Riemann Sum for

$f(x) = 3x + 2x^2$ over $[0,1]$

$\Delta x = \frac{1-0}{n} = \frac{1}{n}$

$c_k = a + k\Delta x = 0 + k\left(\frac{1}{n}\right) = \frac{k}{n}$

$f(c_k) = \frac{3k}{n} + \frac{2k^2}{n^2}$

$S_n = \sum_{k=1}^{n}\left[\left(\frac{3k}{n} + \frac{2k^2}{n^2}\right) \cdot \frac{1}{n}\right]$

$S_n = \sum_{k=1}^{n}\left(\frac{3k}{n^2} + \frac{2k^2}{n^3}\right)$

$S_n = \frac{3}{n^2}\sum_{k=1}^{n} k + \frac{2}{n^3}\sum_{k=1}^{n} k^2$

$S_n = \frac{3}{n^2}\left(\frac{n(n+1)}{2}\right) + \frac{2}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)$

$S_n = \frac{3(n+1)}{2n} + \frac{(n+1)(2n+1)}{3n^2}$

$\text{Area} = \lim_{n\to\infty}\left(\frac{3(n+1)}{2n} + \frac{(n+1)(2n+1)}{3n^2}\right)$

$\lim_{n\to\infty} = \left(\frac{3}{2} + \frac{3}{n^2} + \frac{2}{3} + \frac{1}{n} + \frac{1}{3n^2}\right)$

$\lim_{n\to\infty} = \left(\frac{3}{2} + 0 + \frac{2}{3} + 0 + 0\right) \rightarrow \lim_{n\to\infty} = \boxed{\frac{13}{6}}$

for $f(x)=x^2-x^3$ on $[-1,0]$

$\Delta x = \frac{1}{n}$

$c_k = -1+\frac{k}{n}$

$f(c_k) = \left(-1+\frac{k}{n}\right)^2 - \left(-1+\frac{k}{n}\right)^3$

$$f(c_k) = 2 - \frac{5k}{n} + \frac{4k^2}{n^2} - \frac{k^3}{n^3}$$

$$S_n = \sum_{k=1}^{n}\left(\left(2-\frac{5k}{n}+\frac{4k^2}{n^2}-\frac{k^3}{n^3}\right)\cdot\frac{1}{n}\right)$$

$$S_n = \sum_{k=1}^{n}\left(\frac{2}{n}-\frac{5k}{n^2}+\frac{4k^2}{n^3}-\frac{k^3}{n^4}\right)$$

$$S_n = \frac{2}{n}\sum_{k=1}^{n}1-\frac{5}{n^2}\sum_{k=1}^{n}k+\frac{4}{n^3}\sum_{k=1}^{n}k^2-\frac{1}{n^4}\sum_{k=1}^{n}k^3$$

$$S_n = \frac{2}{n}n-\frac{5}{n^2}\left(\frac{n(n+1)}{2}\right)+\frac{4}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)-\frac{1}{n^4}\left(\frac{n^2(n+1)^2}{4}\right)$$

$$\lim_{n\to\infty} S_n = \left(2-\frac{5(n+1)}{2n}+\frac{2(n+1)}{2n}+\frac{2(n+1)(2n+1)}{3n^2}-\frac{n^2}{4n^2}\right)$$

$\text{Area} = 2-\frac{5}{2}+\frac{4}{3}-\frac{1}{4}$

$\text{Area} = \frac{7}{12}$

for $\int_1^e \ln(x)dx$

$\Delta x = \frac{e-1}{n}$

$c_k = 1 + k\left(\frac{e-1}{n}\right)$

$f(c_k) = \ln\left(1 + k\frac{e-1}{n}\right)$

$S_n = \sum_{k=1}^{n}\left(\ln\left(1+k\frac{e-1}{n}\right)\left(\frac{e-1}{n}\right)\right)$

$$\boxed{S_n = \frac{e-1}{n}\sum_{k=1}^{n}\ln\left(1+\frac{k(e-1)}{n}\right)}$$

Definite Integral for $\lim_{\Delta x \to 0} \sum f(x)dx$

$$\lim_{\Delta x \to 0}\sum_{k=1}^{n} f(c_k)\Delta x = \int_a^b f(x)dx$$

Area $= \lim_{n\to\infty}\sum_{k=1}^{n} f(c_k)\Delta x$

$f(x) = x^2 - x^3 \quad [a,b] = [-1,0]$

$\Delta x = \frac{b-a}{n} = \frac{1}{n}$

$c_k = a + k\Delta x = -1 + k\frac{1}{n} = \frac{k}{n} - 1$

$f(c_k) = (c_k)^2 - (c_k)^3 \rightarrow f(c_k) = \left(\frac{k}{n}-1\right)^2 - \left(\frac{k}{n}-1\right)^3$

$f(c_k) = \left[\frac{k^2}{n^2} - \frac{2k}{n} + 1\right] - \left[\frac{k^3}{n^3} - \frac{3k^2}{n^2} + \frac{3k}{n} - 1\right]$

$$f(c_k) = -\frac{1}{n^3}k^3 + \frac{4}{n^2}k^2 - \frac{5}{n}k + 2$$

$$\sum_{k=1}^{n}\left(-\frac{1}{n^3}k^3 + \frac{4}{n^2}k^2 - \frac{5}{n}k + 2\right)\cdot\frac{1}{n}$$

$$\sum_{k=1}^{n}\left(-\frac{1}{n^4}k^3 + \frac{4}{n^3}k^2 - \frac{5}{n^2}k + \frac{2}{n}\right)$$

$$-\frac{1}{n^4}\sum_{k=1}^{n}k^3 + \frac{4}{n^3}\sum_{k=1}^{n}k^2 - \frac{5}{n^2}\sum_{k=1}^{n}k + \sum_{k=1}^{n}\frac{2}{n}$$

$$-\frac{1}{n^4}\left[\frac{n^2(n+1)^2}{4}\right] + \frac{4}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right] - \frac{5}{n^2}\left[\frac{n(n+1)}{2}\right] + \frac{2}{n}\cdot n$$

$$-\frac{1}{4} \quad + \quad \frac{4}{3} \quad -\frac{5}{2} \quad 2$$

$$\text{Area} = -\frac{1}{4} + \frac{4}{3} - \frac{5}{2} + 2 = \boxed{\frac{7}{12}}$$