Partial Regularization of First-Order Resolution Proofs

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Introduction

- As the ability of automated deduction has improved, it has been applied to new application domains; e.g. Furbach et al. [2] used it in natural language reasoning.
- Resolution proof production is a key feature of modern theorem provers; the best, most-efficient provers do not necessarily generate the best, least redundant proofs.
- For proofs using propositional resolution generated by SAT- and SMT-solvers, there are many proof compression techniques.
- One approach to compressing first-order logic proofs is to lift ideas used in propositional logic.

Propositional RecyclePivotsWithIntersection [1]

- Traverse a proof from the bottom up: store for every node a set of *safe literals*: literals resolved in all paths below the node
- For any node whose resolved literals are safe, replace it with one of its parents (*regularizing* it.)

The set of safe literals for a node η will be denoted $\mathcal{S}(\eta)$.

A Propositional Example

Consider the proof ψ shown below:

$$\dfrac{\dfrac{\eta_1 \quad \dfrac{\eta_2 : a, c, \neg b \quad \dfrac{\eta_1 : \neg a \quad \eta_3 : a, b}{\eta_4 : b}\,a}{\eta_5 : a, c}\,b}{\eta_6 : c}\,a \qquad \dfrac{\dfrac{\eta_4 \quad \eta_7 : a, \neg b, \neg c}{\eta_8 : a, \neg c}\,b \quad \eta_1}{\eta_9 : \neg c}\,a}{\psi : \bot}\,c$$

The algorithm `RPI` assigns $\mathcal{S}(\eta_5) \leftarrow \{a, c\}$, $\mathcal{S}(\eta_8) \leftarrow \{a, \neg c\}$, and $\mathcal{S}(\eta_4) \leftarrow \{a, c, b\} \cap \{a, \neg c, b\} = \{a, b\}$. Since $a \in \mathcal{S}(\eta_4)$ where a is a pivot of η_4, η_4 is detected as a redundant node and regularalized by replacing it by its right parent η_3:

$$\dfrac{\dfrac{\eta_1 : \neg a \quad \dfrac{\eta_2 : a, c, \neg b \quad \eta_3 : a, b}{\eta_5 : a, c}}{\eta_6 : c} \qquad \dfrac{\dfrac{\eta_3 \quad \eta_7 : a, \neg c, \neg b}{\eta_8 : a, \neg c} \quad \eta_1}{\eta_9 : \neg c}}{\psi : \bot}$$

A First-Order Example

Consider the proof ψ below. When computed as in the propositional case, $\mathcal{S}(\eta_3) \leftarrow \{\vdash q(c),\ p(a, X)\}$

$$\dfrac{\dfrac{\dfrac{\eta_1\colon \vdash p(W,X) \quad \eta_2\colon p(W,X) \vdash q(c)}{\eta_3\colon \vdash q(c)} \quad \eta_4\colon q(c) \vdash p(a,X)}{\eta_5\colon \vdash p(a,X)} \quad \eta_6\colon p(Y,b) \vdash}{\psi\colon \bot}$$

Since $p(W, X) \neq p(a, X)$, propositional `RPI` algorithm would not change ψ. However, η_3's left pivot $p(W, X) \in \eta_1$ is unifiable with the safe literal $p(a, X)$. Regularizing η_3, by deleting the edge between η_2 and η_3 and replacing η_3 by η_1, leads to further deletion of η_4 (because it is not resolvable with η_1) and finally to the following proof:

$$\dfrac{\eta_1\colon \vdash p(W,X) \quad \eta_6\colon p(Y,b) \vdash}{\psi'\colon \bot}$$

Unifiability is Not Enough

Consider ψ below. When computed as in the propositional case, $\mathcal{S}(\eta_3) \leftarrow \{\vdash q(c),\ p(a, X)\}$, and as the pivot $p(a, c)$ is unifiable with the safe literal $p(a, X)$, η_3 appears to be a candidate for regularization.

$$\dfrac{\dfrac{\dfrac{\eta_1\colon \vdash p(a,c) \quad \eta_2\colon p(a,c) \vdash q(c)}{\eta_3\colon \vdash q(c)} \quad \eta_4\colon q(c) \vdash p(a,X)}{\eta_5\colon \vdash p(a,X)} \quad \eta_6\colon p(Y,b) \vdash}{\psi\colon \bot}$$

However, if we attempt to regularize the proof, the same series of actions as in the last example would require resolution between η_1 and η_6, which is not possible.

Pre-Regularization Unifiability

Let η be a node with pivot ℓ' unifiable with safe literal ℓ which is resolved against literals $\ell_1, \ldots, \ell_n$ in a proof ψ. η is said to satisfy the *pre-regularization unifiability property* in ψ if $\ell_1, \ldots, \ell_n$, and $\overline{\ell'}$ are unifiable.

