CHAPTER 6

Hypothesis Testing

In Chapter 5, we examined methods for obtaining confidence intervals for means, variances and proportions. In this chapter, we present another type of statistical inference regarding these same parameters.

Quite often, there is a need to make decisions about populations on the basis of sample information. We often use inferential statistics to make decisions or judgments about the value of a parameter, such as a population mean. For example, we might need to decide whether the mean life, μ, of all machine components (such as a shaft) manufactured by a particular company differs from the advertised life of 5 years or we might want to determine whether the mean age, μ, of all automobiles has increased from the 1955 year mean of 8.5 years. In attempting to reach decisions or judgments, often it is necessary to make assumptions or guesses about the population involved. Such assumptions, which may or may not be true, are called *statistical hypothesis* and in general are statements about the probability distributions of the populations.

Decisions made about populations on the basis of sample information are called *statistical decisions*. A *hypothesis* is an assertion made about a population. That is, a hypothesis is a statement that something is true. The assertion concerns a numerical value of some parameter of the population. Procedures which helps us to decide whether to accept or reject hypothesis or to determine whether observed samples differ significantly from expected results are called *test of hypothesis*, *tests of significance* or *results of decision*.

Here, *parameters* are the statistical constants of the population (such as the mean (μ), variance (σ^2), etc.) and the statistical measures computed from the sample observations (data) are called *statistic* (such as the mean $(\overline{x})$, variance s^2, S^2, etc.).

In general, the sample size greater than or equal to 30 ($n \geq 30$) is considered as large sample in statistics whereas the sample size less than 30 ($n < 30$) is regarded as small. Nearly all the distributions can be approximated by a normal distribution.

6.1 NULL HYPOTHESIS AND ALTERNATIVE HYPOTHESIS

A *null hypothesis* is a claim (or a statement) about a population parameter that is assumed to be true until it is declared false. A null hypothesis is a hypothesis to be tested. We use the symbol H_0 to represent the null hypothesis.

An *alternative hypothesis* is a claim about a population parameter that will be true if the *null hypothesis* is false. It is a hypothesis to be considered as an alternative to the null hypothesis. We use the symbol H_1 to represent the alternative hypothesis.

The problem in a *hypothesis test* is to decide whether the null hypothesis should be rejected in favour of the alternative hypothesis. To test a hypothesis, we devise a procedure for taking a random sample, computing an appropriate *test statistic* and then rejecting or failing to reject the null hypothesis H_0. Part of this procedure is specifying the set of values for the test statistic that leads to rejection of H_0. This set of values is called the *critical region* or *rejection region* for the test.

The procedure in setting up a hypothesis test is to decide on the null hypothesis and the alternative hypothesis. For example, for specifying hypothesis test for the population mean, μ, we write

Null Hypothesis: $H_0: \mu = \mu_0$

where μ_0 is some number.

Alternative Hypothesis: The selection of the alternative hypothesis depends greatly on and should reflect the objective of the hypothesis test. Three choices are available for the alternative hypothesis. For example,

1. Whether a population mean, μ, is *different from* a specified value μ_0, we write the alternative hypothesis as

   $H_0: \mu \neq \mu_0$

   A hypothesis test whose alternative hypothesis has the above form is called a *two-tailed test*.
2. Whether a population mean, μ, is *less than* a specified value μ_0, we write the alternative hypothesis as

   $H_1: \mu < \mu_0$

   A hypothesis test whose alternative hypothesis has the above form is called a *left-tailed test.*
3. Whether a population, μ, is *greater than* a specified value μ_0, we express the alternative hypothesis as

   $H_1: \mu > \mu_0$

A hypothesis test whose alternative hypothesis has the above form is called a *right-tailed test*.

A hypothesis test is called a *one-tailed test* if it is either left-tailed or right-tailed.

If H_0 specifies the population completely, then it is known as a *simple hypothesis* otherwise, it is called *composite hypothesis*.

For instance,

$H_0: \mu = \mu_0, \sigma^2 = \sigma_0^2$ is a simple hypothesis.

On the other hand,

$$H_0: \mu < \mu_0, \sigma^2 = \sigma_0^2$$

$$\mathrm{H}_0: \mu > \mu_0, \sigma^2 = \sigma_0^2$$

$$\mathrm{H}_0: \mu = \mu_0, \sigma^2 > \sigma_0^2$$

$$H_0: \mu = \mu_0$$

$$H_0: \sigma^2 = \sigma_0^2$$

are all composite hypothesis.

The basic logic of hypothesis testing is as follows: Select a random sample from the given population. If the sample data are consistent with the null hypothesis, do not reject the null hypothesis; if the sample data are inconsistent with the null hypothesis (in the direction of alternative hypothesis), reject the null hypothesis and conclude that the alternative hypothesis is true.

A simple hypothesis completely determines the distribution of X, whereas a composite hypothesis does not. For instance, the simple and null hypothesis H_0: $\mu = 10$ indicates that the distribution of X is exactly the same as it was before the change, whereas the composite alternative hypothesis H_1: $\mu > 10$ is less specific. It just indicates that the new mean is greater than 10, but does not specify what exactly the new mean is. In general, the null hypothesis will be *simple* and the alternative hypothesis will be composite.

6.2 THE CRITICAL REGION

All values of the test statistic in the direction of the alternative hypothesis with a test statistic value less than or equal to the significance level α define a set called the *critical region* of the test statistic. According to the definition of α, α is the probability that the test statistic will lie in the critical region.

Example E6.1a

If H_0: $\mu = \mu_0$ is true, then the test statistic is the standard normal random variable z. Assume the level of significance α. Determine the critical regions for one-sided and two-sided alternative hypothesis.

SOLUTION:

As shown in Fig. E6.1, for a two-tailed test, the rejection region is on both the left and right. The critical values are $\pm z_{\alpha/2}$; for a left-tailed test, the critical value is $-z_\alpha$; and for a right-tailed test, the critical value is z_α. From the table in Appendix-*E*, using the standard normal table, we obtain the values of z_α corresponding to each of those five tail areas as shown below:

$z_{0.10}$	$z_{0.05}$	$z_{0.025}$	$z_{0.01}$	$z_{0.005}$
1.28	1.645	1.96	2.33	2.575

As an example:

For H_0: $\mu < \mu_0$, the critical region is all values $z \leq -1.28$, since $p(z \leq -1.28) = 0.10$

For H_0: $\mu > \mu_0$, the critical region is all values $z \geq 1.28$, since $p(z \geq 1.28) = 0.10$

For H_0: $\mu \neq \mu_0$, the critical region is all values $z \leq -1.645$ or $z \geq 1.645$,

since $p(z \leq -1.645) + p(z \geq 1.645) = 0.10$ or $[p(z \geq 1.645 = 0.10/2 = 0.05]$.

If the test value lies in the critical region, then the null hypothesis is rejected at the α level of significance. On the other hand, if the test value is not in the critical region, then the null hypothesis is not rejected.

6.3 TYPES OF SAMPLING ERRORS (TYPE I AND TYPE II ERRORS)

There are two important types of errors that can be made when arriving at a decision on the basis of a hypothesis test: They are

1. **Type I error:** If we reject a hypothesis when it should be accepted, we call that a *type I error* has been made. Hence, a type I error occurs when a true null hypothesis is rejected.

The value of α represents the probability of committing this type of error.

That is, the probability of this error is written as: $\alpha = P(\text{Type I Error}) = P(H_0 \mid H_0 \text{ is true})$.

The value of α represents the *significance level* of the test.

Type I error is also called a *rejection error* and Type II error is called *acceptance error*.

2. **Type II error:** A type II error occurs when a false null hypothesis is not rejected. The value of β represents the probability of making a type II error. It represents the probability that H_0 is not rejected when actually H_0 is false. The value of $(1 - \beta)$ is called the *power of the test*. It represents the probability of not making a type II error.

   That is, $\beta = P$ (Type II error) $= P$ (fail to reject $H_0 \mid H_0$ is false).

   Power $= 1 - \beta = P$ (reject $H_0 \mid H_0$ is false).

The two types of errors that occur in test of hypothesis depend on each other. The values of α and β cannot be lowered simultaneously for a test of hypothesis for a fixed sample size. Lowering the value of one of them will raise the value of the other. On the other hand, one can decrease both α and β simultaneously by increasing the sample size.

The Type I and Type II errors are defined in terms of probability numbers and can be controlled to desired values. The results possible in testing a hypothesis are summarised in Table 6.1.

Table 6.1: Type I (α) error and Type II (β) error

If the decision on analysis is:	If H_0 is	
	True	False and H_1 is true
Accept H_0 Do not reject H_0	Correct decision $P = 1 - \alpha$ (probability)	Wrong decision $P = \beta$ (probability)
Reject H_0	Wrong or incorrect decision $P = \alpha$ (probability) $\Sigma P = 1.0$	Correct decision $P = 1 - \beta$ (probability) $\Sigma P = 1.0$

Summarising, we have that when a statistical test procedure based on sample data will lead to precisely one of the following four situations. Two of these situations will entail correct decisions and the other two, incorrect decisions.

1. H_0 is true and H_0 is not rejected – a correct decision.
2. H_0 is true and H_0 is rejected – an incorrect decision.
3. H_0 is false and H_0 is not rejected – an incorrect decision.
4. H_0 is false and H_0 is rejected – a correct decision.

6.4 LEVEL OF SIGNIFICANCE

In testing a given hypothesis, the maximum probability with which one would be willing or prepared to risk a Type I error is called the *level of significance* of the test. This probability often denoted by α is generally specified before any samples are drawn, so that results obtained will not influence the choice made.

The general procedure in hypothesis testing is to specify a value of the probability of type I error α, often called the *significance level* of the test, and then design the test procedure so that the probability of type II error β has a suitably small value.

6.5 TAILS OF A TEST

The rejection region for a hypothesis-testing problem can be on both sides with the non-rejection region in the middle, or it can be on the left side or on the right side of the non-rejection region. A test with two rejection regions is called a *two-tailed test*, and a test with one rejection region is called a *one-tailed test*. The one-tailed test is called a *left-tailed test* if the rejection regions is on the left tail of the distribution curve, and it is called a *right-tailed test* if the rejection region is in the right tail of the distribution curve. In such cases, the critical region is a region to one side of the distribution with area equal to the level of significance.

Table 6.2 gives critical values of z for one-tailed and two-tailed tests at various levels of significance. If under a given hypothesis, the sampling distribution of a statistic S is a normal distribution with mean μ and standard deviation σ. Then, the distribution of a standardised variable (or z-score) is given by $z = (S - \mu)/\sigma$, is the standardised normal distribution (mean 0, variance 1) as shown in Table 6.1. Table 6.3 summarises the relationships between the signs in H_0 and H_1 and the tails of a test.

Table 6.2: Critical value of *z*

Level of significance, z	0.10	0.05	0.01	0.005	0.002
Critical value of z for one-tailed test	–1.28 or 1.28	–1.645 or 1.645	–2.33 or 2.33	–2.58 or 2.58	–2.88 or 2.88
Critical value of z for two-tailed test	–1.645 or 1.645	–1.96 or 1.96	–2.58 or 2.58	–2.81 or 2.81	–3.08 or 3.08

Table 6.3: Summary of the relationships between the signs in H_0 and H_1 and the tails of a test

	Two-tailed test	Left-tailed test	Right-tailed test
Sign in the null hypothesis H_0	=	= or ≥	= or ≤
Sign in the alternative hypothesis H_1	≠	<	>
Rejection region	In both tails	In the left tail	In the right tail

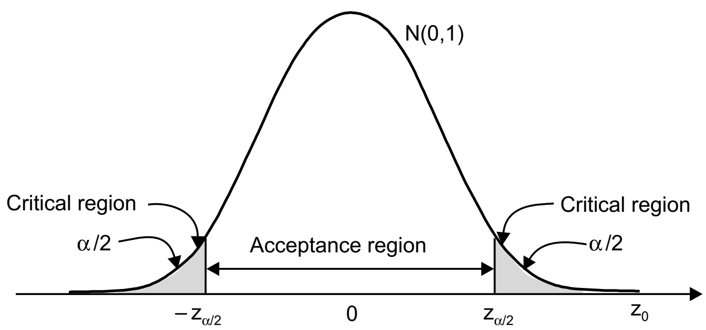


Fig. 6.1: The distribution of Z_0, when H_0: $\mu = \mu_0$ is true

For a fixed sample size, the smaller we specify the significance level, α, the larger will be the probability, β, of not rejecting a false null hypothesis. Suppose that a hypothesis test is conducted at a small significance level, then if the null hypothesis is rejected, we conclude that the alternative hypothesis is true. If the null hypothesis is not rejected, we conclude that the data do not provide sufficient evidence to support the alternative hypothesis.

To test a hypothesis, we devise a procedure for taking a random sample, computing an appropriate *test statistic*, and then rejecting or failing to reject the null hypothesis H_0. Part of this procedure is specifying the set of values for the test statistic that leads to rejection of H_0. This set of values is called the *critical region* or *rejection region* for the test.

The steps involved in performing a test of hypothesis are listed below:

1. State the null hypothesis. Under H_0 give a specific value of the parameter.
2. State the alternative hypothesis H_1. It specifies a range of possible values for the parameter that is being tested. It is important here to decide whether the test is one-tail or two-tail. Rejection of H_0 leads to acceptance of H_1.
3. Select an appropriate test statistic. The selection of the test statistic is determined by the conventional point estimator of the parameter in question.
4. Stipulate the level of significance, α, the probability of rejecting H_0 wrongly. The critical points are determined by the value of α. Formulate the decision rule based on step 2, that is, determine the values of the test statistic that will lead to the rejection of H_0 (the critical region).
5. Take a random sample and compute the value of the test statistic.
6. Make a decision based on the decision rule formulated in step 4. The choice is either reject H_0 or not reject it. Finally translate the conclusions into simple language for the benefit of the uninitiated.

6.6 HYPOTHESIS TEST ON THE POPULATION MEAN, STANDARD DEVIATION KNOWN

A basic assumptions about the population here is that it is normally distributed. In the absence of a normally distributed population, we will require that the sample size be large, that is, at least 30. The level of significance is then approximately 100 α per cent.

Suppose, the random variable X represents the population of interest. We assume that the distribution of X is either normal or that, if it is non-normal, the conditions of the central limit theorem hold.

Consider that the mean of μ of X is unknown but that the variance σ^2 is known. Suppose, we are interested in testing the hypothesis

$$H_0: \mu = \mu_0$$
$$H_1: \mu \neq \mu_0 \tag{6.1}$$

where μ_0 is a specified constant.

A random sample size of n, $X_1, X_2, \ldots, X_n$, is available. Each observation in this sample has unknown mean μ and known variance σ^2. The test procedure for H_0: $\mu = \mu_0$ uses the test statistic

$$Z_0 = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}} \tag{6.2}$$

If the null hypothesis H_0: $\mu = \mu_0$ is true, then $E(\bar{X}) = \mu_0$, and it follows that the distribution of Z_0 is N (0, 1). Consequently, if H_0: $\mu = \mu_0$ is true, the probability is $1 - \alpha$ that a value of the test statistic Z_0 falls between $-Z_{\alpha/2}$ and $Z_{\alpha/2}$, where $Z_{\alpha/2}$ is the percentage point of the standard normal distribution such that $P(Z \geq Z_{\alpha/2}) = \alpha/2$ (i.e., $Z_{\alpha/2}$) is the 100 $\alpha/2$ percentage point of the standard normal distribution). The situation is illustrated in Fig. 6.2.

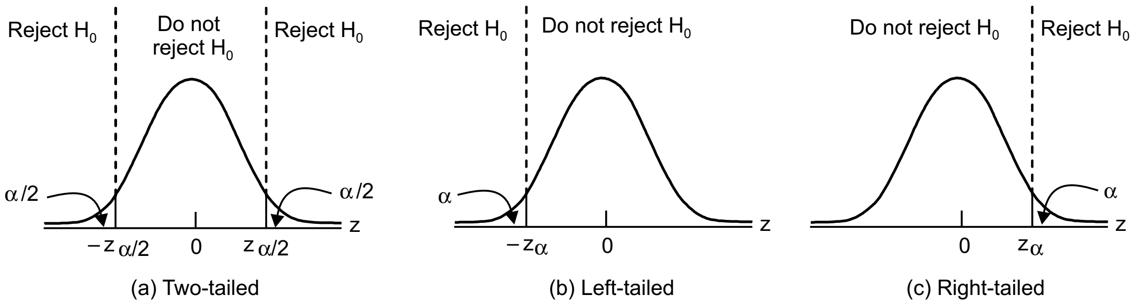


Fig. 6.2

Note that the probability is α that a value of the test statistic Z_0 would fall in the region $Z_0 > Z_{\alpha/2}$ or $Z_0 < -Z_{\alpha/2}$ when H_0: $\mu = \mu_0$ is true. We should reject H_0 if either

$$Z_0 > Z_{\alpha/2} \tag{6.3}$$

or

$$Z_0 < -Z_{\alpha/2} \tag{6.4}$$

and fail to reject H_0 if

$$-Z_{\alpha/2} \leq Z_0 \leq Z_{\alpha/2} \tag{6.5}$$

Equation (6.5) defines the *acceptance region* for H_0 and Eqs. (6.4) and (6.5) defines the *critical region* or *rejection region*. The type II error probability for this test procedure is α.

A summary of the test criteria to test H_0: $\mu = \mu_0$ against the three forms of alternative hypothesis is given in Table 6.4.

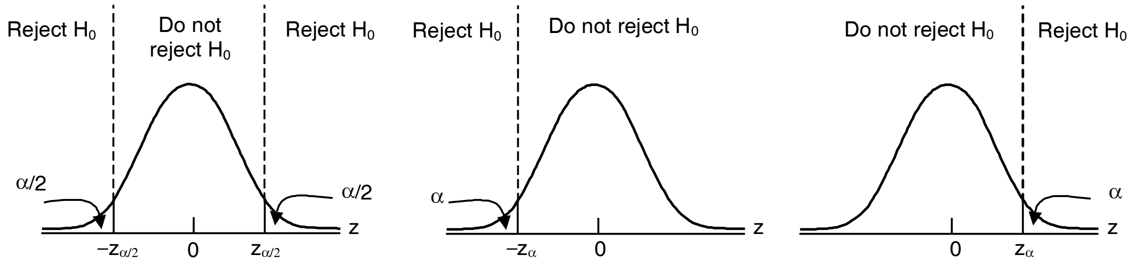
Table 6.4: Procedure to test on the population mean, standard deviation known

Assumptions: 1. Simple random sample 2. Normal population or large sample 3. σ known STEP 1: The null hypothesis is H_0: $\mu = \mu_0$, and the alternative hypothesis is H_α: $\mu \neq \mu_0$ H_α: $\mu < \mu_0$ H_α: $\mu > \mu_0$ (Two-tailed) or (Left-tailed) or (Right-tailed) STEP 2: Decide on the significance level, α. STEP 3: Compute the value of the test statistic $z = \dfrac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$

STEP 4: The critical value(s) are

$\pm z_{\alpha/2}$ (Two-tailed) or $-z_{\alpha}$ (Left-tailed) or z_{α} (Right-tailed)

Use the table in Appendix-*E* to find the critical value(s).



(a) Two-tailed (b) Left-tailed (c) Right-tailed

STEP 5: If the value of the test statistic falls in the rejection region, reject H_0; otherwise, do not reject H_0.
STEP 6: Interpret the results of the hypothesis test.

The hypothesis test is exact for normal populations and is approximately correct for large samples from non-normal populations.

Example E6.1b

The tensile strength of steel bars produced by a manufacturer has a mean of 30 MPa and the standard deviation of 1.5 MPa. By a new technique in the manufacturing process, it is claimed that the tensile strength can be improved. To test this claim, a sample of 50 steel bars is tested and it is found that the mean tensile strength is 30.5 MPa. Can we support the claim at 0.01 level of significance?

