Kacper Kalinowski 19.04.2021 464966

strona 9

1. $68^{-1} \cup 2997$

$997 = 14 \cdot 68 + 45$

$68 = 1 \cdot 45 + 23$

$45 = 1 \cdot 23 + 22$

$23 = 1 \cdot 22 + 1$

$1 = 23 - 22 \cdot 1 = 23 - (45 - 1 \cdot 23) \cdot 1 =$
$-45 + 23 \cdot 2$

$= 23 - (45 - (68 - 1 \cdot 45) \cdot 1) \cdot 1 = 23 - (45 - (68 - (997 - 14 \cdot 68) \cdot 1) \cdot 1) \cdot 1)$
$-68 + 45 \cdot 2 + 23 \cdot 2 \quad -997 + 15 \cdot 68 + 45 \cdot 2 + 23 \cdot 2$

$=$

2. $A = \begin{bmatrix} 0 & 2 & 0 & -400000 \\ 1 & -1 & 7 & 464966 \\ 0 & 3 & 0 & -600000 \end{bmatrix}$ $w_1 \to w_2$, $w_2 \to w_1$

$\begin{bmatrix} 1 & -1 & 7 & 464966 \\ 0 & 2 & 0 & -400000 \\ 0 & 3 & 0 & -600000 \end{bmatrix}$ $w_2 \to w_2 - w_3$

$\begin{bmatrix} 1 & -1 & 7 & 464966 \\ 0 & -1 & 0 & 200000 \\ 0 & 3 & 0 & -600000 \end{bmatrix}$ $w_3 \to w_3 - 3w_2$

$\begin{bmatrix} 1 & -1 & 7 & 464966 \\ 0 & -1 & 0 & 200000 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $w_2 \to w_2 \cdot (-1)$

$\begin{bmatrix} 1 & -1 & 7 & 464966 \\ 0 & 1 & 0 & -200000 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $w_1 \to w_1 + w_2$

Rząd macierzy 2

$\begin{bmatrix} 1 & 0 & 7 & 264966 \\ 0 & 1 & 0 & -200000 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

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3. $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 8 & 5 & 4 & 1 & 6 & 7 & 2 & 9 \end{pmatrix} = (1,3,5)(2,8)$

$= (1,5)(1,3)(2,8) \Rightarrow (-1)^3 = -1$

permutacja nieparzysta

4. $(1,2) \times (2,3) = (1,2,3) \quad w\ S_3$

$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} \times \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$

$a \quad b \quad c$

$a a^{-1} \times b b^{-1} = a^{-1} c b^{-1}$

$a^{-1} = a$

$a^{-1} a = 1$

$x = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} =$

$= \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$

5. $M = \begin{bmatrix} 1 & m - 464966 \\ -1 & m + 464966 \end{bmatrix} \Rightarrow \begin{matrix} w_1 \to w_1 + w_2 \\ w_2 \to -w_1 + w_2 \end{matrix}$

$\begin{bmatrix} 0 & 2m \\ -1 & m + 464966 \end{bmatrix} \quad w_2 = -\frac{1}{2} w_1$

$\begin{bmatrix} 0 & 2m \\ -1 & 464966 \end{bmatrix}$

Dla $m = 464966 \Rightarrow R = 1$

$m \neq 0 \Rightarrow R = 2$

Dla $m = 0 \Rightarrow R = 1$

$m \neq 0 \Rightarrow R = 2$

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6.

$$X = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 2 & 4 \\ 0 & 3 & 7 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ -1 & 2 & 4 & 0 & 1 & 0 \\ 0 & 3 & 7 & 0 & 0 & 1 \end{array}\right] \quad W_2 \to W_2 + W_1$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & 4 & 1 & 1 & 0 \\ 0 & 3 & 7 & 0 & 0 & 1 \end{array}\right] \quad W_2 \to \frac{1}{2} W_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 3 & 7 & 0 & 0 & 1 \end{array}\right] \quad W_3 \to W_3 - 3W_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 & -\frac{3}{2} & -\frac{3}{2} & 1 \end{array}\right] \quad W_2 \to W_2 - 2W_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 4 & 2 & -2 \\ 0 & 0 & 1 & -\frac{3}{2} & -\frac{3}{2} & 1 \end{array}\right]$$

$$A^{-1}$$

Spr. $\begin{bmatrix} 1 & 0 & 0 \\ 4 & 2 & -2 \\ -\frac{3}{2} & -\frac{3}{2} & 1 \end{bmatrix} \cdot \begin{bmatrix} -1 & 0 & 0 \\ -1 & 2 & 4 \\ 0 & 3 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cdot & \cdot \\ 0 & \cdot & \cdot \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & 4 & 1 & 1 & 0 \\ 0 & 3 & 7 & 0 & 0 & 1 \end{array}\right] \quad W_3 = W_3 - W_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & 4 & 1 & 1 & 0 \\ 0 & 1 & 3 & -1 & -1 & 1 \end{array}\right] \quad W_2 = W_2 - W_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 2 & 2 & -1 \\ 0 & 1 & 3 & -1 & -1 & 1 \end{array}\right] \quad W_3 - W_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 2 & 2 & -1 \\ 0 & 0 & 2 & -3 & -3 & 2 \end{array}\right] \quad W_3 - W_2 - W_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & 2 & -1 \\ 0 & 0 & 1 & -\frac{3}{2} & -\frac{3}{2} & 1 \end{array}\right] \quad W_3 \to W$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & -1 \\ -\frac{3}{2} & -\frac{3}{2} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}$$

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4. $(\mathbb{Z}_4, +_4)$

$\{0\}$ $\quad H \leq \mathbb{Z}_4 \quad (H, +_4) \quad H \leq \mathbb{Z}_4$

zał. $1 \in H \Rightarrow 0+1=1 \quad 1+1 \in H = 2$

$2+1 \in H = 3$

$-1 = 3 \in H$
$-2 = 2 \in H$
$-3 = 1 \in H$

$\{0,1,2,3\} = H = \mathbb{Z}_4$

zał. $2 \in H$

$0+2=2 \in H$

$-2=2 \in H$

$\{0,2\}$

Wszystkie podgrupy

$(\{0\}, +_4) \quad (\{0,2\}, +_4) \quad (\{0,1,2,3\}, +_4)$

$D_2 \begin{bmatrix} 1 & m = 464966 \\ -1 & m + 464966 \end{bmatrix}$