Advanced Electromagnetics:
21st Century Electromagnetics

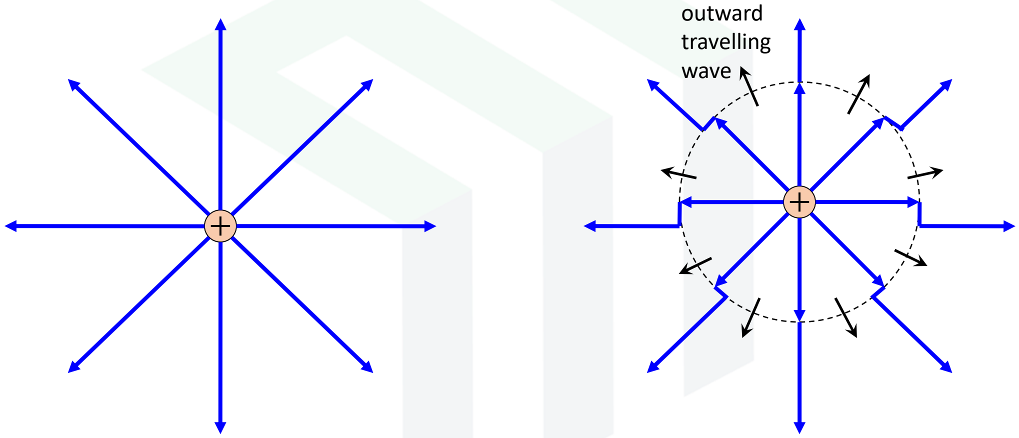
Lorentz Oscillator Model

Lecture Outline

- High level picture of dielectric response
- Qualitative description of resonance
- Derivation of Lorentz oscillator model

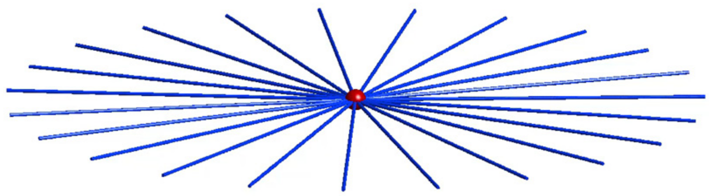
High Level Picture of Dielectric Response

Moving Charges Radiate Waves (1 of 2)



This is called the single-charge radiation model (Heaviside, 1894).

Moving Charges Radiate Waves (2 of 2)



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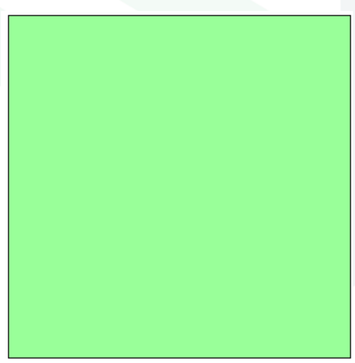
Dielectric Slab

It is desired to understand why a dielectric exhibits an electromagnetic response.



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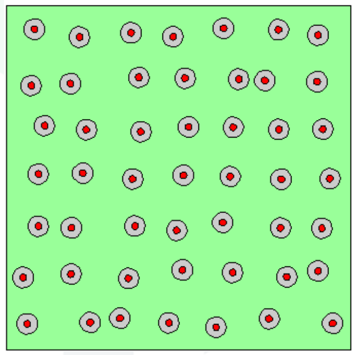
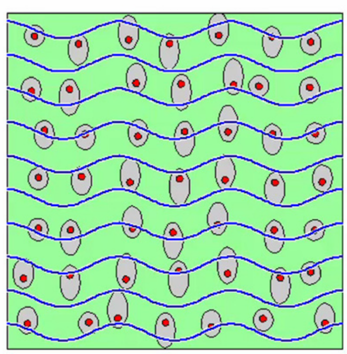
Atoms at Rest

Without an applied electric field $\vec{E}$, the electron "clouds" around the nuclei are symmetric and at rest.



