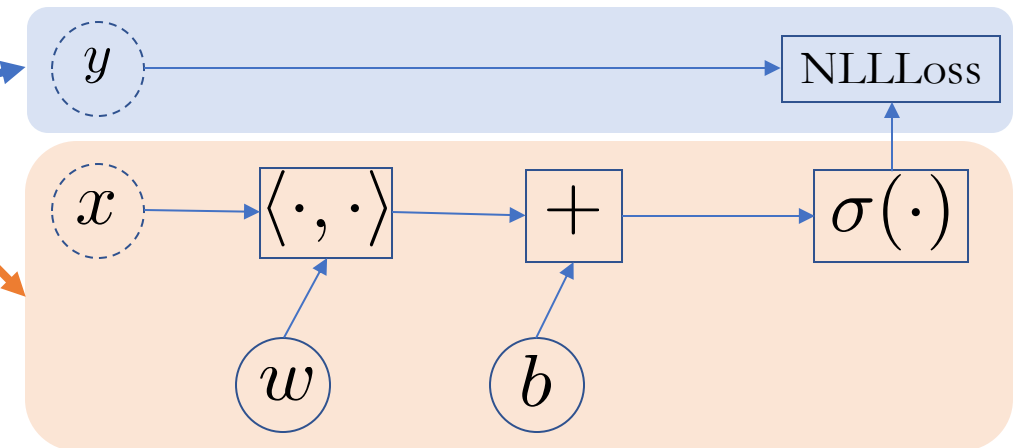
$$L(\theta) = \sum_{n=1}^N l(M_{\theta}(x_n), y_n) = - \sum_{n=1}^N \log p_{\theta}(y_n|x_n)$$

Loss Function – negative log-probability

- Once a neural network outputs a conditional distribution $p_{\theta}(y|x)$, a natural way to define a loss function arises.
- Practical implications
 - An OP node: negative log-probability (e.g., NLLLoss in PyTorch)
 - Inputs: the conditional distribution and the correct output
 - Output: the negative log-probability (a scalar)

Loss Function – negative log-probability

- Once a neural network outputs a conditional distribution $p_{\theta}(y|x)$, a natural way to define a loss function arises.
- Logistic regression
 - Computes a Bernoulli distribution
 - Computes a negative log-probability
 - All in **one directed acyclic graph**
- Forward computation
 - Computes the conditional distribution, and
 - Computes the per-example loss



Supervised Learning

- Three points to consider both in research and in practice
 1. How do we decide/design a **hypothesis set**?
 2. How do we decide a **loss function**?
 3. How do we **optimize** the loss function?

Loss Minimization

- What we now know
 1. How to build a neural network with an arbitrary architecture.
 2. How to define a per-example loss as a negative log-probability.
 3. Define a single directed acyclic graph containing both.
- What we now need to know
 1. Choose an optimization algorithm.
 2. How to use the optimization algorithm to estimate parameters θ .

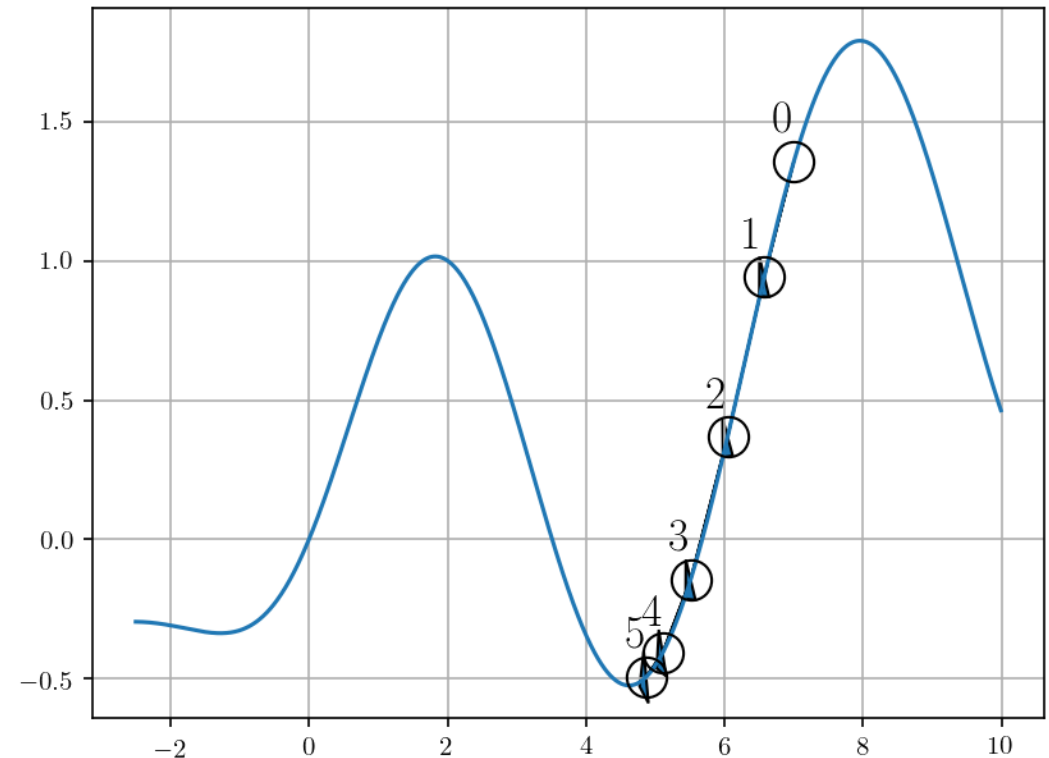
Gradient-based optimization

- A **continuous, differentiable*** function $L : \mathbb{R}^d \rightarrow \mathbb{R}$
- Given the current value θ_0 , how should I move to minimize L ?
- Gradient descent
 - The negative gradient of the function: $-\nabla L(\theta_0)$
 - This is only valid in a local neighbourhood of θ_0 : take a very small step!
$$\theta = \theta_0 - \eta \nabla L(\theta_0)$$
- **Efficient and effective even in the high dimensional space.**
 - Can be improved with the second-order information (Hessian and/or FIM)

* Almost everywhere, but not necessarily everywhere²⁸

Gradient-based optimization

- A **continuous, differentiable** function $L : \mathbb{R}^d \rightarrow \mathbb{R}$
- Given the current value θ_0 , how should I move to minimize L ?
- Gradient descent
 - Efficient and effective even in the high dimensional space.
 - Learning rate must be carefully selected and annealed over time.



Backward Computation – Backpropagation

- How do we compute the gradient of the loss function?

1. Manual derivation

- Relatively doable when the DAG is small and simple.
- When the DAG is larger and complicated, too much hassle.

2. Automatic differentiation (autograd)

- Use the chain rule of derivatives

$$\frac{\partial(f \circ g)}{\partial x} = \frac{\partial f}{\partial g} \frac{\partial g}{\partial x}$$

- The DAG is nothing but a composition of (mostly) differentiable functions.
- Automatically apply the chain rule of derivatives.

Backward Computation – Backpropagation

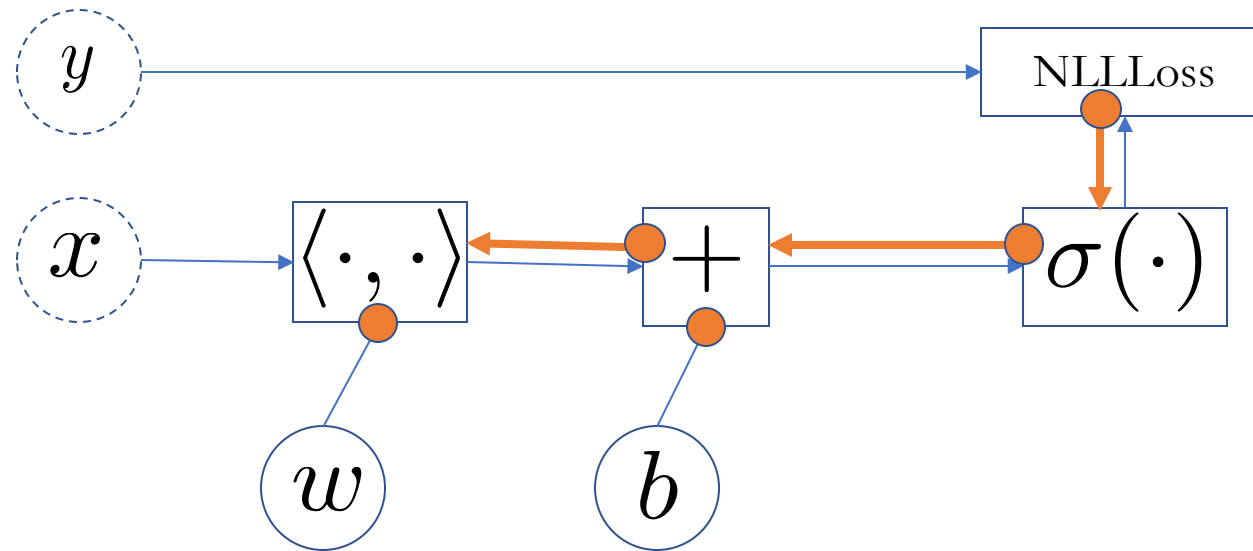
- Automatic differentiation (autograd)
 1. Implement the Jacobian-vector product of each OP node:

$$\begin{bmatrix} \frac{\partial L}{\partial x_1} \\ \vdots \\ \frac{\partial L}{\partial x_d} \end{bmatrix} = \begin{bmatrix} \frac{\partial F_1}{\partial x_1} & \dots & \frac{\partial F_{d'}}{\partial x_1} \\ \vdots & \ddots & \vdots \\ \frac{\partial F_1}{\partial x_d} & \dots & \frac{\partial F_{d'}}{\partial x_d} \end{bmatrix} \begin{bmatrix} \frac{\partial L}{\partial F_1} \\ \vdots \\ \frac{\partial L}{\partial F_{d'}} \end{bmatrix}$$

