BLG 202E Assignment - 1

Due 13.03.2017 23:59

- An e-report should be prepared individually. The written MATLAB codes should be included in the submitted report.
- Plagiarized assignments will be **unacceptable** and **subjected to disciplinary actions.**
- **Do not** miss submission deadline. **Do not** leave your submission until the last minute. The submission system tends to become less responsive due to high network traffic.

Submissions: Please submit your report and your MATLAB codes through Ninova e-Learning System.

If you have any question about the homework, contact the teaching assistant ***Cumali TÜRKMENOĞLU*** *via e-mail (****turkmenogluc@itu.edu.tr****) or in* ***Research Lab 2****.*

1. **(15)** The limit $\boldsymbol{e = \lim_{n\to\infty}(1 + 1/n)\hat{}n}$ defines the number ***e*** in calculus. Write down a **.m** file with MATLAB software in which:

- Estimate ***e*** by taking the value of this expression for ***n* = 8,8^2,8^3,...,8^{10}**.
- Compare with ***e*** obtained from ***e* ← exp*(*1.0*)***, plot relative error obtained for all n's.
- Interpret the results.

2. **(20)** Write a MATLAB program to find two roots of **f(x)** which is twice continuously differentiable.

$$f(x) = 2\cosh(x/4) - x$$

starting your search with **[*a*,*b*] = [0,10]** , ***nprobe* = 10** and **tol = 1.e-8**

Your program should first probe the function ***f* (*x*)** on the given interval to find out where it changes sign. (Thus, the program has, in addition to *f* itself, four other arguments: *a*, *b*, the number *nprobe* of equidistant values between ***a*** and ***b*** at which *f* is probed, and a tolerance **tol**.)
For each subinterval **[a_i ,b_i]** over which the function changes sign, your program should then find a root as follows. Use either Newton's method or the secant method to find the root, monitoring decrease in **| *f* (x_k)|**. The ***i* th** root is deemed "found" as x_k if **| *f* (x_k)| < tol** hold.

3. **(20)** Suppose a computer company is developing a new floating point system for use with their machines. They need your help in answering a few questions regarding their system. The company's floating point system is specified by **(β, t, L,U)** (for more detail info. Check the Section 2.2 or slides of the course). Assume the following:

- All floating point values are normalized (except the floating point representation of zero).
- All digits in the mantissa (i.e., fraction) of a floating point value are explicitly stored.
- The number 0 is represented by a float with a mantissa and an exponent of zeros. (Don't worry about special bit patterns for **$\pm\infty$** and **NaN**.)

Here is your part:

(a) How many different nonnegative floating point values can be represented by this floating point system?

(b) Same question for the actual choice **(β, t, L,U) = (8,5,−100,100)** (in decimal) which the company is contemplating in particular.

(c) What is the approximate value (in decimal) of the largest and smallest positive numbers that can be represented by this floating point system?

(d) What is the rounding unit?

4. **(15)** Suppose a machine with a floating point system **(β, t, L,U) = (10, 8, −50, 50)** is used to calculate the roots of the quadratic equation

$$ax^2 + bx + c = 0$$

where a, b, and c are given, real coefficients.

For each of the following, state the numerical difficulties that arise if one uses the standard formula for computing the roots. <u>Explain how to overcome these difficulties (when possible).</u>

(a) $a = 1$; $b=-10^5$; $c = 1$.

(b) $a = 6 \cdot 10^{30}$; $b = 5 \cdot 10^{30}$; $c=-4 \cdot 10^{30}$.

(c) $a = 10^{-30}$; $b=-10^{30}$; $c = 10^{30}$.

5. **(15)** Consider finding the root of a given nonlinear function ***f*** **(*x*)**, known to exist in a given interval **[*a*,*b*]**, using one of the following three methods: ***bisection, Newton,*** and ***secant***. For each of the following instances, one of these methods has a distinct advantage over the other two. <u>Match problems and methods and justify briefly.</u>

(a) $f(x) = x - 1$ on the interval **[0,2.5]**.
(b) $f(x)$ is given in Figure **1** on **[0,4]**.
(c) $f \in C^5[0.1, 0.2]$, the derivatives of f are all bounded in magnitude by **1**, and $f'(x)$ is hard to specify explicitly or evaluate.

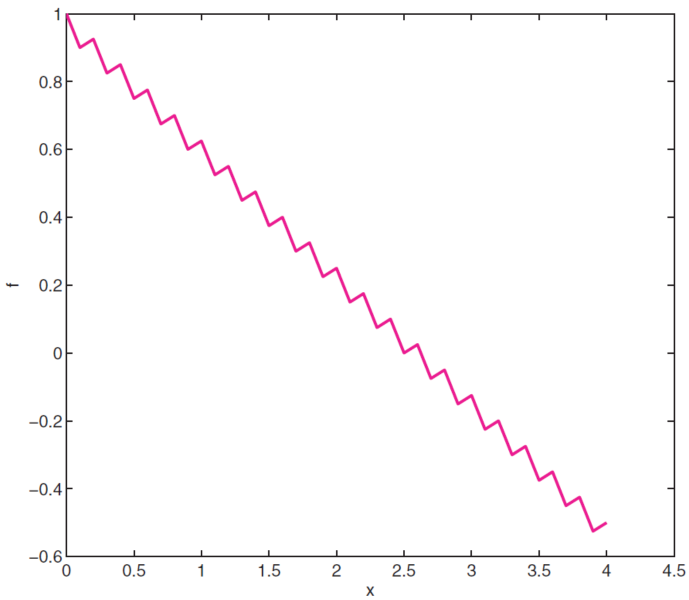

Figure 1: Graph of an anonymous function.

6. **(15)** The function

$$f(x) = (x - 1)^2 e^x$$

has a double root at $x = 1$.

(a) Derive Newton's iteration for this function. Show that the iteration is well-defined so long as $x_k \neq -1$, and that the convergence rate is expected to be similar to that of the bisection method (and certainly not quadratic).
(b) Implement Newton's method and observe its performance starting from $x_0 = 2$.
(c) How easy would it be to apply the bisection method? Explain.

Good Luck