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Bachelor Thesis

Development of a computational tool for turbomachinery Blade Generator

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To my family, friends and J.P.M.

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α	Swirl Angle
c	Absolute Velocity
c_θ	Tangential Absolute Velocity
c_m	Meridional Absolute Velocity
c_r	Radial Absolute Velocity
c_x	Axial Absolute Velocity

3 Introduction

Along this chapter, a brief introduction of turbomachinery is presented (such as definition, the physics behind and other related concepts). Further, related more specifically with this project, the motivation and objectives of this work are specified, together with an explanation of this project's organization.

Generally, a turbomachine refers to those devices in which there is a flowing fluid and its energy is transferred to a rotating blade row. These devices are also referred as rotordynamic devices because the dynamic action of the fluid, which is guided through an annular duct, is generated by the rotor. This transfer changes the stating enthalpy of the fluid by extracting work from the device or from the fluid. According to this definition, two main types of devices can be encountered:

- Those which *absorb* power to increase the fluid pressure - this is the case of fans, compressors and pumps
- and those which *produce* power by decreasing the fluid pressure - gas turbines, for example.

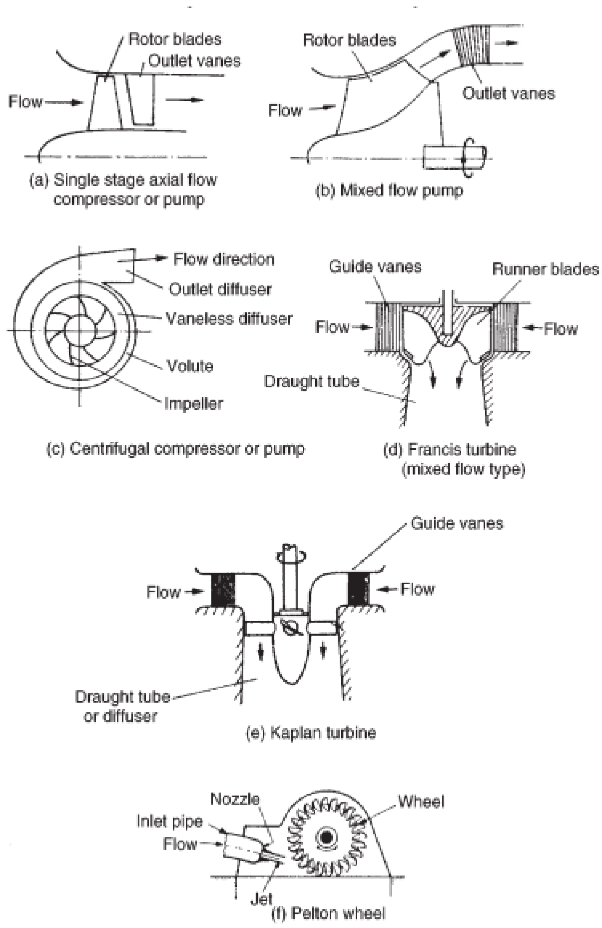


Figure 1: Examples of types of turbomachines: a) and e) are axial devices; c) radial; and b) and d) are mixed flow turbomachines. f) is an impulse turbine

Apart from that, there is another categorization of turbomachines depending on the flow direction; thus, when the flow is parallel to the axis of rotation, the device is called *axial flow turbomachine*; the flow can be also perpendicular to the axis of rotation and in this case is referred to us as a *radial flow turbomachine*; and the last type is *mixed flow turbomachine.*There is a last classification depending on the amount of energy transformed into kinetic energy. That is, if all the hydraulic energy is converted into kinetic energy the machine is called *impulse* and *reaction turbomachine* if only some amount is converted.All these classification is sketched in Figure 1.

Along this project, it is of importance only axial turbines and compressors. The performance of these machines depends not only on the flow conditions, but also on the blade geometry, as it is going to be shown further. Blade geometries can be constructed in many ways, but the most used is the so-called *Direct Method* - Section ?? - which basically consists on, first, design the profile and, secondly, perform the analysis process. Subsequently, there are two sub-processes followed.

Regarding on the organization of the blade row, it is said that they form a *cascade*. Basically, it means that blades are close in terms of proximity. And this implies that the individual behaviour of a blade is affected by the adjacent blades. The dynamic analysis can be performed either by experimental methods or by theoretical means (e.g. Mises - see Section ??).

3.1 Motivation

As said before, one of the variable of the performance of a turbomachine is the geometry of the blade within the cascade. So, having proper tools to determine and analyse easily the performance of the blade is of importance for many industries such as aerospace, and also researchers and students. The tool that was developed with MATLAB has as stakeholders students and researchers from university (Universidad Carlos III de Madrid, for instance). They can use it to generate the blade profile of a turbine and compressor and then run the theoretical analysis with Mises.

As said in the previous paragraph, the user has the opportunity to optimize a process of creation and analysis of a turbomachine blade profile. This optimization, from the first moments, constituted a big challenge because 1. the user has to generate the profile under a user-friendly layout and 2. the analysis process must be run in a totally different Operating System (Linux). So, one of the challenging point of this project is programming.

But not only the previous one, Turbomachinery and Fluid Mechanics, in general, are very interesting subjects. And, thanks to this project, one can acquire lots of knowledges related with these fields.

And last but not least, the fact that this work gives you the opportunity to improve skills such as problem solving, looking for proper references and writing skills. And all these learnings are useful for a future engineer.

3.2 Objectives

vbcvb

3.3 Organization of the document's sections

4 State of the art

5 Basic concepts and principles

The purpose of this chapter is to introduce to the reader an overview related with Turbomachinery fundamentals (laws and principles, velocity triangle, flow angles, among others). Moreover, it is included also an explanation about the mean-line design. And lastly, parametric graphs (and Bézier curves) are also presented in this section.

5.1 Turbomachinery fundamentals

To start with, it is important to give some insights regarding the coordinate system. Since turbomachines rotate about an axis, it may be clear that cylindrical coordinates are the ones that better fit with the problem. Then, three axes are defined: x or axial, r or radial and tangential $r\theta$. Although the velocity field has varying components in all these three coordinates, it is going to be assumed that the tangential component ($r\theta$) does not vary. The meridional velocity, or the velocity along the stream surface, is defined as:

$$c_m = \sqrt{c_x^2 + c_r^2} \tag{1}$$

Then, one can talk about purely *axial* and *radial* turbomachines if $c_r = 0$ and $c_x = 0$, respectively. In the next sketch it can be observed the coordinates system and flow velocities within a turbomachine.

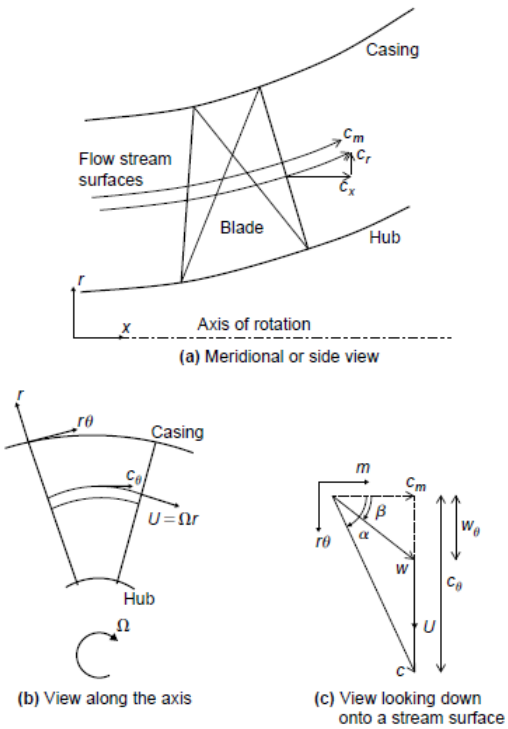


Figure 2: Coordinates system of a turbomachine

The absolute velocity flow is then

$$c = \sqrt{c_x^2 + c_r^2 + c_\theta^2} = \sqrt{c_m^2 + c_\theta^2} \tag{2}$$

As it can be depicted from Figure 2, the swirl or tangential angle is defined as

$$\tan\alpha = \frac{c_\theta}{c_m} \tag{3}$$

The previous concepts are written with respect to a reference frame that is *unsteady* (note that the blades rotated with an angular velocity Ω). A *steady* reference frame solves this problem; then, it is better to work in a relative frame which is stationary with respect to the blades. Bearing in mind that a blade has only tangential velocity because it does not move in the axial nor radial plane (see Figure 2-b), the tangential relative velocity is simply the absolute (c_θ) minus the local ($U = \Omega r$) velocities. Therefore, the relative components are:

$$w_\theta = c_\theta - U = c_\theta - \Omega r \quad w_x = c_x \quad w_r = c_r \tag{4}$$

and the relative flow angle is:

$$\tan\beta = \frac{w_\theta}{c_m} = \frac{c_\theta - U}{c_m} = \tan\alpha - \frac{U}{c_m} \tag{5}$$

Having clear the different reference frames, it is introduced above the fundamentals laws used in turbomachinery:

1. Continuity of flow equation: $\mathrm{d}\dot{m} = \frac{\mathrm{d}m}{\mathrm{d}t} = \rho c \mathrm{d}A_n$, where A_n is the element area normal to the flow direction. Doing a steady 1-D analysis, $\dot{m} = \rho_1 c_1 A_{n1} = \rho_2 c_2 A_{n2} = \rho c A_n$.

2. The First Law of Thermodynamics states that the energy change within a system from state 1 to state 2 is $\mathrm{d}E = \mathrm{d}Q - \mathrm{d}W$, where $E = U + \frac{1}{2}mc^2 + mgz$, $\mathrm{d}Q$ is the heat supplied to the system, $\mathrm{d}W$ is the work done by the system and U the internal energy.