SOLUTION:

A decision has to be made between the two hypotheses

H_0: $\mu = 30$ MPa and there is really no change in tensile strength

H_1: $\mu > 30$ Mpa and there is a change in tensile strength

A one-tailed test should be used here. At a 0.01 level of significance, the decision rule is:

1. If the z-score observed is greater than 2.33, the results are significant at the 0.01 level and H_0 is rejected. The area between 0 and z_1 in Fig. E6.1 is 0.4900, and $z_1 = 2.33$.
2. Otherwise, H_0 is accepted (or the decision is withheld). Under the hypothesis that H_0 is true, we find

$$Z_0 = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{30.5 - 30}{1.5/\sqrt{50}} = 2.357$$

which are greater than 2.33. The decision rule is: Reject H_0 if the computed value of the test statistic is greater than 2.33. Because the computed value 2.357 is greater than 2.33, we reject H_0. Hence, we can conclude that the results are "highly significant" and therefore the claim should be supported.

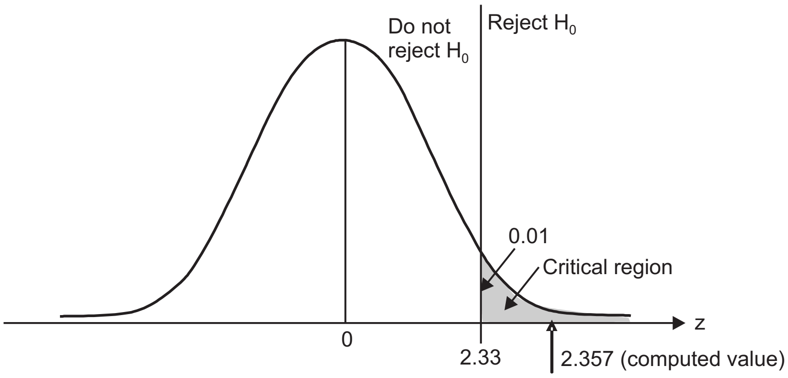


Fig. E6.1

Example E6.2

The manufacturer of a certain brand an automobile component claims that mean life of these components is 65 months. An independent agency wants to check this claim and took a random sample of 36 components and found that the mean life for this sample is 63.75 months with a standard deviation of 4 months.

(a) Using the 2.5% significance level, can we conclude that the mean life of these components is less than 65 months?

(b) Make the test a part of (*a*) using 5% significance level. Would the conclusion be different from the one in part (*a*)?

Solution:

Here, a decision has to be made between two hypotheses.

H_0: $\mu = 65$ months

H_1: $\mu < 65$ months

This is a left-tailed test.

(a) For $\alpha = 0.025$, the critical value of z is –1.96 (From the table in Appendix-*E*).

The test statistic is

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{66.015 - 66}{0.05 / \sqrt{40}} = -1.88$$

The decision rule is: Reject H_0 if the computed value of the test statistic is less than –1.96. Because the computed value –1.88 is greater than –1.96, we do not reject H_0.

Hence, do not reject H_0 and we conclude that the mean life of these components is not less than 65 months (see Fig. E6.2(a)).

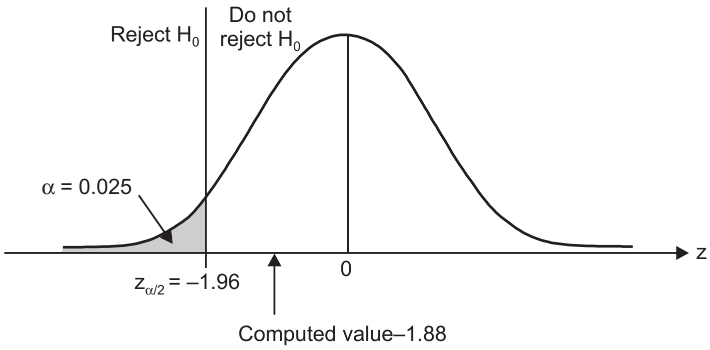


Fig. E6.2(a)

(b) For $\alpha = 0.05$, the critical value of $z = -1.65$ (from table in Appendix-*E*).

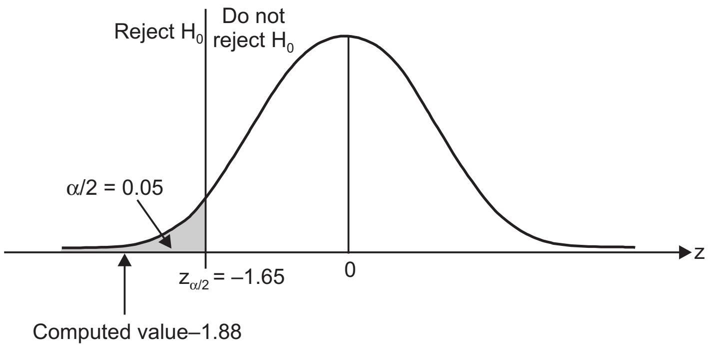


Fig. E6.2(b)

From Part (a), the value of the test statistic is –1.88. The decision rule is: Reject H_0 if the computed value of test statistic is less than –1.65. Because the computed value –1.88 is smaller than –1.65, we reject H_0. Reject H_0, and conclude that the mean life of these components is less than 65 months. The decision in parts (a) and (b) are quite different (see Fig. E6.2(b)).

Example E6.3

ABC company produces steel rods that are supposed to be 66 cm long. The mean length of the rods produced when the machine is working properly is 66 cm. The standard deviation of the length of all rods produced on this machine is 0.05 cm. A sample of 40 such steel rods was taken each week, calculated the mean length of these rods, and tested the null hypothesis against the alternative hypothesis using a 1% significance level. If the null hypothesis is rejected, the machine is stopped and adjusted. Recently a sample of 40 steel rods produced a mean length of 66.015 cm. Based on this sample, can we conclude that the machine needs an adjustment?

SOLUTION:

Here H_0: $\mu = 66$ cm

H_1: $\mu \neq 66$ cm

This is a two-tailed test.

For $\alpha = 0.01$, the critical values of z are –2.58 and 2.58 (From the table in Appendix-*E*).

The test statistic:

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{66.015 - 66}{0.05 / \sqrt{40}} = 1.90$$

The decision rule is: Reject H_0 if the computed value is less than –2.58 or greater than 2.58. Because, the computed value is less than 2.58, we do not reject H_0. Hence, do not reject H_0 and we conclude that the machine does not need to be adjusted (see Fig. E6.3).

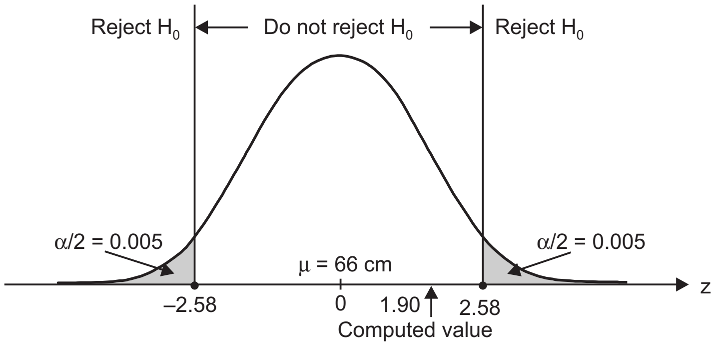


Fig. E6.3

6.7 HYPOTHESIS TEST ON THE POPULATION MEAN, STANDARD DEVIATION UNKNOWN

In tests of hypothesis regarding the population mean, the situation where the population standard deviation is not known is more realistic. The procedure presented here is particularly important when the sample size is small. The following assumptions are made:

1. The observations are independent.
2. The parent population has a normal distribution.

With the above assumptions, the statistic has students t-distribution with $n - 1$ degrees of freedom, if H_0 is true.

Suppose that $X \sim N(\mu, \sigma^2)$ and wish to test H_0: $\mu = \mu_0$, and we do not known μ or σ^2. This test procedure depends on X being normally distributed, although slight departures from normally are not series, especially when the sample size is large. A random sample of size n is taken, and $\bar{X}$ and S^2 are computed.

Now, to test the two-sided hypothesis

$$H_0: \mu = \mu_0$$
$$H_1: \mu \neq \mu_0$$

The test statistic is formed,

$$t_0 = \frac{\bar{X} - \mu_0}{S / \sqrt{n}} \tag{6.5a}$$

which follows the t-distribution with $n - 1$ degrees of freedom if H_0 is true. H_0 is rejected if $|t_0| > t_{\alpha/2,\,v}$ where $v = n - 1$. One-sided alternatives can also be formed in a similar manner.

A summary of the test procedure to test H_0: $\mu = \mu_0$ against the three alternative hypothesis is given in Table 6.5.

Table 6.5: Procedure to test on the population mean, standard deviation known

Assumptions:
1. Simple random sample
2. Normal population or large sample
3. σ unknown

STEP 1: The null hypothesis is H_0: $\mu = \mu_0$, and the alternative hypothesis is

H_α: $\mu \neq \mu_0$ H_α: $\mu < \mu_0$ H_α: $\mu > \mu_0$

(Two-tailed) or (Left-tailed) or (Right-tailed)

STEP 2: Decide on the significance level, α.

STEP 3: Compute the value of the test statistic

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

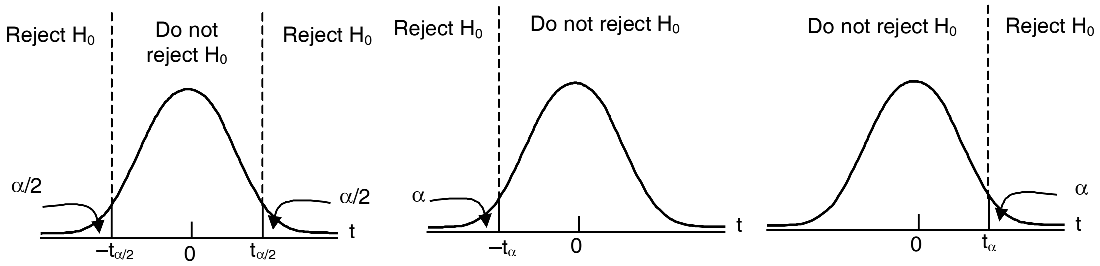
and denote that value t_0.

STEP 4: The critical value(s) are

$\pm t_{\alpha/2}$ $-t_\alpha$ t_α

(Two-tailed) or (Left-tailed) or (Right-tailed)

with $df = n - 1$. Use the table in Appendix-*G* to find the critical value(s).



(a) Two-tailed (b) Left-tailed (c) Right-tailed

STEP 5: If the value of the test statistic falls in the rejection region, reject H_0; otherwise, do not reject H_0.

STEP 6: Interpret the results of the hypothesis test.

The hypothesis test is exact for normal populations and is approximately correct for large samples from non-normal populations.

Example E6.4

A manufacturing company sent ten of its employees for a special training course to increase their productivity. The following table gives the one-week production of these employees before and after they attend this course.

Before	12	10	18	23	25	9	14	16	7	8
After	18	13	24	22	24	14	19	20	10	11

Using the 1% significance level, can a conclusion be made saying that "the mean weekly productions of all the employees increase as a result of attending this course"? Assume the populations of paired differences have a normal distribution.

SOLUTION:

Since the data are for paired samples, we can test the hypothesis about paired differences mean μ_0 of population using the paired differences mean $\bar{x}$ of the sample.

Let x = weekly output before the course – weekly output after the course.

Before	After	Difference	x^2
12	18	–6	36
10	13	–3	9
18	24	–6	36
23	22	1	1
25	24	1	1
9	14	–5	25
14	19	–5	25
16	20	–4	16
7	10	–3	9
8	11	–3	9

$$\sum x = -33 \text{ and } \sum x^2 = 167$$

$$\bar{x} = \frac{\Sigma x}{n} = \frac{-33}{10} = -3.3$$

$$\sigma = \sqrt{\frac{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}{n-1}} = \sqrt{\frac{167 - \frac{(-33)^2}{10}}{10-1}} = 2.541$$

$$\frac{\sigma}{\sqrt{n}} = \frac{2.541}{\sqrt{10}} = 0.8035$$

Let $\mu_d = \mu_1 - \mu_2$

$H_0: \mu_d = 0$ ($\mu_1 - \mu_2 = 0$ or the mean output do not increase)

$H_1: \mu_d < 0$ ($\mu_1 - \mu_2 < 0$ or the mean output do increase)

Since the sample size is small (< 30), the population of paired differences is normal, and μ_d is unknown, we can use t-distribution to conduct the test. Degree of freedom = $n - 1 = 10 - 1 = 9$.

The critical value of t is obtained from the table in Appendix-G, $t_{0.01,9} = 2.821$.

The test statistic t is

$$t = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}} = \frac{-3.3 - 0}{0.8035} = -4.107$$

Since the test statistic $t = -4.107$ for $\bar{X}$ fall in the rejection region, we reject the null hypothesis. Therefore, we conclude that the mean weekly output for all employees increase as a result of the course.

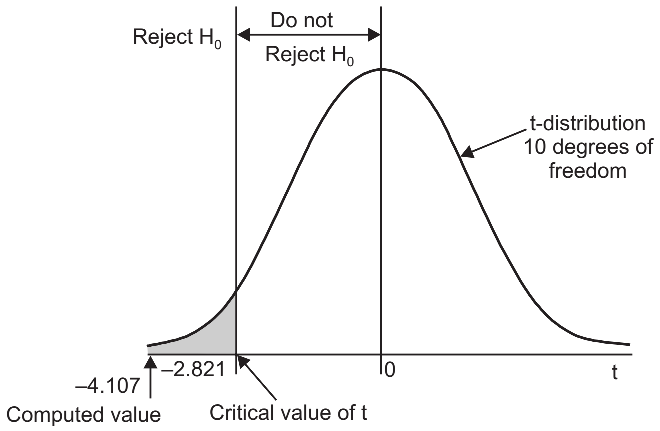


Fig. E6.4

Example E6.5

A manufacturer wants to determine whether the diameter of the steel rings gives satisfactory requirement. The particular ring will be considered satisfactory if the true diameter is greater than 20 cm. A field experiment was conducted in which 10 rings were tested at almost identical conditions and the mean diameter of the steel rings were computed for each. The results (in cm) were as follows:

23, 18, 22.1, 19.1, 19.05, 22.1, 18.2, 18.05, 23.9, and 21.9

Based on the data, is there sufficient evidence for the manufacturer to decide that the rings are satisfactory? The manufacturer is prepared to run a risk of Type I error of 0.05. It may be assumed that the diameter of the steel rings is normally distributed.

Solution:

$$H_0 = \mu = 20$$
$$H_1 = \mu > 20$$

The standard deviation of the distribution is not known. We will have to estimate it from the sample. Hence, the test statistic is

$$\frac{\bar{X} - 20}{S/\sqrt{n}}$$

Here, $\alpha = 0.05$, and since $n = 10$, we have $10 - 1 = 9$ degrees of freedom. Thus, from the table for t-distribution (Appendix-G), we find $t_{9,0.05} = 1.833$. The decision rule is: Reject H_0 if the computed value of the test statistic is greater than 1.833. It can easily verified from the data that

$$\Sigma x = 205.4, \ \bar{X} = 20.54, \ \frac{\Sigma(x - \bar{x})^2}{n-1} = 5.1643$$

$$\sigma^2 = 5.1643 \text{ or } s = 2.2725$$

Hence, the computed value of the test statistic is

$$\frac{\bar{X}-20}{s/\sqrt{n}} = \frac{20.54-20}{2.2725/\sqrt{10}} = 0.7514 \text{ (see Fig. E6.5)}$$

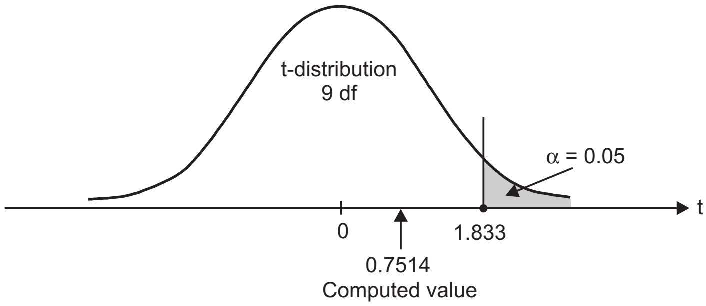


Fig. E6.5

Example E6.6

A machine can adjusted so that when under control, the mean amount of fertilizer filled in a bag is 5 kg. To check if the machine is under control, 6 bags were picked at random and their weight (in kg) were found to be as follows:

5.32, 5.19, 4.78, 5.18, 4.78 and 5.31

At the 5% level of significance, is it true that the machine is not under control? (Assume a normal distribution for the weight of a bag).

SOLUTION:

Here σ is not known and we will estimate it from the given sample.

$$H_0 = \mu = 5$$
$$H_1 = \mu \neq 5$$

The test statistic is

$$\frac{\bar{X}-5}{S/\sqrt{n}}$$

We have a two-tailed test. Given $\alpha = 0.05$ and $n = 6$, $t_{n-1,\,\alpha/2} = t_{5,0.025} = 2.571$ (from the table in Appendix-G). Hence, the decision rule is: Reject H_0 if the computed value is less than –2.571 or greater than 2.571.

It can easily be verified from the given sample data that

$$\bar{X} = 5.0933, \qquad s^2 = \frac{\Sigma(X-\bar{X})^2}{n-1} = 0.0623 \text{ and } s = 0.2496$$

Thus, the computed value of the test statistic is

$$\frac{\bar{X}-20}{S/\sqrt{n}} = \frac{5.1-5}{0.2496/\sqrt{6}} = 0.9813$$

Since the computed value is between –2.571 and 2.571, we do not reject H_0.

Conclusion: There is no significant evidence at the 5% significance level to indicate that the machine is not under control, so there is no need to adjust the machine (See Fig. E6.6).

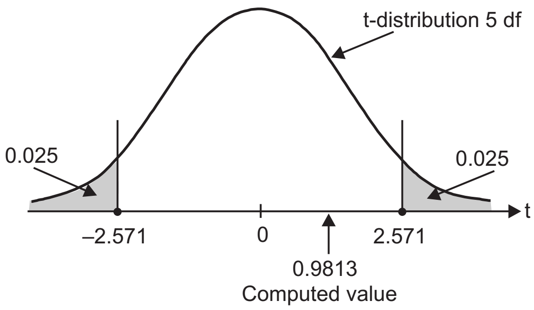


Fig. E6.6

6.8 HYPOTHESIS TEST FOR A POPULATION VARIANCE

The following assumptions are made:

1. The observations are independent.
2. The parent population has a normal distribution.

If the population is normally distributed with variance σ^2, then the distribution of $\dfrac{(n-1)S^2}{\sigma^2}$ is chi-square with $n – 1$ degrees of freedom, where n represents the sample size. Hence, if H_0 is true, that is, if the population variance is σ_0^2, then the distribution of $\dfrac{(n-1)S^2}{\sigma_0^2}$ will be chi-square with $n – 1$ degrees of freedom.

Here, we test the hypothesis that the variance of a normal distribution σ^2 equals a specified value of σ_0^2. Let $X \sim N(\mu, \sigma^2)$, where μ and σ^2 are unknown, and let $X_1, X_2, \ldots, X_n$ be a random sample of n observations from this population. To test

$$H_0: \sigma^2 = \sigma_0^2$$

$$H_1: \sigma^2 \neq \sigma_0^2 \tag{6.6}$$

We use the test statistic

$$\chi_0^2 = \frac{(n-1)S^2}{\sigma_0^2} \tag{6.7}$$

where the S^2 is the sample variance. Now if H_0: $\sigma^2 = \sigma_0^2$ is true, then the test statistic χ_0^2 follow the chi-square distribution with $n – 1$ degrees of freedom. Therefore, H_0: $\sigma^2 = \sigma_0^2$ would be rejected if

$$\chi_0^2 > \chi^2_{\alpha/2,\, n-1} \tag{6.8}$$

$$\chi_0^2 > \chi^2_{1-\alpha/2,\, n-1} \tag{6.9}$$

where $\chi^2_{\alpha/2,\, n-1}$ and $\chi^2_{1-\alpha/2,\, n-1}$ are the upper and lower $\alpha/2$ percentage point of chi-square distribution with $n - 1$ degrees of freedom.

