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Bachelor Thesis

Development of a computational tool for turbomachinery Blade Generator

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To my family, friends and J.P.M.

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3 Introduction

Along this chapter, a brief introduction of turbomachinery is presented (such as definition, the physics behind and other related concepts). Further, related more specifically with this project, the motivation and objectives of this work are specified, together with an explanation of this project's organization.

Generally, a turbomachine refers to those devices in which there is a flowing fluid and its energy is transferred to a rotating blade row. These devices are also referred as rotordynamic devices because the dynamic action of the fluid, which is guided through an annular duct, is generated by the rotor. This transfer changes the stating enthalpy of the fluid by extracting work from the device or from the fluid. According to this definition, two main types of devices can be encountered:

- Those which *absorb* power to increase the fluid pressure - this is the case of fans, compressors and pumps
- and those which *produce* power by decreasing the fluid pressure - gas turbines, for example.

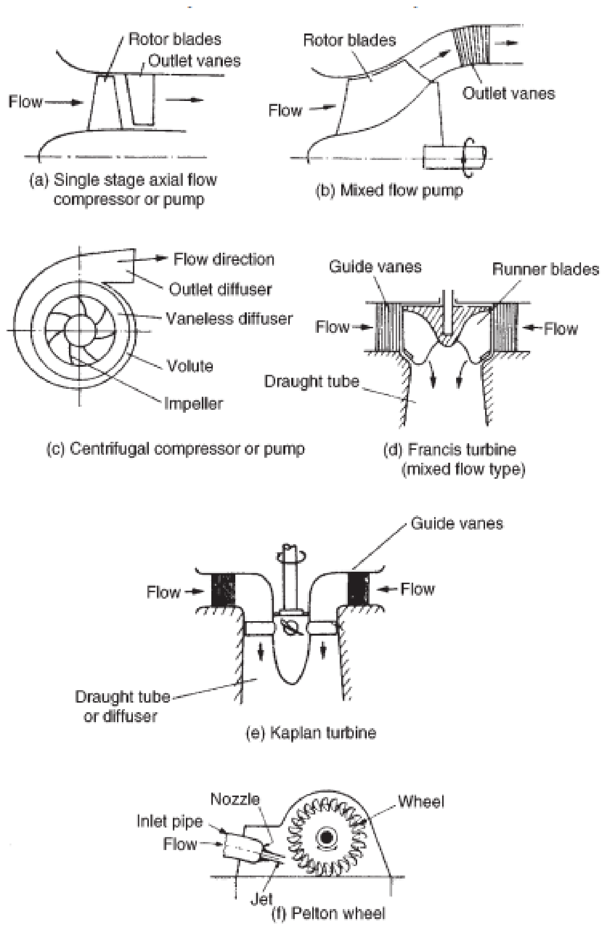

Figure 1: Examples of types of turbomachines: a) and e) are axial devices; c) radial; and b) and d) are mixed flow turbomachines. f) is an impulse turbine

Apart from that, there is another categorization of turbomachines depending on the flow direction; thus, when the flow is parallel to the axis of rotation, the device is called *axial flow turbomachine*; the flow can be also perpendicular to the axis of rotation and in this case is referred to us as a *radial flow turbomachine*; and the last type is *mixed flow turbomachine*.There is a last classification depending on the amount of energy transformed into kinetic energy. That is, if all the hydraulic energy is converted into kinetic energy the machine is called *impulse* and *reaction turbomachine* if only some amount is converted.All these classification is sketched in Figure 1.

Along this project, it is of importance only axial turbines and compressors. The performance of these machines depends not only on the flow conditions, but also on the blade geometry, as it is going to be shown further. Blade geometries can be constructed in many ways, but the most used is the so-called *Direct Method* - Section ?? - which basically consists on, first, design the profile and, secondly, perform the analysis process. Subsequently, there are two sub-processes followed.

Regarding on the organization of the blade row, it is said that they form a *cascade*. Basically, it means that blades are close in terms of proximity. And this implies that the individual behaviour of a blade is affected by the adjacent blades. The dynamic analysis can be performed either by experimental methods or by theoretical means (e.g. Mises - see Section ??).

3.1 Motivation

As said before, one of the variable of the performance of a turbomachine is the geometry of the blade within the cascade. So, having proper tools to determine and analyse easily the performance of the blade is of importance for many industries such as aerospace, and also researchers and students. The tool that was developed with MATLAB has as stakeholders students and researchers from university (Universidad Carlos III de Madrid, for instance). They can use it to generate the blade profile of a turbine and compressor and then run the theoretical analysis with Mises.