3. Çencel and Boles demonstrated how the previous law can be applied to a control volume like the one above:

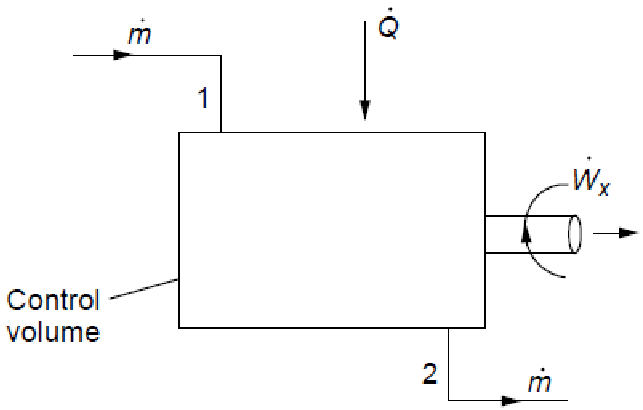


Figure 3: Control Volume of a turbomachine that is supplied with heat and there is a work done by the system

Energy is supplied from the fluid to the blade at a rate of $\dot{W}_x$ and heat is transferred from the surroundings to the control volume at $\dot{Q}$. And noting that the internal energy is related with the change of absolute enthalpy, the First Law of Thermodynamics can be written down as:

$$\dot{Q} - \dot{W}_x = \dot{m}\left[(h_2 - h_1) + \frac{1}{2}(c_2^2 - c_1^2) + g(z_2 - z_1)\right] =$$
$$= \dot{m}\left[(h_{o2} - h_{o1}) + g(z_2 - z_1)\right]$$

where h_o is the stagnation enthalpy $h_o = h + 1/2c^2$. Most machineries are adiabatic ($\dot{Q} = 0$) and the gravitation potential energy is usually ignore because it is small. As a consequence of these assumptions, $\dot{W}_x = \dot{m}\Delta h_o = h_{o1} - h_{o2}$. In the case of a turbine, the work is produced by the turbomachine and then $\dot{W}_x = \dot{W}_t$ but, in a compressor, the work is absorbed: $\dot{W}_c = -\dot{W}_x$.

4. From momentum balance, it is obtained the so-called *Euler work Equation*[1], which states that $h_o = Uc_\theta$ or $\Delta h_o = \Delta(Uc_\theta)\Delta W_x$. This equation is valid for steady problems, viscid and inviscid fluid. From this relation, the *rothalpy* is defined as $I = h_o - Uc_\theta$. This fluid parameter (that it is constant along a streamline) is used in the study of the flow within a rotating frame. Making use of the Equation 2 and rothalpy definition, it can be shown that $I = h_{o,\text{rel}} - \frac{1}{2}U^2$ and it states that, if the radius and Ω are constant (that is, blade speed U is constant), the relative stagnation enthalpy (which is equal to $h_{o,\text{rel}} = h + \frac{1}{2}w^2$) is constant since I is also constant.

Apart from these fundamental laws, it is important also to recall the Bernoulli's Equation whose demonstration is out of the scope of this project. For an adiabatic flow, no work transfer and incompressible:

$$\int_1^2 \frac{1}{\rho}\mathrm{d}p + \frac{1}{2}(c_2^2 - c_1^2) + g(z_2 - z_1) = 0 = \frac{1}{\rho}(p_{o2} - p_{o1}) + g(z_2 - z_1) = 0 \qquad (6)$$

where p_o is the stagnation enthalpy and it is defined as $p_o = p + 1/2\rho c^2$.

Furthermore, let us introduce few compressible flow relations:

- Mach Number: $M = \frac{c}{a} = \frac{c}{\sqrt{\gamma RT}}$. To have a compressible flow, in a turbomachine the Mach number exceed about 0.3. As a result, the density is no longer constant. And nowadays to reach these conditions in inevitably because of high power needed.

- Static and stagnation quantities (assuming perfect gas condition):

$$\frac{T_o}{T} = 1 + \frac{\gamma - 1}{2}M^2$$

$$\frac{p_o}{p} = \left(\frac{T_o}{T}\right)^{\gamma/(\gamma-1)}$$

[1] The demonstration of this equation is out of the scope of this project; further information can be seen in Ref ??

and,

$$\frac{\rho_o}{\rho} = \left(\frac{T_o}{T}\right)^{1/(\gamma-1)}$$

. Applying isentropy between two arbitrary points (1 and 2),

$$\frac{p_{o2}}{p_{o1}} = \left(\frac{T_{o2}}{T_{o1}}\right)^{\gamma/(\gamma-1)}$$

- Another important parameter is the non-dimensional flow rate (or *capacity*).

$$\dot{m} = \frac{\rho_o}{\left(1+\frac{\gamma-1}{2}M^2\right)^{1/(\gamma-1)}} M \sqrt{\gamma R T_o \frac{1}{\left(1+\frac{\gamma-1}{2}M^2\right)}} A_n =$$

$$= \frac{\frac{p_o}{RT_o}}{\left(1+\frac{\gamma-1}{2}M^2\right)^{1/(\gamma-1)}} M \sqrt{\gamma R T_o \frac{1}{\left(1+\frac{\gamma-1}{2}M^2\right)}} A_n$$

and arranging the above expression (and noting that $C_p/C_v = \gamma$ and $C_p - C_v = R$), the final result is:

$$\frac{\dot{m}\sqrt{C_p T_o}}{A_n p_o} = \frac{\gamma}{\sqrt{\gamma-1}} M \left(1 + \frac{\gamma-1}{2} M^2\right)^{-\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)}$$

All these expressions are written in terms of the local Mach Number. And, moreover, they can be also expressed as a function of the relative Mach Number.

The equations showed in this sub-section are of importance for the user in order to calculate the inputs of the blade geometry tool.

5.2 Mean Line Design

For the Mean Line Design, it is assumed that the flow conditions at a mean radius, called *pitchline*, represents the flow at all radii. The approximation is reasonable whenever the ratio between the mean radius and height of the blade is small. If this ratio is large, then *three-dimensional analysis* must be performed.

5.2.1 Axial-Flow Turbines

In the next figure, Figure 4, it can be observed a turbine stage. It is composed by a fixed guided vanes row, nozzles or *stator* and a moving blades row, buckets or *row*. Fluid enters at a flow angles α_1 with an absolute velocity of c_1, and then it accelerates up to c_2 and exits from the stator row at an angle α_2. The rotor inlet *relative* velocity is w_2 and the angle is β_2. These values can be calculates as explained in previous sections. And then, through the rotor, the fluid accelerates to a relative velocity w_3 at an angle β_3. Drawing the velocities as below is very helpful because it prevents errors. Note that the absolute velocities are aligned with the absolute flow angles; and the relative ones with the relative angles.

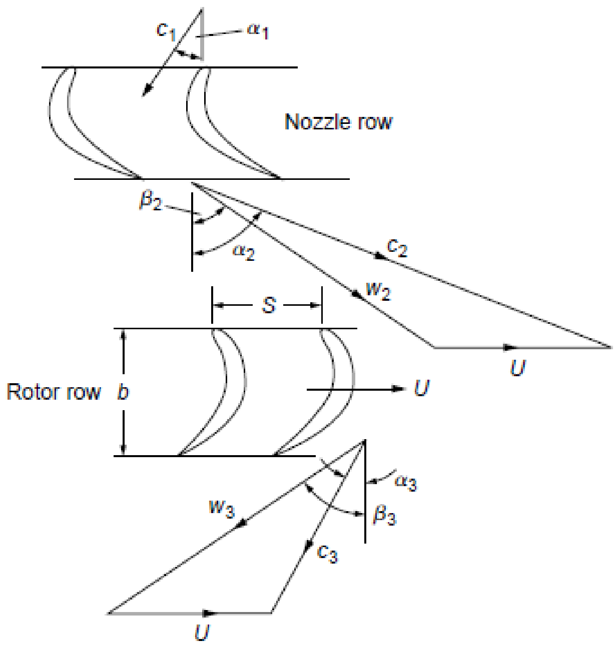


Figure 4: Velocity Triangle of a turbine. Note that all angles are measures with respect to the x angle

There are two main design parameters of a turbine stafe: design flow coefficient (ϕ) and stage loading coefficient (ψ). Following a dimensional analysis process, it can be shown these parameters are equal to:

$$\phi = \frac{c_m}{U} = \frac{c_x}{U} \text{ for purely axial turbine and } \psi = \frac{\Delta h_o}{U^2} = \frac{\Delta c_\theta}{U} \tag{7}$$

Note that for the stage loading coefficient, it has been assumed that Euler equation's conditions are hold. The value of ϕ is related with the stagger angles (parameters that is defined in further sections). High values imply lower stagger angles, and an increment in mass flow (because of continuity equation). And, regarding the stage loading, high values implies large turnings since there is an increment in the tangential component of the velocity.

Another important parameter that the user has to take into account is the stage reaction (R), and is defined as the ratio between the static enthalpy drop in the rotor to the static enthalpy drop across the stage:

$$R = \frac{h_2 - h_3}{h_1 - h_3} \approx \frac{p_2 - p_3}{p_1 - p_3} \tag{8}$$

Note that in the above equation it has been considered that the flow is almost isentropic and the compressibility effects are not taken into account.