- Can be implemented efficiently without explicitly computing the Jacobian.
- The same implementation can be reused every time the OP node is called.

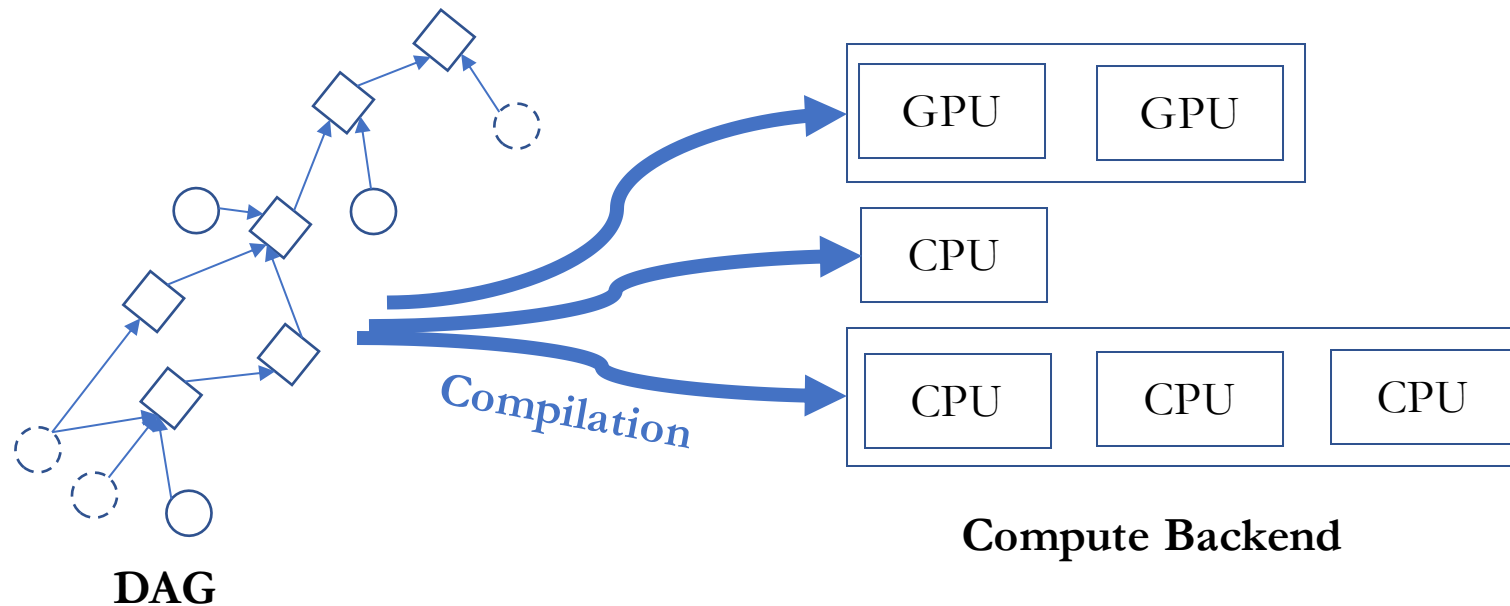
Backward Computation – Backpropagation

- Automatic differentiation (autograd)
 2. Reverse-sweep the DAG starting from the loss function node.
 - Iteratively multiplies the Jacobian of each OP node until the leaf nodes of the parameters.
 - As expensive as forward computation with a constant overhead: $O(N)$, where N : # of nodes.



Backward Computation – Backpropagation

- Practical Implications – Automatic differentiation (autograd)
 - Unless a complete new OP is introduced, no need to manually derive the gradient
 - Nice de-coupling of specification (front-end) and implementation (back-end)
 1. [Front-end] Design a neural network by creating a DAG.
 2. [Back-end] The DAG is “compiled” into an efficient code for a target compute device.



Gradient-based Optimization

- Backpropagation gives us the gradient of the loss function w.r.t. θ
- Readily used by off-the-shelf gradient-based optimizers
 - Gradient descent, L-BFGS, Conjugate gradient, ...
 - Though, most are not applicable in a realistic neural network with 10s or 100s of millions of parameters.
- Stochastic gradient descent
 - Approximate the full loss function (the sum of per-examples losses) using only a small random subset of training examples:

$$\nabla L \approx \frac{1}{N'} \sum_{n=1}^{N'} \nabla l(M(x_{n'}), y_{n'})$$

Stochastic Gradient Descent

- Stochastic gradient descent
 - Approximate the full loss function (the sum of per-examples losses) using only a small random subset of training examples:

$$\nabla L \approx \frac{1}{N'} \sum_{n=1}^{N'} \nabla l(M(x_{n'}), y_{n'})$$

- Unbiased estimate of the full gradient.*
- Learning rate must be annealed appropriately.
- Extremely efficient *de facto* standard practice.

* Under certain conditions

Stochastic Gradient Descent

- Stochastic gradient descent in practice

1. Grab a random subset of M training examples*

$$D' = \{(x_1, y_1), \dots, (x_{N'}, y_{N'})\}$$

2. Compute the minibatch gradient

$$\nabla L \approx \frac{1}{N'} \sum_{n=1}^{N'} \nabla l(M(x_{n'}), y_{n'})$$

3. Update the parameters

$$\theta \leftarrow \theta + \eta \nabla L(\theta; D')$$

4. Repeat until the validation loss stops improving.*

* In practice, sample without replacement until the training set is exhausted (one epoch).

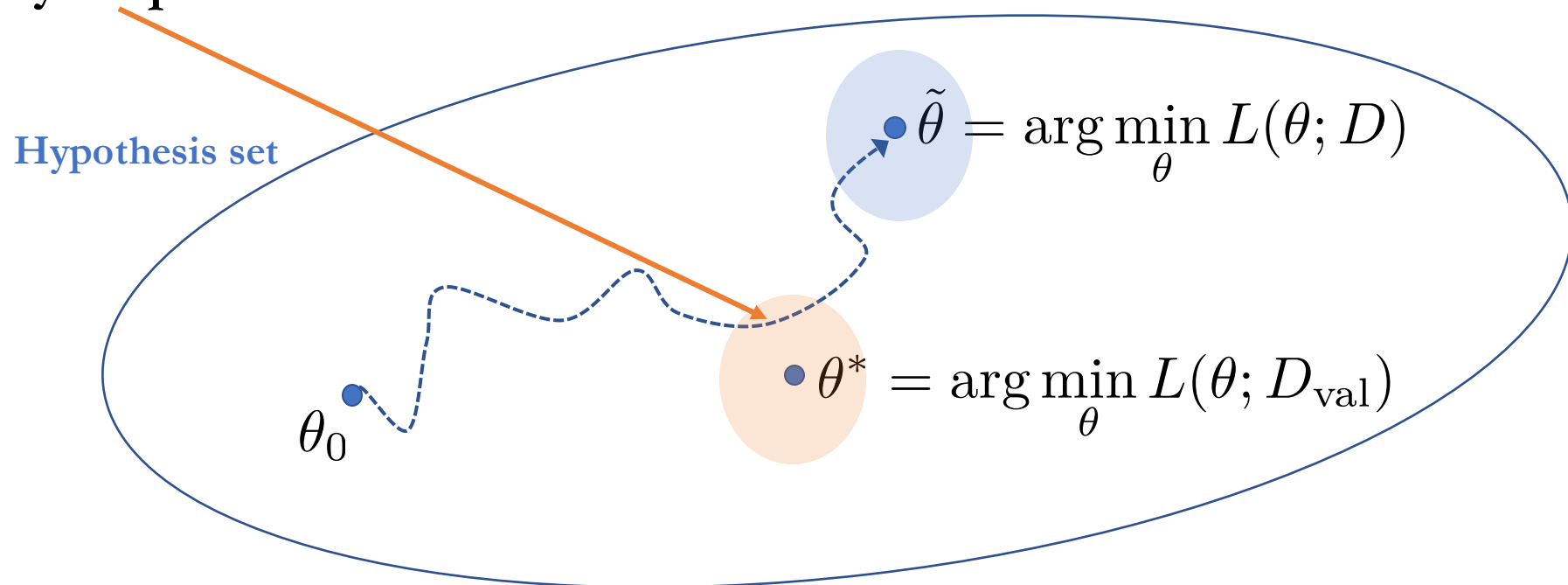
* This is called early-stopping which prevents the neural network from overfitting to training examples.

