

$$1_0 \left[\begin{array}{cccc} 1 & 3 & 5 & 7 \\ 15 & 8 & 11 & 0 \\ 1 & 7 & 5 & 3 \end{array} \right] \xrightarrow{U_2 = 15U_1} \left[\begin{array}{cccc} 1 & 3 & 5 & 7 \\ 0 & -7 & -21 & -35 \\ 1 & 7 & 5 & 3 \end{array} \right] \xrightarrow{U_3 = 1U_1} \left[\begin{array}{cccc} 1 & 3 & 5 & 7 \\ 0 & -7 & -21 & -35 \\ 0 & -9 & -15 & -25 \end{array} \right]$$

$$\xrightarrow{U_3 = -\frac{9}{7}U_2} \left| \begin{array}{cccc} 1 & 3 & 5 & 7 \\ 0 & -7 & -21 & -35 \\ 0 & 0 & 0 & 0 \end{array} \right| \rightarrow \text{rzad 2}$$

$$4_0 \left[\begin{array}{cccc} 7 & 5 & 11 & 18 \\ 5 & 3 & 7 & 9 \\ 6 & 4 & 9 & 13 \\ 10 & 6 & 11 & 19 \end{array} \right] \xrightarrow{\begin{array}{l} U_2 = \frac{5}{7}U_1 \\ U_3 = \frac{6}{7}U_1 \\ U_4 = \frac{10}{7}U_1 \end{array}} \left[\begin{array}{cccc} 7 & 5 & 11 & 18 \\ 0 & -\frac{4}{7} & -\frac{6}{7} & -\frac{27}{7} \\ 0 & -\frac{2}{7} & \frac{3}{7} & -\frac{17}{7} \\ 0 & -\frac{3}{7} & -\frac{23}{7} & -\frac{17}{7} \end{array} \right] \xrightarrow{\begin{array}{l} U_3 = -\frac{1}{2}U_2 \\ U_4 = -2U_2 \end{array}}$$

$$\rightarrow \left| \begin{array}{cccc} 7 & 5 & 11 & 18 \\ 0 & -\frac{4}{7} & -\frac{6}{7} & -\frac{27}{7} \\ 0 & 0 & 0 & -\frac{1}{2} \\ 0 & 0 & -3 & 1 \end{array} \right| \xrightarrow{U_4 = -U_3} \left| \begin{array}{cccc} 7 & 5 & 11 & 18 \\ 0 & -\frac{4}{7} & -\frac{6}{7} & -\frac{27}{7} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & \frac{1}{2} \end{array} \right| =$$

$$= 7 \cdot \left(-\frac{4}{7}\right) \cdot (-3) \cdot \left(\frac{1}{2}\right) = 6 \quad \text{Wyznacznik macierzy przekształceń}$$

jest równy wyznacznikowi macierzy oryginalnej

$$5_0 \quad A = \begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix}$$

$$\lambda_1 = 1 \quad \lambda_2 = 5$$

$$A - \lambda I = \begin{vmatrix} 2-\lambda & 3 \\ 1 & 4-\lambda \end{vmatrix} = \lambda^2 - 6\lambda + 5 = (\lambda - 1)(\lambda - 5) = 0$$

$$\begin{array}{l} \Delta = 36 - 20 \\ \sqrt{\Delta} = 4 \\ x_1 = \frac{6-4}{2} = 1 \\ x_2 = \frac{6+4}{2} = 5 \end{array}$$

$$a) \lambda_1 = 1$$

$$A - \lambda_1 I = \begin{vmatrix} 1 & 3 \\ 1 & 3 \end{vmatrix}$$

$$\left| \begin{array}{cc|c} 1 & 3 & 0 \\ 1 & 3 & 0 \end{array} \right| \rightarrow \left| \begin{array}{cc|c} 1 & 3 & 0 \\ 0 & 0 & 0 \end{array} \right|$$

$$x_1 + 3x_2 = 0$$

$$x_1 = -3x_2$$

$$x_0 = \begin{vmatrix} -3x_2 \\ x_2 \end{vmatrix} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$$

$$\text{dla } x_2 = 1$$

$$v = \begin{vmatrix} -3 \\ 1 \end{vmatrix}$$

wektor własny

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5.

$$b) \lambda_1 = 5 \quad A - \lambda_1 I = \begin{pmatrix} -3 & 3 \\ 1 & -1 \end{pmatrix}$$

$$\begin{pmatrix} -3 & 3 & : 0 \\ 1 & -1 & : 0 \end{pmatrix} \xrightarrow{3W_2} \begin{pmatrix} 1 & -1 & 0 \\ 1 & -1 & 0 \end{pmatrix} \xrightarrow{W_2 - W_1} \begin{pmatrix} 1 & -1 & : 0 \\ 0 & 0 & : 0 \end{pmatrix}$$

$$x - x_2 = 0$$

$$x_1 = x_2$$

$$X = \begin{pmatrix} x_2 \\ x_2 \end{pmatrix}$$

$$\text{da } x_2 = 1 \quad V = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

3.

$$\begin{cases} 4x_1 + 5x_2 + 2x_3 = 1 \\ x_1 + x_2 + 2x_3 = 13 \\ x_1 + 2x_2 + 2x_3 = 4 \end{cases}$$

$$\begin{pmatrix} 4 & 5 & 2 & : & 1 \\ 1 & 1 & 2 & : & 13 \\ 1 & 2 & 2 & : & 4 \end{pmatrix} \xrightarrow{\begin{matrix} W_2 - \frac{1}{4}W_1 \\ W_3 - \frac{1}{4}W_1 \end{matrix}} \begin{pmatrix} 4 & 5 & 2 & : & 1 \\ 0 & -\frac{1}{4} & \frac{3}{2} & : & \frac{51}{4} \\ 0 & \frac{3}{4} & \frac{3}{2} & : & \frac{15}{4} \end{pmatrix} \xrightarrow{W_3 - 3W_2} \begin{pmatrix} 4 & 5 & 2 & : & 1 \\ 0 & -\frac{1}{4} & \frac{3}{2} & : & \frac{51}{4} \\ 0 & 0 & 6 & : & 12 \end{pmatrix}$$

$$\begin{cases} 4x_1 + 5x_2 + 2x_3 = 1 \\ -\frac{1}{4}x_2 + \frac{3}{2}x_3 = \frac{51}{4} \\ 6x_3 = 12 \end{cases} \rightarrow \begin{cases} x_3 = 2 \\ -\frac{1}{4}x_2 = \frac{9}{2} \\ 4x_1 + 5x_2 + 2x_3 = 1 \end{cases} \rightarrow \begin{cases} x_3 = 2 \\ x_2 = -9 \\ 4x_1 + (-45) + 4 = 1 \end{cases}$$

$$\begin{cases} x_1 = 32 \\ x_2 = -9 \\ x_3 = 2 \end{cases} = \begin{cases} x_1 = 8 \\ x_2 = -9 \\ x_3 = 2 \end{cases}$$

$$X = \begin{pmatrix} 8 \\ -9 \\ 2 \end{pmatrix}$$

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$$2_0 \quad T \left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \right) = \begin{bmatrix} 5x_3 \\ -x_1 \\ 2x_3 - x_2 \end{bmatrix} \quad \mathbb{R}^3 \rightarrow \mathbb{R}^3$$

$$T \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$$

$$T \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$$

$$T \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix}$$

$$T \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix}$$