For the one-sided hypothesis

$$H_0\colon \sigma^2 = \sigma_0^2$$

$$H_1\colon \sigma^2 > \sigma_0^2 \tag{6.10}$$

We would reject H_0 if

$$\chi_0^2 > \chi^2_{\alpha,\, n-1} \tag{6.11}$$

For the other one-sided hypothesis

$$H_0\colon \sigma^2 = \sigma_0^2$$

$$H_1\colon \sigma^2 < \sigma_0^2 \tag{6.12}$$

We would reject H_0 if

$$\chi_0^2 < \chi^2_{1-\alpha,\, n-1} \tag{6.13}$$

A summary of the test procedure to test H_0: $\sigma^2 = \sigma_0^2$ against the three alternative hypothesis is given in Table 6.6.

Table 6.6: Procedure to test for a population variance

Assumptions:
1. Simple random sample
2. Normal population

STEP 1: The null hypothesis is H_0: $\sigma^2 = \sigma_0^2$, and the alternative hypothesis is

H_a: $\sigma^2 \neq \sigma_0^2$ H_a: $\sigma^2 < \sigma_0^2$ H_a: $\sigma^2 > \sigma_0^2$

(Two-tailed) or (Left-tailed) or (Right-tailed)

STEP 2: Decide on the significance level, α.

STEP 3: Compute the value of the test statistic

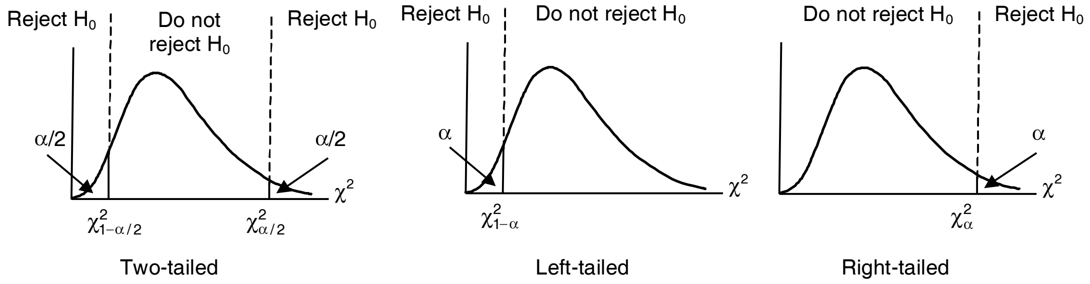
$$\chi^2 = \frac{n-1}{\sigma_0^2} S^2$$

and denote that value χ_0^2 .

STEP 4: The critical value(s) are

$\chi^2_{1-\alpha/2}$ and $\chi^2_{\alpha/2}$ (Two-tailed) or $\chi^2_{1-\alpha}$ (Left-tailed) or χ^2_{α} (Right-tailed)

with $df = n - 1$. Use the table in Appendix-*F* to find the critical value(s).



STEP 5: If the value of the test statistic falls in the rejection region, reject H_0; otherwise, do not reject H_0.

STEP 6: Interpret the results of the hypothesis test.

Example E6.7

The copper content in an alloy material used to make a machine part is normally distributed, and the variance is $\sigma^2 = 20$. Test the hypothesis H_0: $\sigma^2 = 20$ versus H_0: $\sigma^2 \neq 20$, if a random sample of $n = 10$ machine parts yields a sample standard deviation of $s = 4$. Use $\alpha = 0.05$.

Solution:

$$\sigma^2 = 20,\ n = 10, s = 4$$
$$H_0: \sigma^2 = 20$$
$$H_0: \sigma^2 \neq 20$$
$$\alpha = 0.05$$

The test statistic is

$$\chi^2_0 = \frac{(n-1)s^2}{\sigma^2_0} = \frac{(10-1)(4)^2}{20} = 7.2$$

The alternative hypothesis is two-sided so that the left is two-tailed. Now $n = 10$ and $\alpha = 0.05$.

$$\chi^2_{n-1,\alpha/2} = \chi^2_{9,0.025}$$

$$\chi^2_{n-1,1-\alpha/2} = \chi^2_{9,0.975} \quad \text{[from the table in Appendix-}F\text{]}$$

$$\chi^2_{0.05/2,n-1} = \chi^2_{0.025,9} = 19.022 \text{ [from the table in Appendix-}F\text{]}$$

$$\chi^2_{1-\alpha/2,n-1} = \chi^2_{0.975,9} = 2.70 \text{ [From the table in Appendix-}F\text{]}$$

$$\chi^2_{0.975,9} \leq \chi^2_0 \leq \chi^2_{0.025,9}$$

Do not reject H_0, the data provide insufficient evidence to claim that the variance of the copper content of the machine parts is not 20 at $\alpha = 0.05$.

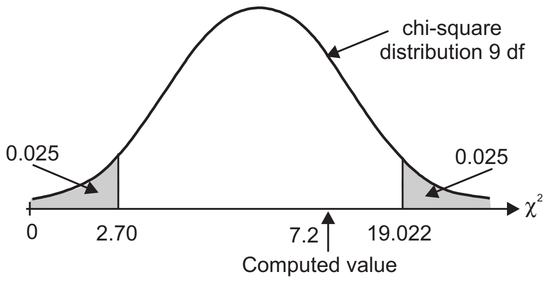


Fig. E6.7

Example E6.8

The following measurements (in kg) were obtained on the weights of 6 manufactured metal parts picked at random: 12.1, 8.1, 7.2, 12.3, 14.05 and 13. At the 5% level of significance, is the population variance of the weight of such metal parts greater than 2.3?

Solution:

$$H_0: \sigma = \sqrt{2.3} = 1.5166$$
$$H_1: \sigma > 1.5166$$

The test statistic is

$$\frac{(n-1)S^2}{2.3}$$

The alternative hypothesis is right-sided. Because $n = 6$, the chi-square has 6 – 1 = 5 degrees of freedom. From the table in Appendix-*F*, $\chi^2_{5,0.05} = 11.07$. Hence, the decision rule is: Reject H_0 if the computed value of the test statistic exceeds 11.07. It can be verified that $\Sigma x = 66.75$, $\overline{X} = 11.125$ and $S^2 = 7.7916$. Therefore, the computed value of the test statistic is

$$\frac{(n-1)S^2}{2.3} = \frac{(6-1)(7.7916)}{2.3} = 16.9382 \text{ (see Fig. E6.8)}$$

Since the computed value 16.9382 is greater than 11.07, we reject H_0. We conclude that at 5% level of significance, there is evidence indicating that the population variance exceeds 2.3.

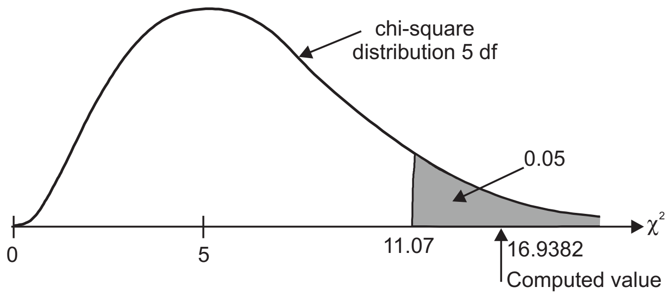


Fig. E6.8

Example E6.9

A restaurant claims that the standard deviation in the length of serving times is less than 3 minutes. A random sample of 23 serving times has a standard deviation of 2.3 minutes. At $\alpha = 0.10$, is there enough evidence to support the restaurant's claim? Assume the population is normally distributed.

SOLUTION:

The claim is "the standard deviation is less than 3 minutes".

$$H_0: \sigma \geq 3 \text{ minutes}$$

and

$$H_1: \sigma < 3 \text{ minutes}$$

The test statistic is

$$\frac{(n-1)S^2}{\sigma^2}$$

Because the test is left-tailed test, the level of significance is $\alpha = 0.10$. There are 23 – 1 = 22 degrees of freedom and the critical value is 14.04 (from the table in Appendix-*F*). Therefore, the decision rule is: Reject H_0 if the computed value of the test statistic is less than 14.04.

$$\frac{(n-1)S^2}{\sigma^2} = \frac{(23-1)(2.3)^2}{3^2} = 12.9311 \text{ (see Fig. E6.9)}$$

Because the computed value is less than 14.04, we reject H_0.

Conclusion: There is enough evidence at the 10% level of significance to support the claim that the standard deviation for the lengths of serving times is less than 3 minutes.

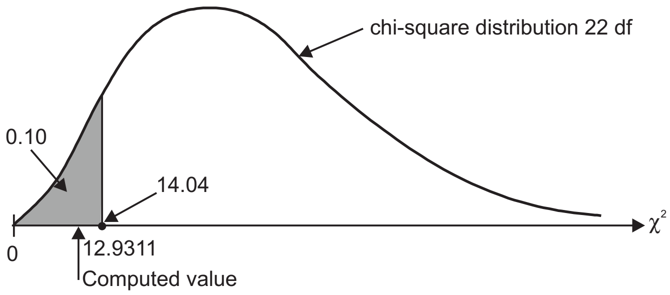


Fig. E6.9

6.9 HYPOTHESIS TEST ON A POPULATION PROPORTION

Here, we treat the case of a qualitative variable, where the data are recorded as black-white, tall-short, successes-failures, defective-nondefective, etc. The purpose will be to test hypothesis regarding the proportion p of a certain attribute in the population.

To carry out the test of hypothesis regarding the population proportion, we select a sample of observations and take the proportion of the attribute in the sample as statistic on which the test is based. We make the following assumptions:

1. The sample consists of n independent observations.
2. The sample size is large.
3. The hypothesised population proportion p_0 is not too close to 0 or 1 and is such that np_0 and $n(1 - p_0)$ are both greater than 5.

If p is the proportion in the population, then the sample proportion $\frac{X}{n}$ has a sampling distribution with mean p and standard deviation $\sqrt{p(1-p)/n}$.

If the sample size is large, both np and $n(1 - p)$ are greater than 5, then the distribution X/n is approximately normal. If the null hypothesis is true, (population proportion, p_0), then X/n will have a distribution that is approximately normal with mean p_0 and standard deviation $\sqrt{p_0(1-p)/n}$, where np_0 and $n(1 - p_0)$ are assumed greater than 5.

Consider a binomial random variable to be tested

$$H_0: p = p_0$$
$$H_0: p^1 = p_0$$

If $0.1 \le p \le 0.9$ and n large, the procedure described is acceptable.

Pick a random sample of size n and compute $\hat{p} = X/n$, where X represents the number of observations that are nonconfirming. If H_0 is true, then $X \sim N[p_0, p_0(1-p_0)/n]$. To test $p = p_0$, calculate

$$Z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \tag{6.14}$$

and reject H_0 if $|Z_0| > Z_{\alpha/2}$. One-sided alternative could also be tested.

The above test procedure is based on the application of the central limit theorem. Therefore, the significance level is approximately 100α per cent.

Assuming of the test procedure to test H_0: $p \ne p_0$ against the three alternative hypotheses is given in Table 6.7.

Table 6.7: Procedure to test for a population proportion

Assumptions: 1. Simple random sample 2. Both np_0 and $n(1 - p_0)$ are 5 or greater STEP 1: The null hypothesis is H_0: $p = p_0$, and the alternative hypothesis is H_a: $p \ne p_0$ H_a: $p < p_0$ H_a: $p > p_0$ (Two-tailed) or (Left-tailed) or (Right-tailed) STEP 2: Decide on the significance level, α.

STEP 3: Compute the value of the test statistic

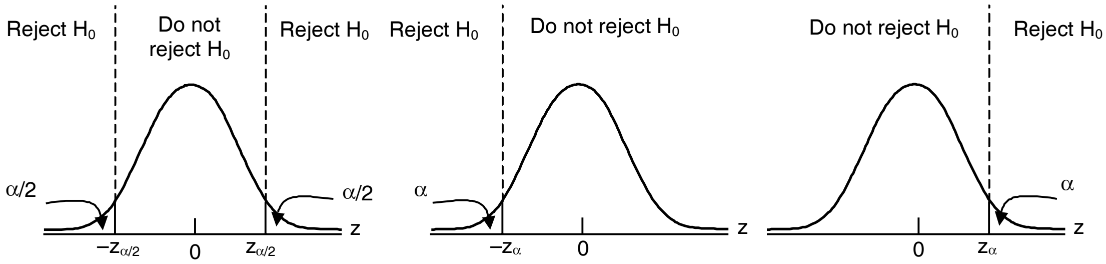
$$z = \frac{\hat{p} - p_0}{\sqrt{p_0(1 - p_0)/n}}$$

and denote that value z_0.

STEP 4: The critical value(s) are

$\pm z_{\alpha/2}$ (Two-tailed) or $-z_{\alpha}$ (Left-tailed) or z_{α} (Right-tailed)

Use the table in Appendix-*E* to find the critical value(s).



(a) Two-tailed (b) Left-tailed (c) Right-tailed

STEP 5: If the value of the test statistic falls in the rejection region, reject H_0; otherwise, do not reject H_0.

STEP 6: Interpret the results of the hypothesis test.

Example E6.10

An internet catalogue ordering company wants to deliver on time at least 90% all the orders received from its customers. A sample of 100 orders taken for inspection showed that 83 of them were delivered on time.

(a) Using the 2% significant level, can we conclude the company's delivery policy is maintained?

(b) What will the decision be if the probability of making a type *I* error is zero?

SOLUTION:

(a)
$$H_0: p \geq 0.90$$
$$H_1: p < 0.90$$

For $\alpha = 0.02$, the critical value of Z is –2.05 (From the table in Appendix-*E*).

$$\hat{p} = \frac{83}{100} = 0.83$$

The test statistic is

$$Z = \frac{\hat{p} - p}{\sqrt{pq/n}} = \frac{0.83 - 0.90}{\sqrt{(0.90)(0.10)/100}} = -2.333$$

Because $\alpha = 0.02$, the decision rule is: Reject H_0 if the computed value is less than $-z_{0.02} = -2.05$. The computed value –2.333 is less than –2.05. Reject H_0. The conclusion is that the company's policy is not maintained.

(b) If $\alpha = 0$, there is no rejection region. Hence, we cannot reject H_0 and cannot conclude that the company's policy is maintained.

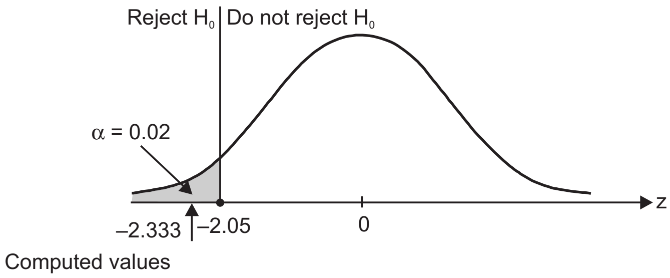


Fig. E6.10

Example E6.11

When a coin is tossed 198 times, it should heads 118 times. Is the coin biased in favour of heads? Conduct the test of hypothesis at the 5% level of significance.

SOLUTION:

Let p = probability of getting a head on a toss. Hence, we can state

$$H_0: p = 0.5 \text{ (really } p \le 0.5)$$
$$H_1: p > 0.5$$

Since $p_0 = 0.5$, the test statistic is

$$\frac{\hat{p} - p_0}{\sqrt{p_0(1 - p_0)/n}}$$

$\alpha = 0.05$, hence the decision rule is: Reject H_0 if the computed value is greater than $z_{0.05} = 1.645$.

Since $x = 198$ and $n = 198$, the computed value of the test statistic is

$$\frac{\hat{p} - p_0}{\sqrt{p_0(1 - p_0)/n}} = \frac{\frac{118}{198} - 0.5}{\sqrt{(0.5)(0.5)/198}} = 2.7 \text{ (see Fig. E6.11)}$$

The computed value 2.7 is greater than 1.645. Hence, we reject H_0. We conclude that at the 5% level of significance, there is strong evidence that the coin is biased in favour of heads.

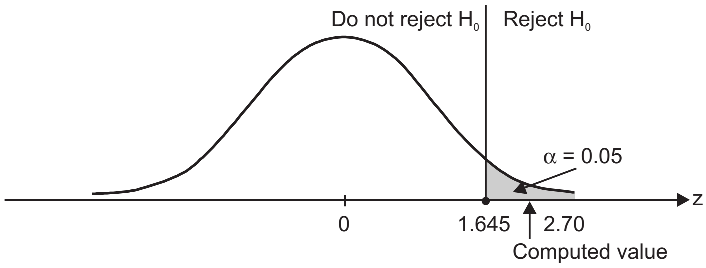


Fig. E6.11

Example E6.12

An independent survey found that 24% of the population are in favour of alcohol prohibition. You decide to test this claim and ask a random sample of 200 persons whether they are in favour of alcohol prohibition of the 200 persons, 28% are in favour. At $\alpha = 0.05$, is there enough evidence to reject the claim?

SOLUTION:

$$H_0: p = 0.24 \text{ (claim)}$$
$$H_1: p \neq 0.24$$

Because $p_0 = 0.24$, the test statistic is

$$\frac{\hat{p} - p_0}{\sqrt{p_0(1 - p_0)/n}}$$

Because the test is a two-tailed test and the level of significance is $\alpha = 0.05$, the critical values are $-z_0 = -1.96$ and $z_0 = 1.96$. Therefore, the decision rule is: Reject H_0 if the computed value is less than -1.96 or greater than 1.96.

$$z = \frac{0.28 - 0.24}{\sqrt{(0.24)(0.76)/200}} = 1.324 \text{ (see Fig. E6.12)}$$

The computed value is between -1.96 and 1.96, and so we do not reject H_0.

Conclusion: At the 5% level of significance, there is not enough evidence to reject the claim that 24% of the population are in favour of alcohol prohibition.

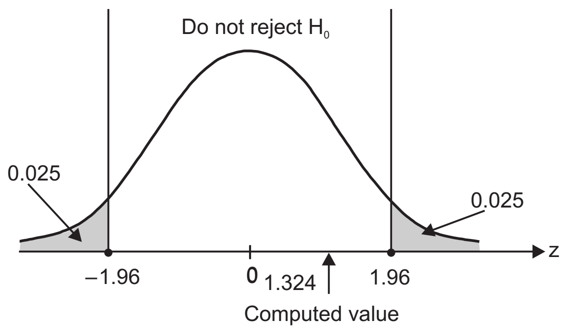


Fig. E6.12

6.10 HYPOTHESIS TEST ON EQUALITY OF TWO MEANS, VARIANCES KNOWN

There are two normal populations (or two populations from which large samples are taken) with unknown means, say μ_1 and μ_2, and known variances σ_1^2 and σ_2^2. To test the equality of the means, the following hypotheses are used:

$$H_0: \mu_1 = \mu_2$$
$$H_1: \mu_1 \neq \mu_2$$

The test procedure consists of taking a random sample of size n_1 from the first population and compute $\bar{X}_1$. Similarly take a sample of size n_2 from the second population and compute $\bar{X}_2$. The test statistic is then given by

$$Z_0 = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \tag{6.15}$$

with critical region $|Z_0| > Z_{\alpha/2}$.

The one-sided alternative hypotheses can also be used to test

$$H_0: \mu_1 = \mu_2$$
$$H_1: \mu_1 > \mu_2$$

The test statistic given by Eq.(6.15) is computed, and H_0 is rejected if $Z_0 > Z_\alpha$.

To test the other one-sided alternative hypotheses

$$H_0: \mu_1 = \mu_2$$
$$H_1: \mu_1 < \mu_2$$

The test statistic given by Eq.(6.15) is also used, but H_0 is rejected if $Z_0 < -Z_\alpha$.

The assumption of normal distribution of population is not important if sample sizes are large, that is, as a general rule, n_1 and n_2 are both at least 30. Also, here, even the population variances need not be known. The sample variance S_1^2 and S_2^2 can be used in place of respective variances σ_1^2 and σ_2^2.

Table 6.8: Procedure to test equality of two means, variances known

Assumptions:
1. Simple random sample
2. Independent samples
3. Normal populations or large samples

STEP 1: The null hypothesis is H_0: $\mu_1 = \mu_2$, and the alternative hypothesis is

H_a: $\mu_1 \neq \mu_2$ (Two-tailed) or H_a: $\mu_1 < \mu_2$ (Left-tailed) or H_a: $\mu_1 > \mu_2$ (Right-tailed)

STEP 2: Decide on the significance level, α.

STEP 3: Compute the value of the test statistic

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{(S_1^2/n_1) + (S_2^2/n_2)}}$$

Denote that value of the test statistic t_0.