As said in the previous paragraph, the user has the opportunity to optimize a process of creation and analysis of a turbomachine blade profile. This optimization, from the first moments, constituted a big challenge because 1. the user has to generate the profile under a user-friendly layout and 2. the analysis process must be run in a totally different Operating System (Linux). So, one of the challenging point of this project is programming.

But not only the previous one, Turbomachinery and Fluid Mechanics, in general, are very interesting subjects. And, thanks to this project, one can acquire lots of knowledges related with these fields.

And last but not least, the fact that this work gives you the opportunity to improve skills such as problem solving, looking for proper references and writing skills. And all these learnings are useful for a future engineer.

3.2 Objectives

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3.3 Organization of the document's sections

jkljk

4 State of the art

5 Basic concepts and principles

The purpose of this chapter is to introduce to the reader an overview related with Turbomachinery fundamentals (laws and principles, velocity triangle, flow angles, among others). Moreover, it is included also an explanation about the mean-line design. And lastly, parametric graphs (and Bézier curves) are also presented in this section.

5.1 Turbomachinery fundamentals

To start with, it is important to give some insights regarding the coordinate system. Since turbomachines rotate about an axis, it may be clear that cylindrical coordinates are the ones that better fit with the problem. Then, three axes are defined: x or axial, r or radial and tangential $r\theta$. Although the velocity field has varying components in all these three coordinates, it is going to be assumed that the tangential component ($r\theta$) does not vary. The meridional velocity, or the velocity along the stream surface, is defined as:

$$c_m = \sqrt{c_x^2 + c_r^2} \tag{1}$$

Then, one can talk about purely *axial* and *radial* turbomachines if $c_r = 0$ and $c_x = 0$, respectively. In the next sketch it can be observed the coordinates system and flow velocities within a turbomachine.

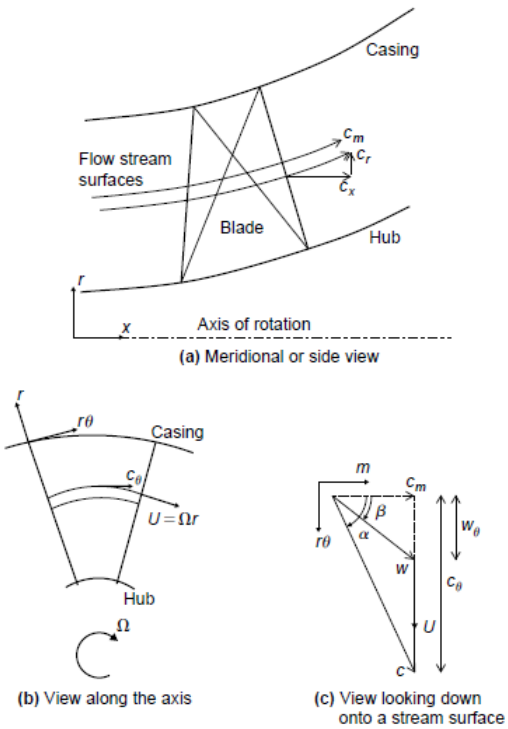

Figure 2: Coordinates system of a turbomachine

The absolute velocity flow is then

$$c = \sqrt{c_x^2 + c_r^2 + c_\theta^2} = \sqrt{c_m^2 + c_\theta^2} \tag{2}$$

As it can be depicted from Figure 2, the swirl or tangential angle is defined as

$$\tan\alpha = \frac{c_\theta}{c_m} \tag{3}$$

The previous concepts are written with respect to a reference frame that is *unsteady* (note that the blades rotated with an angular velocity Ω). A *steady* reference frame solves this problem; then, it is better to work in a relative frame which is stationary with respect to the blades. Bearing in mind that a blade has only tangential velocity because it does not move in the axial nor radial plane (see Figure 2-b), the tangential relative velocity is simply the absolute (c_θ) minus the local ($U = \Omega r$) velocities. Therefore, the relative components are:

$$w_\theta = c_\theta - U = c_\theta - \Omega r \quad w_x = c_x \quad w_r = c_r \tag{4}$$

and the relative flow angle is:

$$\tan\beta = \frac{w_\theta}{c_m} = \frac{c_\theta - U}{c_m} = \tan\alpha - \frac{U}{c_m} \tag{5}$$

Having clear the different reference frames, it is introduced above the fundamentals laws used in turbomachinery:

1. Continuity of flow equation: $\mathrm{d}\dot{m} = \frac{\mathrm{d}m}{\mathrm{d}t} = \rho c \mathrm{d}A_n$, where A_n is the element area normal to the flow direction. Doing a steady 1-D analysis, $\dot{m} = \rho_1 c_1 A_{n1} = \rho_2 c_2 A_{n2} = \rho c A_n$.