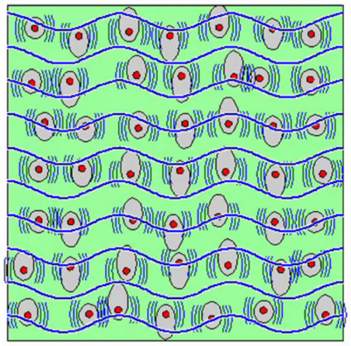
Applied Wave

The electric field $\vec{E}$ of a electromagnetic wave pushes the electrons away from the nuclei producing "clouds" that are offset.



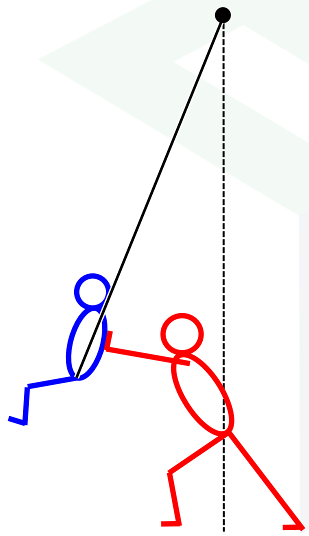
Secondary Waves

The motion of the charges emits secondary waves that interfere with the applied wave to produce an overall slowing effect on the wave.



Qualitative Description of Resonance

Visualizing Resonance – Low Frequency



- Driving force is able to modulate amplitude
- Displacement is in phase with driving force
- There exists a DC offset

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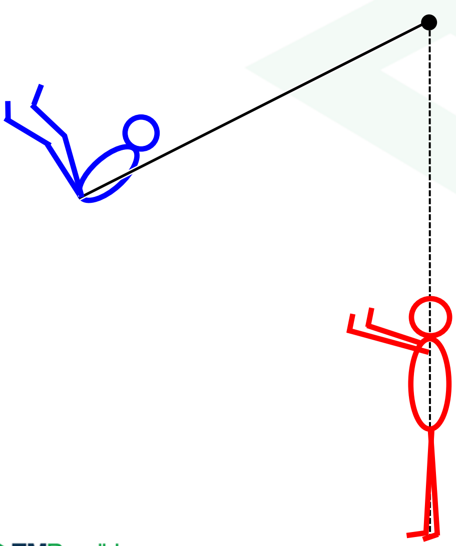
Visualizing Resonance – on Resonance



- Driving force can cause large displacements
- Displacement is 90° out of phase with the driving force (i.e. peaks of push correspond to nulls of displacement)

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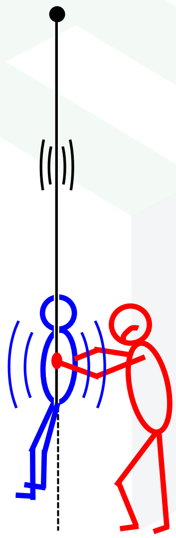
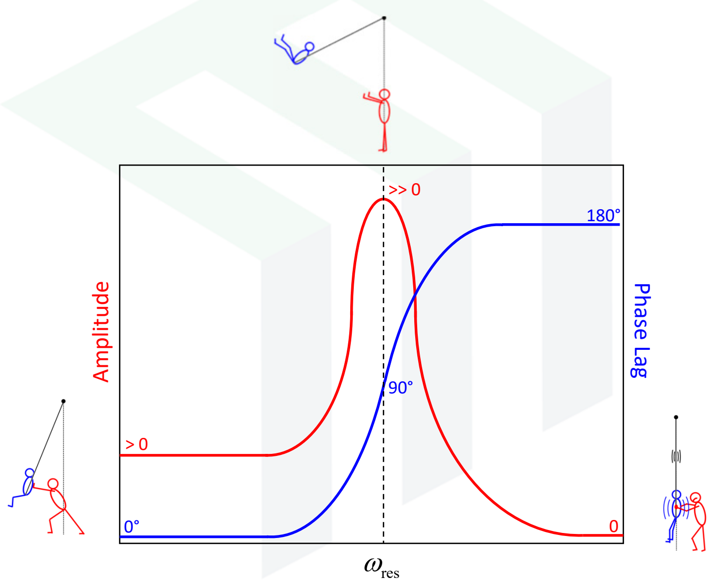
Visualizing Resonance – High Frequency

