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Zadanie 1a

$$\begin{cases} x_2 + x_3 = -1 \\ \frac{1}{2}x_1 + x_3 = 0 \\ 2x_1 + x_2 + 5x_3 = -1 \end{cases}$$

w ciele $\mathbb{Q}$

$$\left[\begin{array}{ccc|c} 0 & 1 & 1 & -1 \\ \frac{1}{2} & 0 & 1 & 0 \\ 2 & 1 & 5 & -1 \end{array}\right] \underset{w_2 = w_1}{\overset{w_1 = w_2}{=\!=}} \left[\begin{array}{ccc|c} \frac{1}{2} & 0 & 1 & 0 \\ 0 & 1 & 1 & -1 \\ 2 & 1 & 5 & -1 \end{array}\right] \overset{w_3 = w_3 - 4w_1}{=\!=}$$

$$\left[\begin{array}{ccc|c} \frac{1}{2} & 0 & 1 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 1 & 1 & -1 \end{array}\right] \overset{w_3 = w_3 - w_2}{=\!=} \left[\begin{array}{ccc|c} \frac{1}{2} & 0 & 1 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{cases} \frac{1}{2}x_1 + x_3 = 0 \\ x_2 + x_3 = -1 \end{cases}$$

$x_2 = -1 - x_3$

$x_1 = -2x_3$

$$\begin{cases} x_1 = -2x_3 \\ x_2 = -1 - x_3 \\ x_3 = x_3 \end{cases}$$

$$X = \begin{bmatrix} -2x_3 \\ -1 - x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Spr.

$$\begin{cases} -1 - x_3 + x_3 = -1 \\ \frac{1}{2}(-2x_3) + x_3 = 0 \\ 2(-2x_3) + (-1 - x_3) + 5x_3 = -1 \end{cases}$$

$$\begin{cases} -1 = -1 \\ 0 = 0 \\ -1 = -1 \end{cases}$$

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Zadanie 2 $\mathbb{R}^4$ $u = \{ |-4,2,0,0|, |0,3,0,1|, |4,0,1,0|, |4,1,1,2| \}$

$$\det A = \begin{vmatrix} -4 & 2 & 0 & 0 \\ 0 & 3 & 0 & 1 \\ 4 & 0 & 1 & 0 \\ 4 & 1 & 1 & 2 \end{vmatrix} \underset{w_4 = w_4 + w_1}{\overset{w_3 = w_3 + w_1}{=}} \begin{vmatrix} -4 & 2 & 0 & 0 \\ 0 & 3 & 0 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 3 & 1 & 2 \end{vmatrix} \underset{w_4 = w_4 - w_2}{\overset{w_3 = w_3 - \frac{2}{3} w_2}{=}}$$

$$\begin{vmatrix} -4 & 2 & 0 & 0 \\ 0 & 3 & 0 & 1 \\ 0 & 0 & 1 & -\frac{2}{3} \\ 0 & 0 & 1 & 1 \end{vmatrix} \overset{w_4 = w_4 - w_3}{=} \begin{vmatrix} -4 & 2 & 0 & 0 \\ 0 & 3 & 0 & 1 \\ 0 & 0 & 1 & -\frac{2}{3} \\ 0 & 0 & 0 & \frac{5}{3} \end{vmatrix} = -4 \cdot 3 \cdot 1 \cdot \frac{5}{3} = -20$$

$-20 \neq 0$

Układ jest liniowo niezależny, więc w tym wypadku stanowi bazę $\mathbb{R}^4$

3.

$$X = \begin{vmatrix} -1 & 0 & 0 \\ 1 & 2 & 4 \\ 0 & 2 & 3 \end{vmatrix} \begin{matrix} -1 & 0 \\ 1 & 2 \\ 0 & 2 \end{matrix}$$

$$X^{-1} = \frac{1}{|X|} \cdot C^{T} = \frac{1}{|X|} \begin{vmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{vmatrix}$$

$\det X = |X| = -1 \cdot (2) \cdot 3 + 0 \cdot 4 \cdot 0 + 0 \cdot 1 \cdot 2 - 0 \cdot 2 \cdot 0 - 2 \cdot 4 \cdot (-1) - 3 \cdot 1 \cdot 0 =$
$= 2$

$C_{11} = (-1)^{1+1} \det \begin{vmatrix} 2 & 4 \\ 2 & 3 \end{vmatrix} = 1 \cdot (2 \cdot 3 - 4 \cdot 2) = -2$

$C_{12} = (-1)^{1+2} \det \begin{vmatrix} 1 & 4 \\ 0 & 3 \end{vmatrix} = -1 \cdot 3 = -3$

$C_{13} = (-1)^{1+3} \det \begin{vmatrix} 1 & 2 \\ 0 & 2 \end{vmatrix} = 1 \cdot 2 = 2$

$C_{21} = (-1)^{2+1} \det \begin{vmatrix} 0 & 0 \\ 2 & 3 \end{vmatrix} = 0$

$C_{22} = (-1)^{2+2} \det \begin{vmatrix} -1 & 0 \\ 0 & 3 \end{vmatrix} = 1 \cdot (-3) = -3$

$C_{23} = (-1)^{2+3} \det \begin{vmatrix} -1 & 0 \\ 0 & 2 \end{vmatrix} = -1 \cdot (-2) = 2$