2. The First Law of Thermodynamics states that the energy change within a system from state 1 to state 2 is $\mathrm{d}E = \mathrm{d}Q - \mathrm{d}W$, where $E = U + \frac{1}{2}mc^2 + mgz$, $\mathrm{d}Q$ is the heat supplied to the system, $\mathrm{d}W$ is the work done by the system and U the internal energy.

3. Çencel and Boles demonstrated how the previous law can be applied to a control volume like the one above:

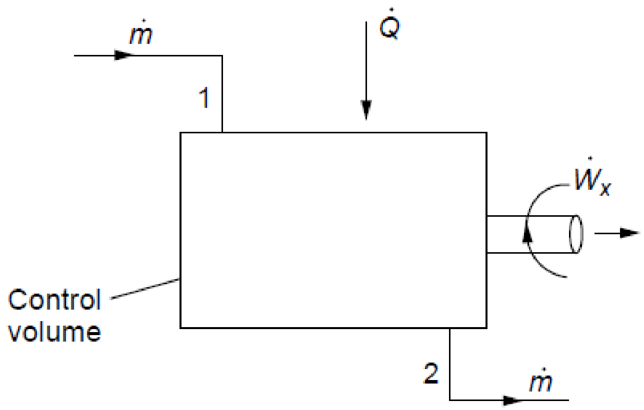

Figure 3: Control Volume of a turbomachine that is supplied with heat and there is a work done by the system

Energy is supplied from the fluid to the blade at a rate of $\dot{W}_x$ and heat is transferred from the surroundings to the control volume at $\dot{Q}$. And noting that the internal energy is related with the change of absolute enthalpy, the First Law of Thermodynamics can be written down as:

$$\dot{Q} - \dot{W}_x = \dot{m}\left[(h_2 - h_1) + \frac{1}{2}(c_2^2 - c_1^2) + g(z_2 - z_1)\right] =$$

$$= \dot{m}\left[(h_{o2} - h_{o1}) + g(z_2 - z_1)\right]$$

where h_o is the stagnation enthalpy $h_o = h + 1/2c^2$. Most machineries are adiabatic ($\dot{Q} = 0$) and the gravitation potential energy is usually ignore because it is small. As a consequence of these assumptions, $\dot{W}_x = \dot{m}\Delta h_o = h_{o1} - h_{o2}$. In the case of a turbine, the work is produced by the turbomachine and then $\dot{W}_x = \dot{W}_t$ but, in a compressor, the work is absorbed: $\dot{W}_c = -\dot{W}_x$.

4. From momentum balance, it is obtained the so-called *Euler work Equation*[1], which states that $h_o = Uc_\theta$ or $\Delta h_o = \Delta(Uc_\theta)\Delta W_x$. This equation is valid for steady problems, viscid and inviscid fluid. From this relation, the *rothalpy* is defined as $I = h_o - Uc_\theta$. This fluid parameter (that it is constant along a streamline) is used in the study of the flow within a rotating frame. Making use of the Equation 2 and rothalpy definition, it can be shown that $I = h_{o,\mathrm{rel}} - \frac{1}{2}U^2$ and it states that, if the radius and Ω are constant (that is, blade speed U is constant), the relative stagnation enthalpy (which is equal to $h_{o,\mathrm{rel}} = h + \frac{1}{2}w^2$) is constant since I is also constant.

Apart from these fundamental laws, it is important also to recall the Bernoulli's Equation whose demonstration is out of the scope of this project. For an adiabatic flow, no work transfer and incompressible:

$$\int_1^2 \frac{1}{\rho}\mathrm{d}p + \frac{1}{2}(c_2^2 - c_1^2) + g(z_2 - z_1) = 0 = \frac{1}{\rho}(p_{o2} - p_{o1}) + g(z_2 - z_1) = 0 \qquad (6)$$

where p_o is the stagnation enthalpy and it is defined as $p_o = p + 1/2\rho c^2$.

Furthermore, let us introduce few compressible flow relations:

- Mach Number: $M = \frac{c}{a} = \frac{c}{\sqrt{\gamma RT}}$. To have a compressible flow, in a turbomachine the Mach number exceed about 0.3. As a result, the density is no longer constant. And nowadays to reach these conditions in inevitably because of high power needed.

