Computability

Assignment 6. Kacper Korban

1.

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$S = \{s_0, s_1\}$ $\Sigma = \{c_1, c_2\}$ $\delta(s_0, c_1) = (s_1, c_1, R)$ $\delta(s_1, c_1) = (s_0, \sqcup, R)$

$s_i = s_0$ $\Gamma = \{c_1, c_2, \sqcup\}$ $\delta(s_0, c_2) = (s_1, c_2, R)$ $\delta(s_1, c_2) = (s_0, \sqcup, R)$

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$\cancel{x s = [c_1, c_1, c_2, c_1, c_2}$ $x s = [c_1, c_1, c_2, c_1, c_2]$

$\text{Result} = [c_1, \sqcup, c_2, \sqcup, c_2]$

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3. Goal:

$$\forall_{f\in\mathbb{N}\to\mathbb{N}}.\left(\exists_{tm\in TM}.\forall_{n\in\mathbb{N}}.\ [\![tm]\!]_{TM}\ \ulcorner n\urcorner_{TM} = \ulcorner f\,n\urcorner_{TM}\right.$$

$$\left.\Rightarrow \exists_{e\in CExp}.\forall_{n\in\mathbb{N}}.\ [\![e\ \ulcorner n\urcorner_{\chi}]\!]_{\chi} = \ulcorner f\,n\urcorner_{\chi}\right)$$

From previous exercise we know that:
(Solution in ex. 2. takes curried arguments instead of a Pair)

$\exists_{evalTM\in CExp}.\ \forall_{tm\in TM}.\ \forall_{xs\in List\,\Sigma_{tm}}$

$$[\![evalTM\ \ulcorner(tm, xs)\urcorner_{\chi}]\!]_{\chi} = \ulcorner [\![tm]\!]_{TM}\ xs\urcorner_{\chi}$$

~~[[evalTM~~

So, for a given function f and Turing machine tm ($f\in\mathbb{N}\to\mathbb{N}$, $tm\in TM$) that implements it, we can construct a closed χ-expression that implements f.

$$\underline{f} = \lambda n.\ decode\,(evalTM\ \ Pair\,(\ulcorner tm\urcorner_{\chi}, encode\,n))$$

where

$$\forall_{n\in\mathbb{N}}.\ [\![encode\ \ulcorner n\urcorner_{\chi}]\!]_{\chi} = \ulcorner\ulcorner n\urcorner_{TM}\urcorner_{\chi}\ \wedge$$

$$\forall_{n\in\mathbb{N}}\ [\![decode\ \ulcorner\ulcorner n\urcorner_{TM}\urcorner_{\chi}]\!]_{\chi} = \ulcorner n\urcorner_{\chi}$$

Proof: $[\![\underline{f}\ \ulcorner n\urcorner_{\chi}]\!]_{\chi} = [\![decode\,(evalTM\ \ Pair\,(\ulcorner tm\urcorner_{\chi}, encode\ulcorner n\urcorner_{\chi}))]\!]_{\chi} =$

$= [\![decode\,(evalTM\ Pair\,(\ulcorner tm\urcorner_{\chi}, \ulcorner\ulcorner n\urcorner_{TM}\urcorner_{\chi}))]\!]_{\chi} =$

$= [\![decode\,(evalTM\ \ulcorner(tm, \ulcorner n\urcorner_{TM})\urcorner_{\chi})]\!]_{\chi} = [\![decode\ \ulcorner [\![tm]\!]_{TM}\ \ulcorner n\urcorner_{TM}\urcorner_{\chi}]\!]_{\chi} =$

$= [\![decode\ \ulcorner\ulcorner f\,n\urcorner_{TM}\urcorner_{\chi}]\!]_{\chi} = \ulcorner f\,n\urcorner_{\chi}$ $\square$