

Biped Patrol

Task 3.3: Think & Answer

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Question No.	Max. Marks	Marks Scored
Q1	10	
Q2	20	
Q3	5	
Q4	5	
Q5	5	
Q6	10	
Q7	15	
Q8	8	
Q9	4	
Q10	8	
Q11	10	
Total	100	

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Task 3.3: Think & Answer

Instructions:

- There are no negative marks.
- Unnecessary explanation will lead to less marks even if answer is correct.
- If required, draw the image in a paper with proper explanation and add the snapshot in your corresponding answer.

Q 1. Describe hardware design for the Medbot, your team is constructing. Describe various parts with well labeled image. Give reasons for selection of design. [10]

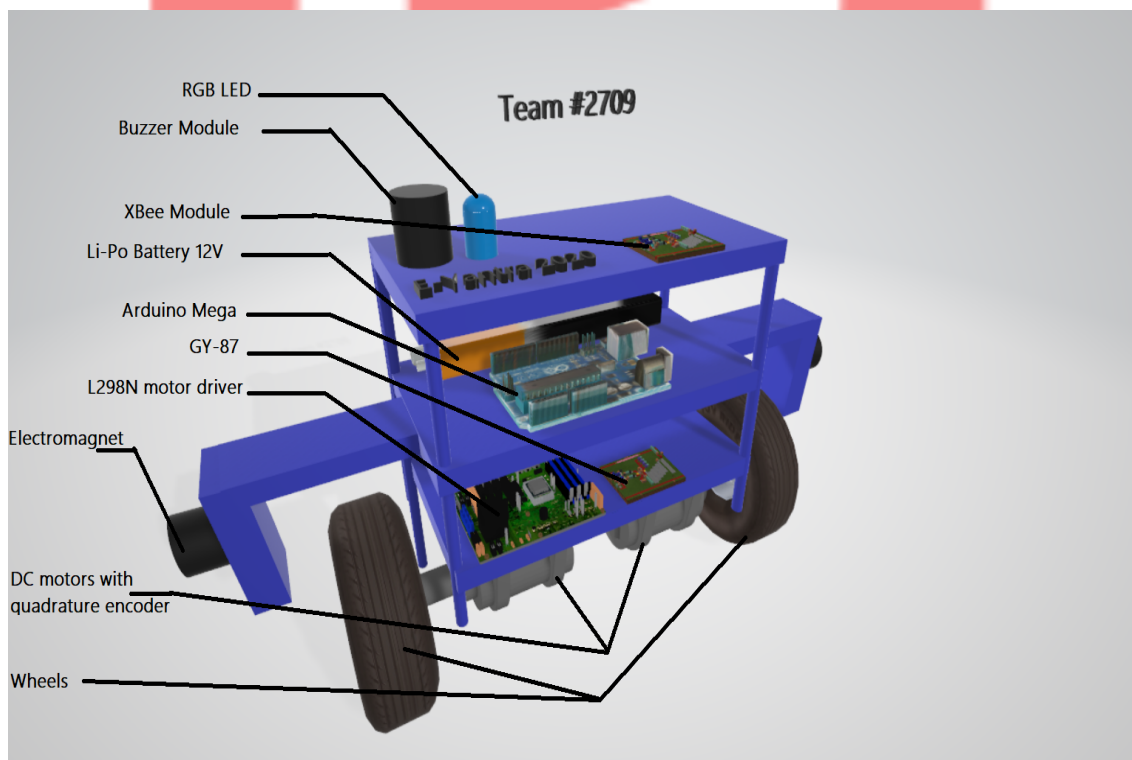


Figure 1: 3D model of the MEDBOT

A 1. We are planning to have a chassis that is compact , light-weight effective to pick supplies

faster, we are thinking of a 3 level design made out of acrylic sheets supported by studs, we plan to place Arduino Mega 2560 in the mid level. We have planned to put motor-driver module and gyroscope together at lowest level, with motors clamped at the bottom. As you can see in the above **Fig.1**. The electromagnet is mounted on the structure above the wheels facing outwards coming out from level 2 as shown in the fig.1. this enables us to pick 2 objects at a time. After the bot balances itself the RGB LED and the buzzer module will turn on which are placed at the top most level.

The main reason to place the battery and arduino module at the second level is to lower the centre of gravity of the Medbot in order to achieve better stability and to optimize balancing as if the battery the heaviest component is placed too higher it will make the system easier to balance but if large deviations in angle θ occur we will require high torque to balance it.