- Static and stagnation quantities (assuming perfect gas condition):

$$\frac{T_o}{T} = 1 + \frac{\gamma - 1}{2}M^2$$

$$\frac{p_o}{p} = \left(\frac{T_o}{T}\right)^{\gamma/(\gamma-1)}$$

[1]The demonstration of this equation is out of the scope of this project; further information can be seen in Ref ??

and,

$$\frac{\rho_o}{\rho} = \left(\frac{T_o}{T}\right)^{1/(\gamma-1)}$$

. Applying isentropy between two arbitrary points (1 and 2),

$$\frac{p_{o2}}{p_{o1}} = \left(\frac{T_{o2}}{T_{o1}}\right)^{\gamma/(\gamma-1)}$$

- Another important parameter is the non-dimensional flow rate (or *capacity*).

$$\dot{m} = \frac{\rho_o}{\left(1+\frac{\gamma-1}{2}M^2\right)^{1/(\gamma-1)}} M \sqrt{\gamma R T_o \frac{1}{\left(1+\frac{\gamma-1}{2}M^2\right)}} A_n =$$

$$= \frac{\frac{p_o}{RT_o}}{\left(1+\frac{\gamma-1}{2}M^2\right)^{1/(\gamma-1)}} M \sqrt{\gamma R T_o \frac{1}{\left(1+\frac{\gamma-1}{2}M^2\right)}} A_n$$

and arranging the above expression (and noting that $C_p/C_v = \gamma$ and $C_p - C_v = R$), the final result is:

$$\frac{\dot{m}\sqrt{C_p T_o}}{A_n p_o} = \frac{\gamma}{\sqrt{\gamma-1}} M \left(1 + \frac{\gamma-1}{2} M^2\right)^{-\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)}$$

All these expressions are written in terms of the local Mach Number. And, moreover, they can be also expressed as a function of the relative Mach Number.

The equations showed in this sub-section are of importance for the user in order to calculate the inputs of the blade geometry tool.

5.2 Mean Line Design

For the Mean Line Design, it is assumed that the flow conditions at a mean radius, called *pitchline*, represents the flow at all radii. The approximation is reasonable whenever the ratio between the mean radius and height of the blade is small. If this ratio is large, then *three-dimensional analysis* must be performed.

5.2.1 Axial-Flow Turbines

In the next figure, Figure 4, it can be observed a turbine stage. It is composed by a fixed guided vanes row, nozzles or *stator* and a moving blades row, buckets or *row*. Fluid enters at a flow angles α_1 with an absolute velocity of c_1, and then it accelerates up to c_2 and exits from the stator row at an angle α_2. The rotor inlet *relative* velocity is w_2 and the angle is β_2. These values can be calculates as explained in previous sections. And then, through the rotor, the fluid accelerates to a relative velocity w_3 at an angle β_3. Drawing the velocities as below is very helpful because it prevents errors. Note that the absolute velocities are aligned with the absolute flow angles; and the relative ones with the relative angles.

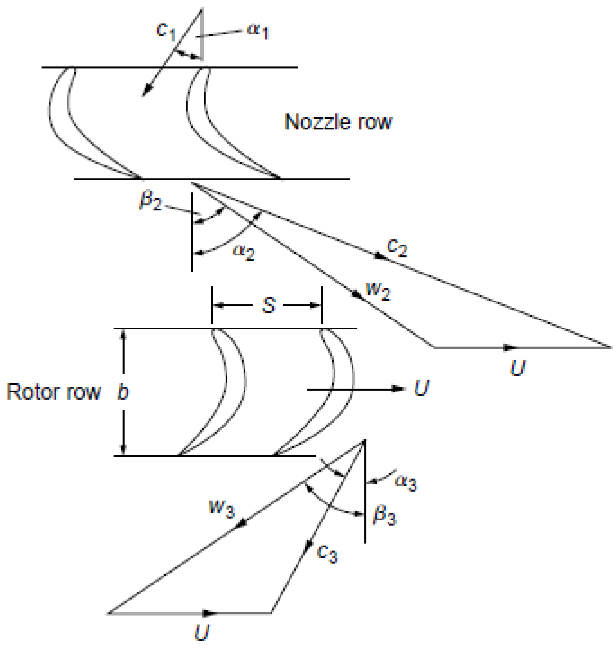

Figure 4: Velocity Triangle of a turbine. Note that all angles are measures with respect to the x angle

There are two main design parameters of a turbine stafe: design flow coefficient (ϕ) and stage loading coefficient (ψ). Following a dimensional analysis process, it can be shown these parameters are equal to:

$$\phi = \frac{c_m}{U} = \frac{c_x}{U} \text{ for purely axial turbine and } \psi = \frac{\Delta h_o}{U^2} = \frac{\Delta c_\theta}{U} \tag{7}$$

Note that for the stage loading coefficient, it has been assumed that Euler equation's conditions are hold. The value of ϕ is related with the stagger angles (parameters that is defined in further sections). High values imply lower stagger angles, and an increment in mass flow (because of continuity equation). And, regarding the stage loading, high values implies large turnings since there is an increment in the tangential component of the velocity.