Stochastic Gradient Descent – Early Stopping

- Stochastic gradient descent in practice
 1. Grab a random subset of M training examples
 2. Compute the minibatch gradient
 3. Update the parameters
 4. **Repeat until the validation loss stops improving.**
- An efficient way to prevent overfitting
 - Overfitting: the training loss is low, but the validation loss is not.
 - The most serious problem in statistical machine learning.
 - Early-stop based on the validation loss

Stochastic Gradient Descent – Early Stopping

- An efficient way to prevent overfitting
 - Overfitting: the training loss is low, but the validation loss is not.
 - The most serious problem in statistical machine learning.
 - **Early-stop** based on the validation loss



Stochastic Gradient Descent

– Adaptive Learning Rate

- Stochastic gradient descent in practice
 1. Grab a random subset of M training examples $D' = \{(x_1, y_1), \dots, (x_{N'}, y_{N'})\}$
 2. Compute the minibatch gradient
 3. Update the per-parameter learning rate η_θ
 4. Update the parameters
$$\theta \leftarrow \theta - \eta_\theta \frac{\partial L'}{\partial \theta}$$
 5. Repeat until the validation loss stops improving.
- Adaptive learning rate: Adam [Kingma&Ba, 2015], Adadelata [Zeiler, 2015], and many more...
 - Approximately re-scale parameters to improve the conditioning of the Hessian.

Supervised Learning with Neural Networks

1. How do we decide/design a **hypothesis set**?
 - Design a network architecture as a directed acyclic graph
2. How do we decide a **loss function**?
 - Frame the problem as a conditional distribution modelling
 - The per-example loss function is a negative log-probability of a correct answer
3. How do we **optimize** the loss function?
 - Automatic backpropagation: no manual gradient derivation
 - Stochastic gradient descent with early stopping [and adaptive learning rate]

Language modeling as supervised learning

On the boundary between unsupervised and supervised learning

Language Modelling

- Input: a sentence
- Output: the probability of the input sentence
- A language model captures the distribution over all possible sentences.

$$p(X) = p((x_1, x_2, \dots, x_T))$$

- It is *unsupervised learning*.
 - We will however turn the problem into a *sequence of supervised learning*.

Autoregressive language modelling

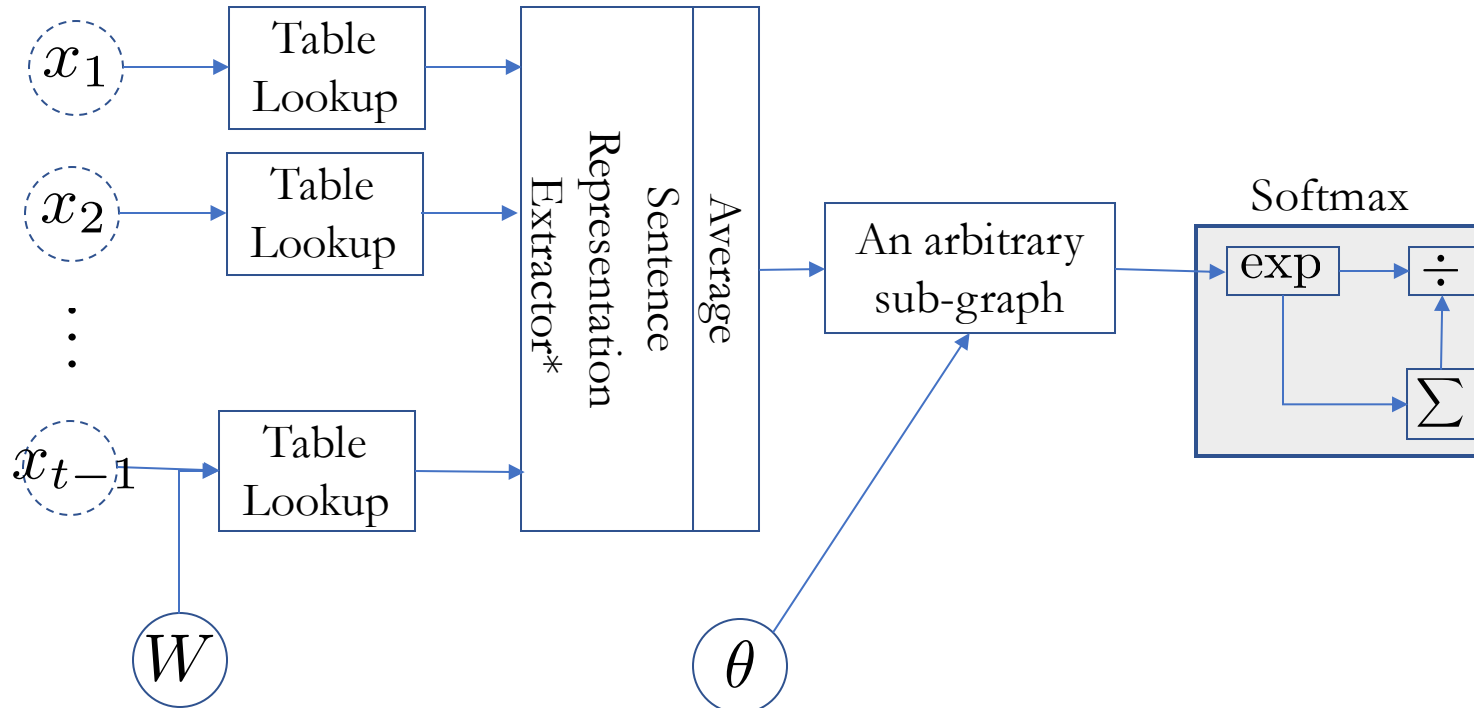
- Autoregressive sequence modelling
 - The distribution over the next token is based on all the previous tokens.
$$p(X) = p(x_1)p(x_2|x_1) \cdots p(x_T|x_1, \dots, x_{T-1})$$
 - This equality holds exactly due to the def. of conditional distribution.
- Unsupervised learning becomes a set of supervised problems.
 - Each conditional is a neural network classifier.
 - Input is all the previous tokens (a partial sentence).
 - Output is the distribution over all possible next tokens (classes).
 - It is a **text classification** problem.

Autoregressive language modelling

- Autoregressive sequence modelling
 - The distribution over the next token is based on all the previous tokens.