Q 2. In Task 1.2, you were asked to model different systems such as Simple Pulley, Complex Pulley, Inverted Pendulum with and without input and stabilizing the unstable equilibrium point using Pole Placement and LQR control techniques. There you had to choose the states; Derive the equations (usually non-linear), find equilibrium points and then linearize around the equilibrium points. You were asked to find out the linear system represented in the form

$$\dot{X}(t) = AX(t) + BU(t) \quad (1)$$

Where $X(t)$ is a vector of all the state, i.e., $X(t) = [x_1(t), x_2(t), \dots, x_n(t)]^T$, and $U(t)$ is the vector of input to the system, i.e. $U(t) = [u_1(t), u_2(t), \dots, u_m(t)]^T$. A is the State Matrix & B is the Input Matrix.

In this question, you have to choose the states for the Medbot you are going to design. Model the system by finding out the equations governing the dynamics of the system using Euler-Lagrange Mechanics. Linearize the system via Jacobians around the equilibrium points representing your physical model in the form given in equation 1.

Note: You may choose symbolic representation such as M_w for Mass of wheel, etc. [20]

A 2. From the following **Fig.**, the medbot can be modelled as an inverted pendulum with mass of body m_B concentrated at centre of gravity (C.O.G) it is the sum of masses of all components that form the body and at axis of rotation above the wheels of mass m_W . Assuming the medbot moves in horizontal x-axis. Now let

τ_W = Torque applied to Wheels.

τ_L = Torque applied to Left Wheel.

τ_R = Torque applied to Right Wheel.

θ = Angle of body w.r.t vertical.

ϕ = Angular displacement of wheels.

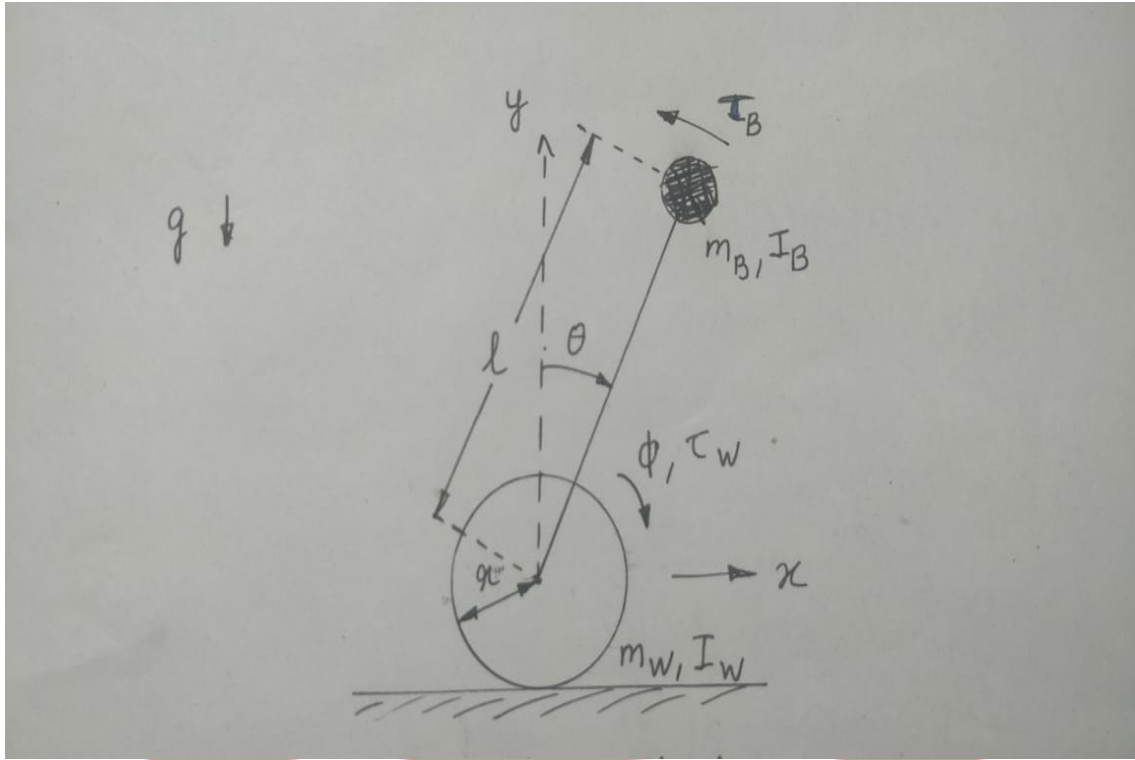


Figure 2: FBD of the MEDBOT

I_B, I_W = Moment of inertia of body and wheels with the axle of wheels as the reference axis.