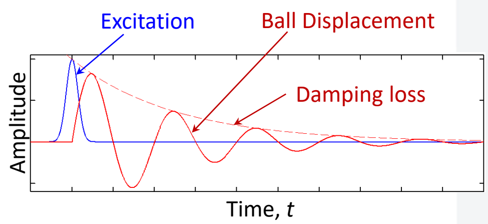


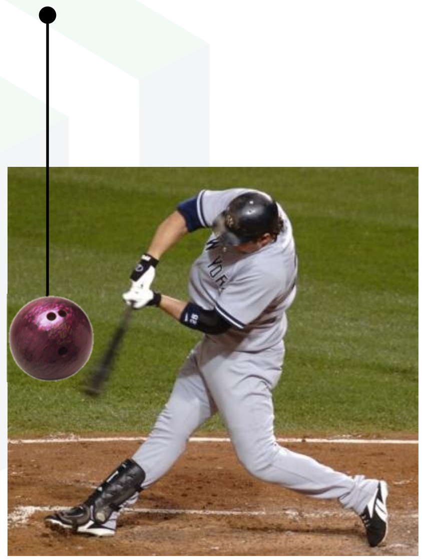
- Displacement has vanishing amplitude
- Displacement is 180° out of phase with driving force in order to perfectly oppose it.

Response of A Harmonic Oscillator



Impulse Response of a Harmonic Oscillator





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Derivation of Lorentz Oscillator Model

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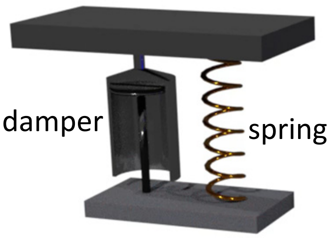
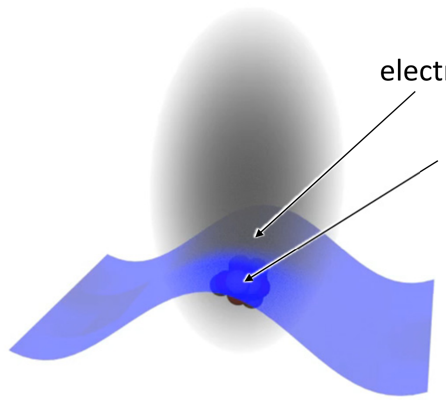
Lorentz Oscillator Model

Mass on a Spring



Atomic Model



Equation of Motion

$$m\frac{\partial^2 \vec{r}}{\partial t^2} + m\Gamma\frac{\partial \vec{r}}{\partial t} + m\omega_0^2 \vec{r} = -q\vec{E}$$

acceleration force

$m \Rightarrow m_e$ mass of an electron

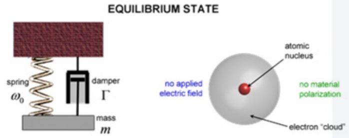
frictional force

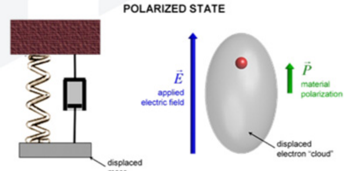
Γ damping rate (loss/sec)

restoring force

$\omega_0 = \sqrt{\frac{K}{m}}$ natural frequency

electric force





Fourier Transform the Equation of Motion

$$m\frac{\partial^2 \vec{r}}{\partial t^2} + m\Gamma\frac{\partial \vec{r}}{\partial t} + m\omega_0^2\vec{r} = -q\vec{E}$$

$$\Big\downarrow \text{Fourier transform}$$

$$m(-j\omega)^2\,\vec{r}(\omega) + m\Gamma(-j\omega)\vec{r}(\omega) + m\omega_0^2\vec{r}(\omega) = -q\vec{E}(\omega)$$

$$\Big\downarrow \text{Simplify}$$

$$\left(-m\omega^2 - j\omega m\Gamma + m\omega_0^2\right)\vec{r}(\omega) = -q\vec{E}(\omega)$$