$$p(X) = p(x_1)p(x_2|x_1) \cdots p(x_T|x_1, \dots, x_{T-1})$$

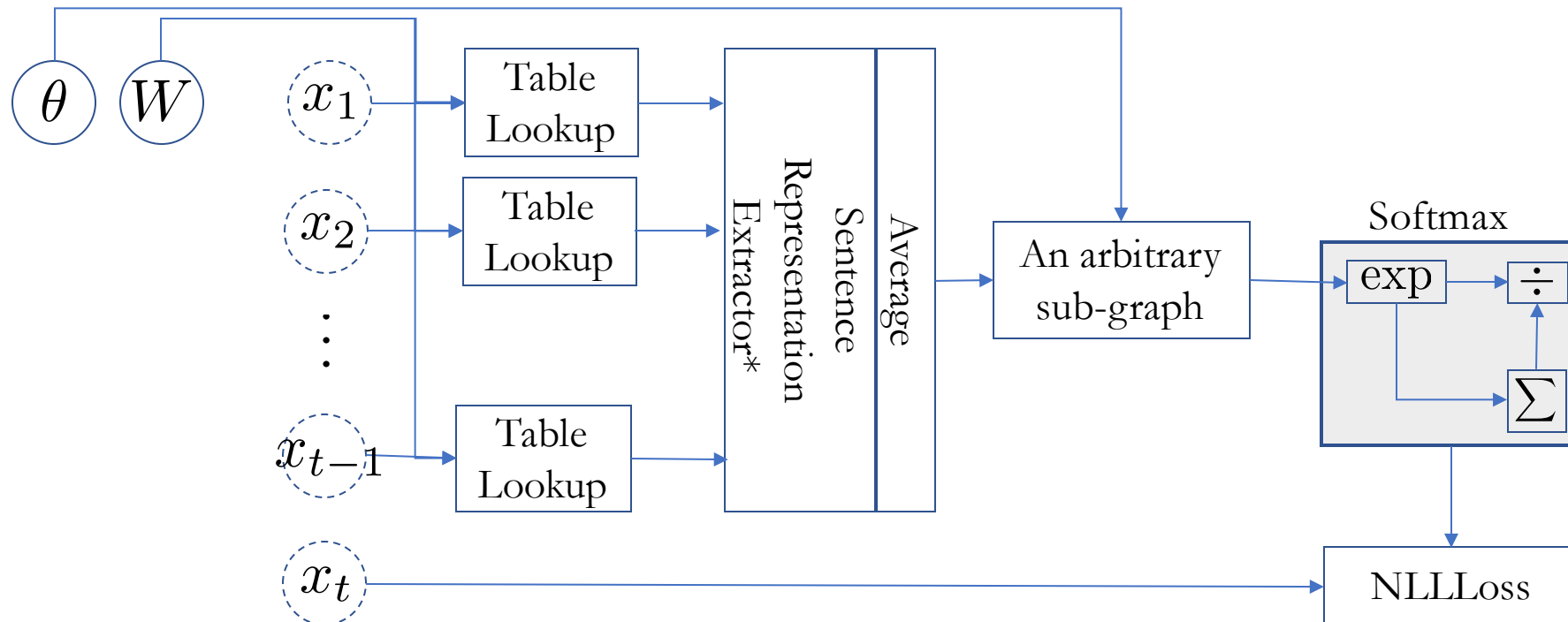
- Each conditional is a sentence classifier:



Autoregressive language modelling

- Autoregressive sequence modelling $p(X) = \prod_{t=1}^T p(x_t|x_{<t})$
- Loss function: the sum of negative log-probabilities

$$\log p_{\theta}(X) = \sum_{n=1}^N \sum_{t=1}^T \log p_{\theta}(x_t|x_{<t})$$



Scoring a sentence

- Autoregressive sequence modelling
 - The distribution over the next token is based on all the previous tokens.

$$p(X) = p(x_1)p(x_2|x_1) \cdots p(x_T|x_1, \dots, x_{T-1})$$

- A natural way to score a sentence:
 - In Korea, more than half of residents speak Korean.
 - “In” is a reasonable token to start a sentence.
 - “Korea” is pretty likely given “In”
 - “more” is okay token to follow “In Korea”
 - “than” is very likely after “In Korea, more”
 - “half” is also very likely after “In Korea, more than”
 - \vdots
- Sum all these scores and get the sentence score.

Scoring a sentence

- Autoregressive sequence modelling
 - The distribution over the next token is based on all the previous tokens.
$$p(X) = p(x_1)p(x_2|x_1) \cdots p(x_T|x_1, \dots, x_{T-1})$$
- A natural way to score a sentence:
 - “In Korea, more than half of residents speak Korean.”
vs.
“In Korea, more than half of residents speak Finnish.”
 - The former is more likely (=higher probability) than the latter.
- This is precisely what NLLLoss computes over the sentence.

N -Gram Language Models

- Let's back up a little...
- What would we do *without* a neural network?
- Assume a Markovian property

$$p(X) = \prod_{t=1}^T p(x_t | x_{<t}) \approx \prod_{t=1}^T p(x_t | x_{t-n}, \dots, x_{t-1})$$

- This turned out to be crucial, and we will discuss why shortly.

N -Gram Language Models

$$p(X) = \prod_{t=1}^T p(x_t | x_{<t}) \approx \prod_{t=1}^T p(x_t | x_{t-n}, \dots, x_{t-1})$$

- We need to estimate n -gram probabilities: $p(x | x_{-N}, x_{-N+1}, \dots, x_{-1})$
- Recall the def. of conditional and marginal probabilities:

$$\begin{aligned} p(x | x_{-N}, x_{-N+1}, \dots, x_{-1}) &= \frac{p(x_{-N}, x_{-N+1}, \dots, x_{-1}, x)}{p(x_{-N}, x_{-N+1}, \dots, x_{-1})} \\ &= \frac{p(x_{-N}, x_{-N+1}, \dots, x_{-1}, x)}{\sum_{x \in V} p(x_{-N}, x_{-N+1}, \dots, x_{-1}, x)} \end{aligned}$$

- V : all possible tokens (=vocabulary)

N -Gram Language Models

- We need to estimate n -gram probabilities:

$$p(x|x_{-N}, x_{-N+1}, \dots, x_{-1}) = \frac{p(x_{-N}, x_{-N+1}, \dots, x_{-1}, x)}{\sum_{x \in V} p(x_{-N}, x_{-N+1}, \dots, x_{-1}, x)}$$

- How do we estimate the probability?
 - I want to estimate the probability of my distorted coin landing head.
 - **Maximum likelihood estimation (MLE):**
toss the coin a lot and look at how often it lands heads.

Data Collection

Estimation

N -Gram Language Models

- We need to estimate n -gram probabilities:

$$p(x|x_{-N}, x_{-N+1}, \dots, x_{-1}) = \frac{p(x_{-N}, x_{-N+1}, \dots, x_{-1}, x)}{p(x_{-N}, x_{-N+1}, \dots, x_{-1})}$$

- Data: all the documents or sentences you can collect
 - e.g., Wikipedia, news articles, tweets, ...
- Estimation:
 1. Count the # of occurrences for the n -gram $(x_{-N}, x_{-N+1}, \dots, x_{-1}, x)$
 2. Count the #'s of occurrences for all the n -grams of the form:
 $(x_{-N}, x_{-N+1}, \dots, x_{-1}, ?)$

N -Gram Language Models

- We need to estimate n -gram probabilities:

$$p(x|x_{-N}, x_{-N+1}, \dots, x_{-1}) = \frac{p(x_{-N}, x_{-N+1}, \dots, x_{-1}, x)}{p(x_{-N}, x_{-N+1}, \dots, x_{-1})}$$

- Estimation:

$$\begin{aligned} p(x|x_{-N}, x_{-N+1}, \dots, x_{-1}) &= \frac{p(x_{-N}, x_{-N+1}, \dots, x_{-1}, x)}{\sum_{x \in V} p(x_{-N}, x_{-N+1}, \dots, x_{-1}, x)} \\ &\approx \frac{c(x_{-N}, x_{-N+1}, \dots, x_{-1}, x)}{\sum_{x' \in V} c(x_{-N}, x_{-N+1}, \dots, x_{-1}, x')} \end{aligned}$$

- *Do you see why this makes sense?*

N-Gram Language Models

- We need to estimate n-gram probabilities:

$$\begin{aligned} p(x|x_{-N}, x_{-N+1}, \dots, x_{-1}) &= \frac{p(x_{-N}, x_{-N+1}, \dots, x_{-1}, x)}{\sum_{x \in V} p(x_{-N}, x_{-N+1}, \dots, x_{-1}, x)} \\ &\approx \frac{c(x_{-N}, x_{-N+1}, \dots, x_{-1}, x)}{\sum_{x' \in V} c(x_{-N}, x_{-N+1}, \dots, x_{-1}, x')} \end{aligned}$$

- How likely is “University” given “New York”?
 - Count all “New York University”
 - Count all “New York ?”: e.g., “New York State”, “New York City”, “New York Fire”, “New York Police”, “New York Bridges”, ...
 - How often “New York University” happens among these?

N-Gram Language Models – Two problems



1. Data sparsity: lack of generalization

- What happens “one” n-gram never happens?

$$\begin{aligned} p(\text{a lion is chasing a llama}) &= p(\text{a}) \times p(\text{lion}|\text{a}) \times p(\text{is}|\text{a lion}) \\ &\quad \times p(\text{chasing}|\text{lion is}) \times p(\text{a}|\text{is chasing}) \\ &\quad \times \underbrace{p(\text{llama}|\text{chasing a})}_{=0} = 0 \end{aligned}$$

2. Inability to capture long-term dependencies

- Each conditional only considers a small window of size n .
- Consider “*the same **stump** which had impaled the car of many a guest in the past thirty years and which **he refused to have removed***”
- It is impossible to tell “removed” is likely by looking at the four preceding tokens.

Traditional Solutions

1. Data Sparsity

- Smoothing: add a small constant to avoid 0.

$$p(x|x_{-N}, x_{-N+1}, \dots, x_{-1}) \approx \frac{c(x_{-N}, x_{-N+1}, \dots, x_{-1}, x) + \epsilon}{\epsilon|V| + \sum_{x' \in V} c(x_{-N}, x_{-N+1}, \dots, x_{-1}, x')}$$

- Backoff: try a shorter window.

$$c(x_{-N}, \dots, x) = \begin{cases} \alpha c(x_{-N+1}, \dots, x) + \beta, & \text{if } c(x_{-N}, \dots, x) = 0 \\ c(x_{-N}, \dots, x), & \text{otherwise} \end{cases}$$

- The most widely used approach: Kneser-Ney smoothing/backoff
- **KenLM** implements the efficient n-gram LM model.

