1.7: Introduction to Proofs

Basic Definitions Needed for Proofs In The Remainder of This Section:

- (1) Given an integer n.
 - n is **even** if there exists an integer k such that n = 2k.
 - n is **odd** if there exists an integer k such that n = 2k+1.
- (2) A number x is *rational* if it can be written in the form x = m/n, where m and n are integers (with $n \ne 0$). x is *irrational* if it is not a rational number.

I. Direct Proof of $P \rightarrow Q$ Suppose P. : Therefore Q. Thus, $P \rightarrow Q$.

Example 1: Prove that if n is odd, then n + 1 is even.

Your strategy for developing a direct proof of an implication should involve the following steps:

- 1. Determine precisely the hypothesis (P) and the conclusion (Q).
- 2. If necessary, replace P (and/or Q) with a more usable, but still equivalent, proposition.
- 3. Develop a chain of statements, each deducible from its predecesors or other known results, that leads from *P* to *Q*.

Example 2: Suppose n is an integer. Prove that n is even if n^2 is even.

Recall that $(P \rightarrow Q) \equiv (\neg Q \rightarrow \neg P)$.

Proof by Contrapositive of $P \rightarrow Q$

Suppose $\neg Q$. \vdots

Therefore $\neg P$.

Thus, $P \rightarrow Q$.

Example 3: Prove that $\sqrt{2}$ is irrational.

Proof by Contradiction of <i>Q</i>	Proof by Contradiction of $P \rightarrow$
	Q
Suppose $\neg Q$.	
:	Suppose P and $\neg Q$.
Therefore <i>R</i> .	:
:	Therefore $\neg P$.
Therefore $\neg R$.	Hence, $P \land \neg P$, a contradiction.
Hence, $R \land \neg R$, a contradiction.	Thus, Q .
Thus, Q .	

Example 4: Let *m* and *n* be positive real numbers. Prove if $m \ge n$, then $m^0 \ge n^0$.

Trivial proof: $P \rightarrow Q$ is automatically true whenever Q is always true.

Example 5: Let m and n be positive real numbers. Prove if mn < 0, then m < 0 or n < 0.

Vacuous proof: $P \rightarrow Q$ is automatically true if P is always false.

SECTION 1.7 – INTRODUCTION TO PROOFS

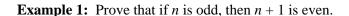
METHODS OF PROVING THEOREMS

Basic Definitions Needed for Proofs In The Remainder of This Section:

- (1) Given an integer n.
 - n is **even** if there exists an integer k such that n = 2k.
 - n is **odd** if there exists an integer k such that n = 2k+1.
- (2) A real number x is *rational* if it can be written in the form x = m/n, where m and n are integers (with $n \ne 0$). x is *irrational* if it is not a rational number.

METHODS OF PROVING THEOREMS

I. Direct Proof of $P \rightarrow Q$	
Suppose <i>P</i> .	
Therefore Q .	
Thus, $P \rightarrow Q$.	-



Your strategy for developing a direct proof of an implication should involve the following steps:

- 1. Determine precisely the hypothesis (P) and the conclusion (Q).
- 2. If necessary, replace P (and/or Q) with a more usable, but still equivalent, proposition.
- 3. Develop a chain of statements, each deducible from its predecesors or other known results, that leads from P to Q.

Example 2: Suppose n is an integer. Prove that n is even if n^2 is even.

Recall that $(P \rightarrow Q) \equiv (\neg Q \rightarrow \neg P)$.

Proof by Contrapositive of $P \rightarrow Q$ $\begin{array}{c} \text{Suppose } \neg Q.\\ & \vdots\\ \text{Therefore } \neg P.\\ \text{Thus, } P \rightarrow Q. \end{array}$

Example 2: Suppose that n is an integer. Prove that n is even if n^2 is even.

Example 3: Prove that $\sqrt{2}$ is irrational.

Proof by Contradiction of <i>Q</i>	Proof by Contradiction of $P \rightarrow Q$
Suppose $\neg Q$. : Therefore R . : Therefore $\neg R$.	Suppose P and $\neg Q$. : Therefore $\neg P$. Hence, $P \land \neg P$, a contradiction. Thus, Q .
Hence, $R \wedge \neg R$, a contradiction.	
Thus, Q .	

Class Notes for Discrete Math I (Rosen)

Example 4: Let *m* and *n* be positive real numbers. Prove if $m \ge n$, then $m^0 \ge n^0$.

This is a *trivial proof*. $P \rightarrow Q$ is automatically true whenever Q is always true.

Example 5: Let m and n be positive real numbers. Prove if mn < 0, then m < 0 or n < 0.

This is a *vacuous proof*. $P \rightarrow Q$ is automatically true if P is always false.