Another important parameter that the user has to take into account is the stage reaction (R), and is defined as the ratio between the static enthalpy drop in the rotor to the static enthalpy drop across the stage:

$$R = \frac{h_2 - h_3}{h_1 - h_3} \approx \frac{p_2 - p_3}{p_1 - p_3} \tag{8}$$

Note that in the above equation it has been considered that the flow is almost isentropic and the compressibility effects are not taken into account.

In order to have high power and high efficiency turbines, multi stages axial turbines are required. In this kind of configuration, it is desired to have similar velocity triangles in all the stages. Regarding the flow, it expands as it passes through the stages and, as a result, the density of the fluid is reduced. To have similar velocity triangle, the mean radius of the blade has to be almost constant, the axial velocity has to be kept constant and the flow angle at the exit of the stages must be the same as the inlet. Applying these assumptions to the previous parameters explained, useful relationships are obtained that are important for the preliminary design of the turbine.

$$R = \frac{h_2 - h_3}{h_1 - h_3} = \frac{h_2 - h_1 + h_1 - h_3}{h_1 - h_3} = 1 - \frac{h_1 - h_2}{h_{o1} - h_{o3}}{}^{2} \tag{9}$$

$$h_1 - h_2 = (h_{o1} - h_{o2}) + \frac{1}{2}(c_2^2 - c_1^2) = \frac{1}{2}c_x^2(\tan^2\alpha_2 - \tan^2\alpha_1)^3 \tag{10}$$

and combining the previous equation with the stage loading definition ($h_{01} - h_{02} = U^2\psi$) and flow coefficient ($\phi = c_x/U$), the reaction definition is then

$$R = 1 - \frac{1}{2}\frac{c_x^2(\tan^2\alpha_2 - \tan^2\alpha_1)}{U^2\psi} = 1 - \frac{\phi^2}{2\psi}(\tan^2\alpha_2 - \tan^2\alpha_1) \tag{11}$$

Equation 11 can be further simplified since the stage loading can be written as follows:

$$\psi = \frac{\Delta c_\theta}{U} = \frac{c_{\theta 2} + c_{\theta 3}}{U} = \frac{c_x(\tan\alpha_2 + \tan\alpha_3)}{U} = \psi(\tan\alpha_2 + \tan\alpha_1) \tag{12}$$

Substituting this in Equation 11,

$$R = 1 - \frac{\phi}{2}(\tan\alpha_2 - \tan\alpha_1) \tag{13}$$

Equation 12 can be used to eliminate α_2 from Equation 13:

$$R = 1 - \frac{\psi - \phi\tan\alpha_1}{2} + \frac{\phi}{2}\tan\alpha_1 = 1 - \frac{\psi}{2} + \phi\tan\alpha_1 \Longrightarrow \psi = 2(1 - R + \phi\tan\alpha_1) \tag{14}$$

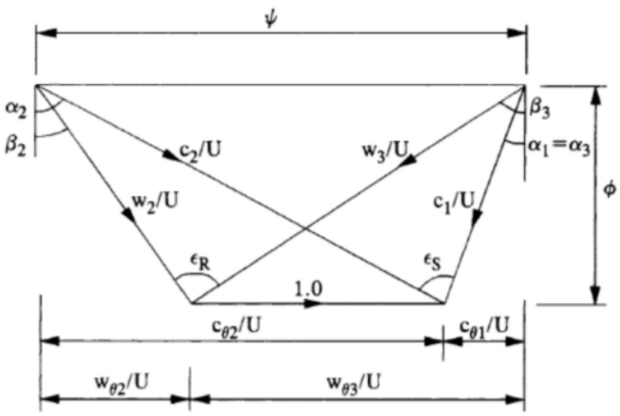

Figure 5: Dimensionless Velocity Triangle of a single turbine stage

[2] $h_{o1} - h_{o3} = h_1 - h_3$ because the inlet and outlet velocities are equal since their flow angles are the same

[3] Through the stator, there is no work done, then the stagnation enthlapy from 1 to 2 is constant

The above expressions shows that for a high stage loading, the reaction should be low and the swirl angle α_1 should be large. It is interesting to have a dimensionless velocity triangle. This is basically obtained by diving all the parameters of Figure 4 by U, as it can be observed in the previous figure.

R value determine the style of the turbine, and there are two possible extremes: zero reaction which leads to very different blade shapes for the stator and rotor; and 50% which leads to symmetrical shapes. Regarding the first one style, high stage loading even if the swirl is low. Since the pressure drop across the rotor is low, the tip leakage is reduced. But, high stage loading implies boundary layer separation and as a consequence the efficiency is lowered. On the other hand, $R = 50\%$ implies symmetrical velocity triangle and therefore blade shapes are similar and this reduces costs. The turning is lower and the losses are also reduced because the flow is more accelerated. As drawbacks, more stages are needed in comparison with lower stages (since the loading coefficient is decreases and with it, the specific work per stage is also reduced according to the Euler equation) and the leakage losses are raised. Note that Reaction values that exceeds unity are not desired since this would mean that the flow is decelerated (since the the out flow angle would be greater than the inlet one). In this cases the flow is said to be *diffused* and *diffusion* implies adverse pressure gradients which, as very well known, lead to flow separation.