Traditional Solutions

2. Long-Term Dependency

- Increase n : not feasible as the data sparsity worsens.
 - # of all possible n -grams grows exponentially w.r.t. n : $O(|V|^n)$
 - The data size does not grow exponentially: many never-occurring n -grams.
-
- These two problems are closely related and cannot be tackled well.
 - To capture long-term dependencies, n must be large.
 - To address data sparsity, n must be small.
 - Conflicting goals..

N-Gram Language Models – Two problems



1. Data sparsity: lack of generalization

- What happens “one” n-gram never happens?

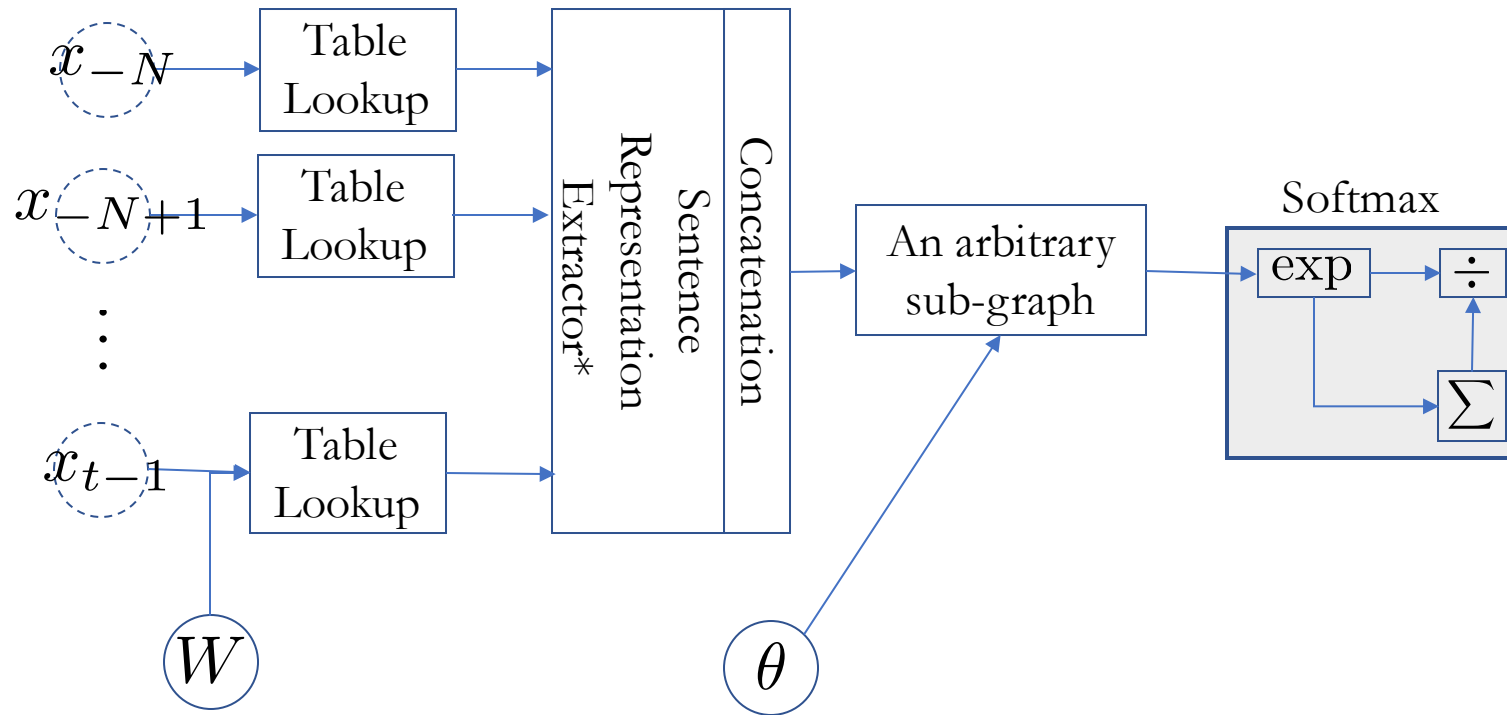
$$\begin{aligned} p(\text{a lion is chasing a llama}) &= p(\text{a}) \times p(\text{lion}|\text{a}) \times p(\text{is}|\text{a lion}) \\ &\quad \times p(\text{chasing}|\text{lion is}) \times p(\text{a}|\text{is chasing}) \\ &\quad \times \underbrace{p(\text{llama}|\text{chasing a})}_{=0} = 0 \end{aligned}$$

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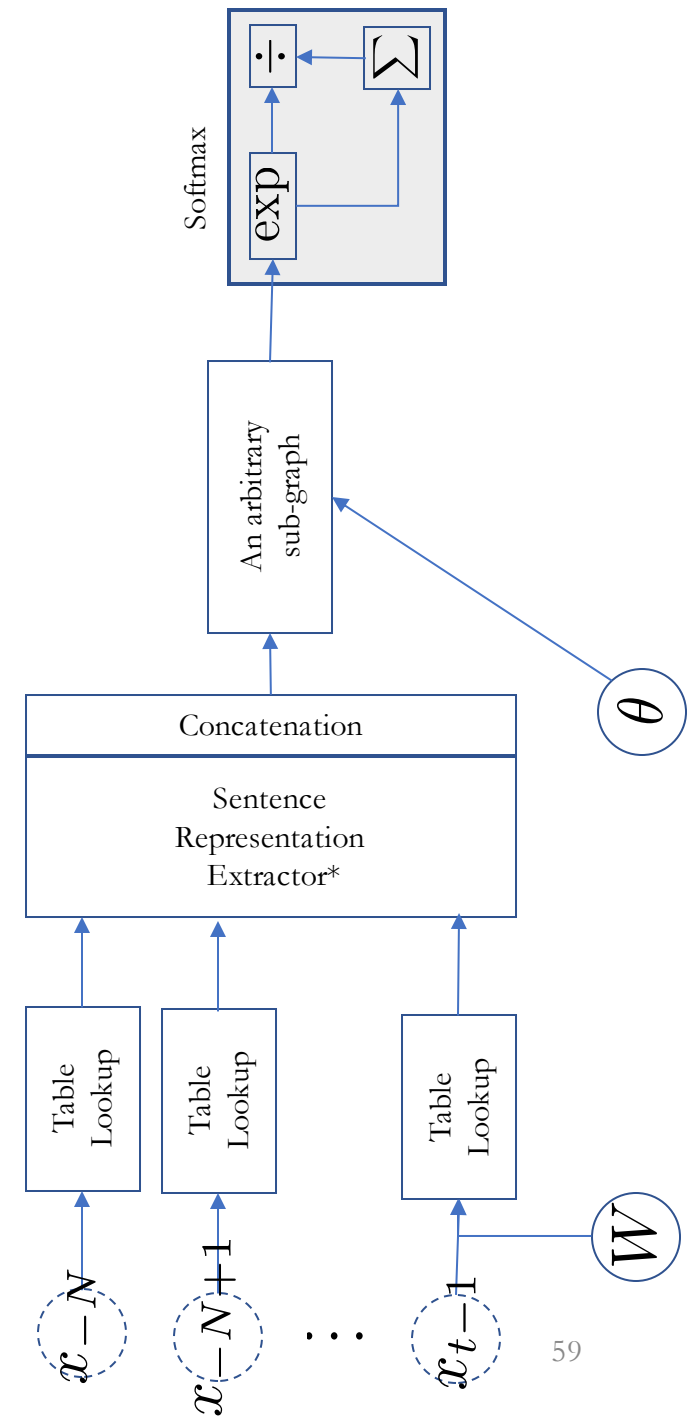
Neural N-Gram Language Model [Bengio et al., 2001]