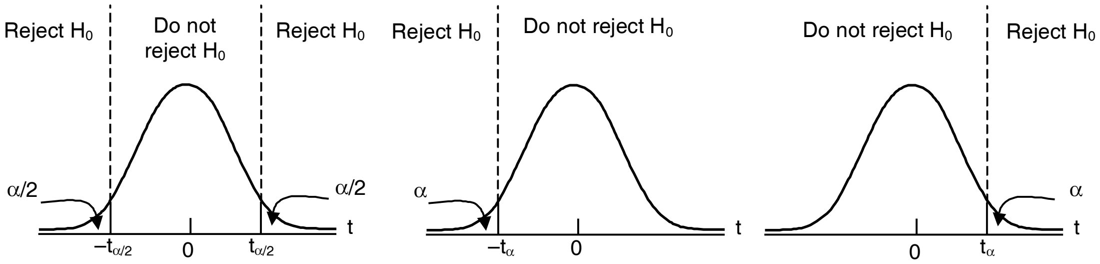
STEP 4: The critical value(s) are

$\pm t_{\alpha/2}$ (Two-tailed) or $-t_\alpha$ (Left-tailed) or t_α (Right-tailed)

with $df = \Delta$, where

$$\Delta = \frac{[(S_1^2/n_1)+(S_2^2/n_2)]^2}{\dfrac{(S_1^2/n_1)^2}{n_1-1}+\dfrac{(S_2^2/n_2)^2}{n_2-1}}$$

rounded down to the nearest integer. Use the table in Appendix-*G* to find the critical value(s).



(a) Two-tailed (b) Left-tailed (c) Right-tailed

STEP 5: If the value of the test statistic falls in the rejection region, reject H_0; otherwise, do not reject H_0.

STEP 6: Interpret the results of the hypothesis test.

Example E6.13

The following is summary statistics for independent simple random samples from 2 populations. Use the nonpooled *t*-test and the nonpooled *t*-interval procedure to conduct the required hypothesis test.

Sample 1	Sample 2
$\bar{X}_1 = 7$	$\bar{X}_2 = 10.14$
$S_1 = 1.22$	$S_2 = 4.59$
$n_1 = 14$	$n_2 = 6$

Is there sufficient evidence to conclude that the mean in sample 1 exceeds that in sample 2? Assume 1% significance level.

Solution:

Given $\bar{X}_1 = 115.8,\ \bar{X}_2 = 25, S_1 = 79.4, S_2 = 10.5, n_1 = 51, n_2 = 19.$

$$H_0: \mu_1 = \mu_2$$
$$H_1: \mu_1 > \mu_2$$
$$\alpha = 0.01$$

The test statistic is

$$t = \frac{115.8-25}{\sqrt{(79.4^2/51)+(10.5^2/19)}} = 7.982$$

$$\Delta = \frac{\left[\left(\frac{S_1^2}{n_1}\right)+\left(\frac{S_2^2}{n_2}\right)\right]^2}{\left[\frac{\left(\frac{S_1^2}{n_1}\right)^2}{n_1-1}\right]+\left[\frac{\frac{S_2^2}{n_2}}{n_2-1}\right]^2} = \frac{\left[\left(\frac{79.4^2}{51}\right)+\left(\frac{10.5^2}{19}\right)\right]^2}{\frac{\left(\frac{79.4^2}{51}\right)^2}{(51-1)}+\frac{\left(\frac{10.5^2}{19}\right)^2}{(19-1)}} = 54.472 \approx 54$$

The critical value = 2.397 (from the table in Appendix-*G*, after interpolation).

The decision rule is: Reject H_0 if the computed value is greater than 2.397. Since the computed value 7.982 is greater than 2.397, we reject H_0 (see Fig. E6.13).

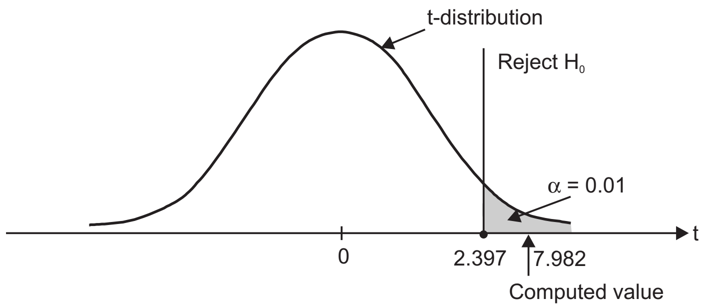


Fig. E6.13

Hence, at 1% significance level, the data do not provide sufficient evidence to conclude the mean from sample 1 exceeds that in sample 2.

Example E6.14

The following is summary statistics for independent simple random samples from 2 populations. Use the nonpooled *t*-test and the nonpooled *t*-interval procedure to conduct the required hypothesis test.

Sample 1	Sample 2
$\bar{X}_1 = 7$	$\bar{X}_2 = 10.14$
$S_1 = 1.22$	$S_2 = 4.59$
$n_1 = 14$	$n_2 = 6$

At the 5% significance level, do the data provide sufficient evidence to conclude that the mean of sample 1 smaller than sample 2?

SOLUTION:

$$H_0: \mu_1 = \mu_2$$
$$H_1: \mu_1 < \mu_2$$
$$\alpha = 0.05$$

The test is a left-tailed test.

The test statistic is

$$t = \frac{7 - 10.14}{\sqrt{\left(\frac{1.22^2}{14}\right) + \left(\frac{4.59^2}{6}\right)}} = -1.652$$

$$\Delta = \frac{\left[\left(\frac{S_1^2}{n_1}\right) + \left(\frac{S_2^2}{n_2}\right)\right]^2}{\frac{\left(\frac{S_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{S_2^2}{n_2}\right)^2}{n_2 - 1}} = \frac{\left[\left(\frac{1.22^2}{14}\right) + \left(\frac{4.59^2}{6}\right)\right]^2}{\frac{\left(\frac{1.22^2}{14}\right)^2}{14 - 1} + \frac{\left(\frac{4.59^2}{6}\right)^2}{6 - 1}} = 5.306 \approx 5$$

The critical value = –2.015 (from the table in Appendix-*G*).

The decision rule is: Reject H_0 if the computed value of the test statistic is less than –2.015. Because the computed value –1.652 is greater than –2.015, we do not reject H_0 (see Fig. E6.14).

At the 5% significance level, the data do not provide sufficient evidence to conclude that the mean is smaller with sample 1 then with sample 2.

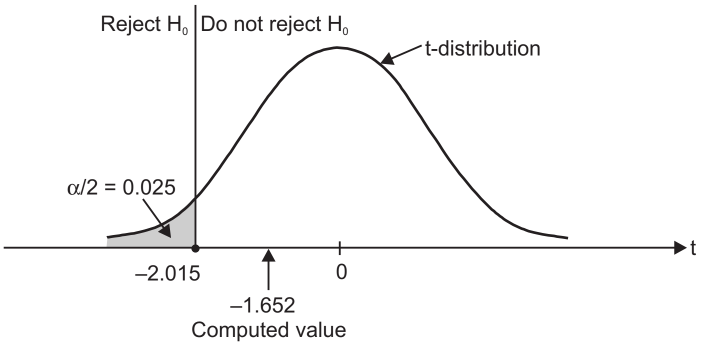


Fig. E6.14

Example E6.15

The following is summary statistics for independent simple random samples from 2 populations. Use the nonpooled *t*-test procedure to conduct the required hypothesis test.

Sample 1	Sample 2
$\bar{X}_1 = 80$	$\bar{X}_2 = 82.8$
$S_1 = 1.501$	$S_2 = 1.698$
$n_1 = 14$	$n_2 = 14$

At the 1% significance level, do the data provide sufficient evidence to conclude the means for the two samples are different?

SOLUTION:

$$H_0: \mu_1 = \mu_2$$
$$H_1: \mu_1 < \mu_2$$
$$\alpha = 0.01$$

The test statistic is

$$t = \frac{80 - 82.8}{\sqrt{\left(\frac{1.501^2}{14}\right) + \left(\frac{1.698^2}{14}\right)}} = -4.62$$

and

$$\Delta = \frac{\left[\left(\frac{S_1^2}{n_1}\right) + \left(\frac{S_2^2}{n_2}\right)\right]^2}{\frac{\left(\frac{S_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{S_2^2}{n_2}\right)^2}{n_2 - 1}} = \frac{\left[\left(\frac{1.501^2}{14}\right) + \left(\frac{1.698^2}{14}\right)\right]^2}{\frac{\left(\frac{1.501^2}{14}\right)^2}{14 - 1} + \frac{\left(\frac{1.698^2}{14}\right)^2}{14 - 1}} = 25.609 \approx 25$$

The critical value = ±2.787 (from the table in Appendix-*G*).

The alternative hypothesis is two-sided so that the test is two-tailed. The decision rule is: Reject H_0 if the computed value of the test statistic is less than –2.787 or greater than 2.787. Since –4.62 < –2.787, we reject H_0 (see Fig. E6.15).

Conclusion: At the 1% significance level, the data provides sufficient evidence to conclude that the mean of the two samples are different.

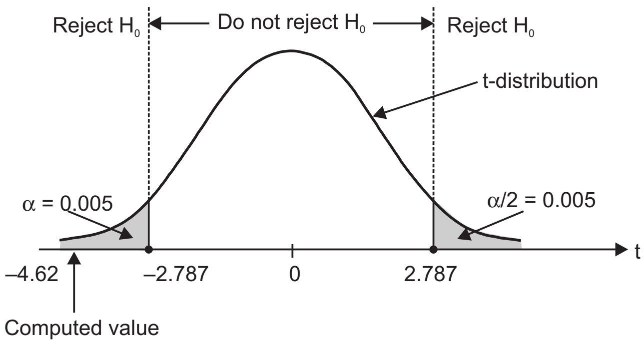


Fig. E6.15

6.11 HYPOTHESIS TEST ON THE MEANS OF TWO NORMAL DISTRIBUTIONS, VARIANCES UNKNOWN

We consider here the tests of hypotheses on the equality of the means μ_1 and μ_2 of two normally distributions where the variances σ_1^2 and σ_2^2 are unknown. A *t*-statistic is used to test the hypotheses.

6.11.1 Case 1: $\sigma_1^2 = \sigma_2^2 = \sigma^2$

The following test procedures are particularly applicable for the case when small independent samples are drawn from normally distributed populations both having the same variance.

Let X_1 and X_2 be two independent normal populations with unknown means μ_1 and μ_2, and unknown but equal variances, $\sigma_1^2 = \sigma_2^2 = \sigma^2$. We wish to test

$$H_0: \mu_1 = \mu_2$$
$$H_1: \mu_1 \neq \mu_2 \tag{6.16}$$

Suppose that $X_{11}, X_{12}, \ldots, X_{1n_1}$ is a random sample of n_1 observations from X_1, and $X_{21}, X_{22}, \ldots, X_{2n_2}$ is a random sample of n_2 observations from X_2.

Let $\bar{X}_1$, $\bar{X}_2$, S_1^2 and S_2^2 be the sample means and sample variances, respectively. S_1^2 and S_2^2 estimate the common variance σ^2 and we can combine them to yield a single estimate

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2} \tag{6.17}$$

To test $H_0: \mu_1 = \mu_2$ in Eq.(6.16), we compute the test statistic

$$t_0 = \frac{\bar{X}_1 - \bar{X}_2}{S_p \sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}} \tag{6.18}$$

If $H_0: \mu_1 = \mu_2$ is true, t_0 is distributed as $t_{n_1 + n_2 - 2}$. Therefore, if

$$t_0 > t_{\alpha/2,\, n_1 + n_2 - 2} \tag{6.19}$$

or if

$$t_0 < -t_{\alpha/2,\, n_1 + n_2 - 2} \tag{6.20}$$

We reject $H_0: \mu_1 = \mu_2$.

The one-sided alternatives are treated similarly. To test

$$H_0: \mu_1 = \mu_2$$
$$H_1: \mu_1 > \mu_2 \tag{6.21}$$

compute the test statistic t_0 in Eq.(6.18) and reject $H_0: \mu_1 = \mu_2$ if

$$t_0 > t_{\alpha,\, n_1 + n_2 - 2} \tag{6.22}$$

For the other one-sided alternative,

$$H_0: \mu_1 = \mu_2$$
$$H_1: \mu_1 < \mu_2 \tag{6.23}$$

calculate the test statistic t_0 and reject $H_0: \mu_1 = \mu_2$ if

$$t_0 < -t_{\alpha,\, n_1 + n_2 - 2} \tag{6.24}$$

A summary of the test procedure to test $H_0: \mu_1 = \mu_2$ against the three alternative hypotheses is given in Table 6.9.

Table 6.9: Procedure to test on the means of two normal distributions, variances unknown

Assumptions:
1. Simple random sample
2. Independent samples
3. Normal populations or large samples
4. Equal population standard deviations

STEP 1: The null hypothesis is H_0: $\mu_1 = \mu_2$, and the alternative hypothesis is

H_a: $\mu_1 \neq \mu_2$ H_a: $\mu_1 < \mu_2$ H_a: $\mu_1 > \mu_2$
(Two-tailed) or (Left-tailed) or (Right-tailed)

STEP 2: Decide on the significance level, α.

STEP 3: Compute the value of the test statistic

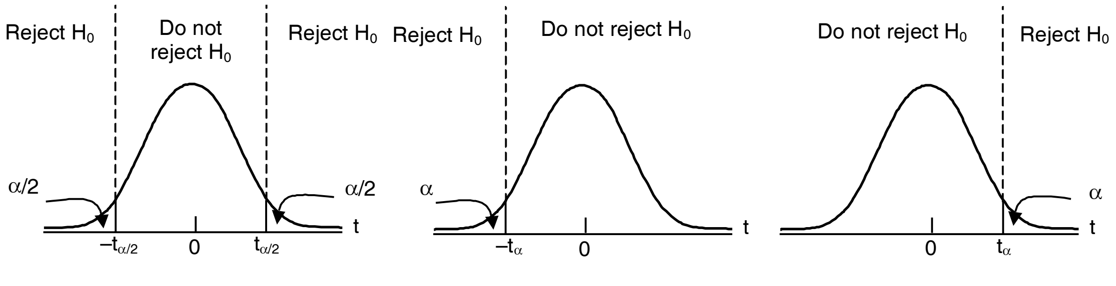
$$t = \frac{\bar{X}_1 - \bar{X}_2}{S_p\sqrt{(1/n_1) + (1/n_2)}}$$

where $$S_p = \sqrt{\frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}}.$$

Denote that value of the test statistic t_0.

STEP 4: The critical value(s) are

$\pm t_{\alpha/2}$ $-t_\alpha$ t_α
(Two-tailed) or (Left-tailed) or (Right-tailed)
with $df = n_1 + n_2 - 2$.
Use the table in Appendix-*G* to find the critical value(s).



(a) Two-tailed (b) Left-tailed (c) Right-tailed

STEP 5: If the value of the test statistic falls in the rejection region, reject H_0; otherwise, do not reject H_0.

STEP 6: Interpret the results of the hypothesis test.

The hypothesis test is exact for normal populations and is approximately correct for large samples from nonnormal populations.

Example E6.16

The brands (*A* and *B*) of fertilizer were used by a company in an experiment. The following is summary statistics for the independent simple random samples from the 2 brands.

Brand	Sample size	Mean yield/plot	Standard deviation
Brand A	10	80	10
Brand B	15	88	20

At the 5% significance level, are the two brands of fertilizers significantly different? It may be assumed that the yield per plot for the two brands of fertilizers are normally distributed with the same variance.

SOLUTION:

$$H_0: \mu_1 = \mu_2$$
$$H_1: \mu_1 \neq \mu_2$$

The test statistic is

$$\frac{\bar{X}_1 - \bar{X}_2}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$n_1 = 10, n_2 = 15, \ \bar{X}_1 = 80, \ \ \bar{X}_2 = 88, S_1 = 10, \text{and } S_2 = 20.$$

The degrees of freedom are $n_1 + n_2 - 2 = 10 + 15 - 2 = 23$; $\alpha = 0.05$. Therefore $t_{23,\,\alpha/2} = t_{23,0.025} = 2.069$ (from the table in Appendix-*G*). The decision rule is: Reject H_0 if the computed value is less than –2.069 or greater than 2.069.

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2} = \frac{(10-1)(10)^2 + (15-1)(20)^2}{10+15-2} = 282.61$$

or $\quad S_p = 16.80$

The test statistic is

$$\frac{80 - 88}{16.8\sqrt{\frac{1}{10} + \frac{1}{15}}} = -1.165 \quad \text{(see Fig. E6.16)}$$

Since the computed value –1.165 is between –2.069 and 2.069, we do not reject H_0.

We conclude that the data do not support the two brands are significantly different.

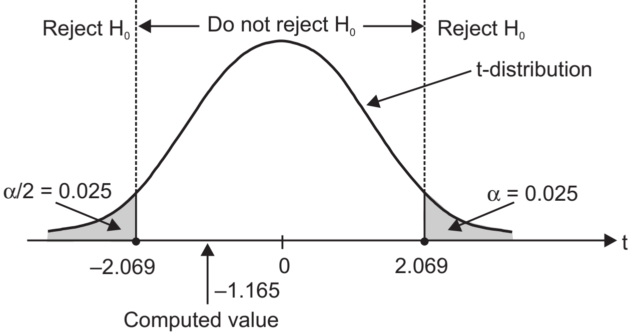


Fig. E6.16

Example E6.17

The mean life (in years) of certain machine part manufactured by two companies *A* and *B* are summarised below:

Company A	Company B
$\bar{X}_1 = 10$	$\bar{X}_2 = 18.66$
$S_1 = 4.9$	$S_2 = 4.64$
$n_1 = 10$	$n_2 = 10$

Use the pooled *t*-procedure to conduct the required hypothesis test. At the 5% significance level, do the data provide sufficient evidence to conclude that the mean life of parts made by company *A* is less than that made by machine *B*?

SOLUTION:

$$H_0: \mu_1 = \mu_2$$
$$H_1: \mu_1 < \mu_2$$
$$\alpha = 0.05$$

The test statistic is

$$\frac{\bar{X}_1 - \bar{X}_2}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{10-18.66}{4.772\sqrt{\frac{1}{10}+\frac{1}{10}}} = -4.06$$

where
$$S_p = \frac{(n_1-1)S_1^2+(n_2-1)S_2^2}{n_1+n_2-2} = \frac{(10-1)(4.9)^2+(10-1)(4.64)^2}{10+10-2} = 4.772$$

$$df = n_1 + n_2 - 2 = 10 + 10 - 2 = 18$$

The critical value is –1.734 (from the table in Appendix-*G*).

The decision rule is: Reject H_0 if the computed value of the test statistic is less than –1.734 (see Fig. E6.17). Because the computed value –4.06 is less than –1.734, we reject H_0. The data do not provide sufficient evidence to consider that, on the average, the mean life for parts made by company *A* is less than that made by Company *B*.

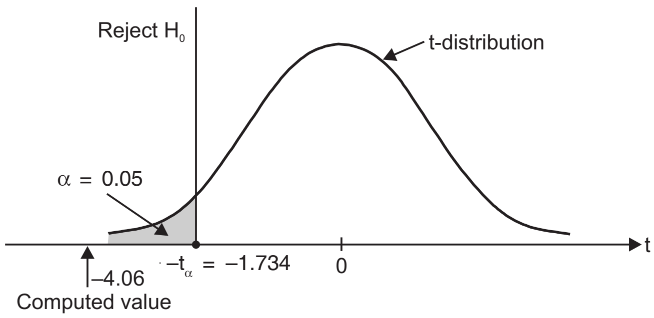


Fig. E6.17

Example E6.18

In a particular company, the mean annual salary of male workers was \$67,750 and that of female workers was \$60,000 in 1999-2000. Assume that these two means are based on random sample of 28 male and 26 female workers. The standard deviations for the two samples are \$3.7 k and \$3.2 k respectively. Using 1% significance level, can we conclude that the mean salary of all male workers in 1999–2000 was higher than that of all female workers? Further assume that the salaries of all male and female workers are both normally distributed with equal standard deviations.