$C_{31} = (-1)^{3+1} \det \begin{vmatrix} 0 & 0 \\ 2 & 4 \end{vmatrix} = 1 \cdot 0 = 0$

$C_{32} = (-1)^{3+2} \det \begin{vmatrix} -1 & 0 \\ 1 & 4 \end{vmatrix} = -1 \cdot (-4) = 4$

$C_{33} = (-1)^{3+3} \det \begin{vmatrix} -1 & 0 \\ 1 & 2 \end{vmatrix} = -2$

$$X^{-1} = \frac{1}{2} \cdot \begin{vmatrix} -2 & 0 & 0 \\ -3 & -3 & 4 \\ 2 & 2 & -2 \end{vmatrix} = \begin{vmatrix} -1 & 0 & 0 \\ -\frac{3}{2} & -\frac{3}{2} & 2 \\ 1 & 1 & -1 \end{vmatrix}$$

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4.

$$B = \left(\begin{vmatrix} 1 \\ 1 \\ 1 \end{vmatrix}, \begin{vmatrix} 0 \\ 1 \\ 1 \end{vmatrix}, \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix} \right), \quad C = \left(\begin{vmatrix} 1 \\ 1 \end{vmatrix}, \begin{vmatrix} 2 \\ 1 \end{vmatrix} \right)$$

$\varphi : R^3 \to R^2$

$$\begin{vmatrix} 0 & -4 & 0 \\ 3 & 3 & -2 \end{vmatrix} \qquad \varphi \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = ?$$

$$\varphi \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{vmatrix} 1 \\ 1 \end{vmatrix} \cdot 0 + \begin{vmatrix} 2 \\ 1 \end{vmatrix} \cdot 3 = \begin{vmatrix} 6 \\ 3 \end{vmatrix}$$

$$\varphi \begin{vmatrix} 0 \\ 1 \\ 1 \end{vmatrix} = \begin{vmatrix} 1 \\ 1 \end{vmatrix} \cdot -4 + \begin{vmatrix} 2 \\ 1 \end{vmatrix} \cdot 3 = \begin{vmatrix} 2 \\ -1 \end{vmatrix}$$

$$\varphi \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix} = \begin{vmatrix} 1 \\ 1 \end{vmatrix} \cdot 0 + \begin{vmatrix} 2 \\ 1 \end{vmatrix} \cdot -2 = \begin{vmatrix} -4 \\ -2 \end{vmatrix}$$

$$\begin{vmatrix} 6x_1 + 8x_2 + 4x_3 \\ 3x_1 + 2x_2 + 0x_3 \end{vmatrix} = \varphi \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix}$$

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5. $A = \begin{vmatrix} 9 & 18 \\ 18 & 9 \end{vmatrix}$

wartości własne

z def. $\det(A - \lambda I) = 0$

$\det(A - \lambda I) = \begin{vmatrix} 9-\lambda & 18 \\ 18 & 9-\lambda \end{vmatrix} = \lambda^2 - 18\lambda - 243$

$\Delta = 324 + 972 = 1296$

$\sqrt{\Delta} = 36$

$\lambda_1 = \frac{18-36}{2} = -9$

$\lambda_2 = \frac{18+36}{2} = 27$

1. $\lambda_1 = -9$

wektor własny:

$A - \lambda_1 I = \begin{vmatrix} 18 & 18 \\ 18 & 18 \end{vmatrix}$

$\left|\begin{matrix} 18 & 18 \\ 18 & 18 \end{matrix}\right| \begin{matrix} 0 \\ 0 \end{matrix} \left| \xrightarrow{w_1 = w_1 \frac{1}{18}} \right| \begin{matrix} 1 & 1 \\ 18 & 18 \end{matrix} \left| \begin{matrix} 0 \\ 0 \end{matrix} \right| \xrightarrow{w_2 = w_2 - 18w_2}$

$\left|\begin{matrix} 1 & 1 \\ 0 & 0 \end{matrix}\right| \begin{matrix} 0 \\ 0 \end{matrix} \Big|$ $\quad x_1 + x_2 = 0$ $\quad x_1 = -x_2$ $\quad X = \begin{vmatrix} -x_2 \\ x_2 \end{vmatrix}$

Przestrzeń własna:

niech: $x_2 = 1$

$V_{-1} = \left\{ X \begin{vmatrix} x_1 \\ x_2 \end{vmatrix} \in R^2 : -x_1 + x_2 \right\} = V_{-1} \begin{vmatrix} -1 \\ 1 \end{vmatrix}$

2. $\lambda_1 = 27$

$\left|\begin{matrix} -18 & 18 \\ 18 & -18 \end{matrix}\right| \begin{matrix} 0 \\ 0 \end{matrix} \Big| = \left|\begin{matrix} 1 & -1 \\ 0 & 0 \end{matrix}\right| \begin{matrix} 0 \\ 0 \end{matrix} \Big|$ $\quad \{ x_1 - x_2 = 0$

$\begin{cases} x_1 = x_2 \\ x_2 = x_2 \end{cases}$

$X = \begin{vmatrix} x_2 \\ x_2 \end{vmatrix}$ niech: $x_2 = 1$

$V_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$