- The first extension of n-gram language models using a neural network



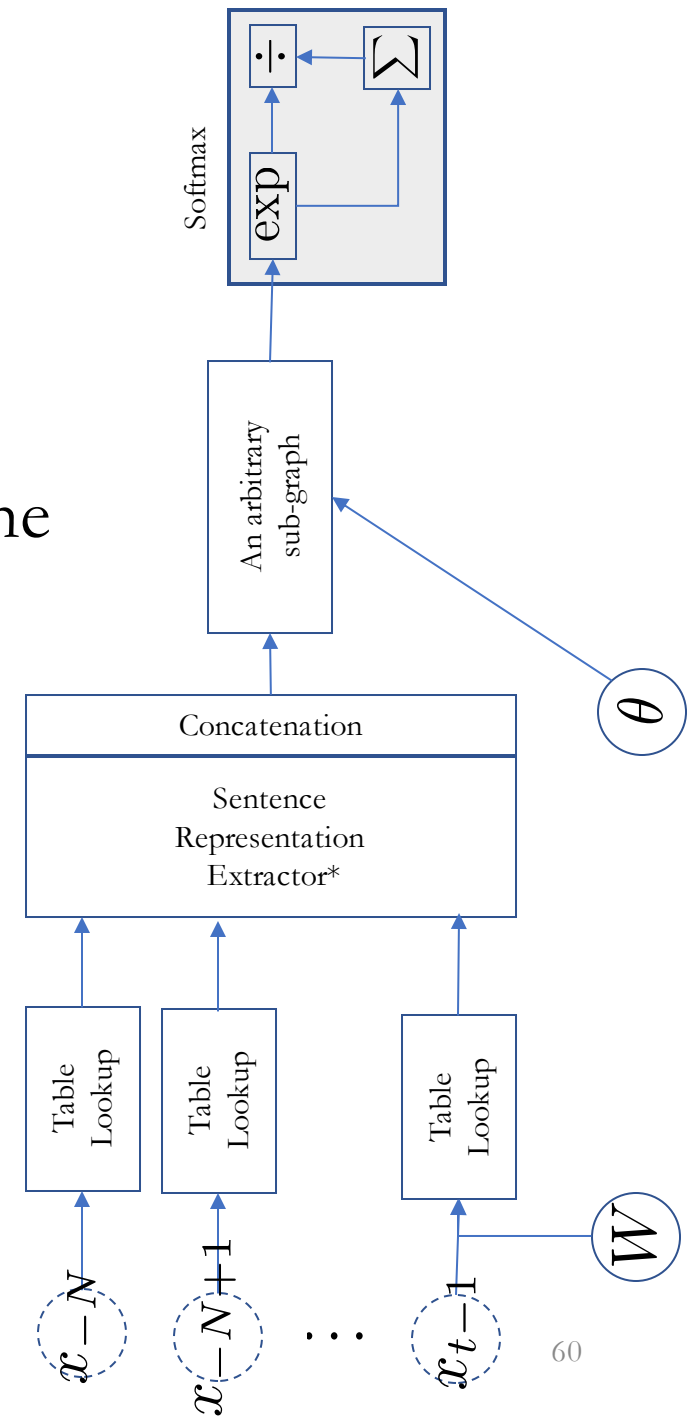
Neural N-Gram Language Model

- The first neural language models
- Trained using backpropagation and SGD
- Generalizes to an unseen n -gram
- **Addresses the issue of data sparsity**
- *How?*



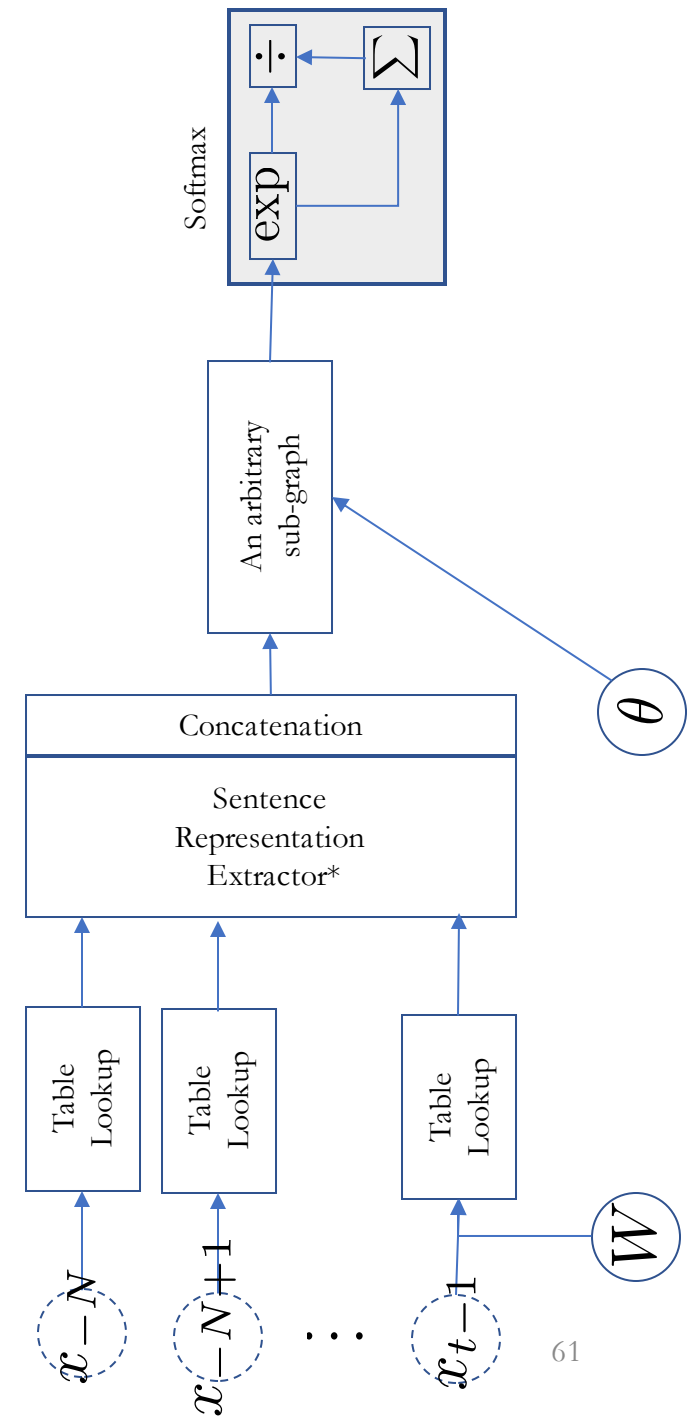
Neural N-Gram Language Model

- Why does the data sparsity happen?
- A “shallow” answer: some n-grams do not occur in the training data, while they do in the test time.
- A “slightly deeper” answer: it is difficult to impose token/phrase similarities in the discrete space.



Neural N-Gram Language Model

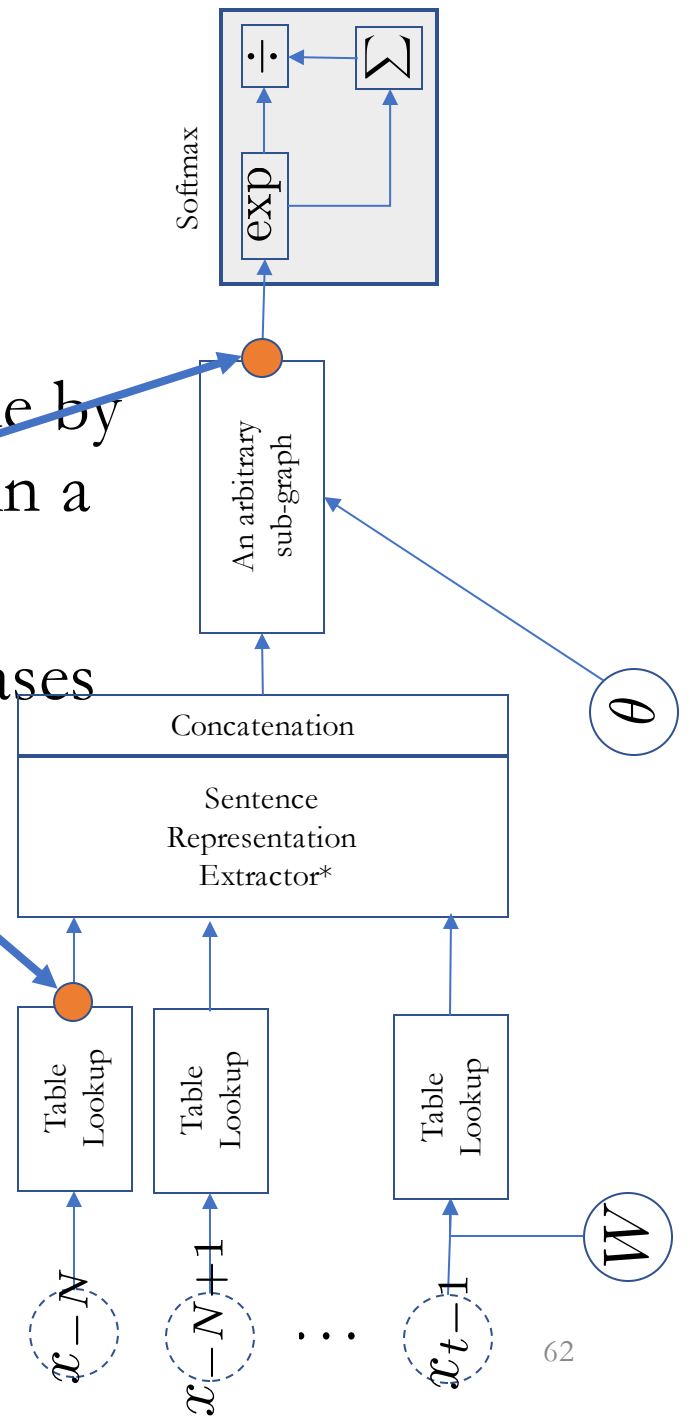
- Why does the data sparsity happen?
- Back to the earlier example
 - Problem: $c(\text{chasing a llama}) = 0$
 - Observation: $c(\text{chasing a cat}) \gg 0$
 $c(\text{chasing a dog}) \gg 0$
 $c(\text{chasing a deer}) \gg 0$
- If the LM knew “llama” is a mammal similar to “cat”, “dog” and “deer”, it would be able to guess “chasing a llama” is as likely as “chasing a cat”, “chasing a dog”, and “chasing a deer”.



