Autoregressive Model

1) The values of exchange rate for TL and USD for the last 10 days of April is given below.

Date	7th	8th	9th	10th	13th	14th	15th	16th	17th	20th
x	1	2	3	4	5	6	7	8	9	10
f(x) (USD)	2.5789	2.5877	2.5941	2.6243	2.6464	2.6725	2.6965	2.6969	2.6821	2.6907

Estimate behavior of the exchange rate by using the defined regression analysis. Estimated value for each day will be represented as a function of the seven predecessor days.

$y'_n = a_1 y_{n-1} + a_2 y_{n-2} + a_3 y_{n-3} + a_4 y_{n-4} + a_5 y_{n-5}$

where y'_n and y_n are the estimated and the actual value of the nth day respectively.

a) Calculate coefficients for the given equation by using Least Square Approach.

b) Predict the value of USD for 21th of April, 2015.

Solution

1) $y'_n = a_1 y_{n-1} + a_2 y_{n-2} + a_3 y_{n-3} + a_4 y_{n-4} + a_5 y_{n-5}$

Applying partial differential on the error function for each coefficient gives out the following equations:

$$\frac{\partial \text{err}}{\partial \text{a1}} = -2 \sum_6^{10} y_{n-1}(y_n - (a_1 y_{n-1} + a_2 y_{n-2} + a_3 y_{n-3} + a_4 y_{n-4} + a_5 y_{n-5})) = 0$$

$$\frac{\partial \text{err}}{\partial \text{a2}} = -2 \sum_6^{10} y_{n-2}(y_n - (a_1 y_{n-1} + a_2 y_{n-2} + a_3 y_{n-3} + a_4 y_{n-4} + a_5 y_{n-5})) = 0$$

$$\frac{\partial \text{err}}{\partial \text{a3}} = -2 \sum_6^{10} y_{n-3}(y_n - (a_1 y_{n-1} + a_2 y_{n-2} + a_3 y_{n-3} + a_4 y_{n-4} + a_5 y_{n-5})) = 0$$

$$\frac{\partial \text{err}}{\partial \text{a4}} = -2 \sum_6^{10} y_{n-4}(y_n - (a_1 y_{n-1} + a_2 y_{n-2} + a_3 y_{n-3} + a_4 y_{n-4} + a_5 y_{n-5})) = 0$$

$$\frac{\partial \text{err}}{\partial \text{a5}} = -2 \sum_6^{10} y_{n-5}(y_n - (a_1 y_{n-1} + a_2 y_{n-2} + a_3 y_{n-3} + a_4 y_{n-4} + a_5 y_{n-5})) = 0$$

In martix form, these equations take the form of AX + B :

$$\begin{bmatrix} \Sigma y_{n-1} * y_{n-1} & \Sigma y_{n-1} * y_{n-2} & \dots & \Sigma y_{n-1} * y_{n-5} \\ \Sigma y_{n-2} * y_{n-1} & \Sigma y_{n-2} * y_{n-2} & \dots & \Sigma y_{n-2} * y_{n-5} \\ \Sigma y_{n-3} * y_{n-1} & \Sigma y_{n-3} * y_{n-2} & \dots & \Sigma y_{n-3} * y_{n-5} \\ \Sigma y_{n-4} * y_{n-1} & \Sigma y_{n-4} * y_{n-2} & \dots & \Sigma y_{n-4} * y_{n-5} \\ \Sigma y_{n-5} * y_{n-1} & \Sigma y_{n-5} * y_{n-2} & \dots & \Sigma y_{n-5} * y_{n-5} \end{bmatrix} * \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} = \begin{bmatrix} \Sigma y_n * y_{n-1} \\ \Sigma y_n * y_{n-2} \\ \Sigma y_n * y_{n-3} \\ \Sigma y_n * y_{n-4} \\ \Sigma y_n * y_{n-5} \end{bmatrix}$$

$\Sigma y_{n-1} * y_{n-1} = (2.6821*2.6821) + (2.6969*2.6969) + (2.6965*2.6965) + (2.6725*2.6725) + (2.6464*2.6464) = 35.8837$

$\Sigma y_{n-1} * y_{n-2} = (2.6821*2.6969) + (2.6969*2.6965) + (2.6965*2.6725) + (2.6725*2.6464) + (2.6464*2.6243) = 35.7294$

$\Sigma y_{n-1} * y_{n-3} = (2.6821*2.6965) + (2.6969*2.6725) + (2.6965*2.6464) + (2.6725*2.6243) + (2.6464*2.5941) = 35.4542$

...

$\Sigma y_n * y_{n-1} = (2.6907*2.6821) + (2.6821*2.6969) + (2.6969*2.6965) + (2.6965*2.6725) + (2.6725*2.6464) = 36.0012$

$\Sigma y_n * y_{n-2} = (2.6907*2.6969) + (2.6821*2.6965) + (2.6969*2.6725) + (2.6965*2.6464) + (2.6725*2.6243) = 35.8458$

...

The resulting matrices are

$$A = \begin{bmatrix} 35.8837 & 35.7294 & 35.4542 & 35.1622 & 35.9508 \\ 35.7294 & 35.5770 & 35.3037 & 35.0129 & 34.7621 \\ 35.4542 & 35.3037 & 35.033 & 34.7443 & 34.4953 \\ 35.1622 & 35.0129 & 34.7443 & 34.4582 & 34.2113 \\ 34.9108 & 34.7621 & 34.4953 & 34.2113 & 33.9667 \end{bmatrix}, B = \begin{bmatrix} 36.0012 \\ 35.8458 \\ 35.5696 \\ 35.2769 \\ 35.0252 \end{bmatrix}$$

and the coefficients are found as

$a_1 = 1.9947$
$a_2 = -2.6789$
$a_3 = 0.7503$
$a_4 = -0.0119$
$a_5 = 0.9727$

which results in the predictive model

$y'_n = 1.9947*y_{n-1} - 2.6789*y_{n-2} + 0.7503*y_{n-3} - 0.0119*y_{n-4} + 0.9727*y_{n-5}$

According to our predictive model, the value of buying USD for 21th of April, 2015 would be

$y'_n = 1.9947*2.6907 - 2.6789*2.6821 + 0.7503*2.6969 - 0.0119*2.6965 + 0.9727*2.6725 = 2.7730$