I_B can be found out by calculating moment of inertia of various components of the body along axis parallel to the reference axis and then using the parallel axis theorem to calculate I_B about the reference axis.

r = Radius of Wheels.

Now, assuming that there is no slip condition between wheels and ground.

$$x = r\phi \quad \dots (1)$$

Displacement of C.O.G is,

$$x_{COG} = x + l \sin(\theta) \quad \dots (2)$$

$$y_{COG} = l \cos(\theta) \quad \dots (3)$$

Velocity of C.O.G is

$$v_{COG} = \sqrt{\dot{x}_{COG}^2 + \dot{y}_{COG}^2} \quad \dots (4)$$

Kinetic Energy of the body is,

$$K.E_B = (1/2)m_B v_{COG}^2 + (1/2)I_B \dot{\theta}^2$$

$$K.E_B = (1/2)m_B (\dot{x}^2 + 2\dot{x}l\cos(\theta)\dot{\theta}^2 + l^2\dot{\theta}^2) + (1/2)I_B \dot{\theta}^2 \quad \dots(5)$$

Kinetic Energy of the wheels is,

$$K.E_W = (1/2)m_W\dot{x}^2 + (1/2)I_W\dot{\phi}^2$$

$$K.E_W = (1/2)m_W\dot{x}^2 + (1/2)I_W\dot{x}^2/r^2 \quad \dots(6)$$

Potential Energy of the body and wheels is,

$$P.E_B = m_B g l \cos(\theta) \quad \dots(7)$$

$$P.E_W = 0 \quad \dots(8)$$

By using Lagrange's equations,

$$L = K.E - P.E$$

$$L = K.E_B + (K.E_W * 2) - P.E_B + (P.E_W * 2) \quad \dots(9)$$

and

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = F = \left(\frac{\tau_L + \tau_R}{r} \right) \quad \dots(10)$$

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}} - \frac{\partial L}{\partial \theta} = \tau_w = -(\tau_L + \tau_R) \quad \dots(11)$$

By using x and θ as system co-ordinates.

$$\text{i.e state vector } X(t) = [x \quad \dot{x} \quad \theta \quad \dot{\theta}]$$

$$L = (1/2)m_B(\dot{x}^2 + 2\dot{x}l\cos(\theta)(\dot{\theta}) + l^2(\dot{\theta})^2) + (1/2)I_B(\dot{\theta})^2 + m_W\dot{x}^2 + I_W(\dot{x}^2/r^2) - m_B g l \cos(\theta)$$

From eqn.9 and eqn.10 we get,

$$\ddot{x}(m_B + 2m_W + 2I_W/r^2) + m_B l \cos(\theta) \ddot{\theta} - m_B l \dot{\theta}^2 \sin(\theta) = \left(\frac{\tau_L + \tau_R}{r} \right) \quad \dots(12)$$

From eqn.9 and eqn.11 we get,

$$m_B \ddot{x} l \cos(\theta) + m_B l^2 \ddot{\theta} + I_B \ddot{\theta} - m_B g l \sin(\theta) = -(\tau_L + \tau_R) \quad \dots(13)$$

Solving equations (12) and (13) to get \ddot{x} and $\ddot{\theta}$ in terms of $x, \theta, \dot{\theta}$

Now,

$$x = x_1 \quad \dots(14)$$

$$\dot{x} = x_2 \quad \dots(15)$$

$$\theta = x_3 \quad \dots(16)$$

$$\dot{\theta} = x_4 \quad \dots(17)$$

$$\ddot{x}_2 = \ddot{x} \text{ and } \ddot{x}_4 = \ddot{\theta} \quad \dots(18)$$

$$\ddot{x} = \frac{m_B l \sin \theta (m_B l \cos \theta g - (m_B l^2 + I_B) \dot{\theta}^2) - (\tau_R + \tau_L) (m_B l \cos \theta + \frac{(m_B l^2 + I_B)}{r})}{m_B^2 l^2 \cos^2 \theta - (m_B l^2 + I_B) (m_B + 2m_W + \frac{2I_W}{r^2})} \dots (19)$$

$$\ddot{\theta} = \frac{m_B l \sin \theta ((m_B + 2m_W + \frac{2I_W}{r^2}) g - m_B l \cos \theta \dot{\theta}^2) - (\tau_L + \tau_R) ((m_B + 2m_W + \frac{2I_W}{r^2}) + \frac{m_B l \cos \theta}{r})}{(m_B l^2 + I_B) (m_B + 2m_W + \frac{2I_W}{r^2}) - m_B^2 l^2 \cos^2 \theta} \dots (20)$$