To sum up, the designer can fix ϕ, ψ and R (e.g., ϕ, ψ and R that minimize losses and maximize the efficiency of the turbine stage), therefore, the velocity triangle is obtained thanks to these parameters and, as a result, it obtains the inputs for the blade shape.

5.3 Axial-Flow Compressors

5.4 Smith Chart

There are large database of measured efficiencies for axial-flow turbomachines as a function of ϕ and ψ, the duty parameters. Smith was in charge of preparing these charts. Regarding the Smith chart for turbines, the experiments where performed for R that were between 0.2 and 0.6. The points of the chart (Figure 6) represent one single test turbine at its best efficiency. Note also that the efficiencies represented are *total-to-total*. This, basically means:

$$\eta_{tt} = \frac{\text{actual work output}}{\text{ideal work output when operating to same back pressure}} = \frac{h_{01} - h_{03}}{h_{01} - h_{03ss}}{}^{4} \tag{15}$$

[4] h_{03ss} referes to the total enthalpy of the point 3ss which has the same isentropy the point 1 from the stage

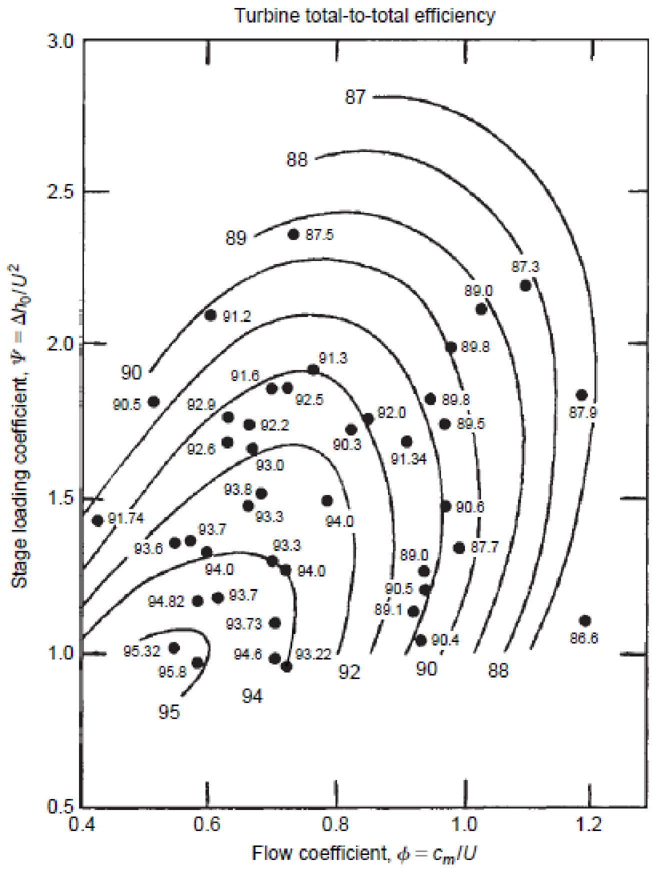

Figure 6: Smith Chart for Turbine Stage Efficiency

Apart from that, Smith developed a theoretical analysis to obtain the optimum stage load coefficient (ψ_{opt}), the stage load coefficient that minimizes blade losses at $R = 0.5$:

$$\psi_{\text{opt}} = \sqrt{4\phi^2 + 1} \tag{16}$$

Based on his analysis, Lewis developed a more accurate expression that include losses:

$$\psi_{\text{opt,exp}} = 0.65\sqrt{4\phi^2 + 1} \tag{17}$$

5.5 Introduction to Parametric graphs and Bézier curves

In computer graphics, it is convenient (because of its easiness) to use the so-called "parametric curves". Therefore, a curve is represented like a n grade polynomial that depends on a parameter t:

$$\vec{c}(t) = \vec{a_0} + \vec{a_1}t + \cdots + \vec{a_n}t^n \tag{18}$$

where the coefficients $\vec{a}_i$ are points of the plane and t is a parameters that goes from 0 to 1: from the starting point of the curve ($\vec{a}_0$) and the final point ($\vec{a}_0$).

This way of representing a curve has one important advantage that is its easiness (as said before) but its main drawback is the fact that coefficients behave in a complex way when, for example, one wants to rotate the curve or translate it. Therefore, it is convenient to use another type of representation instead of polynomial curves (bur following the same *philosophy*). And one of this type of computational representation of curves are *Bézier curves*.