SOLUTION:

$$H_0: \mu_1 - \mu_2 = 0$$
$$H_1: \mu_1 - \mu_2 > 0$$
$$\alpha = 0.01$$

The alternative hypothesis is right-sided. The critical value of t is 2.4 (from the table in Appendix-G after interpolation).

$$S_p = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2} = \frac{(28-1)(3700)^2 + (26-1)(3200)^2}{28+26-2} = 3468.6231$$

$$t = \frac{\bar{X}_1 - \bar{X}_2}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{67,750-60,000}{3468.6231\sqrt{\frac{1}{28}+\frac{1}{26}}} = -8.205$$

The decision rule is: Reject H_0 if the computed value of the test statistic exceeds 2.4 (see Fig. E6.17).

Because the computed value 8.205 is greater than 2.4, we reject H_0. The mean salary of all male workers in 1999–2000 was higher than that of all female workers.

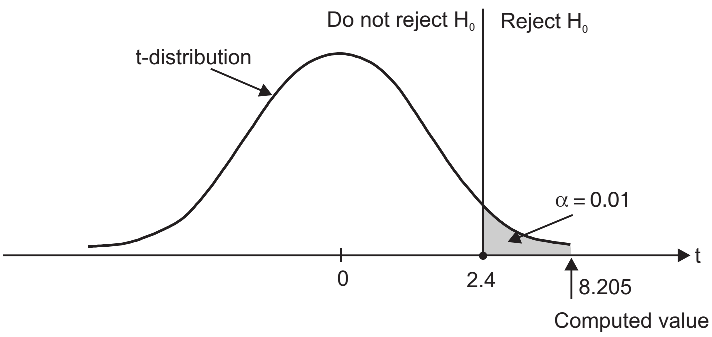


Fig. E6.18

6.11.2 Case 2: $\sigma_1^2 \neq \sigma_2^2$

In some situations, we cannot reasonably assume that the unknown variances σ_1^2 and σ_2^2 are equal. There is not an exact t-statistic available for testing H_0: $\mu_1 = \mu_2$ in this case. However, the statistic

$$t_0^* = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}}} \tag{6.25}$$

is distributed approximately as t with degrees of freedom given by

$$\nu = \frac{\left(\dfrac{S_1^2}{n_1} + \dfrac{S_2^2}{n_2}\right)^2}{\dfrac{(S_1^2/n_1)^2}{n_1 - 1} + \dfrac{(S_2^2/n_2)^2}{n_2 - 1}} \tag{6.26}$$

if the null hypothesis H_0: $\mu_1 = \mu_2$ is true. Therefore, if $\sigma_1^2 \neq \sigma_2^2$, the hypothesis of Eqs. (6.16), (6.21), (6.23) are tested and t_0^* is used as the test statistic and $n_1 + n_2 - 2$ is replaced by n in determining the degrees of freedom in the test.

Example E6.19

A company claims that its high strength steel, type *A* provides higher tensile strength than another company's steel, type *B*. A researcher tested both types of steel on two groups of randomly selected ones. The results of the test are given in the following table. The mean and standard deviation of tensile strength are in MPa.

Type	Sample size	Mean of tensile strengths (MPa)	Standard deviation of tensile strength (MPa)
A	25	40	13
B	22	45	11

(a) construct a 99% confidence interval for the difference between the mean tensile strength for the two types of steel.

(b) test at a 1% significant level if the mean tensile strengths for type *A* is less than that for *B*.

Solution:

(a) Degrees of freedom $= \dfrac{\left[\dfrac{S_1^2}{n_1} + \dfrac{S_2^2}{n_2}\right]^2}{\dfrac{\left(\dfrac{S_1^2}{n_1}\right)^2}{n_1 - 1} + \dfrac{\left(\dfrac{S_2^2}{n_2}\right)^2}{n_2 - 1}}$

$$\text{d.o.f.} = \frac{\left[\dfrac{13^2}{25} + \dfrac{11^2}{22}\right]^2}{\dfrac{\left(\dfrac{12^2}{25}\right)^2}{25 - 1} + \dfrac{\left(\dfrac{11^2}{22}\right)^2}{22 - 1}} = 44$$

The critical t-value (from the table in Appendix-*G* after interpolation), $t_{0.005,44} = 2.692$.

Hence, $$(\bar{X}_1 - \bar{X}_2) \pm t\sqrt{\frac{13^2}{25} + \frac{11^2}{22}}$$

$$(40 - 45) \pm 2.692(3.5014)$$

99% confidence interval for $\mu_1 - \mu_2$ is

$$-5 \pm 9.43 \quad \text{or} \quad -14.42 \text{ to } 4.4179$$

Type A is between 14.42 MPa lesser and 4.42 MPa higher than type B.

(b) H_0: $\mu_1 - \mu_2 = 0$

H_1: $\mu_1 - \mu_2 < 0$

Degrees of freedom = 44

$t_{0.01,44} = -2.414$ (from the table in Appendix-G after interpolation).

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} = \frac{(40 - 45) - 0}{3.5014} = -1.428$$

Do not reject H_0. The mean tensile strength for type A is not less than the mean tensile strength for type B.

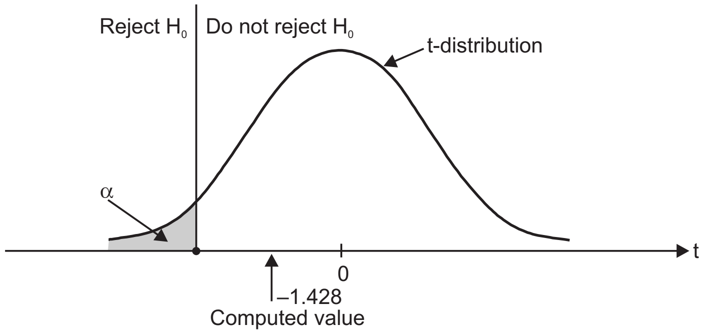


Fig. E6.19

Example E6.20

A manufacturing company is invested in buying one of two different types of machines. The company tested the two machines. The first machine was run for 8 hours and it produced an average of 129 items per hour with a standard deviation of 9 items. The second machine was run for 10 hours and it produced 120 items per hour with a standard deviation of 6 items. Assume that the production per hour for each machine is approximately normally distributed. Further assume that the standard deviations of the hourly production of the two machines are unequal.

Using the 2.5% significance level, can we conclude that the mean number of items produced per hour by the first machine is higher than that of the second machine?

SOLUTION:

$$H_0\colon \mu_1 - \mu_2 = 0$$
$$H_1\colon \mu_1 - \mu_2 > 0$$

The appropriate statistic is

$$\frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\dfrac{S_1^2}{n_1} + \dfrac{S_2^2}{n_2}}}$$

where $$\nu = \frac{\left(\dfrac{S_1^2}{n_1} + \dfrac{S_2^2}{n_2}\right)^2}{\dfrac{\left(\dfrac{S_1^2}{n_1}\right)^2}{n_1} + \dfrac{\left(\dfrac{S_2^2}{n_2}\right)^2}{n_2 + 1}} = 11$$

where $S_1 = 9$, $S_2 = 6$, $n_1 = 8$ and $n_2 = 10$.

The alternative hypothesis is one-sided (right sided). Also, $df = 11$ and $\alpha = 0.025$ so that the critical value of t is 2.201 (from the table in Appendix-*G* after interpolation).

Hence, $$\text{test statistic} = \frac{129 - 120}{\sqrt{\dfrac{9^2}{8} + \dfrac{6^2}{10}}} = 2.429$$

Therefore, the decision rule is: Reject H_0 if the computed value of the test statistic is greater than 2.201. Because the computed value 2.429 is greater than 2.201, we reject H_0 (see Fig. E6.20).

Hence, at the 2.5% significance level, there is evidence that the mean number of items produced per hour by the first machine is higher than that by the second machine.

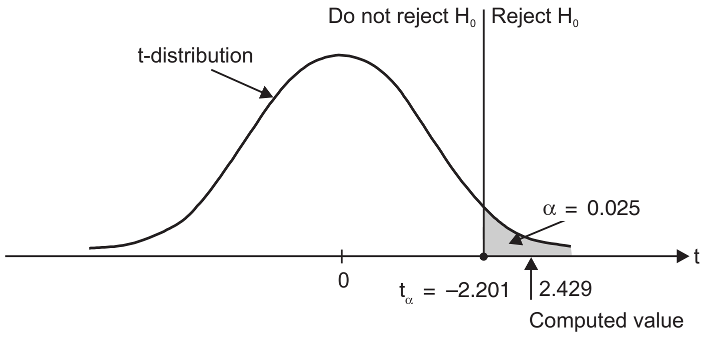


Fig. E6.20

Example E6.21

A sample of 19 cans of Brand *A* soda gave the mean number of calories per can as 15 with a standard deviation of 1.920. Another sample of 16 cans of Brand *B* soda gave the mean number of calories 18.11 per can with a standard deviation of 2.250 calories. Assume that the calories per can of soda are normally distributed for each of these brands and that the standard deviations for the two populations are not equal. Test at the 5% significance level if the mean number of calories per can of soda are different for these two brands.

Solution:

Let μ_1 and μ_2 be the mean number of calories for all cans of soda of brand *A* and brand *B*, respectively. Given

$$n_1 = 8, n_2 = 10, S_1 = 9 \text{ and } S_2 = 6.$$

$$df = \frac{\left(\left(\frac{S_1^2}{n_1}\right) + \left(\frac{S_2^2}{n_2}\right)\right)^2}{\frac{\left(\frac{S_1^2}{n_1}\right)}{(n_1 - 1)} + \frac{\left(\frac{S_2^2}{n_2}\right)}{(n_2 - 1)}} = \frac{\left(\frac{9^2}{8}\right) + \left(\frac{6^2}{10}\right)}{\frac{\left(\frac{9^2}{8}\right)}{(8-1)} + \frac{\left(\frac{6^2}{10}\right)}{(10-1)}} = 29$$

$$H_0: \mu_1 - \mu_2 = 0$$

$$H_1: \mu_1 - \mu_2 > 0$$

$$df = 29$$

Area to the right tail of the *t*-curve = 0.025.

The test statistic is

$$\frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} = \frac{15 - 18.11}{\sqrt{\frac{9^2}{8} + \frac{6^2}{10}}} = -4.353$$

The alternative hypothesis is two-sided. The critical values of *t* are ±2.045 (from the table in Appendix-*G*). Therefore, we have a two-tailed test. The decision rule is: Reject H_0 if the computed value is less than –2.045 or greater than 2.045.

Because the computed value –4.353 falls in the rejection region, we reject H_0 (see Fig. E6.21).

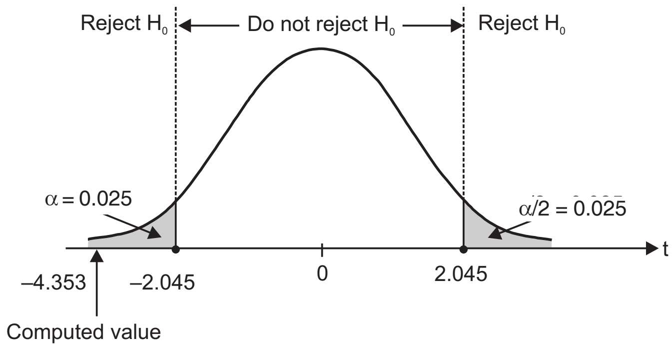


Fig. E6.21

We conclude that there is difference in the mean number of calories per can for the two brands of soda.

6.12 HYPOTHESIS TEST TO COMPARE TWO POPULATION MEANS (PAIRED t-TEST)

The method of a paired t-test reduces the problem of comparing the means of two populations to that of a one-sample t-test. Here, we denote the difference of the pair (x_i, y_i) by $d_i = x_i - y_i$.

The n differences $d_1, d_2, \ldots, d_n$ constitute a sample of n independent observations from a population of "within pair differences D". We assume here that the differences D are normally distributed (especially when n is small). The population mean of paired differences is then equals to $\mu_1 - \mu_2$. We are interested in testing

$$H_0: \mu_1 = \mu_2$$

This is the same as testing the null hypothesis

$$H_0: \mu_D = 0$$

Now, we have a sample of n independent observations $d_1, d_2, d_3, \ldots, d_n$ from a normal population whose variance is unknown, and we wish to test the hypothesis about its mean μ_D.

We can use the t-test with $n - 1$ degrees of freedom to test

$$H_0: \mu_D = 0$$

against the various alternative hypothesis.

The test statistic is

$$\frac{\bar{d} - 0}{s_d \sqrt{n}} \quad \text{or} \quad \frac{\bar{d}\sqrt{n}}{s_d}$$

where $\bar{d} = \frac{\Sigma d_i}{n}$ and $s_d = \sqrt{\frac{1}{n-1}[\Sigma(d_i - \bar{d})^2]}$

The procedure for paired differences test of a hypothesis H_0: $\mu_1 = \mu_2$ is summarised in Table 6.10.

Table 6.10: Procedure to test to compare two population means (paired t-test)

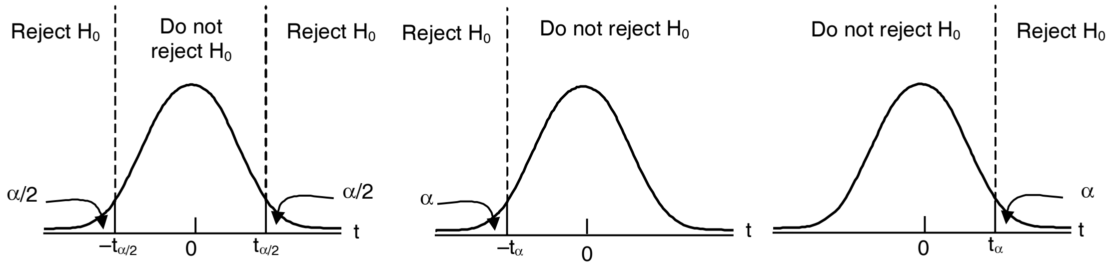
Assumptions: 1. Simple random paired sample 2. Normal differences or large sample
STEP 1: The null hypothesis is H_0: $\mu_1 = \mu_2$, and the alternative hypothesis is H_a: $\mu_1 \neq \mu_2$ H_a: $\mu_1 < \mu_2$ H_a: $\mu_1 > \mu_2$ (Two-tailed) or (Left-tailed) or (Right-tailed)
STEP 2: Decide on the significance level, α.
STEP 3: Calculate the paired differences of the sample pairs.
STEP 4: Compute the value of the test statistic $t = \frac{\bar{d}}{s_d / \sqrt{n}}$ and denote that value to t_0, also $\bar{d} = \frac{\Sigma d_i}{n}$ and $s_d = \sqrt{\frac{1}{n-1}\Sigma(d_i - \bar{d})^2}$

STEP 5: The critical value(s) are

$\pm t_{\alpha/2}$ (Two-tailed) or $-t_\alpha$ (Left-tailed) or t_α (Right-tailed)

with $df = n - 1$.

Use the table in Appendix-*G* to find the critical value(s).



(a) Two-tailed (b) Left-tailed (c) Right-tailed

STEP 6: If the value of the test statistic falls in the rejection region, reject H_0; otherwise, do not reject H_0.

STEP 7: Interpret the results of the hypothesis test.

The hypothesis test is exact for normal differences and is approximately correct for large samples and nonnormal differences.

Example E6.22

Table E6.22 gives the systolic blood pressures of seven adults and after completion of a special dietary plan based on a special dietary plan for three months. Using the 5% significance level, can we conclude that the mean of the paired differences is different from zero? Assume that the population of paired differences is approximately normally distributed.

Before	209	179	195	221	232	199	223
After	192	185	186	223	221	182	232

SOLUTION:

	Before	After	Difference, d	d^2
	209	192	17	289
	179	185	–6	36
	195	186	9	81
	221	223	–2	4
	232	221	11	121
	199	182	17	289
	223	232	–9	81
Σ	1458	1421	37	901

$$\bar{d} = \frac{\Sigma d}{n} = \frac{37}{7} = 5.2857$$

$$s_d = \sqrt{\frac{\Sigma d^2 - \frac{(\Sigma d)^2}{n}}{n-1}} = \sqrt{\frac{901 - \frac{(37)^2}{7}}{7-1}} = 10.8430$$

H_0: $\mu_1 \neq \mu_2$ $(\mu_d = 0)$
H_1: $\mu_d \neq 0$

The test statistic is

$$t = \frac{\bar{d}}{s_d\sqrt{n}} = \frac{5.2857}{10.8430\sqrt{7}} = 1.2898$$

The alternative hypothesis is two-sided. Therefore, we have a two-tailed test. $df = n - 1 = 7 - 1 = 6$ and $\alpha/2 = 0.025$.

The two critical values of t for $df = 6$ and $\alpha = 0.025$ are -2.447 and 2.447 (from the table in Appendix-G). The decision rule is: Reject H_0 if the computed value is less than -2.447 or greater than 2.447.

Because the computed value 1.2898 for $\bar{d}$ falls in the non-rejection region as shown in Fig. E6.22 and we fail to reject the null hypothesis.

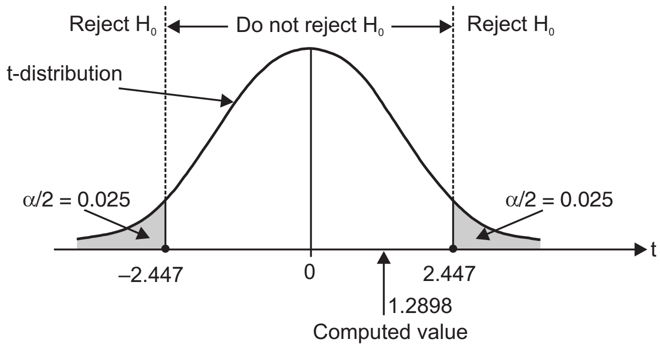


Fig. E6.22

Example E6.23

A college claims that the tutoring service it offers significantly increases the test scores of students in mathematics. The following table gives the scores out of 120 of 8 students before and after they took the tutorial help.

Before	82	75	89	91	66	70	91	69
After	97	72	94	111	80	72	117	76

Using the 5% significance level, can you conclude that taking tutorial service increases the test scores of the students? Assume that the population of paired differences is approximately normally distributed.

SOLUTION:

$n = 8$

	Before	After	Difference, d	d^2
	82	97	15	225
	75	72	3	9
	89	94	–5	25
	91	111	–20	400
	66	80	–14	196
	70	72	2	4
	91	117	–26	676
	69	76	–7	49
Σ	633	719	–86	1584

$$\bar{d} = \frac{\Sigma d}{n} = \frac{-86}{8} = -10.75$$

$$s_d = \sqrt{\frac{\Sigma d^2 - \frac{(\Sigma d)^2}{n}}{n-1}} = \sqrt{\frac{1584 - \frac{(86)^2}{8}}{8-1}} = 9.7064$$

$H_0\text{: } \mu_d = 0$

$H_1\text{: } \mu_d < 0$

The population statistic is

$$\frac{\bar{d}}{s_d\sqrt{n}} = \frac{-10.75}{9.7064\sqrt{8}} = -3.1325$$

The alternative hypothesis is one-sided (right sided). Also $\alpha = 0.05$. The critical value of $t = -1.895$ (from the table in Appendix-G).

Therefore, the decision rule is: Reject H_0 if the computed value of the test statistic is greater than –1.895.

Because the computed statistic –3.1325 is greater than –1.895, we reject H_0 (see Fig. E6.23). We conclude that attending the tutoring service increases the mean test scores.

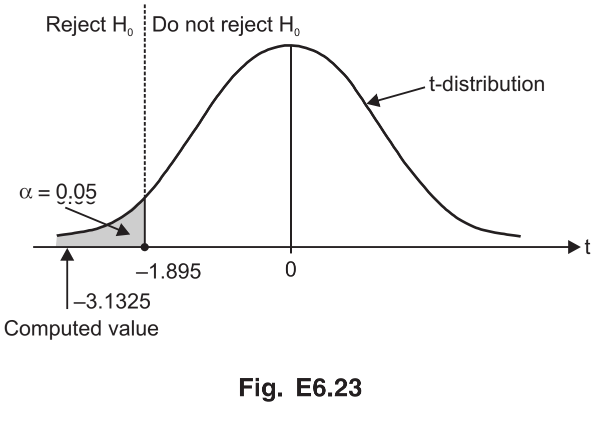


Fig. E6.23

Example E6.24

A medical agency measured the corneal thickness of 8 patients who had glaucoma in one eye but not in the other. The following are the data on corneal thickness, in microns.