Using Jacobian to linearize these system equations about the equilibrium points

$$\begin{bmatrix} x & \dot{x} & \theta & \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}$$

Now from the EQN, $\dot{X}(t) = AX(t) + BU(t)$

We have,

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & h & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ j & j \\ 0 & 0 \\ k & k \end{bmatrix} \begin{bmatrix} \tau_L \\ \tau_R \end{bmatrix} \dots (21)$$

where,

$$i = \frac{m_B^2 l^2 g}{m_B^2 l^2 - (m_B l^2 + I_B) (m_B + 2m_W + \frac{2I_W}{r^2})}$$

$$h = \frac{m_B l (m_B + 2m_W + \frac{2I_W}{r^2}) g}{(m_B + 2m_W + \frac{2I_W}{r^2}) (m_B l^2 + I_B) - m_B^2 l^2}$$

$$j = \frac{-(m_B l + \frac{m_B l^2 + I_B}{r})}{m_B^2 l^2 - (m_B l^2 + I_B) (m_B + 2m_W + \frac{2I_W}{r^2})}$$

$$k = \frac{-[(m_B + 2m_W + \frac{2I_W}{r^2}) + \frac{m_B l}{r}]}{(m_B + 2m_W + \frac{2I_W}{r^2}) (m_B l^2 + I_B) - m_B^2 l^2}$$

Q 3. Equation 1 represents a continuous-time system. The equivalent discrete time system is represented as:

$$X(k+1) = A_d X(k) + B_d U(k) \quad (2)$$

Where $X(k)$ is a measure of the states at k_{th} sampling instant, i.e., $X(k) = [x_1(k), x_2(k), \dots, x_n(k)]^T$, and $U(k)$ is the vector of input to the system at k_{th} sampling instant, i.e. $U(k) = [u_1(k), u_2(k), \dots, u_m(k)]^T$. A_d is the Discrete State Matrix & B_d is the Discrete Input Matrix.

What should be the position of eigen values of A_d for system to be stable.

Hint: In frequency domain, continuous-time system is represented with Laplace transform and discrete-time system is represented with Z transform. [5]

A 3. In the continuous time domain we found the determinant of $|sI - A|$ and found its roots by equating it to 0 i.e. the poles of the system and checked if these poles lied

- (a) In the left half of the s-plane then the system is stable.
- (b) On the imaginary axis then it is marginally stable.
- (c) In the right half of the s-plane then the system is unstable.

While in the Discrete time domain we will find the determinant of $|zI - A_d|$ and find its roots by equating it to 0 i.e. the poles of the system and if these poles lie

- (a) Inside the unit circle then the system is stable.
- (b) On the unit circle then the system is marginally stable.
- (c) Outside the unit circle then the system is unstable.

Q 4. Will LQR control always works? If No, then why not? and if Yes, Justify your answer.

Hint: Take a look at definition of Controllable System. What is controllability? [5]

A 4. Yes, LQR method will always work in case of a controllable system as it calculates the value of 'K' (Feedback gain) in such a way that it optimizes value of 'K' to get the desired performance measure from the system. Here we choose the weights 'Q' of greater value for θ with the vertical and the angular velocity $\dot{\theta}$ as these state variables are more important to balance our bots.

Controllability is the ability to drive a state from any initial value to a final value in finite amount of time by providing a suitable input.

Q 5. For balancing robot on two wheel i.e. as inverted pendulum, the center of mass should be made high or low? Justify your answer. [5]

A 5. Higher the centre of mass the easier it will be to balance the bot in the upright position when the deviation of the angle from vertical is small. The downside to keeping the centre of mass higher up is that recovering from large deviations of angle will require much greater motion or acceleration. While keeping the centre of mass at lower position stabilizes the bot i.e. if a large deviation occurs the Med-bot won't require much area to stabilize.

So it is necessary to find an optimum position for centre of mass to get better stability and more controllability.

Q 6. Why do we require filter? Do we require both the gyroscope and the accelerometer for measuring the tilt angle of the robot? Why? [10]

A 6. The filter removes noise from sensor output data. As tilt angle from accelerometer data has slow response time i.e. it is low pass in nature so we need to pass this signal from a low

pass filter. While the integrated tilt angle from the gyro data is subjected to drift over a period of time thus it is high frequency signal and we need to pass it through High pass filter.