In order to have high power and high efficiency turbines, multi stages axial turbines are required. In this kind of configuration, it is desired to have similar velocity triangles in all the stages. Regarding the flow, it expands as it passes through the stages and, as a result, the density of the fluid is reduced. To have similar velocity triangle, the mean radius of the blade has to be almost constant, the axial velocity has to be kept constant and the flow angle at the exit of the stages must be the same as the inlet. Applying these assumptions to the previous parameters explained, useful relationships are obtained that are important for the preliminary design of the turbine.

$$R = \frac{h_2 - h_3}{h_1 - h_3} = \frac{h_2 - h_1 + h_1 - h_3}{h_1 - h_3} = 1 - \frac{h_1 - h_2}{h_{o1} - h_{o3}}\text{[2]} \tag{9}$$

$$h_1 - h_2 = (h_{o1} - h_{o2}) + \frac{1}{2}(c_2^2 - c_1^2) = \frac{1}{2}c_x^2(\tan^2\alpha_2 - \tan^2\alpha_1)\text{[3]} \tag{10}$$

and combining the previous equation with the stage loading definition ($h_{01} - h_{02} = U^2\psi$) and flow coefficient ($\phi = c_x/U$), the reaction definition is then

$$R = 1 - \frac{1}{2}\frac{c_x^2(\tan^2\alpha_2 - \tan^2\alpha_1)}{U^2\psi} = 1 - \frac{\phi^2}{2\psi}(\tan^2\alpha_2 - \tan^2\alpha_1) \tag{11}$$

Equation 11 can be further simplified since the stage loading can be written as follows:

$$\psi = \frac{\Delta c_\theta}{U} = \frac{c_{\theta 2} + c_{\theta 3}}{U} = \frac{c_x(\tan\alpha_2 + \tan\alpha_3)}{U} = \psi(\tan\alpha_2 + \tan\alpha_1) \tag{12}$$

Substituting this in Equation 11,

$$R = 1 - \frac{\phi}{2}(\tan\alpha_2 - \tan\alpha_1) \tag{13}$$

Equation 12 can be used to eliminate α_2 from Equation 13:

$$R = 1 - \frac{\psi - \phi\tan\alpha_1}{2} + \frac{\phi}{2}\tan\alpha_1 = 1 - \frac{\psi}{2} + \phi\tan\alpha_1 \Longrightarrow \psi = 2(1 - R + \phi\tan\alpha_1) \tag{14}$$

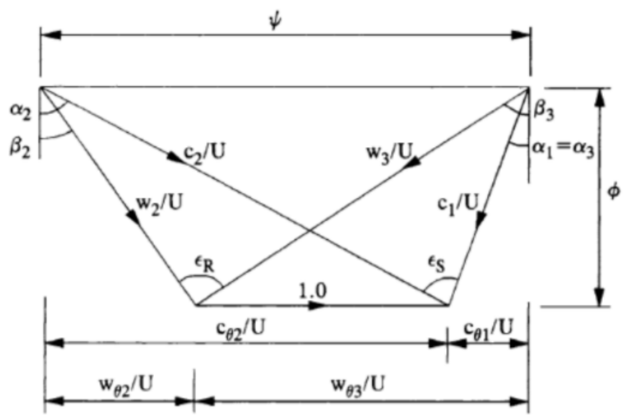


Figure 5: Dimensionless Velocity Triangle of a single turbine stage

[2] $h_{o1} - h_{o3} = h_1 - h_3$ because the inlet and outlet velocities are equal since their flow angles are the same

[3] Through the stator, there is no work done, then the stagnation enthlapy from 1 to 2 is constant

From this velocity triangle, $\tan\alpha_2 = \frac{c_{\theta 2}/U}{\phi} = \frac{w_{\theta 2}/U+1}{\phi} = \tan\beta_2 + \frac{1}{\phi}$ and similarly, $\tan\alpha_1 = \tan\beta_3 - \frac{1}{\phi}$. Substituting these expressions into Equation 13:

$$R = \frac{\phi}{2}(\tan\beta_3 - \tan\beta_2) \tag{15}$$

The above expressions shows that for a high stage loading, the reaction should be low and the swirl angle α_1 should be large. It is interesting to have a dimensionless velocity triangle. This is basically obtained by diving all the parameters of Figure 4 by U, as it can be observed in the previous figure.

Havakechian and Greim summarised the advantages and disadvantages of two extreme turbine designs: zero and 50% reactions. While for the first one the blades are different for stator and rotor, for the second style, they are almost of the same shape. $R = 0$ has as advantage the fact that the Stage Loading, ψ, is large even if the inlet flow angle, α_1 is low (Equation 14). But also this implies an early flow separation resulting in a reduction of the turbine efficiency. $R = 0$ means that the pressure drop through the rotor $(p_2 - p_3)$ is small and as a consequence the tip leakage is reduced.

A 50% Reaction Stage implies firstly a symmetric velocity triangle (as it can be get from the above equations). Consequently, the blade shapes for the rotor and stator are similar, as said previously, and this implies the cost is therefore reduced. Moreover, the flow is highly accelerated and this brings to a loss reduction. All these behaviours together with the flow expansion make the 50% Reaction turbine to work over a range of conditions. By contrast, the work per stage is lowered and as a consequence more stages are needed. In addition, the greater expansion leads to leakage losses.

It is important to talk also about *Diffusion*. In turbines it is not desired because it is accompanied by adverse pressure gradients and hence flow separation. This phenomenon is also helped by the fact that deflection of the blade row is large. Then, negative R drives to a rotor diffusion since $w_3 < w_2$ (Equation 15) and $R > 1$ goes along with stator diffusion because the absolute velocity is reduced ($c_2 < c_1$ - Equation 13).

To sum up, the designer can fix ϕ, ψ and R (e.g., ϕ, ψ and R that minimize losses and maximize the efficiency of the turbine stage), therefore, the velocity triangle is obtained thanks to these parameters and, as a result, it obtains the inputs for the blade shape.

5.2.2 Axial-Flow Compressors

For these turbomachines it is going to be followed the same procedure as done in the previous section with turbines. As before, are not considered into account three-dimensional effects. On contrary with respect to turbines, the layout of a compressor stage is firstly composed by a rotor row and then by a stator one.

Compressors have usually guide vanes at the inlet of the stage whose main function is to accelerate the flow, not to diffuse it as the rotor and stator do. Then, it is not considered as part of the compressor stage.

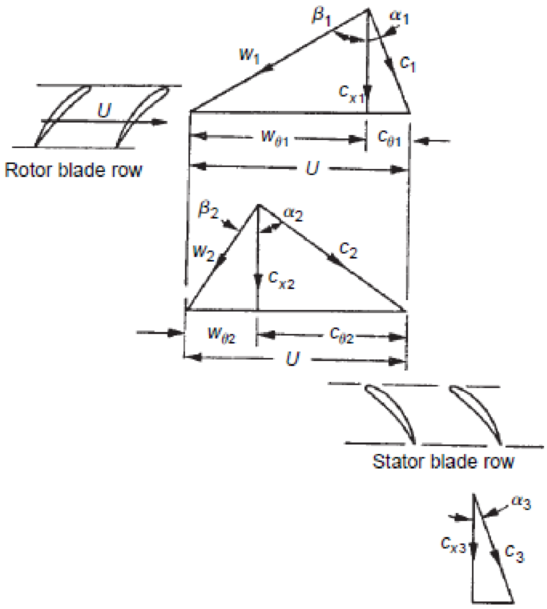


Figure 6: Velocity Triangle of a compressor stage

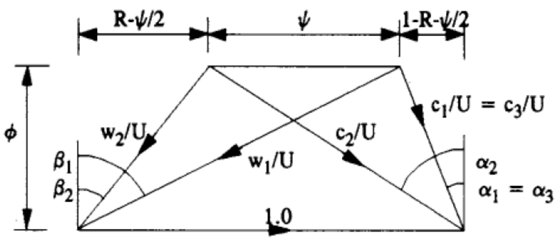


Figure 7: Non-dimensional Velocity Triangle of a compressor stage

The flow enters with a velocity of c_1 and at α_1 from a previous stage or from the guide vanes. Subtracting vectorially the blade speed U, the relative inlet velocity w_1 and the angle β_1 are obtained. Relative to the rotor row, the flow is turned from β_1 to β_2 with a relative speed of w_2. By adding vectorially the blade speed, the absolute parameters at point 2 are obtained; and then through the stator, the flow is deflected from α_2 to α_3 and the outlet velocity (which is the same as the inlet stage velocity because of repeating stage) is c_3.

As it happens with turbines, setting the flow coefficient ϕ, the stage loading ψ and reaction R, all the parameters from the velocity triangle can be calculated. Of course, these values have to be chosen in such a way that there is an adequate stability margin. It is important to not to enter in *stall* or *surge*.

The main function of the compressor blades is to slow down the flow, that is, to diffuse the fluid. This diffusion is limited; boundary layer separation may appear. And to not fall into these unacceptable regime, it has to be limited the pressure rise and the maximum stage loading. The stage loading is defined as

$$\psi = \frac{\Delta h_0}{U^2} = \frac{h_{03} - h_{01}}{U^2} = \frac{\Delta c_\theta}{U} = \frac{c_{\theta 2} - c_{\theta 1}}{U} = \frac{c_x}{U}(\tan\alpha_2 - \tan\alpha_1) = \phi(\tan\alpha_2 - \tan\alpha_1) \tag{16}$$

Note that this equation can be written down in terms of the rotor relative angles, instead of the absolute ones (by inspecting the velocity triangle Figure 6):

$$\psi = \phi(\tan\beta_1 - \tan\beta_2) = 1 - \phi(\tan\alpha_1 + \tan\beta_2) \tag{17}$$

As said before, the chosen value for the flow coefficient is limited, which is about 0.4. A low value implies a large number of stages and a high one it leads to flow separation. The limitation is set by Lieblein's diffusion factor, DF, which establishes the blade pitch-chord ratio (s/l) needed for acceptable performance as a function of the flow turning (Δc_θ) and the rotor relative speeds:

$$DF = \left(1 - \frac{w_2}{w_1}\right) + \frac{\Delta c_\theta}{2w_1}\frac{s}{l} \tag{18}$$

Getting back to Equation 17, it can be observed that, if ψ is fixed, diffusion (or flow turning) is reduced as the flow coefficient increases. On the other hand, if diffusion is kept constant, stage loading increases with flow coefficient. Therefore, it may be straight forward that large values for the flow coefficient are desired. And large flow coefficients means large flow masses that enters in the machine which is a significant advantage. However, compressors need to remain stable and higher ϕ are related with chocking and shock waves, phenomena that are not desired. In order to not avoid this, ϕ normally goes from 0.4 to 0.8.

