

SLAJD 1

Hello everybody, today we will talk about random walks and how they correspond with diff model. We had prepared a short presentation that will hopefully bring you closer to the topic.

SLAJD 2

First we will talk about random walks, what are they, what are real life examples of them and some math behind it. Next we will tell how random walks are applicable in diffusion and similarity between them. There will be graphs and pictures too, so don't worry about walls of text.

In the end we will show u how we can display it and program it all in python

SLAJD 3

1. What is a random walk?

A random walk is the process by which randomly-moving objects wander away from where they started. An elementary example of a random walk is the random walk on the integer number line \mathbb{Z} which starts at 0, and at each step moves $+1$ or -1 with equal probability

2. Example of the random walk in real life?

Random walks and the mathematics that govern them are found everywhere in nature. When gas particles bounce around in a room, changing direction every time they collide with another particle ([wskazanie na obrazek z cząsteczkami](#)), it is random walk mathematics that determines how long it will take them to travel from one location to another.

3. Math behind the random walk?

The simplest random walk to understand is a 1-dimensional walk. Suppose that the black dot below is sitting on a number line. ([wskazanie na obrazek z osią](#)) The black dot starts in the center. Then, it takes a step, either forward or backward, with equal probability. It keeps taking steps either forward or backward each time. The picture shows a black dot that has taken 5 steps and ended up at -1 on the number line.

4. Variable step size of the random walk?

([wskazanie na gif z slajdu nr 2](#)) Now that we know a little more about random walks, let's take another look at the video we saw before. The video shows the paths of 7 black dots undergoing a random walk in 2D. All of the dots start out in the same place and then start to wander around. Notice that the steps they take are not all the same length. By the end of the video, some of the dots ended up above where they started, some below, some to the right, and some to the left.

5. Not all random walks are "random".

So far all of the random walks we have considered allowed an object to move with equal probability in any direction. But not all random walks follow this rule. A *biased random walk* is a random walk that is biased in one direction, leading to a net drift on average of particles in one specific direction.

One of the ways that a random walk can be biased is an equal probability of moving left to right. In other words, the random walk is *biased* toward the right. The video below shows a biased random walk in 2D. ([wskazanie na gif na slajdzie nr 4](#))

6. Random walks in more than one dimension

Of course the 1-dimensional random walk is easy to understand, but not as commonly found in nature as the 2D and 3D random walk, in which an object is free to move along a 2D plane or a 3D space instead of a 1D line. We won't work out the math here, but it turns out to be governed by the same rule as a 1D random walk -- the total distance traveled from where it started is approximately \sqrt{N} where N is the number of steps taken. ([wskazanie na obrazek "Random walk" na slajdzie nr 3](#))

SLAJD 4

In the top left we have an example of a random walk in 1 dimensional plane, under him in 2 dimensions, and to the right in 3 dimensions. Important detail is that there is only one "walker" here. On the right side we can see the Biased walk. It's

biased toward the right side as you can see. We have chosen to show the second type of biased walk, one that has equal probability of choosing either direction, but has a larger step size if going right.

SLAJD 5

How can we relate random walks to the number of atoms/particles in a given region in space? Top left pic shows random walkers on a coarse grained grid on the left. There is initially a number $N_{i,j}$ walkers in the center grid box, but after a few time steps, corresponding to some steps of the random walkers, some of the walkers have walked from $N_{i,j}$ and to its surrounding boxes, and some walkers have walked from the surrounding boxes and into $N_{i,j}$.

(2 approaches) We may consider an ensemble of many particles that diffuse individually, and then count how many particles have ended up in a particular box/position after a given time, or we can describe the number of particles per box, and then calculate how this number changes with time.

(comparing) We can map the random walk model directly onto the diffusion equation. We know that the probability for a particle to move a distance Δx , which is the box size, over a time Δt is $p = R\Delta t$. This allows us to solve the random walk problem or the diffusion problem and compare the results directly. We compare the two descriptions directly by describing the diffusion processes by the number of walkers, N_i , in each box. We start the system by all M walkers starting from $x = 0$. Each time step, Δt , each walker has a probability p to move to the box to its left, and p to move to the box to its right. (It means that it has the probability $1 - 2p$ not to move at all). Notice that we here describe this as a single step for the random walker, but we may think of this process as a result of many small steps that lead to the walker moving out of the box (with probability p) or remaining in the box (with probability $1 - 2p$).

We have diff equation on the bottom of the slide

Solving the diffusion equation is efficient. Solving the diffusion equation is an efficient way to model the changes in concentration — more efficient than using random walkers. We can solve it using the explicit scheme. This is not the most robust or efficient method to solve the diffusion equation, but it is simple and easy to understand.

SLAJD 6

On the left side of the slide we have examples of 2d and 3d random walks with multiple different colored walkers. On the right side we have Illustration of the distribution $N(x)$ of walkers for the diffusion model (red) and the random walker model (blue).

SLAJD 7

Python code for comparing distribution of walkers for the diff model and random walkers in 1d
Which result we saw in the former slide

SLAJD 8

This is a link to our code on github, u can scan qr code for easier access.

SLAJD 9

and these are our sources, all accessed in the last week.

SLAJD 10

Thanks for listening to us, and if you have any questions feel free to ask them.