BLG456E
Robotics
Intro to mobile robot geometry & kinematics

Lecture Contents:

- Differential drive robots.
- Robot abstractions:
  - Point robot.
  - Rigid robot.
    - Rotation vectors.
    - Twists.
- Introduction to reference frames.

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Schedule:	http://djduff.net/my-schedule
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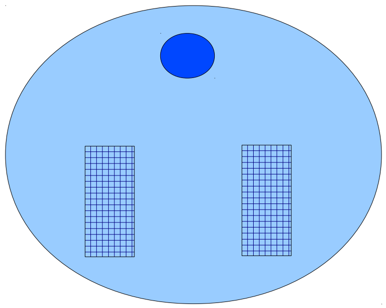
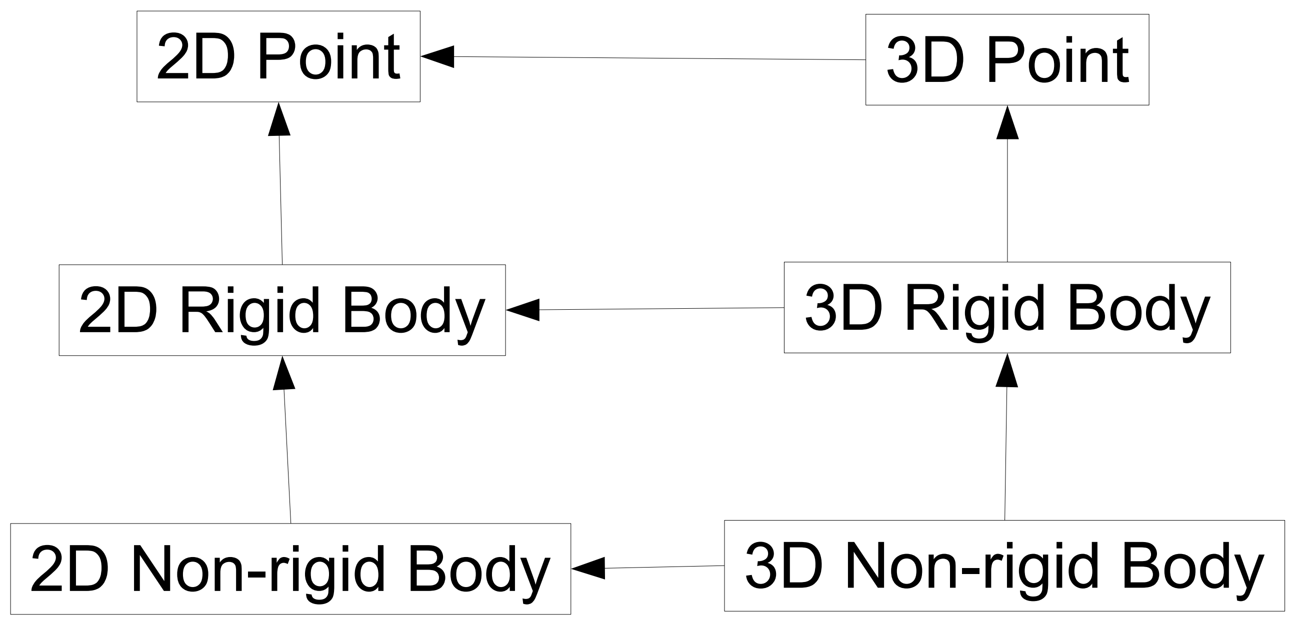
Differential Drive

- 2 wheels on common axis.
- Wheels rotate different speeds – “differentially”.
- Usually 3rd passive wheel.
- Can:
  - Rotate in-place.
  - Move forward or back.
  - Move on curve.



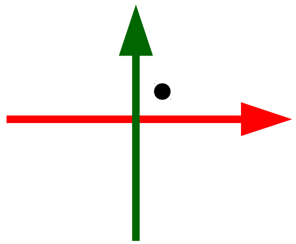
<u>Question</u>: What motions can this robot **not** make?

Abstractions



2D point geometry/kinematics

Robot as a point in Cartesian space.



State (position): $\chi = \begin{bmatrix} x \\ y \end{bmatrix}$

State (position & velocity): $\mathbf{X} = \begin{bmatrix} \chi \\ \dot{\chi} \end{bmatrix} = \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix}$

Note:
$\dot{a} \equiv \frac{da}{dt}$

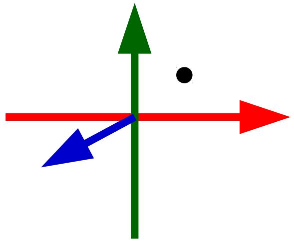
Basic kinematics: $\chi_{t_1} = \chi_{t_0} + \int_{t_0}^{t_1} \dot{\chi}_t \, dt$

Question: How would this look in 1D?