Last duty parameter is the reaction whose expression is

$$R = \frac{h_2 - h_1}{h_3 - h_1} = \frac{h_2 - h_1}{h_{03} - h_{01}} = \frac{1/2(w_1^2 - w_2^2)}{2U(c_{\theta 2} - c_{\theta 1})}{}^{4} \tag{19}$$

Noting that $w_1^2 - x_2^2 = w_{\theta 1}^2 + c_x^2 - w\theta 2^2 - c_x^2 = w_{\theta 1}^2 - w_{\theta 2}^2 = (w_{\theta 1} + w_{\theta 2})(w_{\theta 1} - w_{\theta 2}) = (w_{\theta 1} + w_{\theta 2})(c_{\theta 1} - c_{\theta 2})$, the above expression is simplified as

$$R = \frac{w_{\theta 1} + w_{\theta 2}}{2U} = \frac{1}{2}\phi(\tan\beta_1 + \tan\beta_2) \tag{20}$$

or, since $w_{\theta 1} = c_{\theta 1} - U$, another version of the Equation 20 is

$$R = \frac{1}{2} + \frac{1}{2}\phi(\tan\beta_2 - \tan\alpha_2) \tag{21}$$

From this equation, it can be solved for $\phi\tan\beta_2$ ($\phi\tan\beta_2 = 2R - 1 + \phi\tan\alpha_1$) and substitute in Equation 17 resulting in

$$\psi = 2(1 - R - \phi\tan\alpha_1) \tag{22}$$

[4]Note that through the rotor the relative stagnation enthalpy is kept constant and Euler's equation is applied

According to Cumpsty and its parametric design studies, the reaction is not a critical parameter for the design of compressor stages since it can be determined by the other duty parameters (ϕ and ψ) if the inlet swirl is fixed, for example. If $R = 50\%$, as it happens with turbines, stator and rotor have similar blade shapes and also the pressure gradient is equally shared between stator and rotor. Normally, for jet engines high reaction (between 0.5 and 0.8) are preferred for compressors.

Another interesting fact to take into account when designing the blade shape of a compressor is that positive inlet swirl (in accordance with sign criterion stated in Figure 6) helps to reduce the loading of the stage. Typically, the inlet swirl angle is between 20 and 30 degrees.

5.2.3 Smith chart for axial turbine and compressor stages

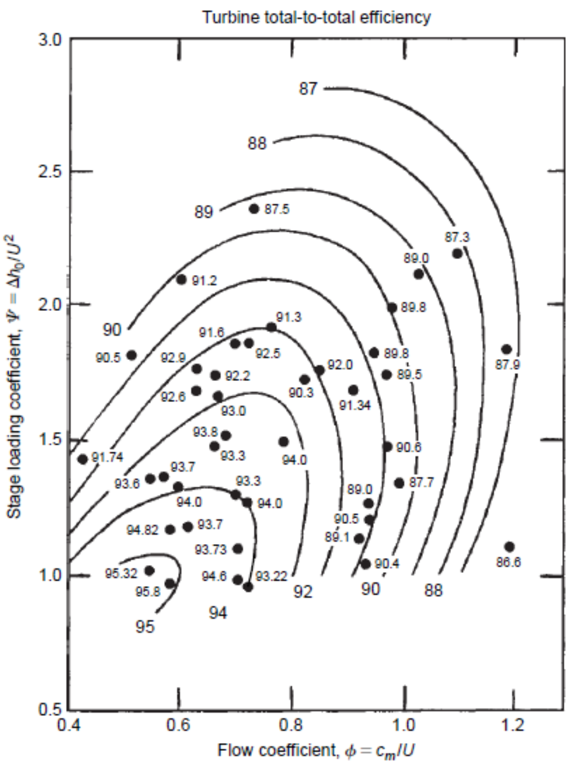


Figure 8: Smith Chart for Turbine Stage Efficiency

There are large database of measured efficiencies for axial-flow turbomachines as a function of ϕ and ψ, the duty parameters. Smith was in charge of created a correlation between the efficiency and the duty parameters for turbines, and it is known as *Smith Chart*. Regarding this chart, the experiments where performed for R that were between 0.2 and 0.6. The points of the chart (Figure 8) represent one single test turbine at its best efficiency. Note also that the efficiencies represented are *total-to-total*. This, basically means:

$$\eta_{tt} = \frac{\text{actual work output}}{\text{ideal work output when operating to same back pressure}} = \frac{h_{01} - h_{03}}{h_{01} - h_{03ss}}{}^{5} \tag{23}$$

Apart from that, Smith developed a theoretical analysis to obtain the optimum stage load coefficient (ψ_{opt}), the stage load coefficient that minimizes blade losses at $R = 0.5$:

$$\psi_{\text{opt}} = \sqrt{4\phi^2 + 1} \tag{24}$$

Based on his analysis, Lewis developed a more accurate expression that included losses. He observed that the losses and hence efficiency are dependent on the product of two factors: blade row aerodynamics and by the velocity triangle environment (which is determined by the duty coefficients). And the experimental optimum flow coefficient is:

$$\psi_{\text{opt,exp}} = 0.65\sqrt{4\phi^2 + 1} \tag{25}$$

M. V. Casey published a performance prediction method for axial compressor. Additionally, he provided diagrams to those that are similar with 'Smith' charts for turbines. There are presented (in Figure 9) three charts for three different reactions: 50%, 70% and 90%.

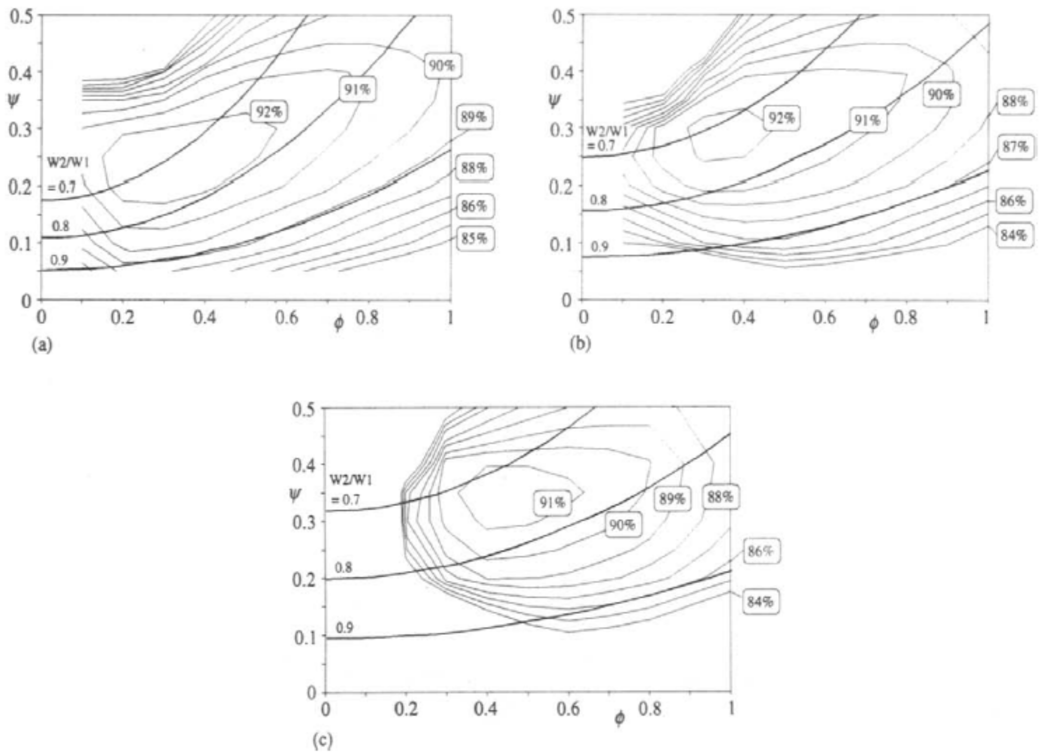


Figure 9: 'Smith' chart for axial compressor stages with a) R=50%, b) R=70% and c) R=90%

[5] h_{03ss} referes to the total enthalpy of the point 3ss which has the same isentropy the point 1 from the stage

As in turbines, there is are theoretical relations that give the optimum and the maximum stage loading as a function of flow coefficient:

$$\psi_{\text{opt}} = 0.185\sqrt{4\phi^2 + 1}$$
$$\psi_{\text{max}} = 0.32 + 0.2\phi$$

And typically, as already said in previous sections, ϕ takes values from 0.5 to 0.9 with work coefficients as great as $\psi = 0.4$ to 0.45.

5.3 Turbine and Compressor cascades

Axial turbomachines designers have treated the three-dimensional flow of these machines as a superposition of two-dimensional flows. This is better since it is easier to choose a blade design. Let us take the example of a turbomachine whose casing and hub are cylinders. Then, it can be assumed that the stream surfaces remain cylindrical in the turbomachine. To make the design process easier, one can project the blade shapes (for certain radial position r) along the a cylindrical blade-to-blade section in $x - r\theta$ coordinate. And finally, this can be transformed into a rectangular *cascade* plane. This unwrapping can be seen in the next figure:

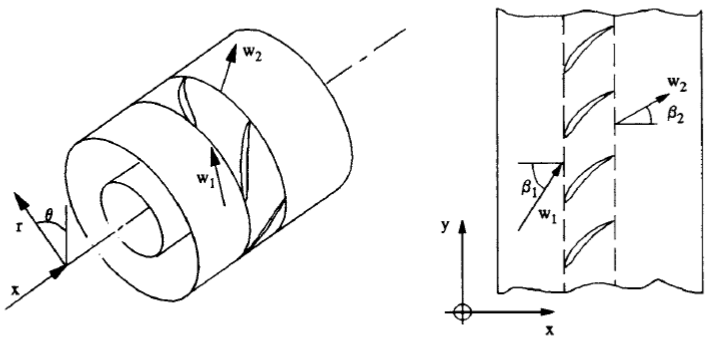


Figure 10: Development process of a cylindrical blade-to-blade section into a rectangular cascade plane

The main advantage of this method is that equations that were previously developed for axial compressors and turbines can be applied to each cascade section independently. In this way, one can find the velocity triangle for that particular blade section. So, the designer must fulfil when choosing the blade shape certain flow conditions such as a correct flow deflection and a smooth inlet flow around the leasing edge.

Since the dynamic analysis of a cascade is out of the scope of this project, let us describe the geometry of a blade section. In the next figure, there are presented the main geometric parameters of interest.