The accelerometer data is useful for long term while the gyro data is useful for short term. Thus the data from both gyro and accelerometer is essential to get an accurate tilt angle.

Idea behind a complementary filter is to take slow moving signals from accelerometer and fast moving signals from a gyroscope and combine them. Complementary filter is designed in such a way that the strength of one sensor will be used to overcome the weakness of the other sensor which is complementary to each other.

Accelerometer gives a good indicator of orientation in static conditions. Gyroscope gives a good indicator of tilt in dynamic conditions. So the idea is to pass the accelerometer signals through a low-pass filter and the gyroscope signals through a high-pass filter and combine them to give the final rate.

Q 7. What is Perpendicular and Parallel axis theorem for calculation of Moment of Inertia? Do you require this theorem for modelling the Medbot? Explain Mathematically. [15]

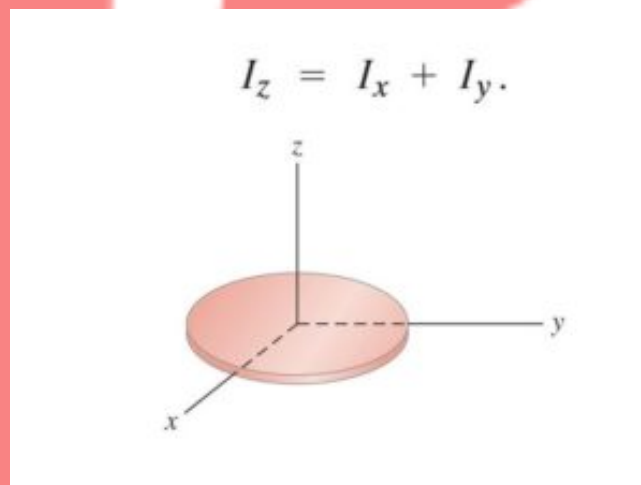
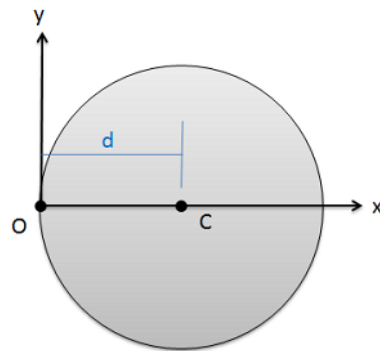


Figure 3: Perpendicular axis theorem

A 7. Perpendicular axis theorem states that the moment of inertia of a plane lamina about an axis perpendicular to the plane of the lamina is equal to the sum of the moments of inertia of the lamina about the two axes at right angles to each other, in its own plane intersecting each other at the point where the perpendicular axis passes through it.

Parallel axis theorem can be used to determine the moment of inertia or the second moment of area of a rigid body about any axis, given the body's moment of inertia about a parallel axis through the object's center of gravity and the perpendicular distance between the axes. Suppose a body of mass m is rotated about an axis z passing through the body's centre of



$$I_O = I_C + m d^2$$

Moment of Inertia of shape about point O

Moment of Inertia of Shape about Centroid (C)

Added Moment of Inertia due to distance between O and C

Figure 4: **Parallel axis theorem**

gravity. The body has a moment of inertia I_{cm} with respect to this axis. The parallel axis theorem states that if the body is made to rotate about a new axis z which is parallel to the first axis and displaced from it by a distance d , then the moment of inertia I with respect to the new axis is related to I_{cm} by

$$I = I_{cm} + m d^2$$

We need perpendicular axis theorem to find the moment of inertia of various components along a axis parallel to our reference axis (which is along the wheel axle) using the moment of inertia of some standard masses like rod, plane, disc, etc. and then we require the parallel axis theorem to find these moment of inertias about our reference axis.