Question: If velocity is constant, what is the simpler version of the kinematic equation?

3D point geometry/kinematics

Robot as a point in Cartesian space.



State (position): $\chi = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

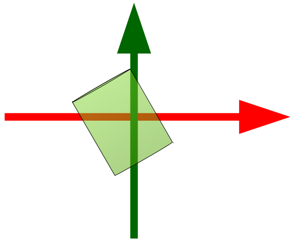
State (position & velocity): $\mathbf{X} = \begin{bmatrix} \chi \\ \dot{\chi} \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}$

Basic kinematics:

$$\chi_{t_1} = \chi_{t_0} + \int_{t_0}^{t_1} \dot{\chi}_t \, dt$$

2D geometry/kinematics with rotations

Robot has many points - is **extended**.

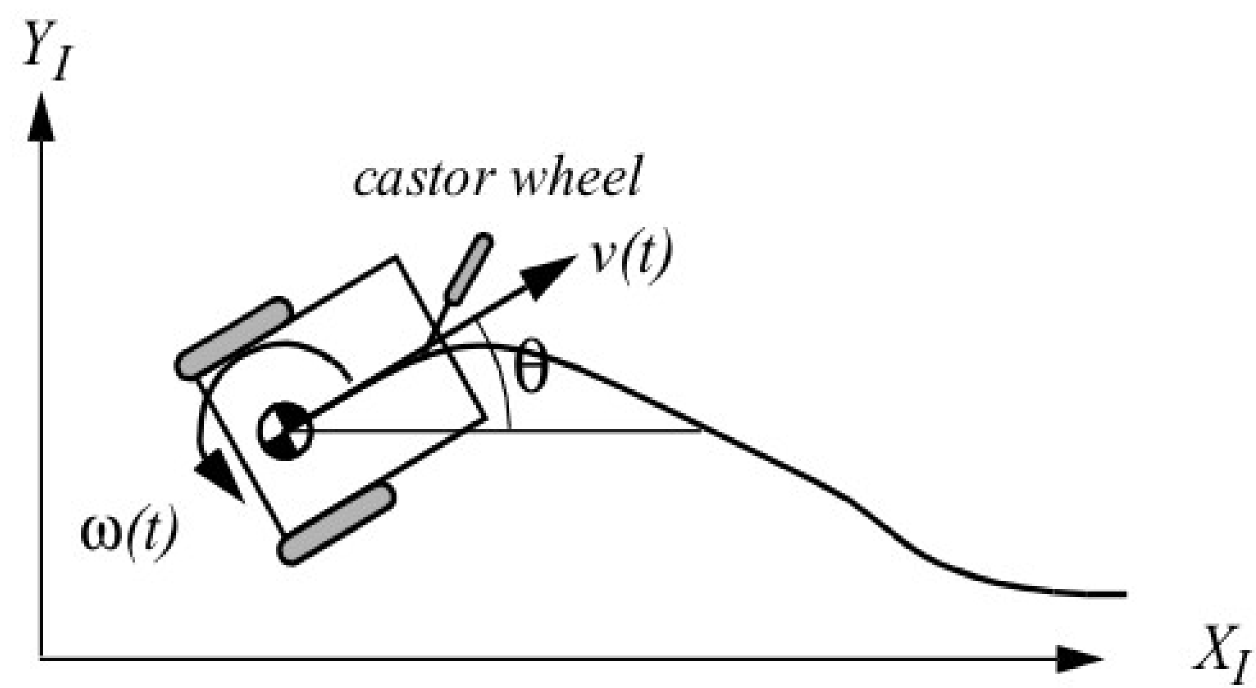


State (pose): $\chi = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$

State (position & velocity): $\mathbf{X} = \begin{bmatrix} \chi \\ \dot{\chi} \end{bmatrix} = \begin{bmatrix} x \\ y \\ \theta \\ \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}$

Kinematics: (will investigate later)

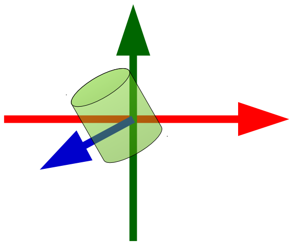
Motion of all points in a rigid body is captured by its linear and angular velocities.