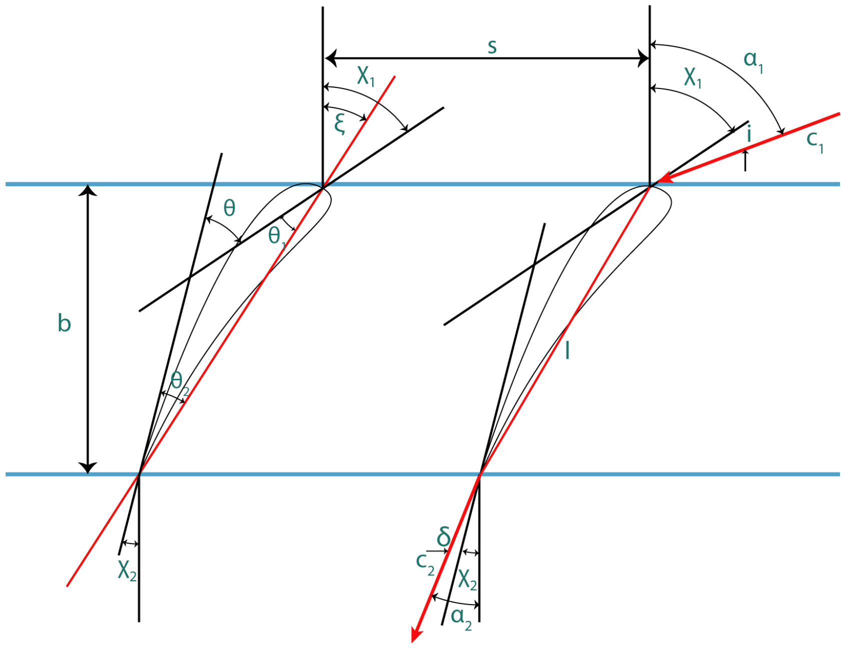


Figure 11: Compressor cascade

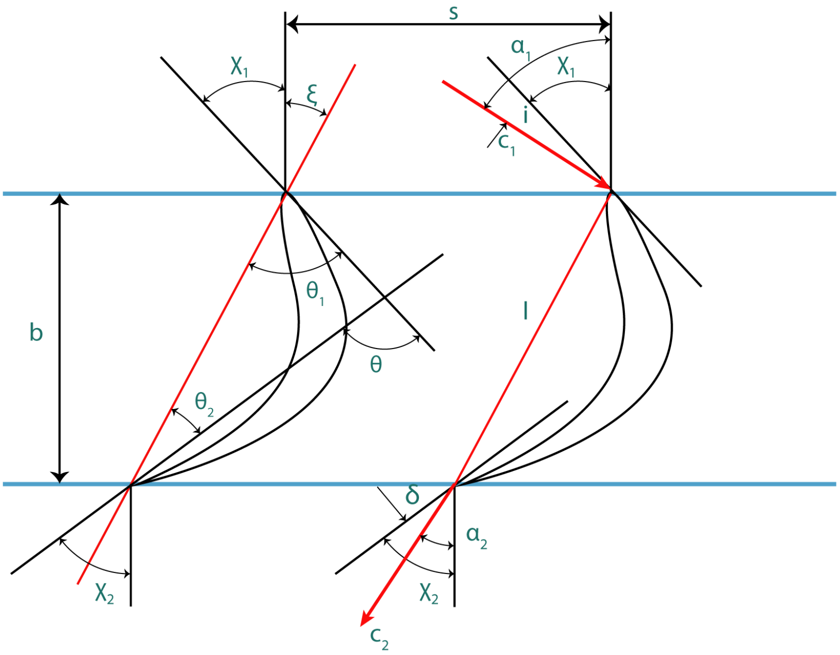


Figure 12: Turbine cascade

As already presented in previous section, sub-index 1 refers to inlet conditions and 2, to outlet conditions, and α (or β) represents the flow angle (or the relative one in case of a rotor) and it is oriented with the absolute (or relative) velocity c (or w in the case of the rotor). Regarding the geometric parameters, let us define ξ as the *stagger* angle and it is the orientation of the camber line with respect to the axial axis; χ is the metal or the blade angle and it is the angle between the axial axis and the camber line at the inlet (χ_1) and at the outlet (χ_2); θ_1 and θ_2 are the camber angle at the inlet and outlet sections, respectively, and, as observed above, it is the angle between the metal and the stagger; finally, the last parameters are the chord l, the pitch s (which is the spacing between two consecutive blade within a cascade) and the axial chord (the projection of the chord onto the axial axis - $b = l\cos\xi$). There are two angles that relate metal and flow angles: incidence i and deviation δ.

From Figures 11 and 12, there can be obtained the next geometric relationships:

- Turbine
  - $\alpha_1 = \chi_1 + i$
  - $\alpha_2 = \chi_2 - \delta$
  - $\chi_1 = \theta_1 - \xi$
  - $\chi_2 = \theta_2 + \xi$
  - $\theta = \theta_1 + \theta_2$ or $\theta = \chi_1 + \chi_2$
- Compressor
  - $\alpha_1 = \chi_1 + i$
  - $\alpha_2 = \chi_2 - \delta$
  - $\chi_1 = \theta_1 + \xi$
  - $\chi_2 = \xi - \theta_2$
  - $\theta = \theta_1 + \theta_2$ or $\theta = \chi_1 - \chi_2$

One important hint that the designer must take into account is that, in order to have properly designed the blade shapes of the axial turbomachines, the sign criteria established above must be fulfilled. So, all the camber angles (θ_1, θ_2 and θ) have to be positive.

In the later chapters, it is discussed how to obtain the blade shape as a function of these cascade parameters.

Include Mach s/l and o/l and include effect on the incidence angle

5.4 Introduction to Parametric graphs and Bézier curves

In computer graphics, it is convenient (because of its easiness) to use the so-called "parametric curves". Therefore, a curve is represented like a n grade polynomial that depends on a parameter t:

$$\vec{c}(t) = \vec{a_0} + \vec{a_1}t + \cdots + \vec{a_n}t^n \tag{26}$$

where the coefficients $\vec{a}_i$ are points of the plane and t is a parameters that goes from 0 to 1: from the starting point of the curve ($\vec{a}_0$) and the final point ($\vec{a}_0$).

This way of representing a curve has one important advantage that is its easiness (as said before) but its main drawback is the fact that coefficients behave in a complex way when, for example, one wants to rotate the curve or translate it. Therefore, it is convenient to use another type of representation instead of polynomial curves (bur following the same *philosophy*). And one of this type of computational representation of curves are *Bézier curves*.

Bézier curves are very used in computer graphics because they are smooth. In few words, they are constructed by a set of control points, and these points can be displaced in order to modify the plot. As previously mentioned, parametric curves can be transformed (namely, they can be rotated and/or rotated) and in the case of Bézier curves, one has just only to transform the control points properly in order to have the line transformed.

The degree of a Bézier curve is defined by the set of control points. If $\vec{c}(t)$ is only defined by $\vec{a}_0$ and $\vec{a}_1$, the curve is linear. If it is defined by three points, then it is quadratic. And the degree can be increased up to n. The computational complexity increases with n. In the next figure (Figure 13) , one can see these two types of curves in a 2D plane:

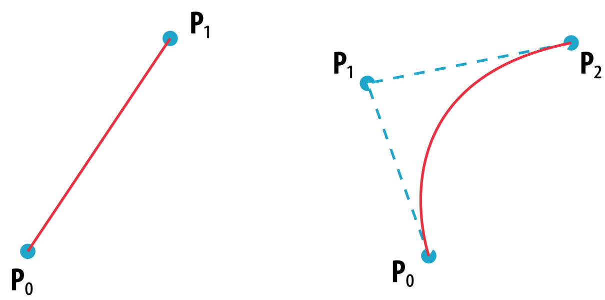


Figure 13: Two examples of Bézier curves. Left side - linear cuve; Right side - quadratic curve

Next figure shows how the curve is constructed depending on the value of the parameter t:

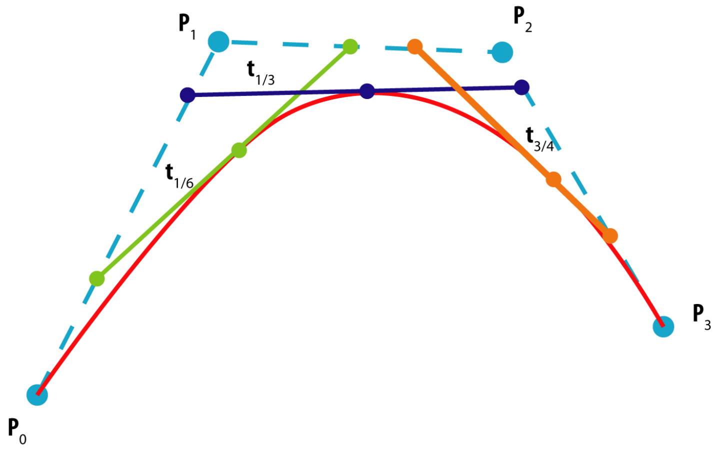


Figure 14: Building process of a parametric Bézier graph

For a linear curve, the line is expressed as a straight line:

$$\vec{c}(0) = P_0 + t(P_1 - P_0) = (1-t)P_0 + tP_1 \tag{27}$$

For higher orders of n, the process is more complex. Let us develop the Bézier curve for a quadratic curve. For this kind of lines, implicitly, the control point P_1 depends on the tangent line of the finishing points.