Neural N-Gram Language Model

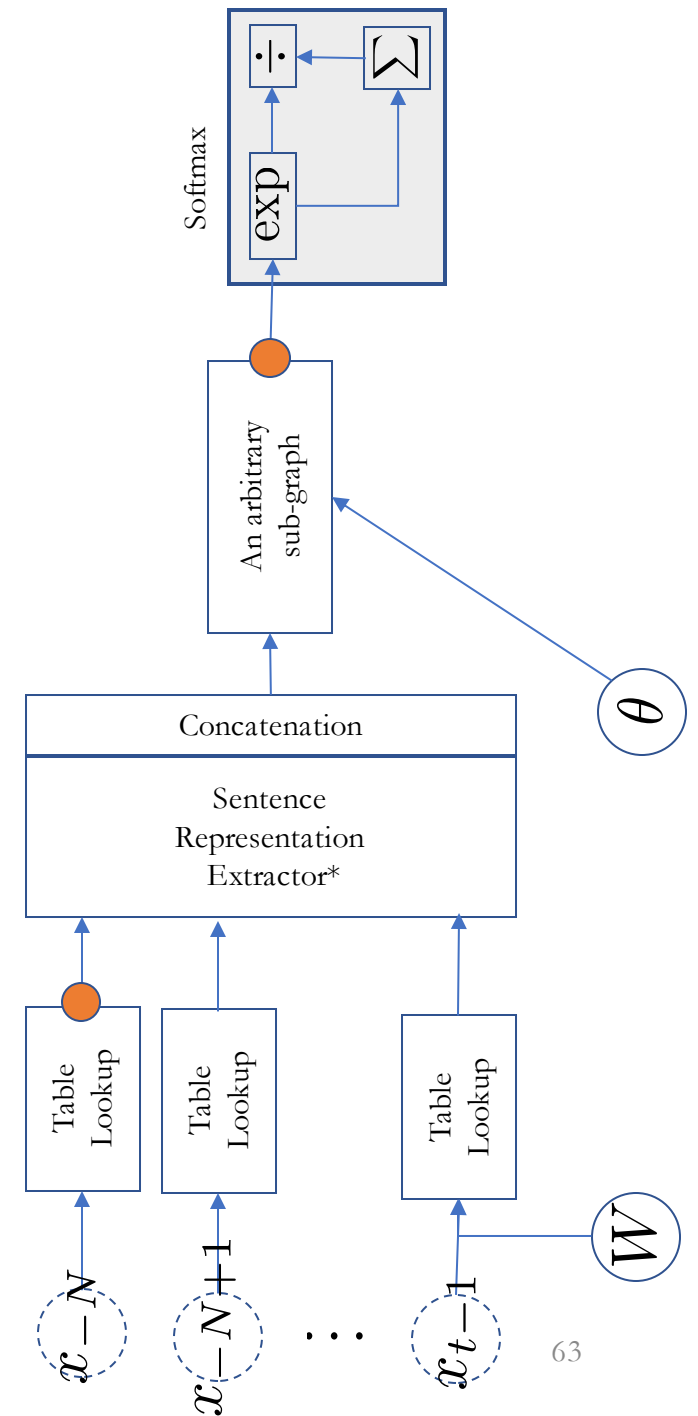
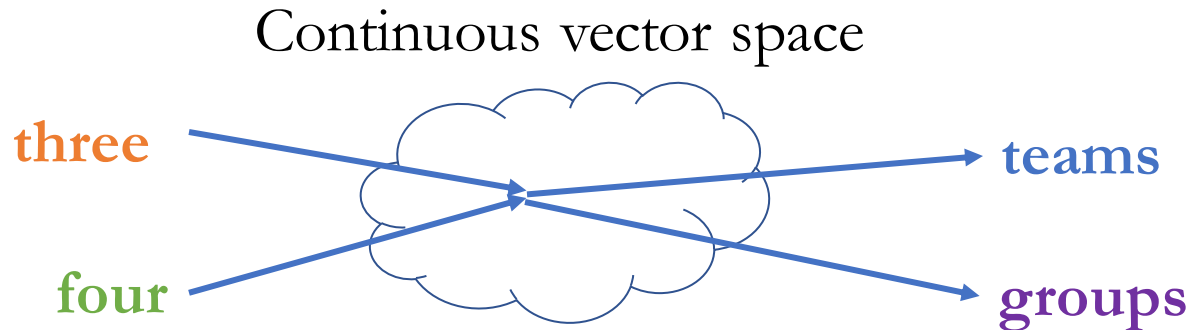
- The neural n-gram language model addresses this issue by “learning the similarities” among tokens and phrases in a “continuous vector space”.
- In the “continuous vector space”, similar tokens/phrases are nearby: e.g., word2vec [Mikolov et al., 2013; Pennington et al., 2014], doc2vec [Le&Mikolove, 2014], sentence-to-vec [Hill et al., 2016 and ref’s therein]
- Then, similar input n-grams lead to similar output:

$$D(x_t | x_{t-N}, \dots, x_{t-1} || x_t | x'_{t-N}, \dots, x'_{t-1}) < \epsilon$$



Neural N-Gram Language Model

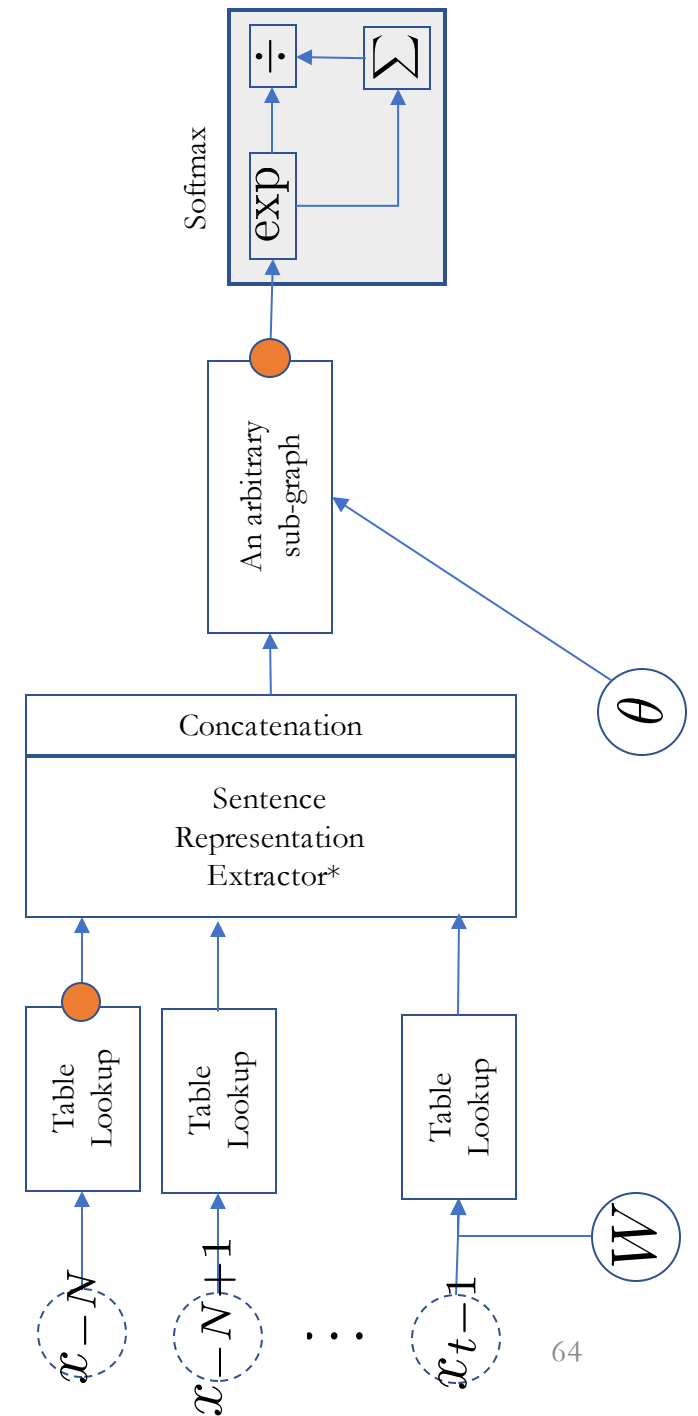
- Training examples
 - there are **three** **teams** left for qualification.
 - **four** **teams** have passed the first round.
 - **four** **groups** are playing in the field.
- Q: how likely is “groups” followed by “three”?