Bézier curves are very used in computer graphics because they are smooth. In few words, they are constructed by a set of control points, and these points can be displaced in order to modify the plot. As previously mentioned, parametric curves can be transformed (namely, they can be rotated and/or rotated) and in the case of Bézier curves, one has just only to transform the control points properly in order to have the line transformed.

The degree of a Bézier curve is defined by the set of control points. If $\vec{c}(t)$ is only defined by $\vec{a}_0$ and $\vec{a}_1$, the curve is linear. If it is defined by three points, then it is quadratic. And the degree can be increased up to n. The computational complexity increases with n. In the next figure (Figure 7) , one can see these two types of curves in a 2D plane:

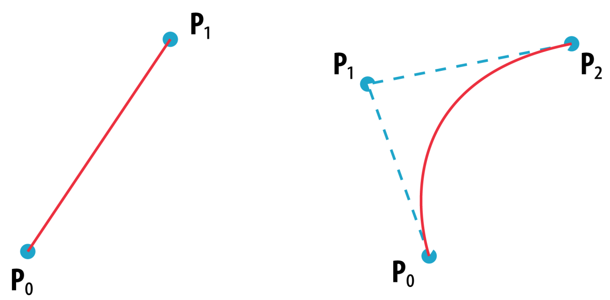

Figure 7: Two examples of Bézier curves. Left side - linear cuve; Right side - quadratic curve

Next figure shows how the curve is constructed depending on the value of the parameter t:

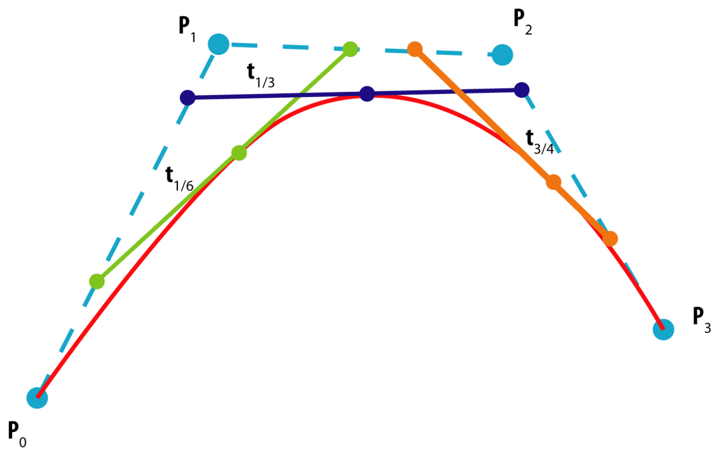

Figure 8: Building process of a parametric Bézier graph

For a linear curve, the line is expressed as a straight line:

$$\vec{c}(0) = P_0 + t(P_1 - P_0) = (1-t)P_0 + tP_1 \tag{19}$$

For higher orders of n, the process is more complex. Let us develop the Bézier curve for a quadratic curve. For this kind of lines, implicitly, the control point P_1 depends on the tangent line of the finishing points.

Making use of Equation 18 and defining $\vec{c}(t)$ as a $(x_B(t), y_B(t))$,

$$x_B(t) = a_{0x} + a_{1x}t + a_{2x}t^2$$

$$y_B(t) = a_{0y} + a_{1y}t + a_{2y}t^2$$

or in a matrix form:

$$\vec{c}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{Bmatrix} t^2 & t & 1 \end{Bmatrix} \begin{bmatrix} a_{2x} & a_{2y} \\ a_{1x} & a_{1y} \\ a_{0x} & a_{0y} \end{bmatrix} = TC_B = TM_BG_B \tag{20}$$

From Equation 20, T, G_B and M_B are defined as:

- $T = \begin{Bmatrix} t^2 & t & 1 \end{Bmatrix}$
- $G = \begin{bmatrix} P_{2x} & P_{2y} \\ P_{1x} & P_{1y} \\ P_{0x} & P_{0y} \end{bmatrix}$
- Matrix M_B is the unknown of the equation and it relates matrices G_B (which depends on the control points) and C_B (which is the coefficients matrix)

In order to determine M_B, let us state the next conditions:

1. $\vec{c}(0) = P_0 = \begin{Bmatrix} 0 & 0 & 1 \end{Bmatrix} M_BG_B$
2. $\vec{c}(1) = P_2 \begin{Bmatrix} 1 & 1 & 1 \end{Bmatrix} M_BG_B$
3. $R_1 = \frac{P_2 - P_1}{t_2 - t_1} = \frac{P_1 - P_0}{1 - 1/2} = 2(P_2 - P_1) = \vec{c}\ '(1) = T'(1)M_BG_B = \begin{Bmatrix} 2 & 1 & 0 \end{Bmatrix} M_BG_B$