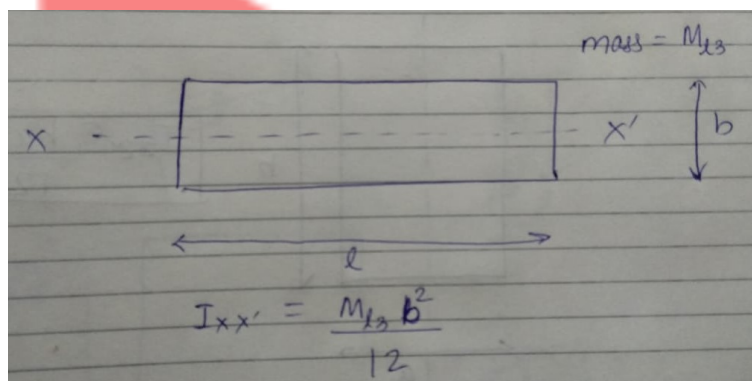


Figure 5: **Moment of Inertia of acrylic sheet**

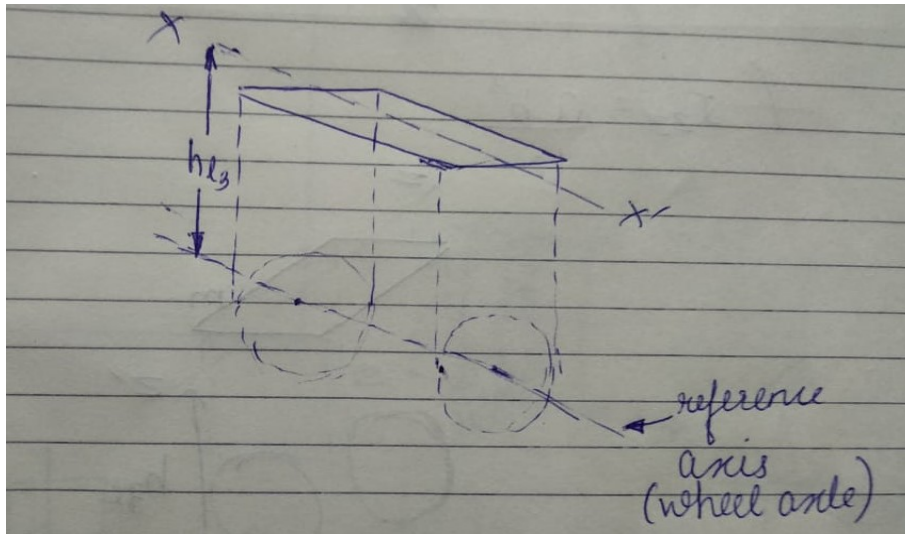


Figure 6: Moment of Inertia of acrylic sheet about the reference axis using parallel axis theorem

For eg. An acrylic sheet which makes up a level can be considered as a rectangular plane and its moment of inertia about the reference axis can be found as follows as shown in fig.5 and fig.6

$$I_{XX} = M_{l3}b^2/12$$

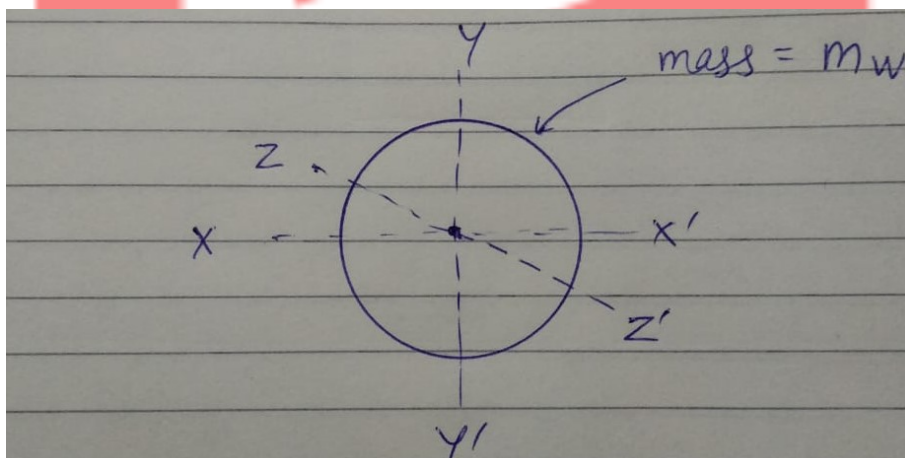


Figure 7: Moment of Inertia of wheel of radius 'r' about reference axis using perpendicular axis theorem

$$I_{ref} = M_{l3}b^2/12 + M_{l3}h_{l3}^2$$

Similarly the moment of inertia of the wheel about reference axis can be found using the perpendicular axis theorem as shown in fig.7

$$I_{xx} = I_{yy} = m_w r^2/4$$

$$I_{ref} = I_{zz} = I_{xx} + I_{yy},$$

$$I_{ref} = mr^2/2$$

Q 8. What will happen in the following situations:

- (a) Medbot picks a First-Aid Kit from the shelf of Medical Store but the First-Aid Kit falls inside the store. Will there be any penalty imposed, points awarded? Will the First-Aid Kit be repositioned? [2]
- (b) Medbot picks a First-Aid Kit from the shelf of Medical Store but the First-Aid Kit falls outside the store. Will there be any penalty imposed, points awarded? Will the First-Aid Kit be repositioned? [2]
- (c) Medbot picks a First-Aid Kit from the shelf of Medical Store but the First-Aid Kit and the Medbot both fall inside the store. Will there be any penalty imposed, points awarded? Will the First-Aid Kit be repositioned? [2]
- (d) Medbot picks a First-Aid Kit from the shelf of Medical Store but the First-Aid Kit and the Medbot both fall outside the store. Will there be any penalty imposed, points awarded? Will the First-Aid Kit be repositioned? [2]

A 8. Considering the Run is a valid run and all repositions have not been exhausted:

- (a) No penalty will be applied. There are no points awarded and as the reposition is allowed only when the Medbot falls so FAK won't be repositioned.
- (b) No, there will be no penalty applied. As the First-Aid kit falls outside the store, it will be considered as a successful pick up as the FAK was taken out of the medical store. Hence points will be awarded for pick up. The FAK is not repositioned.
- (c) Yes, there will be fall penalty applied. As the First-Aid kit and the Medbot falls inside the store, it will be considered as a unsuccessful pick up. There are no points awarded. Also the First-Aid Kit and Medbot are repositioned at their respective positions.
- (d) Yes, there will be fall penalty applied. As the First-Aid kit and the Medbot falls outside the store, it will be considered as a successful pick. There are points awarded for the pick up. Also the First-Aid Kit and Medbot are repositioned.

Q 9. What will be the points awarded if Medbot picks only one of the item from the medical store and repeatedly moves back and forth around the gravel pathway or the bridge for the entire run. [4]

A 9. The run won't be considered a valid run as a triplet is not delivered and no points will be awarded in this case.

Q 10. What are the different communication protocols you'll be using? Name the hardware interfaced related to each of the communication protocols. Explain how these communication protocols works and what are the differences between them. [8]

A 10. Different types of communication protocols used are

1. UART(Universal Asynchronous Receiver Transmitter)
2. Inter Integrated Circuit(I2C)
3. Zigbee Protocol

(1) Arduino Mega uses UART to communicate with PC.

To upload the code(.ino file) into the Arduino Mega we use UART. On Mega pins 0 and 1 are used for communication with the computer. Connecting anything to these pins can interfere with that communication, including causing failed uploads to the board. You can use the Arduino environment's built-in serial monitor to communicate with an Arduino board. Click the serial monitor button in the toolbar and select the same baud rate used in the call to begin(). Serial communication on pins TX/RX uses TTL logic levels (5V or 3.3V depending on the board).

Also Arduino Mega and XBee S2C module communicate with each other using UART.

(2) GY-87 communicates with Arduino Mega using I2C.

I2C is a serial protocol for two-wire interface to connect low-speed devices like microcontrollers, EEPROMs, A/D and D/A converters, I/O interfaces and other similar peripherals in embedded systems. The two lines used are SDA (Data line) and SCL (Clock Line). In normal state both lines (SCL and SDA) are high. The communication is initiated by the master device. It generates the Start condition followed by the address of the slave device. If the bit 0 of the address byte was set to 0 the master device will write to the slave device. Otherwise, the next byte will be read from the slave device. Once all bytes are read or written the master device generates Stop condition.

(3) XBee uses Zigbee Protocol.

XBee S2C is a RF module designed for wireless communication or data exchange and it works on ZigBee mesh communication protocols that sit on top of IEEE 802.15.4. In this protocol we have two modes namely:

1. AT Mode (Transparent Mode)
2. API Mode- Application Programming Interface. We have used API mode which is frame based method for sending and receiving data to and from a radio's serial UART.

Communication Protocols Used		
UART	I2C	Zigbee
1. UART uses two lines namely transmit and receive. 2. Only two devices can communicate at a time 3. UART can have a range of 1.2km using the RS485 at 100kbps speed 4. UART is half duplex communication	1. I2C uses two lines SDA and SCL. 2. I2C can handle upto 128 devices 3. I2C can be extended upto 100m 4. I2C is also Half duplex communication	1. Zigbee is Wireless communication Protocol. 2. 2 raise to 64 devices can be connected if 64 bit address mode is used. 3. Zigbee can be used upto 40m indoors and in line of sight upto 120m 4. Zigbee is also Half Duplex communication

Q 11. Why do we require IRF540N? Provide circuit diagram for interfacing IRF540N with the microcontroller. [5+5]

A 11. As we know IRF540N is a power MOSFET and it is acting as a power switch. IRF540N is used to drive a high power circuit (Electromagnets) with a low power circuit (Arduino Mega). Thus IRF540N acts as MOSFET switch which provides isolation between the high power and low power circuit. The reason to use this MOSFET is that it has low threshold voltage of 4V, which means it can easily be driven by low voltages like 5V and it has a high current rating of upto 33A and power rating of 120W. Hence it is used with Arduino Mega development board.