Making use of Equation 26 and defining $\vec{c}(t)$ as a $(x_B(t), y_B(t))$,

$$x_B(t) = a_{0x} + a_{1x}t + a_{2x}t^2$$

$$y_B(t) = a_{0y} + a_{1y}t + a_{2y}t^2$$

or in a matrix form:

$$\vec{c}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{Bmatrix} t^2 & t & 1 \end{Bmatrix} \begin{bmatrix} a_{2x} & a_{2y} \\ a_{1x} & a_{1y} \\ a_{0x} & a_{0y} \end{bmatrix} = TC_B = TM_BG_B \tag{28}$$

From Equation 28, T, G_B and M_B are defined as:

- $T = \begin{Bmatrix} t^2 & t & 1 \end{Bmatrix}$
- $G = \begin{bmatrix} P_{2x} & P_{2y} \\ P_{1x} & P_{1y} \\ P_{0x} & P_{0y} \end{bmatrix}$
- Matrix M_B is the unknown of the equation and it relates matrices G_B (which depends on the control points) and C_B (which is the coefficients matrix)

In order to determine M_B, let us state the next conditions:

1. $\vec{c}(0) = P_0 = \begin{Bmatrix} 0 & 0 & 1 \end{Bmatrix} M_BG_B$
2. $\vec{c}(1) = P_2 \begin{Bmatrix} 1 & 1 & 1 \end{Bmatrix} M_BG_B$

3. $R_1 = \frac{P_2-P_1}{t_2-t_1} = \frac{P_1-P_0}{1-1/2} = 2(P_2 - P_1) = \vec{c}\ '(1) = T'(1)M_BG_B = \begin{Bmatrix} 2 & 1 & 0 \end{Bmatrix} M_BG_B$

R_2 is defined as the tangent at $t = 1$, that is $\frac{d(P(t))}{dt}$. Note that at P_2, $t = 1$ and at P_1, $t = 1/2$. In a matrix form, these conditions are rewritten as follow:

$$G_H = \begin{Bmatrix} P_0 \\ P_2 \\ R_1 \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 2 & -2 & 0 \end{bmatrix} G_B = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 2 & 1 & 0 \end{bmatrix} M_BG_B \tag{29}$$

From this last equation, M_B can be calculated and it is equal to:

$$M_B = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 2 & -2 \\ 0 & 0 & 1 \end{bmatrix} \tag{30}$$

and since $\vec{c} = TM_BG_B$,

$$\vec{c} = (t-1)^2P_0 + 2t(t-1)P_1 + t^2P_2 \tag{31}$$

A similar procedure can be followed for a cubic Bézier curve. Apart from the initial and final point conditions, for cubic expression, it may be also note that (as similar for the quadratic form) $R(0) = 3(P_1 - P_0)$ and $R(1) = 3(P_2 - P_1)$. And doing similar computations as before, the Bézier equation is expressed as:

$$\vec{c}(t) = (1-t)^3P_0 + 3(1-t)^2tP_1 + 3(1-t)t^2 + t^3P_3 \tag{32}$$

If the process is doing for higher degrees, it can be observed Bézier functions have the shape of *Bernstein basis polynomial*[6] therefore, the explicit equation of order n is:

$$\vec{c}(t) = \sum_{i=0}^{n} \begin{pmatrix} n \\ i \end{pmatrix} (1-t)^{n-1}t^iP_i \tag{33}$$

From the practical point of view, only quadratic form of Bézier parametric graphs is the one that it is used for this work.

5.5 MISES

Hablar de: - Introducción y principios (uno o dos párrafos) - Flow chart de Mises y resumir un poco de qué va cada archivo - Hablar de los archivos ises y blade (y qué se modifica) y de las condiciones que se tienen en cuenta para compresor y turbina

[6]Bernstein basis polynomial is defined explicitly as $B_n(x) = \sum_{i=0}^{n} \beta_\nu \begin{pmatrix} n \\ \nu \end{pmatrix} x^\nu(1-x)^{n-\nu}$ and few examples of this polynomial basis are $b_{0,0} = 1$, $b_{0,1} = 1-x$, $b_{1,1} = x$, $b_{0,2} = (1-x)^2$, $b_{1,2} = 2x(1-x)$ and $b_{2,2} = x^2$, and so on

6 Blade profile geometry

Having in mind all the principles and fundamentals that were described in the previous chapter, in this section it is expose a way to get the blade shape. And this is further use for the dynamic analyses.

According to Lewis (Ref ??), there are two ways to design a blade for a compressor and a turbine. These approaches are called:

- *Direct (or analysis) method*: it is basically a geometrical technique. Generated cascades are further analysed by experimental test or theoretical approaches in order to find the performance desired.
- *Inverse (or synthesis) method*: this is an advanced method and it is the opposite than the previous approach; in this case, the user specifies the velocity and pressure distribution along the surface of the blade and then this method generates the blade profile from these distributions. These methods are often refereed to as *Prescribed Velocity Distribution* or *PVD* analysis.

It may be thought that inverse method is the one that gives the best results but in fact, from a practical point of view, direct method is widely used by engineers. And therefore, it is the one that was preferred for this work. In the next sections, it is explained how these profiles are obtained.

6.1 Direct Method

As explained previously, with this method, the engineer elaborate a cascade family and then it is analysed experimentally or by theoretical means (e.g. MISES software). There are distinguished two ways:

1. Related Cascade families: already analysed thickness families are used for the construction of the blade shapes
2. High parameters design (or *manual*): user has full control on the design by setting a large bunch of parameters

In some way, both algorithms are similar in the way that the software asks the user about certain inputs and, making use of geometrical techniques, the software gives as output the blade profile. It is important to recall the importance of having clear the dynamics explained in the previous sections because, in this way, the designing process is easier (??).

6.1.1 Related Cascade families

The procedure followed, as shown in the next flow chart (Figure 15), is:

1. Build the Camber Line

   (a) Ask the user for the camber line inputs: metal and stagger angles, χ_1, χ_2 and ξ, respectively. Metal angles are obtained with the velocity triangle and by setting certain incidence i and deviation δ as explained in previous chapters

(b) Build the Camber Line with Bézier Theory

2. Add perpendicularly (to the camber line) the thickness family. It is assumed that the thickness distribution is symmetric with respect to the camber line

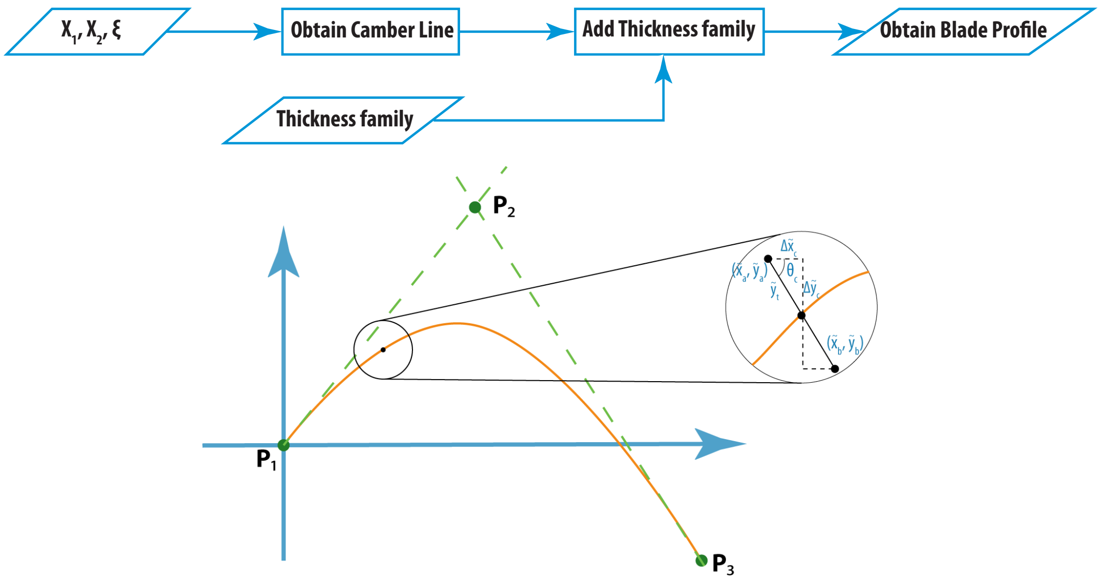


Figure 15: Direct method flow chart for a prescribed thickness family

As explained in Section ??, to obtain the camber line using Bézier method, there are needed three control point. To have in mind that these points are normalized with respect to the chord length:

1. First point is the initial point of the camber line. For instance, it can be the origin $\mathbf{P}_0 = (0, 0)$

2. Third point is the ending point of the camber. Since the stagger is set by the user, this point is easily get using the next expression:

$$\mathbf{P}_3 = (\cos\xi, \sin\xi)$$

Dimensionally, $\mathbf{P}_3$ is equal to

$$\mathbf{P}_3 = (l\cos\xi, l\sin\xi)$$

. Note that for compressor, according to the sign criteria defined in previous figures, is positive and then, for turbines is negatives. In Blade Designer next expressions are taken into consideration but, implicitly, the sign criterion is taken into account:

$$\mathbf{P}_3 = (\cos|\xi|, \sin|\xi|) \tag{34}$$

for compressors and

$$\mathbf{P}_3 = (\cos|\xi|, -\sin|\xi|) \tag{35}$$

3. The second control point is obtained from the metal angles and the other two control points:

$$P_{2x} = \frac{P_{3y} + P_{3x} \tan|\chi_2|}{\tan|\chi_1| + \tan|\chi_2|} \tag{36}$$

$$P_{2x} = \frac{P_{3y} - P_{3x} \tan|\chi_2|}{\tan|\chi_1| - \tan|\chi_2|} \tag{37}$$

and the y-coordinate of this control point is simply

$$P_{2y} = P_{2x} \tan|\chi_1| \tag{38}$$

Once the control points are properly calculated, the camber line is obtained by substituting these coordinates in Equation 32.

Regarding the thickness families, already studied databases are employed. Blade Designer include several thickness families. They are taken from Lewis book:

c4

$\tilde{x}_t$	0.00	1.25	2.50	5.00	7.50	10.00	15.00	20.00	30.00	40.00	50.00
$\tilde{y}_t$	0.00	1.65	2.27	3.08	3.62	4.02	4.55	4.83	5.00	4.89	4.57

$\tilde{x}_t$	60.00	70.00	80.00	90.00	95.00	100.00
$\tilde{y}_t$	4.05	3.27	2.54	1.60	1.06	0.00

NACA 0012

$\tilde{x}_t$	0.00	1.25	2.50	5.00	7.50	10.00	15.00	20.00	25.00	30.00
$\tilde{y}_t$	0.00	1.894	2.615	3.555	4.200	4.683	5.345	5.737	5.941	6.002

$\tilde{x}_t$	40.00	50.00	60.00	70.00	80.00	90.00	95.00	100.00
$\tilde{y}_t$	5.803	5.294	4.563	3.664	2.623	1.448	0.807	0.00

NACA 0015

$\tilde{x}_t$	0.00	1.25	2.50	5.00	7.50	10.00	15.00	20.00	25.00	30.00
$\tilde{y}_t$	0.000	2.367	3.268	4.443	5.250	5.853	6.682	7.172	7.427	7.502

$\tilde{x}_t$	40.00	50.00	60.00	70.00	80.00	90.00	95.00	100.00
$\tilde{y}_t$	7.254	6.617	5.704	4.580	3.273	1.810	1.008	0.000