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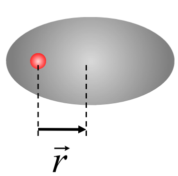
Charge Displacement $\vec{r}(\omega)$

$$\left(-m\omega^2 - j\omega m\Gamma + m\omega_0^2\right)\vec{r}(\omega) = -q\vec{E}(\omega)$$

$$\Big\downarrow \text{Solve for } \vec{r}(\omega)$$

$$\vec{r}(\omega) = -\frac{q}{m_e}\frac{\vec{E}(\omega)}{\omega_0^2 - \omega^2 - j\omega\Gamma}$$



The displacement $\vec{r}(\omega)$ describes how far charge is displaced from its equilibrium position.

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Electric Dipole Moment $\vec{\mu}(\omega)$

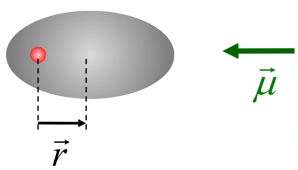
Definition of Electric Dipole Moment: $\vec{\mu}(\omega) = -q\vec{r}(\omega)$

charge

distance from center

** Sorry for the confusing notation, but μ here is NOT permeability.

$$\vec{\mu}(\omega) = \frac{q^2}{m_e} \frac{\vec{E}(\omega)}{\omega_0^2 - \omega^2 - j\omega\Gamma}$$



The electric dipole moment $\vec{\mu}(\omega)$ is a measure of the strength and separation of positive and negative charges.

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Lorentz Polarizability $\alpha(\omega)$

Definition of Lorentz Polarizability: $\vec{\mu}(\omega) = \left[\alpha(\omega)\right]\vec{E}(\omega)$

** Sorry for the confusing notation, but α here is NOT absorption.

$[\alpha(\omega)]$ is a tensor quantity for anisotropic materials.
For simplicity, the scalar form will be adopted here.
This is the Lorentz polarizability for a single atom.

$$\alpha(\omega) = \frac{q^2}{m_e} \frac{1}{\omega_0^2 - \omega^2 - j\omega\Gamma}$$

The Lorentz polarizability $[\alpha(\omega)]$ is a measure of how easily electrical charges are displaced. Charge may be more easily displaced in some directions that others.

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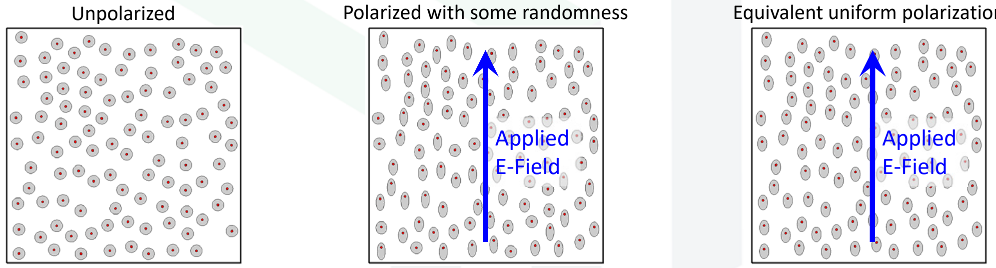
Polarization Per Unit Volume $\vec{P}(\omega)$

Definition: $\vec{P}(\omega)=\frac{1}{V}\sum_{V}\vec{\mu}_i(\omega)$

Average dipole moment over all atoms in a material. All billions and trillions of them!!!