From Siegwart & Nourbaksh.

3D geometry/kinematics with rotations

3D rotation is stranger.



Pose (translation vector, rotation vector): $\chi = \begin{bmatrix} x \\ y \\ z \\ \theta_x \\ \theta_y \\ \theta_z \end{bmatrix}$

Velocity (**twist**): $V = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$

$$\boxed{\dot{\theta}_x \neq \omega_x}$$

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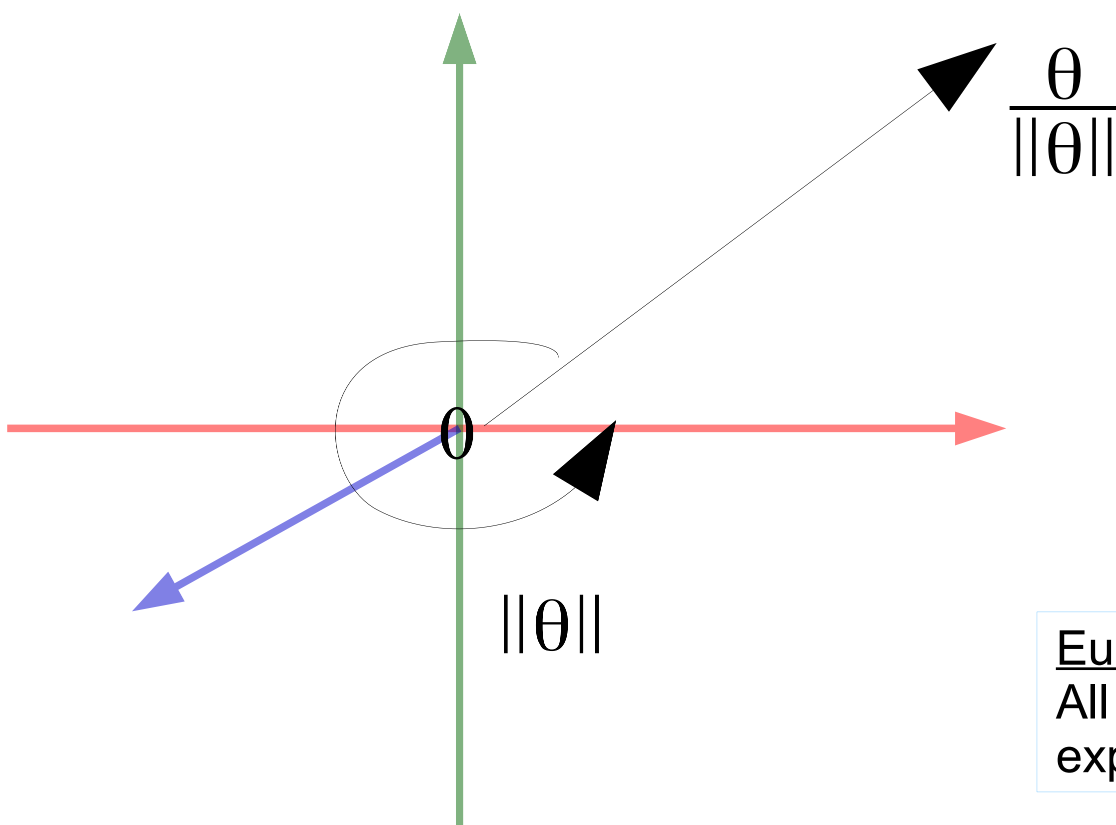
Rotation Vector $\boldsymbol{\theta} = \begin{bmatrix} \theta_x \\ \theta_y \\ \theta_z \end{bmatrix}$

- Length is amount of rotation.
- Direction is axis of rotation.