Patient	Normal	Glaucoma
1	484	488
2	479	479
3	493	481
4	445	427
5	437	441
6	399	409
7	464	459
8	477	460

At the 10% significance level, do the data provide sufficient evidence to conclude the mean corneal thickness is greater in normal eyes than in eyes with glaucoma?

Solution:

$n = 8$

Patient	Normal	Glaucoma	Difference, d	d^2
1	484	488	–4	16
2	479	479	0	0
3	493	481	12	144
4	445	427	18	324
5	437	441	–4	16
6	399	409	–10	100
7	464	459	5	25
8	477	460	17	289
Σ	3678	3644	34	914

$$\bar{d} = \frac{\Sigma d}{n} = \frac{34}{8} = 4.25$$

$$s_d = \sqrt{\frac{\Sigma d^2 - \frac{(\Sigma d)^2}{n}}{n-1}} = \sqrt{\frac{914 - \frac{(34)^2}{8}}{8-1}} = 10.4847$$

The test statistic is

$$t = \frac{\bar{d}}{s_d\sqrt{n}} = \frac{4.25}{10.4847\sqrt{8}} = 1.1465$$

H_0: $\mu_1 = \mu_2$

H_1: $\mu_1 > \mu_2$

$\alpha = 0.10$, $df = 7$, critical value = 1.415 (from the table in Appendix-*G*).

The alternative hypothesis is one-sided (right sided). Also, the decision rule is: Reject H_0 if the computed value of the test statistic is greater than 1.415.

Because the computed value 1.1465 is less than 1.415, do not reject H_0 (see Fig. E6.24).

Hence, the data do not provide sufficient evidence at the 10% significance level that the mean corneal thickness is greater in normal eyes than in eyes with glaucoma.

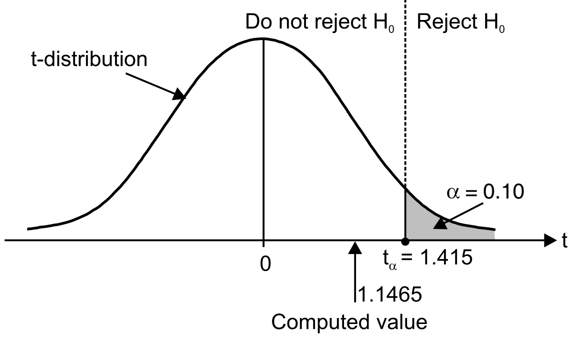


Fig. E6.24

6.13 HYPOTHESIS TEST ON THE EQUALITY OF TWO VARIANCES

The two independent populations are $X_1 \sim N(\mu_1, \sigma_1^2)$ and $X_2 \sim N(\mu_2, \sigma_2^2)$ where μ_1, σ_1^2 and μ_2, σ_2^2 are unknown. We wish to test hypotheses about the equality of the two variances, say H_0: $\sigma_1^2 = \sigma_2^2$. Assume that two random samples of size n_1 from population 1 and of size n_2 from population 2 are available, and let s_1^2 and s_2^2 be the sample variances. To test the two-sided alternative

$$\begin{aligned} H_0&: \sigma_1^2 = \sigma_2^2 \\ H_1&: \sigma_1^2 \neq \sigma_2^2 \end{aligned} \tag{6.27}$$

The statistic

$$F_0 = \frac{S_1^2}{S_2^2} \tag{6.28}$$

is distributed as F, with $n_1 - 1$ and $n_2 - 1$ degrees of freedom, if the null hypothesis H_0: $\sigma_1^2 = \sigma_2^2$ is true. Hence, we would reject H_0 if

$$F_0 > F_{\alpha/2,\, n_1-1,\, n_2-1} \tag{6.29}$$

or if

$$F_0 < F_{1-\alpha/2,\, n_1-1,\, n_2-1} \tag{6.30}$$

where $F_{\alpha/2,\, n_1-1,\, n_2-1}$ and $F_{1-\alpha/2,\, n_1-1,\, n_2-1}$ are the upper and lower $\alpha/2$ percentage points of the F-distribution with $n_1 - 1$ and $n_2 - 1$ degrees of freedom. Appendix-*G* gives only the upper tail points of F, so to find $F_{1-\alpha/2,\, n_1-1,\, n_2-1}$ we must use

$$F_{1-\alpha/2, n_1-1, n_2-1} = \frac{1}{F_{\alpha/2, n_1-1, n_2-1}} \tag{6.31}$$

To test one-sided alternative hypothesis, let X_1 denotes the population that may have the largest variance, then the one-sided alternative hypothesis is

$$H_0:\ \sigma_1^2 = \sigma_2^2$$

$$H_1:\ \sigma_1^2 > \sigma_2^2 \tag{6.32}$$

If $$F_0 > F_{\alpha, n_1-1, n_2-1} \tag{6.33}$$

We would reject $H_0:\ \sigma_1^2 = \sigma_2^2$.

A summary of the test procedure to test $H_0:\ \sigma_1^2 = \sigma_2^2$ against the three alternative hypothesis is given in Table 6.11.

Table 6.11: Procedure to test on the equality of two variances

Assumptions:
1. Simple random sample
2. Independent samples
3. Normal population

STEP 1: The null hypothesis is H_0: $\sigma_1 = \sigma_2$, and the alternative hypothesis is
H_a: $\sigma_1 \neq \sigma_2$ H_a: $\sigma_1 < \sigma_2$ H_a: $\sigma_1 > \sigma_2$
(Two-tailed) or (Left-tailed) or (Right-tailed)

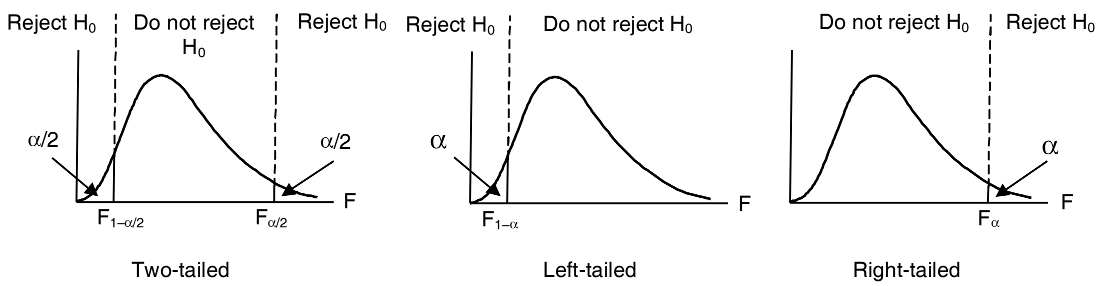
STEP 2: Decide on the significance level, α.

STEP 3: Compute the value of the test statistic

$$F = \frac{S_1^2}{S_2^2}$$

and denote that value F_0.

STEP 4: The critical value(s) are
$F_{1-\alpha/2}$ and $F_{\alpha/2}$ $F_{1-\alpha}$ F_α
(Two-tailed) or (Left-tailed) or (Right-tailed)
with $df = (n_1 - 1, n_2 - 1)$. Use the table in Appendix-*H* to find the critical value(s).



STEP 5: If the value of the test statistic falls in the rejection region, reject H_0; otherwise, do not reject H_0.

STEP 6: Interpret the results of the hypothesis test.

Example E6.25

A company has two branch operations, A and B, producing a particular product. Branch A has 15 employees while branch B has 25 employees on any given month, although there was no significant difference in mean production, branch A had a standard deviation of 8 while branch B had a standard deviation of 10. Can we conclude at the (a) 0.01, (b) 0.05 level of significance that the variability of branch B is greater than that of branch A?

Solution:

$$n_1 = 15, n_2 = 25$$

(a) We have, on using subscripts 1 and 2 for branches A and B respectively, $S_1 = 8$, $S_2 = 10$, so that

$$\hat{S}_1^2 = \frac{n_1}{n_1 - 1} S_1^2 = \frac{15}{(15-1)}(8^2) = 68.571$$

$$\hat{S}_2^2 = \frac{n_2}{n_2 - 1} S_2^2 = \frac{25}{(25-1)}(10^2) = 104.167$$

We have to decide between the hypotheses

H_0: $\sigma_1 = \sigma_2$, and any observed variability is due to chance.

H_2: $\sigma_2 > \sigma_1$, and the variability of branch B is greater than that of A.

The decision must, therefore, be based on a one-tailed test of the F-distribution. Hence,

$$F = \frac{\hat{S}_2^2}{\hat{S}_1^2} = \frac{104.167}{68.571} = 1.519$$

$v_2 = 25 - 1 = 24$; $v_1 = 15 - 1 = 14$. At 0.01 level for 24, 14 degrees of freedom, we have from Appendix-H, $F_{0.99} = 3.43$.

Then, since $F < F_{0.99}$, we cannot reject H_0 at the 0.01 level.

(b) $F_{0.95} = 2.35$ for 24, 14 degrees of freedom (see Appendix-H).

We must note again that $F < F_{0.95}$. Thus, we cannot reject H_0 at the 0.05 level.

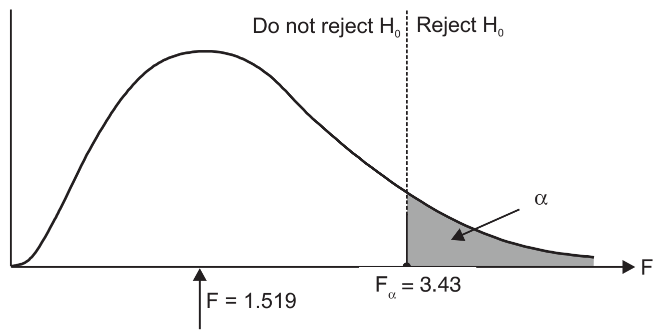


Fig. E6.25

Example E6.26

Patients who undergo chronic hemodialysis often experience severe anxiety. Videotapes of progressive relaxation exercises were shown to one group of 31 patients and neutral videotapes to another group of 25 patients. A psychiatric questionnaire was used to measure anxiety where higher scores correspond to higher anxiety. The standard deviations from the anxiety measurement were 11.0403 and 10 respectively. Do the data provide sufficient evidence to conclude that variation in anxiety-test scores differs between patients who are shown videotapes of progressive relaxation exercises and those who are shown neutral viedeotapes? Perform an F-test at the 10% significance level.

SOLUTION:

$$n_1 = 31 \text{ and } n_2 = 25$$
$$H_0: \sigma_1 = \sigma_2$$
$$H_0: \sigma_1 \neq \sigma_2$$
$$\alpha = 0.10$$

and test statistic

$$F = \frac{S_1^2}{S_2^2} = \frac{11.0413^2}{10^2} = 1.2189$$

The critical values are $F_{0.05}$ with df = (30, 24) or 1.94, and $F_{0.95} = \dfrac{1}{F_{0.05}}$ with df = (24, 30) = $\dfrac{1}{1.89}$ = 0.53

[see the table in Appendix-H].

The alternative hypothesis is two-sided. Therefore, we have a two-tailed test. The decision rule is: Reject H_0 if the computed value is less than 0.53 and greater than 1.94.

Because the computed value 1.2189 is 0.53 < 1.2189 < 1.94, do not reject H_0 (see Fig. E6.26).

Hence, there is not sufficient evidence at the 0.10 significance level to obtain that the variation in anxiety test scores for patients seeing videotapes showing progressive relaxation exercises is different from that in patients seeing neutral videotapes

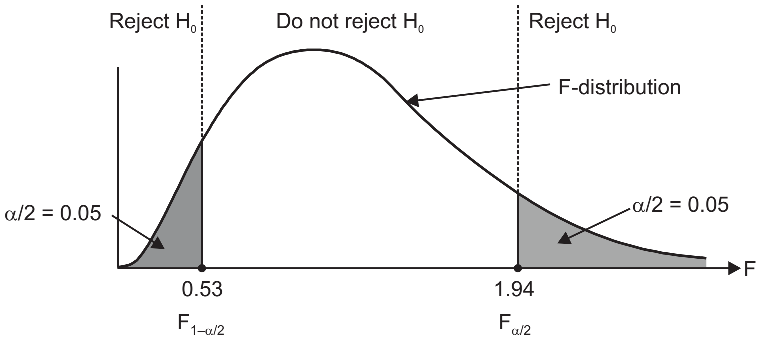


Fig. E6.26

Example E6.27

An independent golf equipment testing facility compared the differences in the performance of golf balls hit off a regular wooden tee to those hit off a brand *A* golf tee. Tests were conducted and a robot swung the club head at approximately 150 kilometer per hour. Data on ball velocity (in kilometers per hour) with each type of tee are as follows:

Brand *A*	Regular
$\bar{X}_1 = 204$	$\bar{X}_2 = 196$
$S_1 = 0.368$	$S_2 = 0.8$
$n_1 = 30$	$n_2 = 30$

At the 1% significance level, do the data provide sufficient evidence to conclude that the standard deviation of velocity is less with Brand *A* tee than with the regular tee?

SOLUTION:

$$H_0: \sigma_1 = \sigma_2$$
$$H_0: \sigma_1 < \sigma_2$$

The test statistic is

$$F = \frac{0.368^2}{0.8^2} = 0.211$$

The critical value is $F_{0.99}$ with $df = (29, 29)$ or $\frac{1}{2.42} = 0.41$ [Note that $F_{0.01} = 2.42$] from the table in Appendix-*H*. Because the alternative hypothesis is one-sided (on the left), the decision rule is: Reject H_0 if the computed value of the test statistic is less than 0.41 (see Fig. E6.27).

Because the computed value 0.211 is < 0.41, we reject the null hypothesis H_0.

There is sufficient evidence at the 0.01 significance level to claim that the standard deviation of ball velocity is less with the Brand *A* tee than with the regular tee.

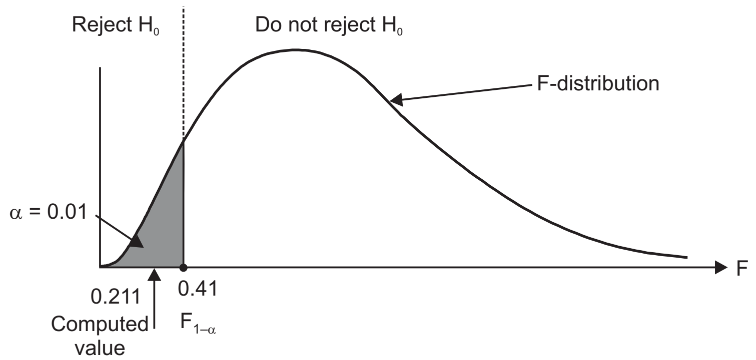


Fig. E6.27

6.14 HYPOTHESIS TEST ON TWO PROPORTIONS

Quite often, we are required to decide whether the observed differences between two sample proportions is due to sampling error or due to the fact that the proportions in the parent populations from which the samples are drawn are inherently different. Let p_1 and p_2 represent the proportions of those infected in the two populations.

The two binomial parameters are p_1 and p_2 and we with to test that they are equal. That is, H_0: $p_1 = p_2$.

If the two random samples of sizes n_1 and n_2 are taken from two populations, and X_1 and X_2 represent the number of observations that belong to the class of interest in sample 1 and 2, respectively, then assume that the normal approximation to the binomial applies to each population, so that the estimators of the population proportions $\hat{p}_1 = X_1/n_1$ and $\hat{p}_2 = X_2/n_2$ have approximate normal distribution. Now, if the null hypothesis H_0: $p_1 = p_2$ is true, then using the fact that $p_1 = p_2 = p$, the random variable

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{p(1-p)\left[\dfrac{1}{n_1} + \dfrac{1}{n_2}\right]}} \tag{6.34}$$

is distributed approximately $N(0,1)$. An estimate of the common parameter p is

$$\hat{p} = \frac{X_1 + X_2}{n_1 + n_2}$$

The test statistic for H_0: $p_1 = p_2$ is then

$$Z_0 = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left[\dfrac{1}{n_1} + \dfrac{1}{n_2}\right]}} \tag{6.35}$$

If $$Z_0 > Z_{\alpha/2} \text{ or } Z_0 < -Z_{\alpha/2} \tag{6.36}$$

the null hypothesis is rejected.

A summary of the test procedure to test H_0: $p_1 = p_2$ against the three alternative hypotheses is given in Table 6.12.

Table 6.12: Procedure to test for comparison of two population proportions

Assumptions:
1. Simple random samples
2. Independent samples
3. x_1, $n_1 - x_1$, and $n_2 - x_2$ are all 5 or greater
STEP 1: The null hypothesis is H_0: $p_1 = p_2$, and the alternative hypothesis is H_a: $p_1 \neq p_2$ H_a: $p_1 < p_2$ H_a: $p_1 > p_2$ (Two-tailed) or (Left-tailed) or (Right-tailed)
STEP 2: Decide on the significance level, α.

STEP 3: Compute the value of the test statistic

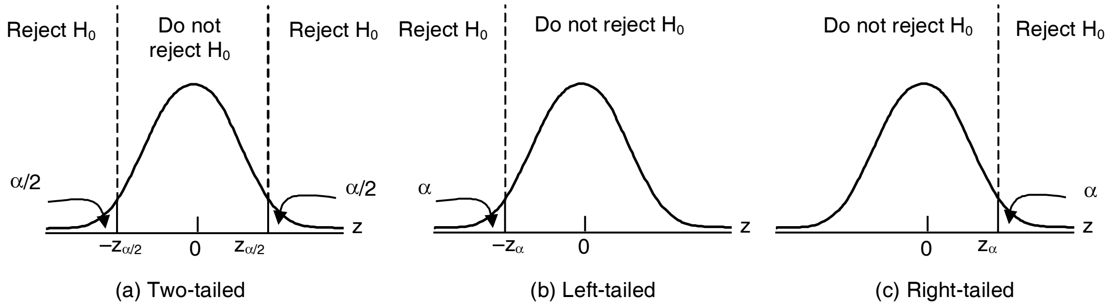
$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})}\sqrt{(1/n_1) + (1/n_2)}}$$

where $\hat{p} = (x_1 + x_2)/(n_1 + n_2)$. Denote the value of the test statistic z_0.

STEP 4: The critical value(s) are

$\pm z_{\alpha/2}$	$-z_\alpha$	z_α
(Two-tailed) or	(Left-tailed) or	(Right-tailed)

Use the table in Appendix-*E* to find the critical value(s).



STEP 5: If the value of the test statistic falls in the rejection region, reject H_0; otherwise, do not reject H_0.

STEP 6: Interpret the results of the hypothesis test.

Example E6.28

In an independent study it was found that a sample of 500 male registered voters showed that 60% of them voted in the last general elections. Another sample of 400 female registered voters showed that 50% of them voted in the same election. Test at the 1% significance level if all male voters who voted in the last general election is different from that of all female voters.

Solution:

$$H_0: p_1 - p_2 = 0$$
$$H_1: p_1 - p_2 \neq 0$$

$$\hat{p}_1 = 0.60, \ \hat{p}_2 = 0.50, n_1 = 500 \text{ and } n_2 = 400$$

For $\alpha = 0.01$, the critical values of z are –2.58 and 2.58 (from the table in Appendix-*E*).

$$\hat{p} = \frac{500(0.60) + 400(0.50)}{500 + 400} = 0.5556, \ \hat{p} = 1 - \hat{p} = 1 - 0.5556 = 0.4444$$

$$\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = \sqrt{(0.5556)(0.4444)\left(\frac{1}{500} + \frac{1}{400}\right)} = 0.03333$$

Hence $$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.60 - 0.50}{0.03333} = 3.0$$

Hence, we reject H_0 and conclude that the proportion of all male voters who voted in the last general elections is different from that of all female voters who voted in the same election (see Fig. E6.28).

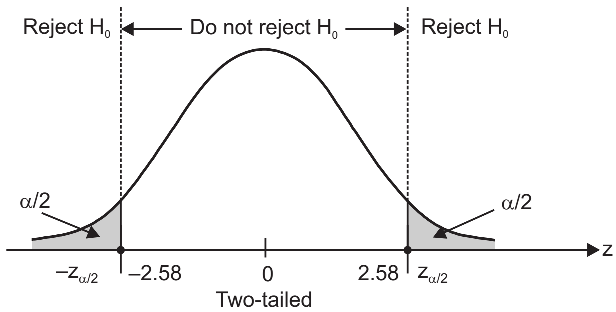


Fig. E6.28

Example E6.29

A survey found that 44% male students and 31% female students in a particular college had more than one hour of physical exercise time per day. Suppose that these percentages are based on a random sample of 395 male students and 410 female students. At the 1% significance level, can we conclude that the percentage of all male students and of all female students who have more than one hour of physical exercise time per day are different?