Pre-Regularization Unifiability: Still Not Enough

Consider the proof ψ below. After collecting the safe literals, $\mathcal{S}(\eta_3) \leftarrow \{q(T, V), p(c, d) \vdash q(f(a, e), c)\}$.

$$\dfrac{\eta_8\colon q(f(a,e),c) \vdash \quad \dfrac{\eta_6\colon \vdash p(c,d) \quad \dfrac{\dfrac{\eta_1\colon p(U,V) \vdash q(f(a,V),U) \quad \eta_2\colon q(f(a,X),Y), q(T,X) \vdash q(f(a,Z),Y)}{\eta_3\colon p(U,V), Q(T,V) \vdash q(f(a,Z),U)} \quad \eta_4\colon \vdash q(R,S)}{\eta_5\colon p(U,V) \vdash q(f(a,Z),U)}}{\eta_7\colon \vdash q(f(a,Z),c)}}{\psi\colon \bot}$$

η_3's pivot $q(f(a, V), U)$ is unifiable to (and even more general than) the safe literal $q(f(a, e), c)$. Attempting to regularize η_3 would lead to the removal of η_2, the replacement of η_3 by η_1 and the removal of η_4 (because η_1 does not contain the pivot required by η_5), with η_5 also being replaced by η_1. Then resolution between η_1 and η_6 results in η_7', which cannot be resolved with η_8, as shown below.

$$\dfrac{\eta_8\colon Q(f(a,e),c) \vdash \quad \dfrac{\eta_6\colon \vdash P(c,d) \quad \eta_1\colon P(U,V) \vdash Q(f(a,V),U)}{\eta_7'\colon \vdash Q(f(a,d),c)}}{\psi'\colon ??}$$

η_1's literal $q(f(a, V), U)$, which would be resolved with η_8's literal, was changed to $Q(f(a, d), c)$ due to resolution between η_1 and η_6.

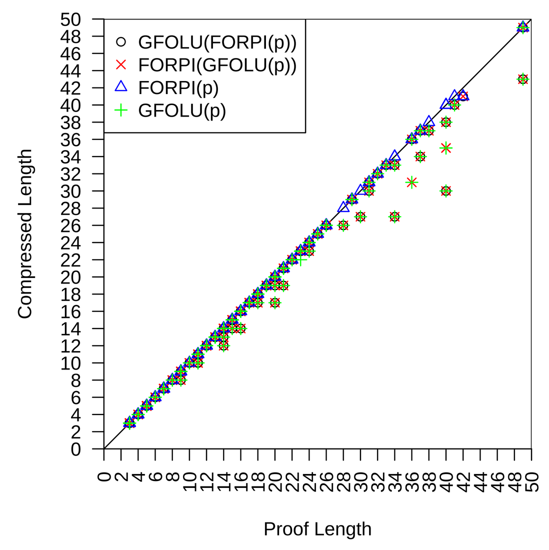
Regularization Unifiability

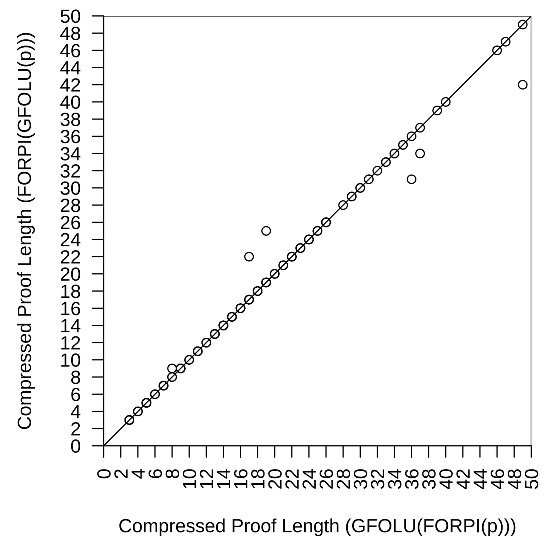
Let η be a node with safe literals $\mathcal{S}(\eta) = \phi$ that is marked for regularization with parents η_1 and η_2, where η_2 is marked as a `deletedNode` in a proof ψ. η is said to satisfy the *regularization unifiability property* in ψ if there exists a substitution σ such that $\eta_1\sigma \subseteq \phi$.

The First-Order Algorithm

- Similar idea to the propositional case, but with care taken to ensure proofs satisfy the last two properties.
- First order *factoring* also employed to reduce proof size further, e.g. if $\eta_1 : p(X), p(Y) \vdash$, factor to $\eta_1' : p(X) \vdash$ before performing resolution.
- Intersection of safe literals must also employ unification.
- Does not compress all first-order proofs (yet).

Preliminary Results





- First-order proofs generated by SPASS show some compression by Scala based implementation.
- Data set is small: only 308 proofs, all of which are short
- Evaluation with `GFOLU` [3], a first-order variant of `LowerUnits`, another propositional compression algorithm lifted to first-order logic.
  - Recycle pivots compresses 10x more than lower units in the propositional case
- Algorithm composition may matter less in the first-order case when compared to the propositional case.
- Quick compression: 40 minutes to generate all proofs, 8 seconds to compress all proofs.

Future Directions

- Larger evaluation - more proofs, bigger proofs
- Identify properties that would enable all irregular first-order proofs to be compressed
- Is it possible to lift other propositional proof compression techniques to first-order logic?

References

[1] P. Fontaine, S. Merz, and B. Woltzenlogel Paleo. Compression of propositional resolution proofs via partial regularization. In *CADE-23*. Springer, 2011.

[2] Ulrich Furbach, Björn Pelzer, and Claudia Schon. Automated reasoning in the wild. In *CADE-25*. Springer, 2015.

[3] J. Gorzny and B. Woltzenlogel Paleo. Towards the compression of first-order resolution proofs by lowering unit clauses. In *CADE-25*. Springer, 2015.