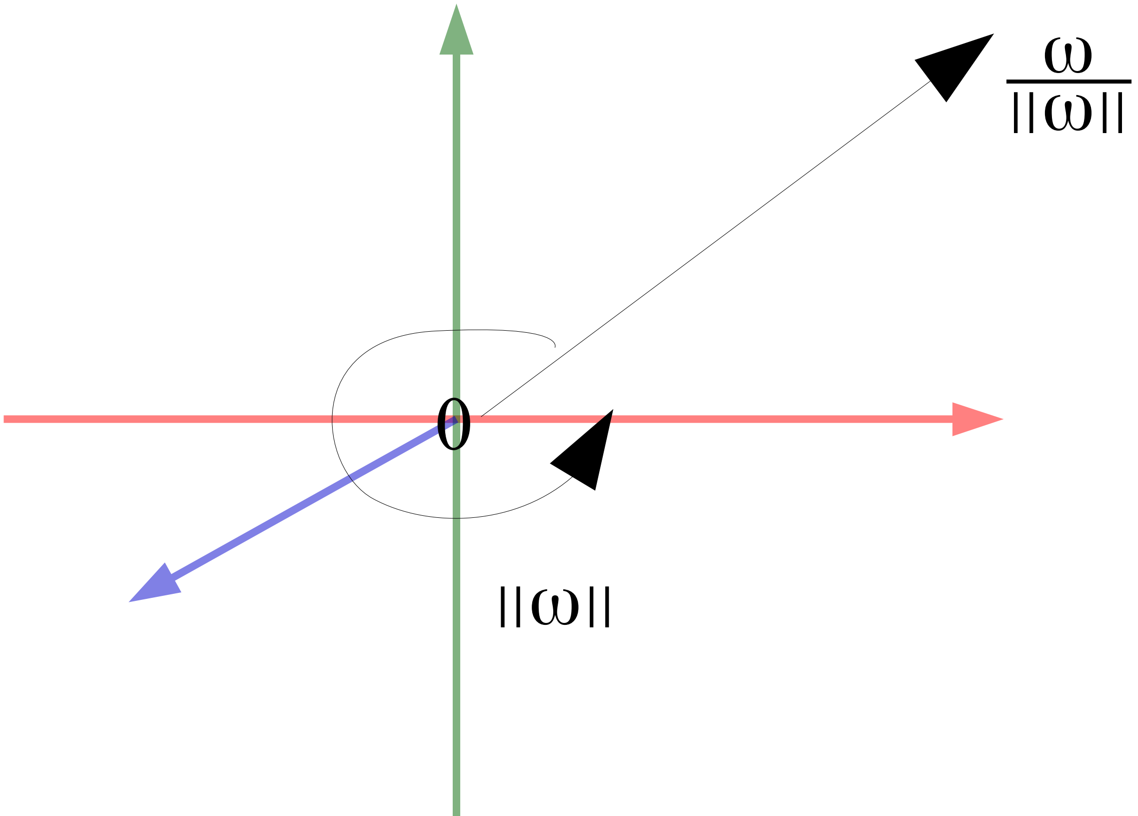


<u>Euler's rotation theorem</u>:
All rotations can be expressed like this.

Twist (Rotation Part)

$$\boldsymbol{\omega} = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

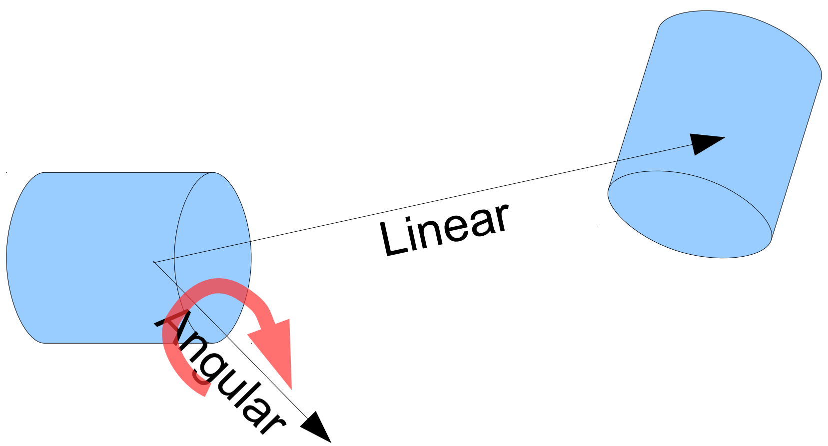
- Length is speed of rotation.
- Direction is axis of rotation.



Twists

Twist = a pair:

- 3D Linear Velocity (vector).
  - **Direction** = direction of travel.
  - **Magnitude** = speed of body.
- 3D Angular Velocity (vector).
  - **Direction** = axis of rotation.
  - **Magnitude** = speed of rotation (angular velocity).



Twists in two dimensions

2D case (body constrained to XY plane):

- Linear velocity vector is in the plane.
  - $z = 0$.
- Direction of angular velocity vector is vertical.
  - $\omega_x = 0$
  - $\omega_y = 0$

For differential drive robots:

- Robot cannot move sideways:
  - $y = 0$.

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Introduction to physical Degrees Of Freedom: poses and velocities

Spatial Dimens.	Quantity	DOF
2	Linear Displacement	2
2	Linear Velocity	2
2	Orientation	1
2	Rotational velocity	1
2	Pose	3
2	Rigid motion	3
2	Non-rigid pose/motion	?