Neural N-Gram Language Model

- In practice,
 1. Collect all n-grams from the corpus.
 2. Shuffle all the n-grams to build a training set
 3. Train the neural n-gram language model using stochastic gradient descent on minibatches containing 100-1000 n-grams.
 4. Early-stop based on the validation set.
 5. Report perplexity on the test set.

$$\text{ppl} = b^{\frac{1}{|D|} \sum_{(x_1, \dots, x_N) \in D} \log_b p(x_N | x_1, \dots, x_{N-1})}$$



Infinite context $n \rightarrow \infty$

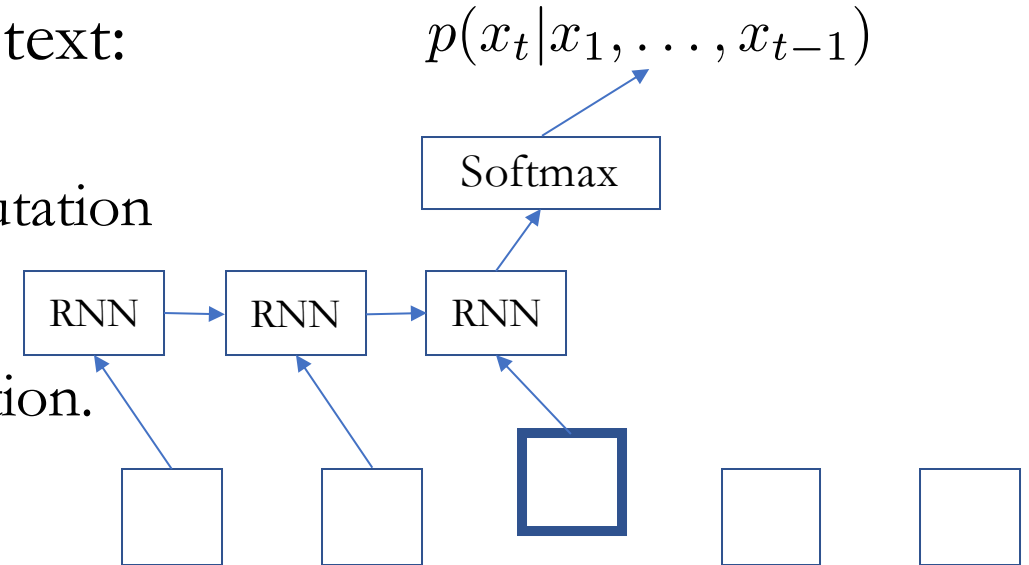
– CBoW Language Models

- Equivalent to the neural LM after replacing “concat” with “average”
 - “Averaging” allows the model to consider the infinite large context window.
- Extremely efficient, but a weak language model
 - Ignores the order of the tokens in the context windows.
 - Any language with a fixed order cannot be modelled well.
 - Averaging ignores the absolute counts, which may be important:
 - If the context window is larger, “verb” becomes less likely in SVO languages.

Infinite context $n \rightarrow \infty$

– Recurrent Language Models [Mikolov et al., 2010]

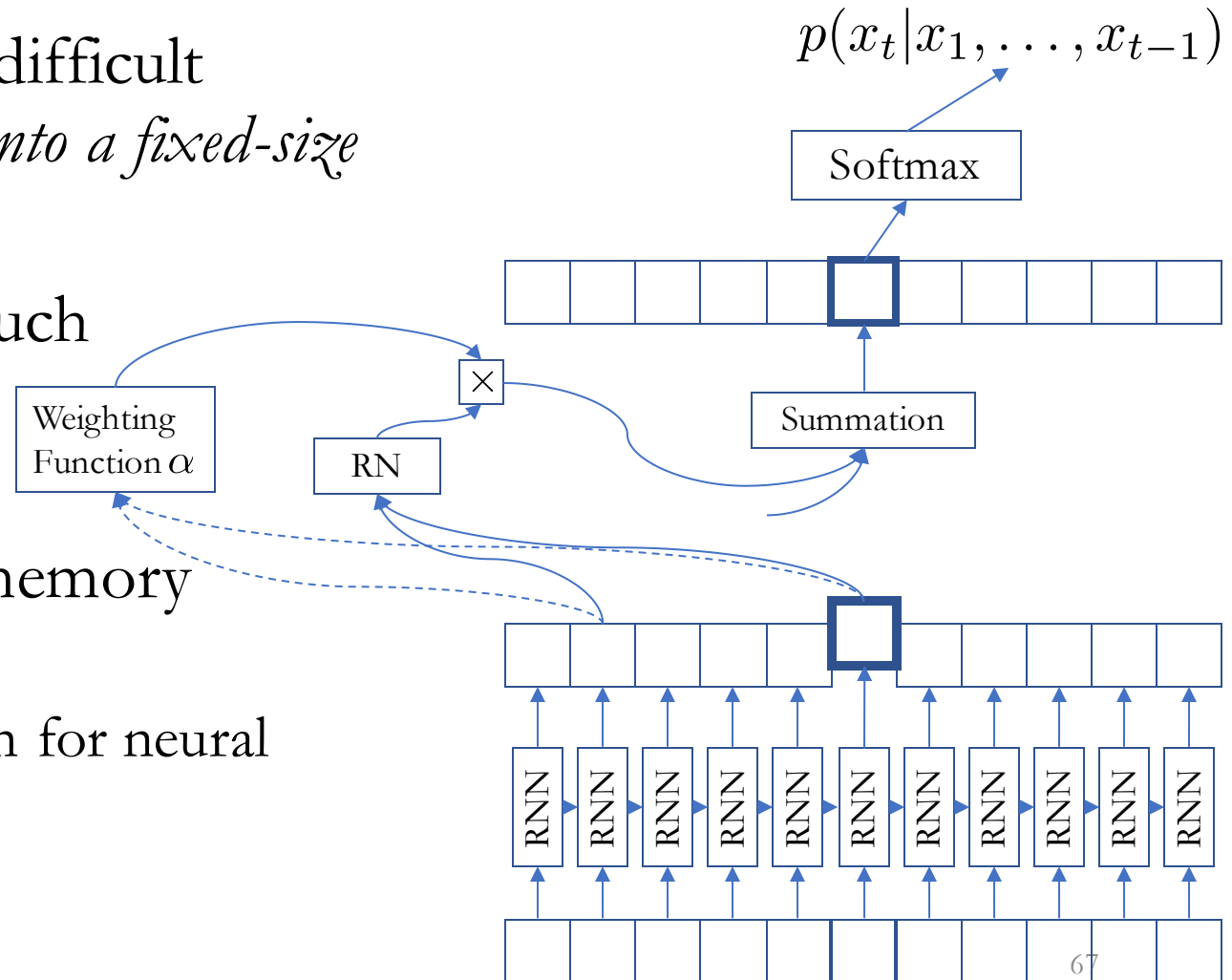
- A recurrent network summarizes all the tokens so far.
- Use the recurrent network's memory to predict the next token.
- Efficient online processing of a streaming text:
 - Constant time per step.
 - Constant memory throughout forward computation
- Useful in practice:
 - Useful for autocomplete and keyword suggestion.
 - Scoring partial hypotheses in generation.



Infinite context $n \rightarrow \infty$

– Recurrent Memory Networks [Tran et al., 2016]

- The **recurrent network** solves a difficult problem: *compress the entire context into a fixed-size memory vector*.
- **Self-attention** does not require such compression but still can capture long-term dependencies.
- Combine these two: a recurrent memory network (RMN) [Tran et al., 2016]
 - RNMT+: a similar, recent extension for neural machine translation



In this lecture, we learned

- What autoregressive language modelling is:

$$p(X) = p(x_1)p(x_2|x_1) \cdots p(x_T|x_1, \dots, x_{T-1})$$

- How autoregressive language modelling transforms unsupervised learning into a series of supervised learning:
 - It is a series of predicting the next token given previous tokens.
- How neural language modelling improves upon n-gram language models:
 - Continuous vector space facilitates generalization to unseen n-grams.
 - Infinitely large context window
- How sentence representation extraction is used for language modelling:
 - Convolutional language models, recurrent language models and self-attention language models..