Solution:

Here H_0: $p_1 - p_2 = 0$

H_1: $p_1 - p_2 \neq 0$

For $\alpha = 0.01$, the critical values of z are –2.58 and 2.58 (from the table in Appendix-*E*).

$$\hat{p}_1 = 0.44,\ \hat{p}_2 = 0.31,\ n_1 = 395, \text{ and } n_2 = 410$$

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\hat{p}(1-\hat{p})}} = \frac{(0.44 - 0.31) - 0}{\sqrt{(0.374)(0.626)\left(\frac{1}{395} + \frac{1}{410}\right)}} = 3.81$$

where $$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{0.44(395) + 0.31(410)}{395 + 410} = 0.374$$

Hence, $z = 3.81$ and we reject H_0 and conclude that the percentage of all male students and all female students who have more than one hour of physical exercise time per day are different as shown in Fig. E6.29.

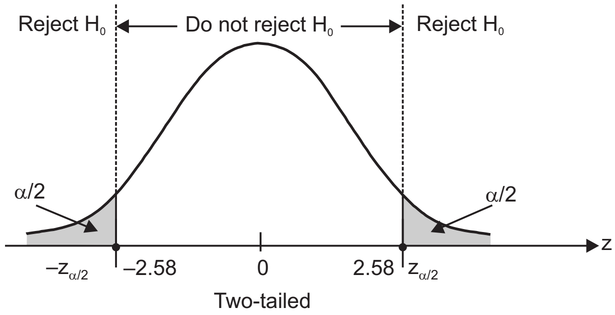


Fig. E6.29

Example E6.30

An independent medical research team conducted a study to test the effect of a cholesterol reducing medication. At the end of study, the team found that of the 9400 subjects who took the medication, 602 died of heart disease. Of the 8600 persons who took brand *A*, 714 died of heart disease. At $\alpha = 0.01$, can we conclude that the death rate is lower for those who took the medication than for those who took the brand *A*?

Solution:

$$H_0: p_1 \ge p_2$$
$$H_1: p_1 < p_2 \text{ (claim)}$$

Because the test is left-tailed and the level of significance is $\alpha = 0.01$, the critical value is –2.33 (from the table in Appendix-*E*). The decision rule is: Reject H_0 if the computed value of the test statistic is less than –2.33.

$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{602}{9400} = 0.064$$

$$\hat{p}_2 = \frac{x_2}{n_2} = \frac{714}{8600} = 0.083$$

$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{602 + 714}{9400 + 8600} = 0.073$$

$$\hat{q} = 1 - \hat{p} = 1 - 0.073 = 0.927$$

The test statistic is

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{0.064 - 0.083}{\sqrt{(0.073)(0.927)\left(\frac{1}{9400} + \frac{1}{8600}\right)}} = -4.8989$$

Because the computed value –4.8989 is less than –2.33, we reject H_0 (see Fig. E6.30). At 1% level, there is enough evidence to conclude that the death rate is lower for those who took the medication than for those who took the brand *A*.

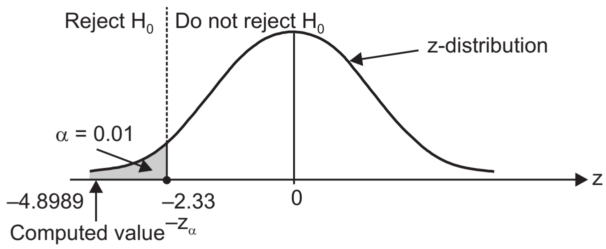


Fig. E6.30

Example E6.31

A survey of 872 workers showed that 384 of them said that it was seriously unethical to monitor employee e-mail. When 242 senior-level managers were surveyed, 80 of them said that it was seriously unethical to monitor employee e-mail. Use a 0.05 significance level to test the claim that for those saying monitoring e-mail is seriously unethical, the proportion of employee is greater than the proportion of managers.

Solution:

$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{384}{872} = 0.4404$$

$$\hat{p}_2 = \frac{x_2}{n_2} = \frac{80}{242} = 0.3306$$

$$\hat{p} = \frac{384+80}{872+242} = 0.4165$$

$$H_0: p_1 - p_2 = 0$$
$$H_1: p_1 - p_2 > 0$$
$$\alpha = 0.05$$

The critical values of $z_\alpha = 1.645$ (from the table in Appendix-*E*).

The alternative hypothesis is one-sided (right-side). Therefore, the decision rule is Reject H_0 if the computed value of the test statistic is greater than 1.645.

The test statistic is

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{0.4404 - 0.3306}{\sqrt{0.4165(0.5835)\left(\frac{1}{872}+\frac{1}{242}\right)}} = 3.0654$$

Because the computed value 3.0654 is greater than 1.645, we reject H_0 (see Fig. E6.31).

Hence, there is sufficient evidence to conclude that $p_1 - p_2 > 0$.

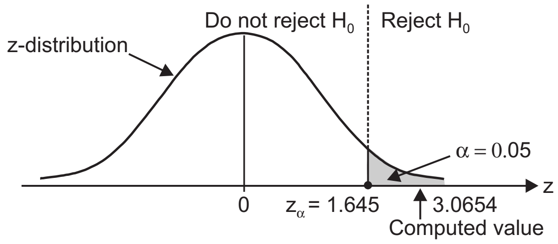


Fig. E6.31

Procedures for testing hypotheses on means, variances, and proportions for a single population and two populations are summarised in Tables 6.13 and 6.14 respectively.

Table 6.13: Summary of tests of hypotheses (single population)

No.	Nature of the population	Parameter	Null hypothesis	Alternative hypothesis	Test statistic	Decision rule: Reject the computed value
1.	Quantitative data; variance σ^2 is known; population normally distributed	μ	$\mu = \mu_0$	1. $\mu > \mu_0$ 2. $\mu < \mu_0$ 3. $\mu \neq \mu_0$	$\dfrac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$	1. greater than z_α 2. less than $-z_\alpha$ 3. less than $-z_{\alpha/2}$ or greater than $z_{\alpha/2}$
2.	Quantitative data; variance σ^2 is not known; population normally distributed; sample size small ($n < 30$)	μ	$\mu = \mu_0$	1. $\mu > \mu_0$ 2. $\mu < \mu_0$ 3. $\mu \neq \mu_0$	$\dfrac{\bar{X} - \mu_0}{S / \sqrt{n}}$	1. greater than $t_{n-1,\alpha}$ 2. less than $-\ t_{n-1,\alpha}$ 3. less than $-\ t_{n-1,\alpha/2}$ or greater than $t_{n-1,\alpha/2}$
3.	Quantitative data; variance σ^2 is not known; population not necessarily normal; sample large ($n \geq 30$)	μ	$\mu = \mu_0$	1. $\mu > \mu_0$ 2. $\mu < \mu_0$ 3. $\mu \neq \mu_0$	$\dfrac{\bar{X} - \mu_0}{S / \sqrt{n}}$	1. greater than z_α 2. less than $-z_\alpha$ 3. less than $-z_{\alpha/2}$ or greater than $z_{\alpha/2}$ The level of significance approximately 100α per cent
4.	Quantitative data; population normally distributed	σ	$\sigma = \sigma_0$	1. $\sigma > \sigma_0$ 2. $\sigma < \sigma_0$ 3. $\sigma \neq \sigma_0$	$\dfrac{(n-1)S^2}{\sigma_0^2}$	1. greater than $\chi^2_{n-1,\alpha}$ 2. less than $\chi^2_{n-1,1-\alpha}$ 3. less than $\chi^2_{n-1,1-\alpha/2}$ or greater than $\chi^2_{n-1,\alpha/2}$
5.	Quantitative data; binomial case	p	$p = p_0$	1. $p > p_0$ 2. $p < p_0$ 3. $p \neq p_0$	$\dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$ Here n is assumed large.	1. greater than z_α 2. less than $-z_\alpha$ 3. less than $-z_{\alpha/2}$ or greater than $z_{\alpha/2}$ The level of significance approximately 100α per cent

Table 6.14: Summary of tests of hypotheses (two populations)

No.	Nature of the population	Parameter	Null hypothesis	Alternative hypothesis	Test statistic	Decision rule: Reject H_0 if the computed value is
1.	Quantitative data; variances σ_1^2 and σ_2^2 are known; both populations normally distributed	$\mu_1 - \mu_2$	$\mu_1 = \mu_2$	1. $\mu_1 > \mu_2$ 2. $\mu_1 < \mu_2$ 3. $\mu_1 \neq \mu_2$	$\dfrac{\bar{X}_1 - \bar{X}_2}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$	1. greater than z_α 2. less than $-z_\alpha$ 3. less than $-z_{\alpha/2}$ or greater than $z_{\alpha/2}$
2.	Quantitative data; σ_1^2, σ_2^2 unknown; populations not necessarily normal; sample sizes are large	$\mu_1 - \mu_2$	$\mu_1 = \mu_2$	1. $\mu_1 > \mu_2$ 2. $\mu_1 < \mu_2$ 3. $\mu_1 \neq \mu_2$	$\dfrac{\bar{X}_1 - \bar{X}_2}{\sqrt{\dfrac{S_1^2}{n_1} + \dfrac{S_2^2}{n_2}}}$	1. greater than z_α 2. less than $-z_\alpha$ 3. less than $-z_{\alpha/2}$ or greater than $z_{\alpha/2}$
3.	Quantitative data; σ_1^2 and σ_2^2 are not known, assumed to be equal; Both populations are normally distributed.	$\mu_1 - \mu_2$	$\mu_1 = \mu_2$	1. $\mu_1 > \mu_2$ 2. $\mu_1 < \mu_2$ 3. $\mu_1 \neq \mu_2$	$\dfrac{\bar{X}_1 - \bar{X}_2}{\sqrt{\dfrac{S_1^2}{n_1} + \dfrac{S_2^2}{n_2}}}$ $df = \dfrac{\left[\left(S_1^2/n_1\right) + \left(S_2^2/n_2\right)\right]^2}{\dfrac{(S_1^2/n_1)^2}{n_1 - 1} + \dfrac{(S_2^2/n_2)^2}{n_2 - 1}}$	1. greater than $t_{m+n-2,\alpha}$ 2. less than $-t_{m+n-2,\alpha}$ 3. less than $-t_{m+n-2,\alpha/2}$ or greater than $t_{m+n-2,\alpha/2}$
4.	Quantitative data; σ_1^2 and σ_2^2 are not known but are assumed to be equal; both populations are normally distributed.	$\mu_1 - \mu_2$	$\mu_1 = \mu_2$	1. $\mu_1 > \mu_2$ 2. $\mu_1 < \mu_2$ 3. $\mu_1 \neq \mu_2$	$\dfrac{\bar{X}_1 - \bar{X}_2}{S_p\sqrt{\dfrac{S_1^2}{n_1} + \dfrac{S_2^2}{n_2}}}$ where $S_p = \sqrt{\dfrac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}}$	1. greater than $t_{m+n-2,\alpha}$ 2. less than $-t_{m+n-2,\alpha}$ 3. less than $-t_{m+n-2,\alpha/2}$ or greater than $t_{m+n-2,\alpha/2}$

5.	Paired observations	$\mu_1 - \mu_2$ $(=\mu_D)$	$\mu_D = 0$ i.e., $\mu_1 = \mu_2$	1. $\mu_D > 0$ 2. $\mu_D < 0$ 3. $\mu_D \neq 0$	$\dfrac{\bar{d}\sqrt{n}}{s_d}$ where $\bar{d} = \Sigma d_i / n$ and $s_d = \sqrt{\dfrac{1}{n-1}\Sigma(d_i - \bar{d})^2}$	1. greater than $t_{n-1,\alpha}$ 2. less than $-t_{n-1,\alpha}$ 3. less than $-t_{n-1,\alpha/2}$ or greater than $t_{n-1,\alpha/2}$
6.	Quantitative data; μ_1, σ_1^2, μ_2 and σ_2^2 are unknown; Both populations are normally distributed.	$\sigma_1^2 - \sigma_2^2$	$\sigma_1^2 = \sigma_2^2$	1. $\sigma_1^2 > \sigma_2^2$ 2. $\sigma_1^2 < \sigma_2^2$ 3. $\sigma_1^2 \neq \sigma_2^2$	$F_0 = \dfrac{S_1^2}{S_2^2}$	$F_0 > F_{\alpha/2, n_1-1, n_2-1}$ $F_0 < F_{1-\alpha/2, n_1-1, n_2-1}$ $F_0 > F_{\alpha, n_1-1, n_2-1}$
7.	Qualitative data; binomial case	$p_1 - p_2$	$p_1 = p_2$	1. $p_1 > p_2$ 2. $p_1 < p_2$ 3. $p_1 \neq p_2$	$\dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}}$ where $\hat{p} = \dfrac{x_1 + x_2}{n_1 + n_2}$ Both n_1 and n_2 are assumed large $\hat{p}_1 = \dfrac{x_1}{n_1}$ $\hat{p}_2 = \dfrac{x_2}{n_2}$	1. greater than z_α 2. less than $-z_\alpha$ 3. less than $-z_{\alpha/2}$ or greater than $z_{\alpha/2}$ The level of significance approximately 100α per cent

6.15 SUMMARY

In this chapter, we introduced the second topic in inferential statistics: tests of hypotheses. In a test of hypothesis, we test a certain given theory or belief about a population parameter. Based on some sample proportions, we find out whether or not a given claim (or a statement) about a population parameter is true.

We discussed how to make such tests of hypotheses about the population mean, μ and population proportion, p, for a single population. The hypotheses-testing procedures have been extended to the difference between two population means and the difference between two population proportions. Constructing confidence intervals and testing hypotheses about population parameters are referred to as *making inferences*.

PROBLEMS

P6.1 Determine the critical values for a hypothesis test at the 1% significance level ($\alpha = 0.01$) if the test is

(a) two-tailed
(b) left-tailed
(c) right-tailed

P6.2 Determine the critical values for a hypothesis test at the 5% significance level ($\alpha = 0.05$) if the test is

(a) two-tailed
(b) left-tailed
(c) right-tailed

P6.3 Determine the critical values for a hypothesis test at the 0.5% significance level ($\alpha = 0.005$) if the test is

(a) two-tailed
(b) left-tailed
(c) right-tailed

P6.4 Determine the critical values for a hypothesis test at the 0.2% significance level ($\alpha = 0.002$) if the test is

(a) two-tailed
(b) left-tailed
(c) right-tailed

P6.5 The breaking strength of a machine part is required to be at least 75 Pa. Based on previous test data, the standard deviation of breaking strength is 6 Pa. A random sample of 16 specimens is tested and the average value is found to be 73 Pa.

(a) state the hypotheses that need to be tested
(b) test the hypotheses using $\alpha = 0.05$.

P6.6 The diameter of copper shafts produced by a certain manufacturing company process should have a mean diameter of 0.51 mm. The diameter is known to have a standard deviation of 0.0002 mm. A random sample of 40 shafts has an average diameter of 0.509 mm. Test the hypotheses on the mean diameter using $\alpha = 0.05$.

P6.7 Refer to Problem P6.7. Determine the sample size required to construct a 95% confidence interval on the mean that has a total width of 2.0.

P6.8 The mean life span of population of light bulbs manufactured by a particular company is found to be 2400 hours with a standard deviation of 250 hours. A sample of 100 light bulbs produced in a lot is found to have a mean life span of 2300 hours. Test whether the difference between the population and sample means is statistically significant. Assume $\alpha = 0.05$.

P6.9 *(a)* In an experimental lab, 10 students are selected at random from a class and were assigned to set up an experiment. Their time to complete the experimental set up are recorded (in minutes) is 51, 55, 54, 50, 56, 59, 57, 59, 58, 60. Would it be appropriate to suggest that the mean completion time for the given experimental set up in the student population as 58.2 minutes? Given $\alpha = 0.05$.

(b) The mean weight of all basketball players in a particular college is 69.3 kg. A random sample of 20 such players produced a mean weight of 70.0 kg with a standard deviation of 2.1 kg. Assuming that the weights of all college basketball players are normally distributed, test at the 1% significance level if their mean weight is different from.

P6.10 A study claims that students in a particular college spend an average of 18 hours per week on leisure activities. The administration wanted to test this claim. A sample of 10 students selected at random gave their responses as follows (in hours):

14, 16, 19, 22, 23, 25, 26, 33, 38, 41

Assume that the time spend on leisure activities by all students is normally distributed. Using the 5% significance level, can the administration conclude that the claim of the earlier study is true?

P6.11 A manufacturing company claims that the mean production time for their product is not more than 45 minutes. A random sample of 20 products selected from the production line of the company showed that the mean production time for this sample is 50 minutes with a standard deviation of 3 minutes. Using 1% significance level, can we conclude that the company's claim is true?

P6.12 The mean height of 45 students who showed an above-average participation in college sports was 68.1 inches with a standard deviation of 2.42 inches, while 45 other students who showed no interest in such sports participation and a mean height of 67.67 inches with a standard deviation of 2.76 inches.

(a) test the hypothesis that the students who participate in sports activities are taller that the other students.

(b) by how much the sample size of each of the two groups be increased in order that the observed difference of 0.75 inches in the mean heights be significant at the level of significance (*i*) 0.05 and (*ii*) 0.01.

P6.13 A garage has a line of electronic tune-up systems. Another line has been installed and the manager would like to find out if the output of the new line is greater than that of the old one. Twelve days of data was gathered at random from the old line and 10 days of data are gathered at random from the new line, with $\bar{X}_1 = 1{,}150$ cases and $\bar{X}_2 = 1{,}175$ cases. It is known that $\sigma_1^2 = 50$ and $\sigma_2^2 = 60$. Test the appropriate hypotheses at $\alpha = 0.05$ assuming that the outputs are normally distributed.

P6.14 A company tested two different machines in order to buy the best one out of the two machines. Machine 1 was run for 8 hrs and it produced an average of 120 parts per hour with a standard deviation of 9 parts. Machine 2 was run for 10 hrs and it produced an average of 111 parts per hour with a standard deviation of 6 parts. The production per hour of both machines is assumed as normally distributed. The standard deviation of the hourly productions of the two populations are unequal. Using a 2.5% significance level, can we conclude that the mean number of parts produced per hour by machine 1 is higher than that of machine 2?

P6.15 To compare two brands of machine parts, Brand *A* and Brand *B*, for their carbon content, a sample of 60 was inspected from Brand *A* and a sample of 40 from Brand *B*. The results of the inspection are given below:

Brand A	$\bar{x} = 16$	$s_1^2 = 3$
Brand B	$\bar{y} = 17.4$	$s_2^2 = 4$

At the 5% significance level, do the two brands differ in their mean carbon content?

P6.16 The time required to change flat tires in a garage is a normal distributed random variable. The tire changing time for 15 automobiles selected at random are as follows:

5 9 7 6 24 11 4 13 10 9 20 8 19 17 25

(a) at $\alpha = 0.05$, does it seem reasonable that the mean tire changing time is 10 hours?

(b) does it seem reasonable that the mean tire changing time is less than 10 hours?

(c) does it seem reasonable that the mean tire changing time is greater than 10 hours?

P6.17 The results of tensile tests on parts made of steel specimens are as follows. The load at which specimen failure in MPa are: the sample mean $\bar{X} = 12$, and the sample standard deviation is $s = 3$. The numbers of samples are 25. Do the data suggest that the mean load at failure exceeds 10 MPa? Assume that the load at failure has normal distribution, and uses $\alpha = 0.05$.

P6.18 The maximum baking temperature for a certain component is supposed to be 90ºC. The oven temperature have a standard deviation of 10ºC at the maximum baking temperature. A random sample of 228 readings is taken and the average baking temperature is found to be 94ºC. Do the data suggest that the mean temperature at baking process exceeds 90ºC? Assume that the baking process has a normal distribution and use $\alpha = 0.05$.