Spatial Dimens.	Quantity	DOF
3	Linear Displacement	3
3	Linear Velocity	3
3	Orientation	3
3	Rotational velocity	3
3	Pose	6
3	Rigid motion	6
3	Non-rigid pose/motion	?

Constraints can reduce the degrees of freedom.

Exercise

- My robot is at position $(x,y) = (3,5)$

  (displacement from world origin).

  - 1. Plot this point against x,y axes.

- Its orientation is $\boldsymbol{\theta}_z = \boldsymbol{\pi}/2$ rad

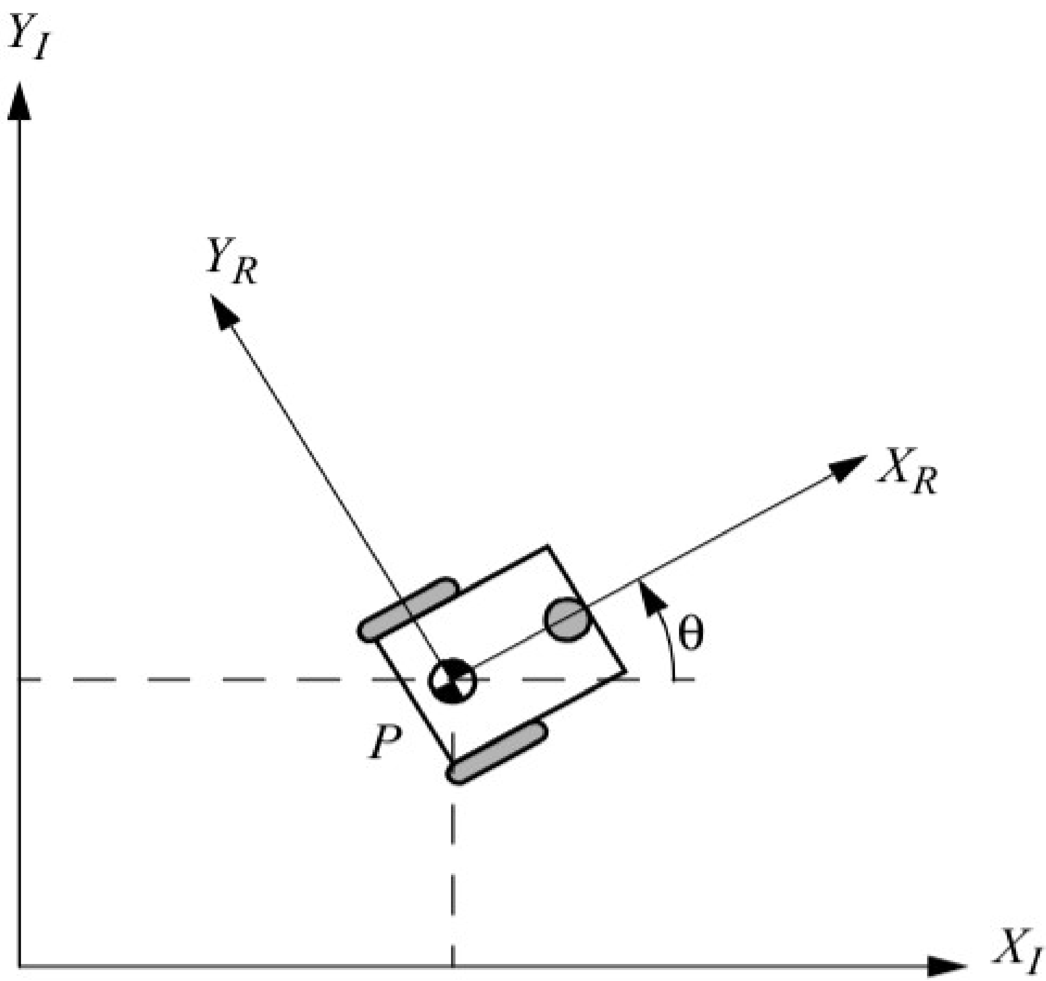
  (rotation from the x-axis).

  - 2. Draw the robot.

- 3. From the *robot's* perspective, **what are the coordinates of the origin?**

  (forward is positive x axis, left is positive y axis).

Introduction to reference frames



World reference frame:

$x_I \quad y_I \quad \theta_I$

Robot reference frame:

$x_R \quad y_R \quad \theta_R$

Provocation: What is the general relationship between θ_I and θ_R?