NACA 66-010

$\tilde{x}_t$	0.00	0.50	0.75	1.25	2.50	5.00	7.50	10.00	15.00	20.00
$\tilde{y}_t$	0.000	0.759	0.913	1.141	1.516	2.087	2.536	2.917	3.530	4.001
$\tilde{x}_t$	25.00	30.00	35.00	40.00	45.00	50.00	55.00	60.00	65.00	70.00
$\tilde{y}_t$	4.363	4.636	4.832	4.953	5.000	4.971	4.865	4.665	4.302	3.787

$\tilde{x}_t$	75.00	80.00	85.00	90.00	95.00	100.0
$\tilde{y}_t$	3.176	2.494	1.773	1.054	0.408	0.000

NGTEmod

$\tilde{x}_t$	0.00	1.25	2.50	5.00	7.50	10.00	15.00	20.00	30.00	40.00
$\tilde{y}_t$	0.000	1.375	1.910	2.680	3.195	3.600	4.180	4.550	4.950	4.820

$\tilde{x}_t$	50.00	60.00	70.00	80.00	85.00	90.00	92.50	95.00	97.50	100.0
$\tilde{y}_t$	3.980	3.250	2.450	1.740	1.500	1.270	1.170	1.080	0.980	0.000

Table 1: Sample of base profile thickness used in Blade Designer for compressors

T4

$\tilde{x}_t$	0.00	1.25	2.50	5.00	7.50	10.00	15.00	20.00	30.00	40.00	50.00
$\tilde{y}_t$	0.00	1.17	1.54	1.99	2.37	2.74	3.40	3.95	4.72	5.00	4.67

$\tilde{x}_t$	60.00	70.00	80.00	90.00	95.00	100.00
$\tilde{y}_t$	3.70	2.51	1.42	0.85	0.72	0.00

A3K7

$\tilde{x}_t$	0.00	1.25	2.50	5.00	10.00	15.00	20.00	25.00	30.00	35.00
$\tilde{y}_t$	0.000	3.469	4.972	6.918	9.007	9.827	10.000	9.899	9.613	9.106

$\tilde{x}_t$	40.00	45.00	50.00	55.00	60.00	65.00	70.00	75.00	80.00
$\tilde{y}_t$	8.594	7.913	7.152	6.339	5.500	4.661	3.848	3.087	2.406

$\tilde{x}_t$	85.00	90.00	95.00	100.0
$\tilde{y}_t$	1.830	1.387	1.101	0.000

Table 2: Sample of base profile thickness used in Blade Designer for turbines

In Tables 1 and 2, $\tilde{x}_t$ is the non-dimensional axial coordinate (represented in %) and it is defined as x_t/l; similarly, $\tilde{y}_t$ is defined as y_t/l and it is represented in %.

Furthermore, the *resolution* (that is, the number of points) of these samples is low therefore these distributions must be interpolated. In order to have a proper analysis in MISES, it may be interesting to approximate the inlet of the sample profile elliptically:

$$\tilde{x}_i = a - a\cos\phi \qquad \tilde{y}_i = b\sin\phi \tag{39}$$

where a is the semi-major axis (which in this case $\tilde{x}_t(2)$), b is half of the minor axis ($\tilde{y}_t(2)$) and ϕ goes from 0 to $1/2\pi$.

Once the thickness profile is chosen by the user (depending on if a compressor or a turbine blade is derided), these values are added perpendicularly to the camber line in order to obtain the final blade profile. Therefore, the upper blade profile is determined by this general expression:

$$\tilde{x}_a = \tilde{x}_c - \tilde{y}_t\cos\theta_c \qquad \tilde{y}_a = \tilde{y}_c + \tilde{y}_t\sin\theta_c \tag{40}$$

and for the lower side of the blade

$$\tilde{x}_a = \tilde{x}_c + \tilde{y}_t\cos\theta_c \qquad \tilde{y}_a = \tilde{y}_c - \tilde{y}_t\sin\theta_c \tag{41}$$

In Equations 40 and 41, θ_c angle is defined (in concordance Figure 15) as:

$$\tan\theta_c = \frac{\mathrm{d}\tilde{y}_c}{\mathrm{d}\tilde{x}_c} \approx \frac{\Delta\tilde{y}_c}{\Delta\tilde{x}_c} \tag{42}$$

Although in this section only few thickness families are specified, the software Blade Designer has the option that the user can introduce more samples. After that, the tool generates all needed files.

As a last remark, once the blade profile is obtained, for MISES purposes, it should be interpolated again but with much less points, 300 for instance because of MISES limitations. It is interesting to develop a method with which the interpolation is performed depending on the curvature of the profile. Lewis (Ref ??) proposes the next one:

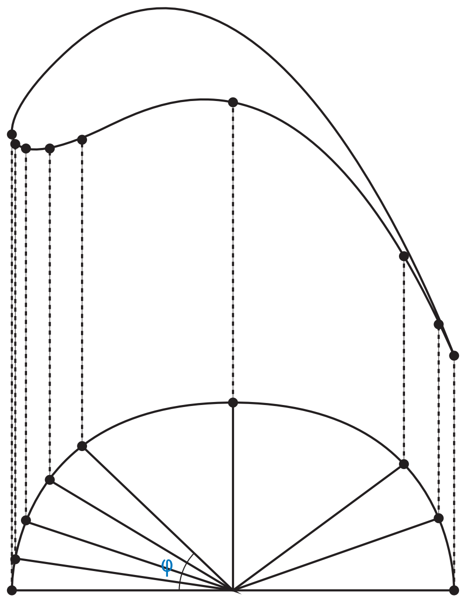
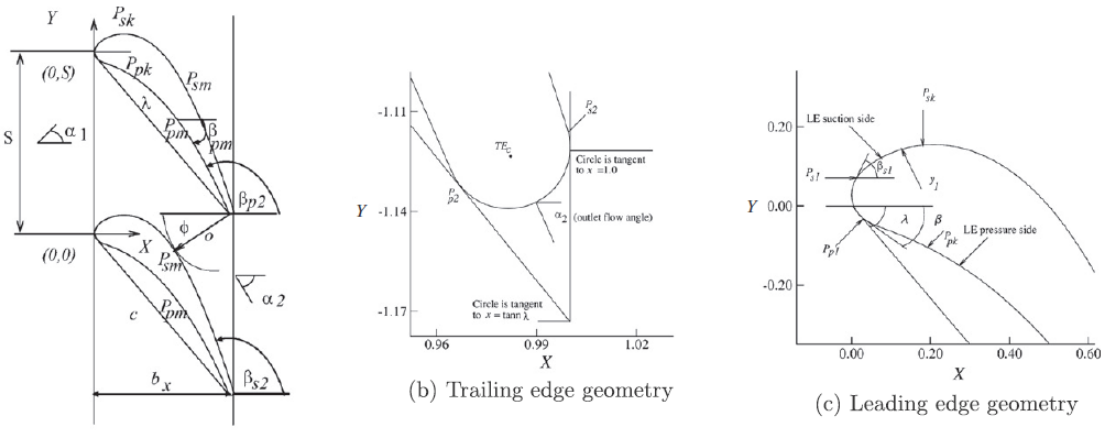


Figure 16: Interpolation method depending on the curvature of the profile

As it can be depicted on Figure 16, angle ϕ is defined as π/N, where N is an arbitrary number of points ($N = 150$ for example). And, according to this plot, the projection of the $\cos\phi$ over the blade profile is less spaced in the area where the curvature is more pronounced as in the area where it is not. The interpolation x-coordinates, therefore, is defined as $x_{\text{interpolation}} = l - l\cos\phi$ and the y-coordinates are obtained by using, for instance, MATLAB interpolation functions. Although this method is defined in this section, it is used in the *Higher parameters design* method.

6.1.2 Higher parameters design

In this section it is shown the methodology followed in this option of the blade design tool. And, in this case, there not any prescribed thickness family. Therefore, more inputs are needed to be asked to the user, apart from the metal angles (χ_1 and χ_2) and the stagger (ξ).
This method is based on the one proposed by T. Korakianitis, I.A. Hamakhan, M.A. Rezaienia, A.P.S. Wheeler and E.J. Avital, J.J.R. Williams (Ref ??) and it is named as CIRCLE which stands for *presCribed suRface Curvature distribution bLade dEsign*. It basically consists on a tool that joins the trailing edge and the leading edge circles or ellipses by means of a set of curves that are continuous between these areas.



(a) Main blade geometry (b) Trailing edge geometry (c) Leading edge geometry

Figure 17: CIRCLE blade geometry definition

As it can be seen in Figure 17, there are a set of variables that fully define the blade geometry. These parameters are grouped into a) Main blade geometry parameters, b) TE geometry and c) Leading edge geometry. Key variables are the LE and TE radii (in case if they are assumed to be circles), but their shapes can be generalised to elliptic ones; flow angles (or the metal - to be kept in mind the relationships between the metal and flow angles); the stagger; and finally the pitch of the cascade. As already know, the blade shape is defined by a suction and a pressure surface. In order to define the upper surface (and as depicted from this figure) (**hablar de esto en la geometría de una blade!!**), it is needed to take into account the throat o and its angle angle ϕ. These two parameters, from a physical point of view,define the minimum area flow passage along the turbomachine blade. The throat is the distance from the point of the suction surface where the Mach Number is 1 to the TE of the previous stage (**Recordad: incluir esta última frase en la sección de dinámica de una cascada!!**). Once this suction surface control point is obtained, arbitrarily, user defines the pressure surface control point.At TE, the blade shape is defined by a circle or an ellipse. The joining points between the mean and TE geometry are P_{s2} and P_{p2}, parameters set arbitrarily by the designer. And, similarly, the LE is defined. Likewise to the tool developed in this project, CIRCLE uses parametric graphics such as Bézier splines or NURBS, which are more advanced methods. **Hablar de por qué el throat se define en esas zonas**