Unpolarized | Polarized with some randomness | Equivalent uniform polarization



There is some randomness to the polarized atoms so a statistical approach is taken to compute the average.

$$\vec{P}(\omega)=N\langle\vec{\mu}(\omega)\rangle=\frac{Nq^2}{m_e}\frac{\vec{E}(\omega)}{\omega_0^2-\omega^2-j\omega\Gamma}$$

$N \equiv$ Number of atoms per unit volume $\quad \langle\ \rangle \equiv$ Statistical volume average

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Electric Susceptibility $\chi_e(\omega)$ (1 of 2)

A material becomes polarized $\vec{P}$ in the presence of an electric field $\vec{E}$ according to

$$\vec{P}(\omega)=\varepsilon_0\chi_e(\omega)\vec{E}(\omega)$$

$\chi_e(\omega)$ is called the *electric susceptibility* and is a measure of how easily an electric field $\vec{E}$ can polarize a material.

This leads to an expression for the electric susceptibility:

$$\chi_e(\omega)=\frac{N\alpha(\omega)}{\varepsilon_0}=\left(\frac{Nq^2}{\varepsilon_0 m_e}\right)\frac{1}{\omega_0^2-\omega^2-j\omega\Gamma}$$

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Electric Susceptibility $\chi_\mathrm{e}(\omega)$ (2 of 2)

The electric susceptibility of a dielectric material is:

$$\chi_\mathrm{e}(\omega) = \frac{\omega_\mathrm{p}^2}{\omega_0^2 - \omega^2 - j\omega\Gamma} \qquad \omega_\mathrm{p}^2 = \frac{Nq^2}{\varepsilon_0 m_\mathrm{e}} \quad \text{plasma frequency}$$

$$q = 1.60217646\times10^{-19}\ \mathrm{C}$$
$$\varepsilon_0 = 8.8541878176\times10^{-12}\ \mathrm{F/m}$$
$$m_\mathrm{e} = 9.10938188\times10^{-31}\ \mathrm{kg}$$

- Note this is the susceptibility of a dielectric which has only *one* resonance.
- The location of atoms is important because they can influence each other. This was ignored.
- Real materials have many sources of resonance and all of these must be added together.
- Electric susceptibility is the *transfer function* of the oscillator system.

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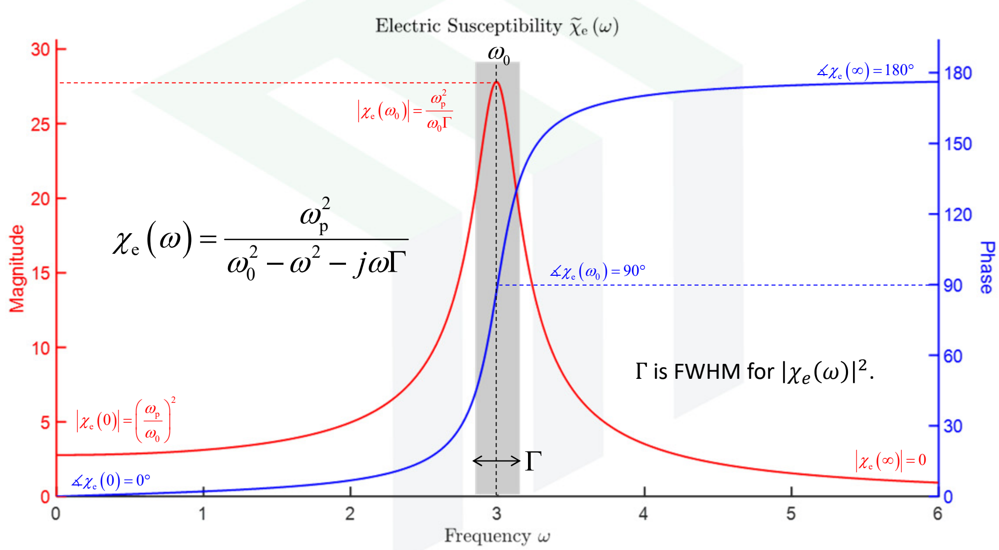
Plot of Electric Susceptibility $\chi_\mathrm{e}(\omega)$



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