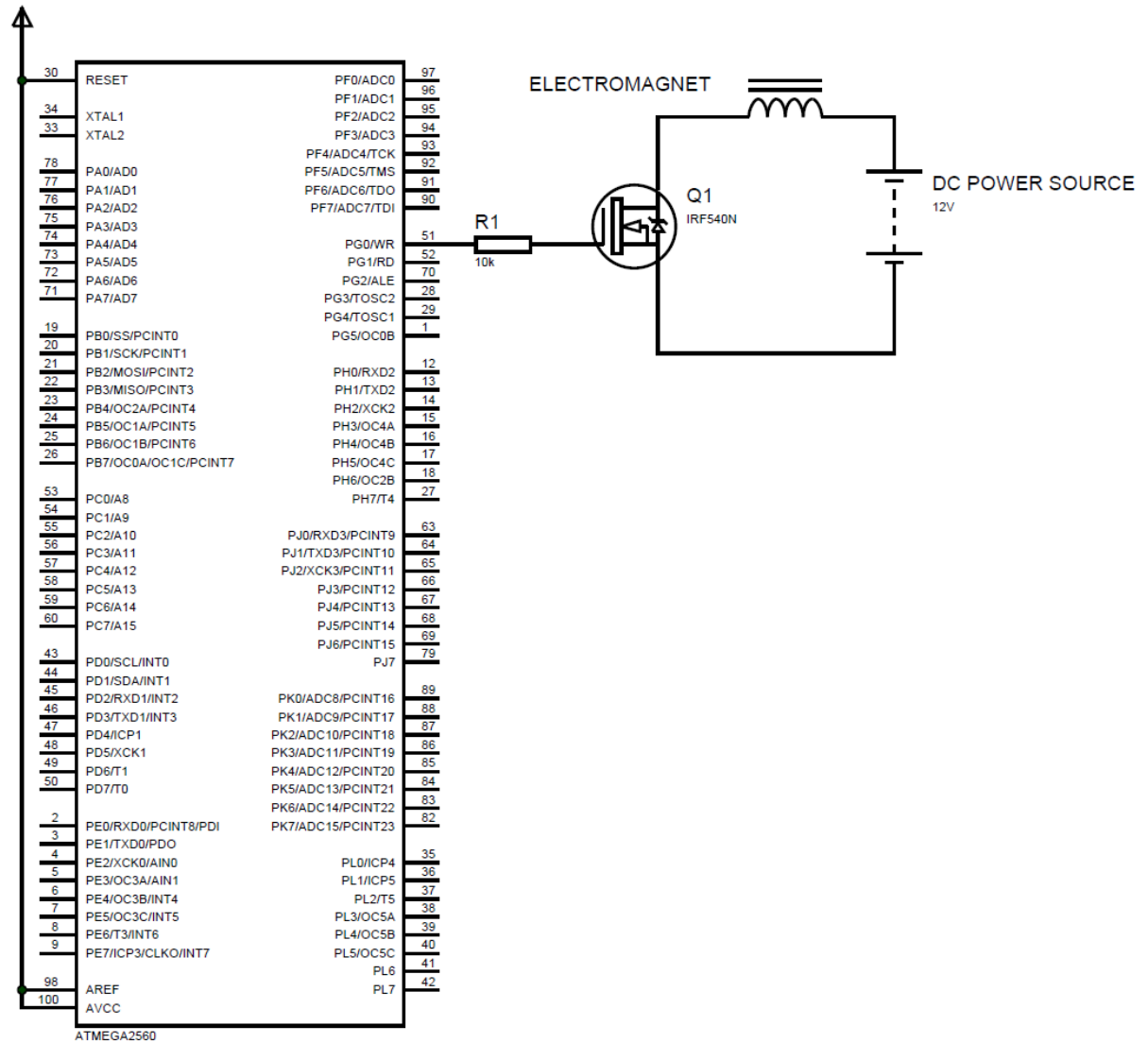


Figure 8: IRF540N interfaced with ATmega 2560 microcontroller