A related technique it has been developed in the following way. The geometry is defined, as above, in three sections: Leading Edge, Trailing Edge and the Main geometry which is based on the Bézier theory and creates the curves that join LE and TE. Firstly, the mean camber line is obtained as explained in previous sections based on the metal and stagger angles. Moreover, the LE is defined by an ellipse and, therefore, the semi-axes a and b are introduced by the user. Further, this ellipse is rotated counter-clockwise (for both, compressor and turbine) at the inlet metal angle:

$$\begin{Bmatrix} x_i \\ y_i \end{Bmatrix} = \begin{bmatrix} \cos|\chi_1| & -\sin|\chi_1| \\ \sin|\chi_1| & \cos|\chi_1| \end{bmatrix} \begin{Bmatrix} x_j \\ y_j \end{Bmatrix} \tag{43}$$

where x_j and y_j are the coordinates with zero metal inlet angle and x_i and y_j are the coordinates after rotating the ellipse. Apart from that, the joining points between the mean and LE geometries are limited by two angles $\phi_{\mathrm{LE,SS}}$ (for the suction surface) and $\phi_{\mathrm{LE,PS}}$ (for the pressure surface). Note that apart from that, these orientations set the position of the next control points. And, moreover, these angles are defined relative to major-axis of the ellipse. These parameters that define the LE are shown in the next figure:

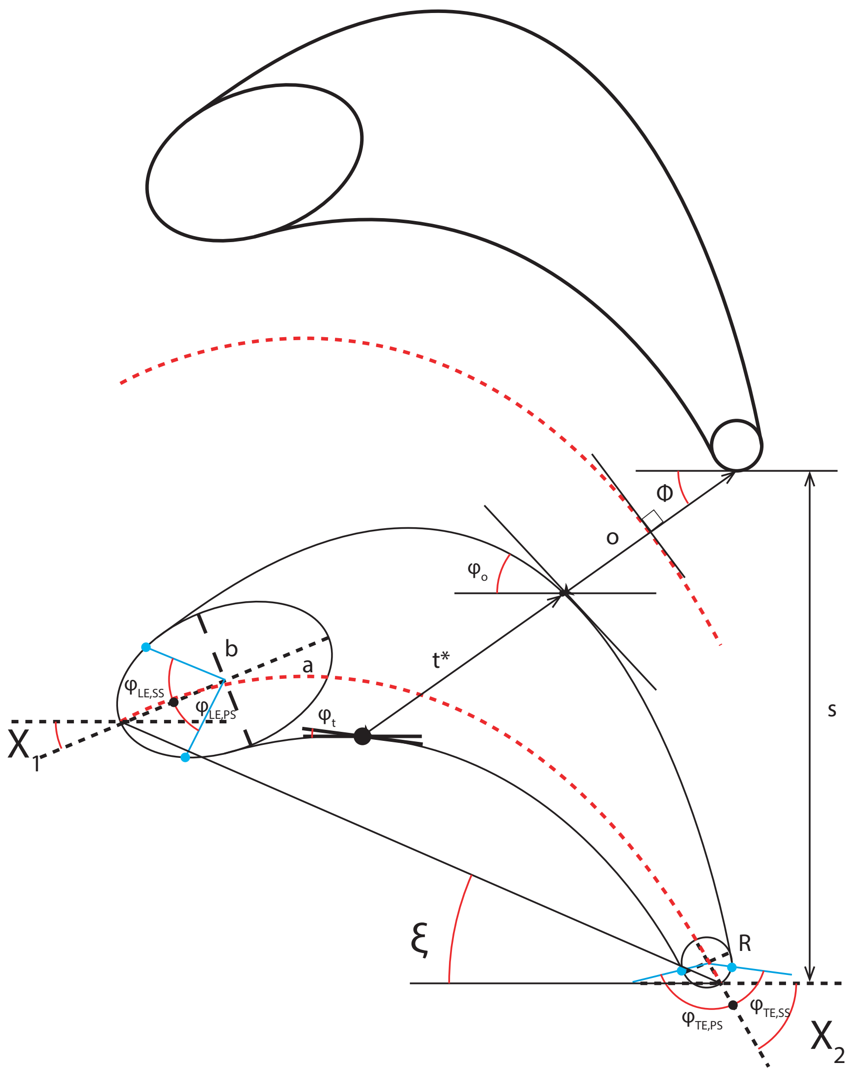


Figure 18: High parameters design

According to the same figure (Figure 18), the TE is modelled as a circle with radius R. This value, in fact, does not need to be a higher one [7] in order to fulfil Kutta condition:

[7] And, apart from that, blade profile has to be *open* when using MISES

The condition in steady streamlined flow whereby the stagnation streamline leaves the wing at the sharp trailing edge, thereby separating the flows from both sides of the wing –¿ Ref ??

Likewise to LE, there are two angles which define two control points (and which intersect with the main blade geometry): $\phi_{\mathrm{TE,SS}}$ (for the suction surface) and $\phi_{\mathrm{TE,PS}}$ (for the pressure surface). To be notices that the rotation for the circle at TE for a turbine blade is clockwise, therefore, the rotation matrix is defined as

$$R_{\chi_2}\begin{bmatrix} \cos|\chi_2| & \sin|\chi_2| \\ -\sin|\chi_2| & \cos|\chi_2| \end{bmatrix}$$

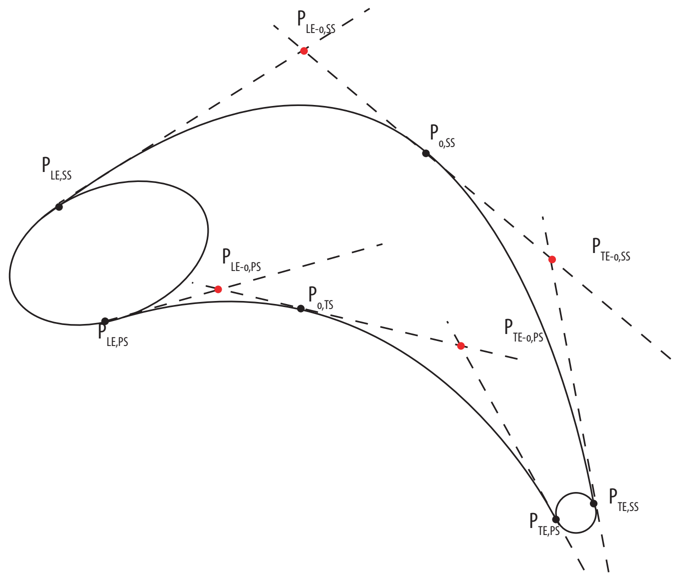


Figure 19: Control Points of a turbine blade

About the main blade geometry, firstly, it has to be said that Bézier is used and that, two control points per surface are already defined from the LE and TE geometries. The construction of the whole curve is divided into parts: one from point $P_{\mathrm{LE,SS}}$ to $P_{o,\mathrm{SS}}$ and from $P_{o,\mathrm{SS}}$ to $P_{\mathrm{LE,SS}}$. In the case of the pressure surface, the same procedure is followed. The reason behind is that the curves must pass through these points and, as a result, the quadratic Bézier equation is used four times (Figure 19).

In the previous paragraph, a new set of points have been mentioned: $P_{o,\mathrm{SS}}$ and $P_{o,\mathrm{PS}}$. $P_{o,\mathrm{SS}}$ is determined by the throat distance o, the throat angle ϕ and the upwards TE of the cascade. The location of this point is

$$(\tilde{x}'_{TE}, \tilde{y}'_{TE}) = (\tilde{x}_{TE}, \tilde{y}_{TE} + s/l) \tag{44}$$

where $(\tilde{x}'_{TE}, \tilde{y}'_{TE})$ are the upwards TE coordinates. The throat distance is a parameter that can be calculated from the formulae of cascade dynamics (or set by the user arbitrarily) and, while in the CIRCLE method ϕ angle is set by the designer, with this method it can be calculated. The procedure (according to

Ref ??) consists on translate the mean camber line at $\tilde{s}/2$ above and then it is known that the line that joins TE' and $P_{o,\mathrm{SS}}$ is perpendicular to the tangent of the translated camber line at certain point. This point (namely $P_{\mathrm{d,min}}$) can be obtained numerically knowing that the distance from TE' to this point (that, remember, belongs to the translated camber line) is minimum:

$$P_{\mathrm{d,min}} = \min(\tilde{x}'_{TE} - \tilde{x}'_c, \tilde{y}'_{TE} - \tilde{y}'_c) \tag{45}$$

In the above equation, $(\tilde{x}'_c, \tilde{y}'_c)$ represent the non-dimensional translated camber line coordinates. One this point is determined, ϕ is equal to

$$\tan\phi = \frac{\tilde{y}'_{TE} - P_{\mathrm{d,min,y}}}{\tilde{x}'_{TE} - P_{\mathrm{d,min,x}}} \tag{46}$$

Once $P_{o,\mathrm{SS}}$ is calculated, $P_{o,\mathrm{PS}}$ is obtained by subtracting t^* distance along ϕ direction. All this procedure can be better understood by inspecting Figure 18.

As said many times, to construct a quadratic Bézier curve there are needed three control points. $P_{\mathrm{LE-o,SS}}$, $P_{\mathrm{TE-o,SS}}$, $P_{\mathrm{LE-o,PS}}$ and $P_{\mathrm{TE-o,PS}}$ are determined by intersection the tangent line that passes through the previous defined control points (Figure 19):

$$P_{\mathrm{LE-o,SS}} = ... \tag{47}$$

$$P_{\mathrm{TE-o,SS}} = ... \tag{48}$$

$$P_{\mathrm{LE-o,PS}} = ... \tag{49}$$

$$P_{\mathrm{TE-o,PS}} = ... \tag{50}$$

All theses developed steps are defined for a turbine blade and, in the case of a compressor, is similar although a bit different. Since the throat is located in the LE area,

$$P_{\mathrm{d,min}} = ... \tag{51}$$

and

$$\phi = ... \tag{52}$$

The rest of control points are also slightly different:

$$P_{\mathrm{LE-o,SS}} = ... \tag{53}$$

$$P_{\mathrm{TE-o,SS}} = ... \tag{54}$$

$$P_{\mathrm{LE-o,PS}} = ... \tag{55}$$

$$P_{\mathrm{TE-o,PS}} = ... \tag{56}$$

Incluir plot de un compresor con los ángulos y definir algunos detalles más

Incluir flow chart a modo de resumen

7 Results

7.1 Design

7.1.1 Create a Turbine Blade profile

7.1.2 Create a Compressor Blade profile

7.1.3 Higher parameters design

Turbine

Compressor

7.1.4 Other features of the software

7.2 Analysed designs

7.2.1 Blade Profiles

7.2.2 Analysis of MISES results

8 Possible Improvements

9 Conclusion

10 Bibliography