Computability

Assignment 4. Kacper Korban

1. A function $f \in \mathbb{N} \to \mathbb{N}$ is χ-computable if

$$\exists e \in CExp.\ \forall a \in \mathbb{N}.\ e\ \ulcorner a \urcorner \Downarrow \ulcorner fa \urcorner$$

i.e. there exists a closed expression e, such that for every natural number a, given $\ulcorner a \urcorner$ as ~~input~~ input evaluates to $\ulcorner fa \urcorner$.

2. If f was χ-computable, there would have to be an $f \in CExp$, such that $\forall a \in CExp.\ f\ \ulcorner a \urcorner \Downarrow \ulcorner fa \urcorner$ then we can reduce "intensional" halting problem to f.

$$\underline{halts} = \lambda p.\ f\ \ulcorner (\lambda\_.\ Zero())\ p \urcorner$$

Then:

$$[\![halts\ \ulcorner e \urcorner]\!] = [\![f\ \ulcorner (\lambda\_.\ Zero())\ \ulcorner e \urcorner \urcorner]\!] = [\![f\ \ulcorner (\lambda\_.\ Zero())\ e \urcorner]\!] =$$

$$= \ulcorner f((\lambda\_.\ Zero())\ e) \urcorner = \ulcorner \text{if } ((\lambda\_.\ Zero())\ e) \text{ terminates with } Zero() \text{ then true else false} \urcorner =$$

$$= \ulcorner \text{if } e \text{ terminates then true else false} \urcorner = \ulcorner halts\ e \urcorner$$