R_2 is defined as the tangent at $t = 1$, that is $\frac{d(P(t))}{dt}$. Note that at P_2, $t = 1$ and at P_1, $t = 1/2$. In a matrix form, these conditions are rewritten as follow:

$$G_H = \begin{Bmatrix} P_0 \\ P_2 \\ R_1 \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 2 & -2 & 0 \end{bmatrix} G_B = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 2 & 1 & 0 \end{bmatrix} M_BG_B \tag{21}$$

From this last equation, M_B can be calculated and it is equal to:

$$M_B = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 2 & -2 \\ 0 & 0 & 1 \end{bmatrix} \tag{22}$$

and since $\vec{c} = TM_BG_B$,

$$\vec{c} = (t-1)^2P_0 + 2t(t-1)P_1 + t^2P_2 \tag{23}$$

A similar procedure can be followed for a cubic Bézier curve. Apart from the initial and final point conditions, for cubic expression, it may be also note that (as similar for the quadratic form) $R(0) = 3(P_1 - P_0)$ and $R(1) = 3(P_2 - P_1)$. And doing similar computations as before, the Bézier equation is expressed as:

$$\vec{c}(t) = (1-t)^3P_0 + 3(1-t)^2tP_1 + 3(1-t)t^2 + t^3P_3 \tag{24}$$

If the process is doing for higher degrees, it can be observed Bézier functions have the shape of *Bernstein basis polynomial*[5] therefore, the explicit equation of order n is:

$$\vec{c}(t) = \sum_{i=0}^{n} \binom{n}{i} (1-t)^{n-1}t^iP_i \tag{25}$$

From the practical point of view, only quadratic form of Bézier parametric graphs is the one that it is used for this work.

6 Blade profile geometry

There are two ways to design a blade for a compressor and a turbine. These approaches are called:

- *Direct (or analysis) method*: it is basically a geometrical technique. The cascades generated are further analysed by experimental test or theoretical approaches in order to find performance desired.
- *Inverse (or synthesis) method*: this is an advanced method and it is the opposite than the direct approach; in this case, the user specifies the velocity and pressure distribution along the surface of the blade. The method that generate the blade profile from these distribution is called *Prescribed Velocity Distribution* or *PVD* analysis.

It may be thought that inverse method is the one that gives the best approach but, from a practical point of view, direct method is widely used by engineers. And this method is used for this project. In the next sections, it is explained how these profiles are obtained.

6.1 Direct Method

As explained previously, with this method, the engineer elaborate a cascade family and then it is analysed experimentally or by theoretical means (e.g. MISES software). There are distinguished two ways: one given the thickness family; and

[5]Bernstein basis polynomial is defined explicitly as $B_n(x) = \sum_{i=0}^{n} \beta_\nu \binom{n}{\nu} x^\nu(1-x)^{n-\nu}$ and few examples of this polynomial basis are $b_{0,0} = 1$, $b_{0,1} = 1-x$, $b_{1,1} = x$, $b_{0,2} = (1-x)^2$, $b_{1,2} = 2x(1-x)$ and $b_{2,2} = x^2$, and so on

another, that was named as *Higher parameters design*, the user has fully control about the design. Both algorithms are similar in the way that the software ask the user about certain input and, making use of geometrical techniques, the software gives as output the blade profile.

Related Cascade families

The procedure follow, as shown in the next flow chart (Figure 9), is:

1. Build the Camber Line
   (a) Ask the user for the Camber Line inputs: metal and stagger angles
   (b) Build the Camber Line with Bézier Theory
2. Add perpendicularly (to the Camber Line) the thickness family

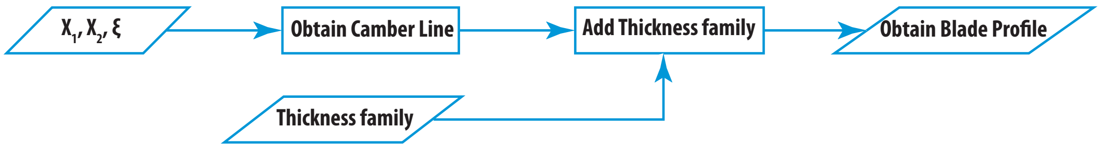

Figure 9: Direct method flow chart for a prescribed thickness family

The Camber Line is obtain using three control points:

Higher parameters design

6.2 MISES

7 Results

7.1 Design

7.1.1 Create a Turbine Blade profile

7.1.2 Create a Compressor Blade profile

7.1.3 Higher parameters design

Turbine

Compressor

7.1.4 Other features of the software

7.2 Analysed designs

7.2.1 Blade Profiles

7.2.2 Analysis of MISES results

8 Possible Improvements

9 Conclusion

10 Bibliography