P6.19 A company sent six of its sales employees to attend a special course in order to improve the sales of the company's product. The one-week sales of these salespersons before and after attended the course are given below:

Before	9	12	14	16	18	25
After	14	18	19	20	24	24

Using 1% significance level, can we conclude that the mean weekly sales for all salespersons increase as a result of attending the course? Assume the population of paired differences has a standard normal distribution.

P6.20 Table P6.20 gives the systolic blood pressure of seven adults before and after the completion of a special dietary plan for 3 months.

Table P6.20

Before	210	180	195	220	231	199	224
After	192	185	185	223	220	185	230

Consider μ_d as the mean difference between systolic blood pressure before and after completing this special dietary plan of all adults. Using the 5% significance level, can we conclude that the mean of the paired difference μ_d is different from zero? Assume that the population of paired difference is approximately normally distributed.

P6.21 A random sample of 6 automobiles was selected and these automobiles were driven for one week without the gasoline additive and then for one week with the gasoline additive. The manufacturer of the gasoline additive claims that the use of this additive increases gasoline mileage. The following table gives miles per gallon for these automobiles without and with the gasoline additive.

Without	25	28	19	23	15	29
With	27	29	21	24	17	31

Using the 2.5% significance level, can you conclude that the use of the gasoline additive increases the gasoline mileage? Assume that the population of paired differences is approximately normally distributed.

P6.22 A manufacturing company was testing two machines for production automation purpose. The first machine was run for 8 hours and it produced an average of 136 items per hour with a standard deviation of 9 items. The second machine was run for 10 hours and produced an average of 127 items per hour with a standard deviation of 6 items. Assume that the production rates of both machines are normally distributed.

(a) make a 95% confidence interval for the difference between the population means

(b) using a 2.5% significance level, can it be concluded that the mean number of items produced per hour by the first machine is higher than that of the second machine?

P6.23 Refer to problem 6.9. Assume that σ_1^2 and σ_2^2 are unknown and not assumed to be equal. However $S_1^2 = 65$ and $S_2^2 = 75$. Test the appropriate hypotheses at $\alpha = 0.05$.

P6.24 Refer to Problem 6.3. Assume that σ_1^2 and σ_2^2 are unknown but assumed to be equal with sample variances as given. Test the appropriate hypotheses at $\alpha = 0.05$.

P6.25 The annual salaries, in thousands of dollars, of 10 men in middle management at a given corporation are: 55, 65, 68, 70, 52, 56, 60, 72, 71, 73, while those for 8 women are: 56, 49, 58, 51, 61, 55, 57, 59. Let X and Y denote the salaries of men and women, respectively; assuming normal distribution and equal standard deviations, test H_0: $\mu_x = \mu_y$ against H_0: $\mu_x > \mu_y$ at the 0.05 significance level by constructing a critical region for the tests.

P6.26 The following information was obtained from two independent samples selected from two normally distributed populations with unknown but equal standard deviation.

Sample 1	5	8	9	9	10	10	11	12	12	13	13	
Sample 2	11	12	13	14	14	16	16	17	17	18	19	19

Test at the 2.5% significance level if μ_1 is lower than μ_2, where μ_1 and μ_2 are the means of populations 1 and 2 respectively.

P6.27 A consulting agency was asked by a large health insurance company to investigate if business majors were better salespersons. A sample of 40 salespersons with a business degree showed that they sold an average of 10 insurance policies per week with a standard deviation of 1.80 policies. Another sample of 45 salespersons with a degree other than business showed that they sold an average of 8.5 insurance policies per week with a standard deviation of 1.35 policies. At 1% significance level, can we conclude that the persons with a business degree are better salespersons than those who has a non-business degree?

P6.28 The administration wants to test if the mean GPAs (grade point averages) of all male and female college students who actively participate in sports are different. A random sample of 28 male and 24 female students who actively participate in sports was selected. The mean GPAs of the two groups are found to be 2.62 and 2.74 out of 4.0 respectively, with the corresponding standard deviation equal to 0.43 and 0.38. Assume that the GPAs of all male and female students have a normal distribution with the same standard deviation. Test at the 5% significance level if the mean GPAs of the two populations are different.

P6.29 The percentage of manganese in an alloy used in metal castings is measured in 50 randomly selected parts. The sample standard deviation is $s = 0.4$. Test the hypothesis H_0: $\sigma = 0.25$ versus H_1: $\sigma \neq 0.25$ using $\alpha = 0.05$.

P6.30 The thickness of a machine part used in a system is its critical dimension and that measurements of the thickness of a random sample of 18 such parts have a variance of $s^2 = 0.6$, where the measurements are in thousandths of an inch. The process is considered good if the variation of the thickness is given by a variance not greater than 0.3. Assuming that the measurements contribute a random sample from a normal population, test the null hypothesis $\sigma^2 = 0.3$ against the alternative hypotheses $\sigma^2 > 0.3$ at the 0.05 level of significance.

P6.31 In a random sample, the weights of 24 machined parts of a certain process have a standard deviation of 3 grams. Assume that the weights constitute a random sample from a normal population, test the null hypothesis $\sigma = 4$ grams against the two-sided alternative $\sigma \neq 4$ grams at the 0.01 level of significance.

P6.32 The variance of scores on a standardised engineering test for all seniors was 150 in 2003. A sample of scores for 20 seniors who took this test in 2007 gave a variance of 170. Test at the 5% significance level if the variance of current seniors on this test is different from that of 2003. Assume that the scores of all seniors on this test are approximately normally distributed.

P6.33 The gaskets manufactured by a particular company must have a variance of 0.003 mm for acceptance by the customer. A random sample of 29 such gaskets gave a variance of 0.0058 mm. Test at the 1% significance level if the variance of all such gaskets is greater than 0.003 mm. Assume that the thickness of all such gaskets produced by this company are approximately normally distributed.

P6.34 The manufacturer of a certain brand of fluorescent lights claims that the variance of the lives of these is 4000 hours. A consumer agency took a random sample of 25 such lights and tested them. The variance of the lives of these lights was found to be 4800 hours. Assume that the lives of all such lights are approximately normally distributed. Test at the 5% significance level if the variation of such light is different from 4000 hours.

P6.35 A random sample of 15 light bulbs produced by a certain company was selected and each bulb was tested for the number of hours before it burned out. If $s = 13$ hours at the 5% level of significance, is the population standard deviation different from 10 hours?

P6.36 A manufacturing company has two production lines that turn parts to a predetermined length. From a sample of 21 items taken from production line 1 and 30 items taken from production line 2, it was found that $s_1 = 15$ and $s_2 = 12$. Test the hypothesis that the two production lines have the same variance from the predetermined length. Take $\alpha = 0.05$.

P6.37 A new voltage-regulating device is installed in an electromechanical system. Before its installation, a random sample yielded the following information about the percentage of breakdown of the system: $\bar{X}_1 = 8$, $s_1^2 = 36$ and $n_1 = 8$. After installation, a random sample yielded $\bar{X}_2 = 7$, $s_2^2 = 25$ and $n_2 = 9$.

(*a*) can you conclude that the variance are equal? Use $\alpha = 0.05$

(*b*) has the new voltage-regulating device reduced the machine system breakdowns significantly? Use $\alpha = 0.05$.

P6.38 In bolted joints using gaskets the variability in the thickness of the gasket material is a critical characteristic of the bolted joint and a low variability is desirable for better performance of the bolted joint. Two different materials are being considered to determine whether one is superior in reducing the variability of the thickness. Twenty gaskets are selected. The sample standard deviation of gasket thickness are $s_1 = 2.96$ mm and $s_2 = 3.05$ mm, respectively. Is there any evidence to indicate that either gasket material is preferable? Use $\alpha = 0.05$.

P6.39 Two independent random samples are drawn from two normal populations. Test $H_0 : \sigma_1^2 = \sigma_2^2$ at 5% level of significance.

Sample 1	10	8	18	12	9	14	16	9				
Sample 2	8	13	21	15	11	15	18	9	21	22	11	16

P6.40 A random sample of six values of a random variable *X* is 24, 25, 28, 31, 32, 40 and independently obtained eight sample values of a random value *Y* are 12, 14, 15, 16, 16, 17, 17, 19. Assume that *X* and *Y* are normally distributed, test the null hypothesis $H_0 : \sigma_X^2 = \sigma_Y^2$ against the alternative hypothesis $H_1 : \sigma_X^2 > \sigma_Y^2$ at the 0.05 significance level by finding the critical region for the alternative hypothesis.

P6.41 Refer to Problem P6.40. Check if $H_0 : \sigma_x^2 = \sigma_y^2$ be rejected in favour of $H_1 : \sigma_x^2 > \sigma_y^2$ at the 0.01 significance level.

P6.42 Two different manufacturing processes are used by a company to produce turbine blades with identical mean surface roughness. It is desirable to select the process having the least variability in surface roughness. A random sample of $n_1 = 11$ parts from the first process results in a sample standard deviation $s_1 = 5.1$ micro centimeters, and a random sample of $n_2 = 16$ parts from the second process results in a sample standard deviation of $s_2 = 4.7$ micro centimeters. Find at 90% confidence interval on the ratio of the two standard deviations, s_1/s_2.

P6.43 A manufacturing firm promises a distributor that not more than 0.10 of those delivered toys will be blemished. A random sample of 500 toys delivered over the past 6 months yields 75 blemished toys. Should the toy manufacturer be declared in default at $\alpha = 0.05$?

P6.44 In a survey of 1,000 random samples of machined parts, 700 were found to be non-defective. The manufacturing company wants at least 60% of all parts it produces are non-defective. Using a 0.05 significance level, can we conclude that the company's manufacturing policy is maintained?

P6.45 A political analyst knows that a certain district voted 70% Republican in the last Congressional election. He also knows, however, that drastic changes in this district have occurred since the last election. He believes that these changes may have changed the percentage of votes that will vote for the Republican candidate for Congress, but doesn't know which way it will be changed. He polls 2,000 voters in the district and finds that 960 of these plan to vote for the Democratic candidate. Is his belief supported, at 0.01 level of significance?

P6.46 A sample survey shows that 615 of 820 people surveyed prefer to live in single homes. Test the hypothesis that the true proportion of people who prefer to live in single homes is 0.70. Given $\alpha = 0.05$.

P6.47 A machine is found to produce 32 per cent defective bolts. After repairing the machine, it was found it produced 24 defective bolts in the first production of 100. Is the true proportion of defective tubes reduced after the repairs? Use $\alpha = 0.01$.

P6.48 In a certain city only 400 men out of 900 were found to drink alcohol. Determine whether this information supports the view that majority of men in that city are non-alcohol drinkers. (Use 5% level of significance).

P6.49 It was found that the proportion of families in the U.S. who took vacation of at least 1 week last year was 20 per cent. In order to find the attitude of families on traveling this year, 100 families selected at random and were interviewed and, of these, 15 said they would take such a vacation. Has the attitude changed from last year? Use $\alpha = 0.10$.

P6.50 Two different polishing materials are being evaluated for use in machining operations in a manufacturing company. One hundred machine parts were polished using polishing material *A* and of this number 87 had no polishing-included defects. Another 100 machine parts were polished using polishing material *B* and 75 parts were satisfactory upon completion. Is there any reason to believe that the two polishing materials differ? Use $\alpha = 0.01$.

P6.51 Two different types of grinding machines are used to finish metal parts. A part is considered defective if it has excessive surface roughness or variation. Two random samples, each of size 100 are selected and 5 defective parts are found in the sample from machine 1, while 3 defective parts are found in the sample from machine 2. Is it reasonable to conclude that both machines produce the same fraction of defective parts, using $\alpha = 0.05$?

P6.52 A sample of 500 observations taken from the first population gave $\bar{X}_1 = 400$. Another sample of 600 observations taken from the second population gave $\bar{X}_2 = 420$.

(a) find the point estimate of $\hat{p}_1 - \hat{p}_2$

(b) make a 97% confidence interval for $\hat{p}_1 - \hat{p}_2$

(c) show the rejection and non-rejection regions on the sampling distribution of $\hat{p}_1 - \hat{p}_2$ for $H_0: p_1 = p_2$ versus $H_1: p_1 > p_2$. Use a significance level of 2.5%

(d) find the value of the test statistic *z* for the test of part *(c)*

(e) do you reject the null hypothesis mentioned in part *(c)* at a significance level of 2.5%?

P6.53 According to a particular research study, 24% of children in the age group 3 to 17 were overweight in 1994 and 31% were considered overweight in 2003. These percentages are based on random samples of 400 and 500 children in the given age group in 1994 and 2003, respectively. Test at the 1% significance level if the proportion of overweight children in the age group was less than in 1994 than in 2003.

P6.54 A sample survey conducted in a city results show that out of 800 men 480 are employed whereas out of 600 women only 350 were employed. Can this difference between two proportions of employed persons be ascribed due to sampling fluctuations? Use $\alpha = 0.05$.

P6.55 An insecticide of Brand *A* was sprayed into a jar containing 12 insects and it was found 95 of the insects were killed. When another jar containing 145 insecticides of the same type was sprayed with Brand *B*, 124 insects were killed. Do the two brands differ in their effectiveness at 2% level of significance?

P6.56 A study polled independent random samples of 747 men and 434 women in a particular city. Of those sampled, 276 men and 195 women responded that they frequently ordered pure vegetarian meals. Do the data poll provide sufficient evidence to conclude that the percentage of men who sometimes order pure vegetarian meals is smaller than the percentage of women who sometimes order vegetarian meals? Use 5% significance level.

REVIEW QUESTIONS

1. Describe the meaning of each of the following terms:
   (*a*) null hypothesis (*b*) alternate hypothesis
   (*c*) critical points (*d*) significance level
   (*e*) rejection region (*f*) nonrejection region
   (*g*) tails of a test (*h*) two types of errors
2. Describe briefly Type I and Type II errors.
3. Describe the general steps involved in testing a hypothesis.
4. Explain the difference between the critical value of *z* and observed value of *z*.
5. What does the level of significance represent in a test of hypothesis?
6. Describe the conditions that must hold true to use in *t*-distribution to make a test of hypothesis about the population mean.
7. Explain when a sample is large enough to use the normal distribution to make a test of hypothesis about the population proportion.
8. Describe the following terms:
   (*a*) α (*b*) β
   (*c*) left-tailed test (*d*) right-tailed test
   (*e*) one-tailed test (*f*) test statistic
9. Explain the meaning of independent and dependent samples.
10. Describe the sampling distribution of $\bar{x}_1 - \bar{x}_2$ for large and independent samples. What are the mean and standard deviation of this sampling distribution?
11. Describe the conditions that must hold true to use the *t*-distribution to make a confidence interval and to test a hypothesis about $\mu_1 - \mu_2$ for two independent samples selected from two populations with unknown but equal standard deviations.

12. Explain when the paired sample procedures are used to make confidence intervals and test hypothesis.
13. Describe the shape of the sampling distribution of $\hat{p}_1 - \hat{p}_2$ for two large samples. What are the mean and standard deviation of this sampling distributions?
14. Define the following terms:
    (a) paired or matched samples (b) μ_d
    (c) s_d (d) σ_d

STATE TRUE OR FALSE

1. Null hypothesis is a hypothesis to be tested. (True/False)
2. Alternative hypothesis is a hypothesis to be considered as an alternative to the null hypothesis. (True/False)
3. The problems in a hypothesis test is to decide whether the null hypothesis should be accepted in favour of the alternative hypothesis. (True/False)
4. A hypothesis test whose alternative hypothesis has the form H_1: $\mu \neq \mu_0$ is called a one-tailed test. (True/False)
5. A hypothesis test whose alternative hypothesis has the form H_1: $\mu < \mu_0$ is called a right-tailed test. (True/False)
6. A hypothesis test whose alternative hypothesis has the form H_1: $\mu > \mu_0$ is called a right-tailed test. (True/False)
7. A hypothesis is a statement that something is true. (True/False)
8. The decision criterion provides an objective method for deciding whether the null hypothesis should be rejected in favour of the alternative hypothesis. (True/False)
9. Test statistic is used as a basis for deciding whether the null hypothesis should be rejected. (True/False)
10. Rejection region is the set of values for the test statistic that leads to acceptance of the null hypothesis. (True/False)
11. Nonrejection region is the set of values for the test statistic that leads to non rejection of the null hypothesis. (True/False)
12. Critical values are the values of the test statistic that separate the rejection and nonrejection regions. (True/False)
13. A critical value is considered part of the nonrejection region. (True/False)
14. Type I error is accepting the null hypothesis when it is in fact true. (True/False)
15. Type II error is not rejecting the null hypothesis when it is in fact false. (True/False)
16. The probability of making a Type I error, that is, of rejecting a true null hypothesis, is called the significance level, α, of a hypothesis test. (True/False)
17. For a fixed-sample size, the smaller we specify the significance level, α, the large will be the probability, β, of not rejecting a false null hypothesis. (True/False)
18. If a null hypothesis is not rejected, we conclude that the alternative hypothesis is true. (True/False)

19. If the null hypothesis is not rejected, we conclude that the data do not provide sufficient evidence to support the alternative hypothesis. (True/False)
20. If it is important not to reject to a true null hypothesis, the hypothesis test should be performed at a small significance level. (True/False)
21. For a fixed sample size, decreasing the significance level of a hypothesis test results in an increase in the probability of making a Type II error. (True/False)
22. Hypothesis tests and confidence intervals are not closely related. (True/False)
23. The power of a hypothesis test is the probability of not making a Type II error, that is, the probability of rejecting a false null hypothesis. (True/False)
24. Type I error is rejecting a true null hypothesis. (True/False)
25. Type II error not rejecting a true null hypothesis. (True/False)
26. Significance level is the probability of rejecting a true null hypothesis, that is, of making a Type I error. (True/False)
27. In the context of hypothesis testing, α is the probability of making a Type II error (rejecting a true null hypothesis) also known as the significance level of the hypothesis test. (True/False)
28. In the context of hypothesis testing β is the probability of making a Type II error (not rejecting a false null hypothesis). (True/False)
29. In the context of hypothesis testing, $(1 - \beta)$ is the power of the hypothesis test (the probability of not making a Type II error or equivalently) of rejecting a false null hypothesis. (True/False)
30. For a fixed significance level, increasing the sample size decreases the power. (True/False)
31. Decreasing the significance level of a hypothesis test without changing the sample size decreases the probability of a Type II error, equivalently, decreases the power. (True/False)
32. The *P*-value of a hypothesis test equals the smallest significance level at which the null hypothesis can be rejected, that is, the smallest significance level for which the observed sample data results in rejecting of H_0. (True/False)
33. If the *P*-value is less than or equal to the specified significance level, reject the null hypothesis, otherwise, do not reject the null hypothesis. (True/False)
34. Independent sample means that the sample selected from one of the populations has no effect or bearing on the sample selected from the other population. (True/False)
35. Population proportion is the proportion (percentage) of the entire population that has the specified attribute. (True/False)
36. Sample proportion is the proportion (percentage) of a sample from the population that has the specified attribute. (True/False)
37. A population proportion is a parameter because it is a descriptive measure for a population. (True/False)
38. A sample proportion is a statistic because it is a descriptive measure for a sample. (True/False)
39. Number of successes is the number of members in the sample that have the specified attribute. (True/False)
40. Number of failures is the number of members in the sample that have the specified attribute. (True/False)

41. The mean of all possible sample proportions is equal to the population proportion. (True/False)
42. For large sample, the possible sample proportions have approximately a chi-square distribution. (True/False)
43. A rule of thumb for using a normal distribution to approximate the distribution of all possible sample proportions is that both np, $n(1 - p)$ are 5 or greater. (True/False)

ANSWERS TO STATE TRUE OR FALSE

1. **True**	2. **True**	3. **False**	4. **False**	5. **False**	6. **True**	7. **True**	8. **True**	9. **True**	10. **False**
11. **True**	12. **True**	13. **False**	14. **False**	15. **True**	16. **True**	17. **True**	18. **True**	19. **True**	20. **True**
21. **True**	22. **False**	23. **True**	24. **True**	25. **False**	26. **True**	27. **True**	28. **True**	29. **True**	30. **False**
31. **False**	32. **True**	33. **True**	34. **True**	35. **True**	36. **True**	37. **True**	38. **True**	39. **True**	40. **False**
41. **True**	42. **False**	43. **True**							