1: The first Problem

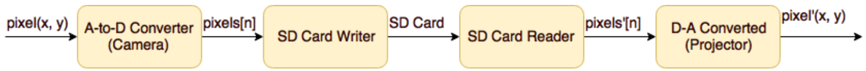

Figure 1: Block diagram for a system composed of a digital camera and a projector

2: The second problem

Roland Priemer defines signal briefly: "A signal is a function that conveys information about the behavior of a system or attributes of some phenomenon" [1]

Figure 1: A heartbeat record is a signal because it conveys the heartbeat information.

Figure 2: A voice record is also a signal because voice can be represented numerically and transfered between systems.

Figure 3: An image is also a signal because image can be represented as 2 by 2 matrix and conveyed easily.

3: The third problem

$$z^4 = j$$
$$z = r \cdot e^{j \cdot Q} = cos(Q) + j \cdot sin(Q), \text{ where } Q = \frac{\pi}{2}$$
$$z^4 = e^{j \cdot \left(\frac{\pi}{2} + 2 \cdot \pi \cdot k\right)}$$
$$z_i = e^{j \cdot \pi \cdot \left(\frac{1+4 \cdot k}{8}\right)}, \text{ where } k = \{0, 1, 2, 3\}$$

$$\begin{aligned} z_1 &= e^{j \cdot \frac{\pi}{8}}, \text{where k} = 0 \\ z_2 &= e^{j \cdot \frac{5 \cdot \pi}{8}}, \text{where k} = 1 \\ z_3 &= e^{j \cdot \frac{9 \cdot \pi}{8}}, \text{where k} = 2 \\ z_4 &= e^{j \cdot \frac{13 \cdot \pi}{8}}, \text{where k} = 3 \end{aligned}$$

4: The fourth problem

Taylor's Formulas:

$$\begin{aligned} e^x &= \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \\ \sin x &= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots \\ \cos x &= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots \end{aligned}$$

We can express the equation, $e^{j\theta}$, in terms of the Taylor's Formulas:

$$\begin{aligned}
e^{j\theta} &= 1 + (j\theta) + \frac{(j\theta)^2}{2!} + \frac{(j\theta)^3}{3!} + \frac{(j\theta)^4}{4!} + \cdots \\
e^{j\theta} &= 1 + (j\theta) + \frac{-\theta^2}{2!} + \frac{-j\theta^3}{3!} + \frac{\theta^4}{4!} + \cdots \\
e^{j\theta} &= (1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \cdots) + j(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots) \\
e^{j\theta} &= cos(\theta) + jsin(\theta)
\end{aligned}$$

5: The fifth problem

(a) A function can be called as **odd** if and only if it satisfies $f(-x) = -f(x)$ These functions are typically symmetric with respect to the origin.

Ex: $f(x) = x^3$

(b) A function can be called as **even** if and only if it satisfies $f(-x) = f(x)$ These functions are typically symmetric with respect to y-axis.

Ex: $f(x) = x^4$

(c)

- **a.** $sin(\theta) \to cos(\theta - \pi/2)$
- **b.** $cos(\theta + 2\pi k) \to cos(\theta)$, when k is an integer
- **c.** $cos(-\theta) \to cos(\theta)$
- **d.** $sin(-\theta) \to -sin(\theta)$
- **e.** $sin(\pi k) \to 0$, when k is an integer
- **f.** $cos(2\pi k) \to 1$, when k is an integer
- **g.** $cos\left[2\pi\left(k + 1/2\right)\right] \to -1$, when k is an integer

(d)

i.

$$\begin{aligned}
e^{j\theta} &= cos(x) + jsin(x), (Euler'sFormula) \\
e^{-j\theta} &= cos(x) - jsin(x) \\
e^{j\theta} \cdot e^{-j\theta} &= cos^2(x) + sin^2(x) \\
e^0 = 1 &= cos^2(x) + sin^2(x)
\end{aligned}$$

ii.

$$\begin{aligned}
cos(2\theta) &= Re\left\{e^{j2\theta}\right\}\\
cos(2\theta) &= Re\left\{(e^{j\theta})^2\right\}\\
cos(2\theta) &= Re\left\{(cos(\theta)+jsin(\theta))^2\right\}\\
cos(2\theta) &= Re\left\{cos^2(\theta)+2jsin(\theta)cos(\theta)-sin^2(\theta)\right\}\\
cos(2\theta) &= cos^2(\theta)-sin^2(\theta)
\end{aligned}$$

iii.

$$\begin{aligned}
sin(2\theta) &= Im\left\{e^{j2\theta}\right\}\\
sin(2\theta) &= Im\left\{(e^{j\theta})^2\right\}\\
sin(2\theta) &= Im\left\{(cos(\theta)+jsin(\theta))^2\right\}\\
sin(2\theta) &= Im\left\{cos^2(\theta)+2jsin(\theta)cos(\theta)-sin^2(\theta)\right\}\\
sin(2\theta) &= 2sin(\theta)cos(\theta)
\end{aligned}$$

6: The sixth problem

$$\begin{aligned}
\sum_{k=1}^{N} A_k cos(\omega_0 t+\varPhi_k) &= Acos(\omega_0 t+\varPhi)\\
\sum_{k=1}^{N} A_k e^{j\varPhi_k} &= Ae^{j\varPhi}\text{(The essense of the phasor addition rule) [2]}\\
Acos(\omega_0 t+\varPhi) &= Re\left\{Ae^{j(\omega_0 t+\varPhi)}\right\} = Re\left\{Ae^{j\varPhi}e^{j\omega_0 t}\right\}\\
Re\left\{\sum_{k=1}^{N} X_k\right\} &= \sum_{k=1}^{N} Re\left\{X_k\right\}\\
\sum_{k=1}^{N} A_k cos(\omega_0 t+\varPhi_k) &= \sum_{k=1}^{N} Re\left\{A_k e^{j(\omega_0 t+\varPhi_k)}\right\}\\
&= Re\left\{\left(\sum_{k=1}^{N} A_k e^{j\varPhi_k}\right)e^{j\omega_0 t}\right\}\\
&= Re\left\{\left(Ae^{j\varPhi}\right)e^{j\omega_0 t}\right\}\\
&= Re\left\{Ae^{j(\omega_0 t+\varPhi)}\right\{\\
&= Acos(\omega_0 t+\varPhi)
\end{aligned}$$

7: The seventh problem

8: The eighth problem

9: The ninth problem

10: The tenth problem

11: The eleventh problem

12: The twelfth problem

13: The thirteenth problem

References

[1] Roland Priemer *Introductory Signal Processing.* (English) [*World Scientific. p.1*]. ISBN 9971509199, 2013.

[2] James H. McClellan., Ronald W. Schafer, Mark A. Yoder *Signal Processing First.* (English) [*Phasor Addition Rule 2-6.2*]. ISBN 0-13-120265-0, 2003.