1.8: PROOF METHODS AND STRATEGY

Example 1: For a real number x, prove that $-|x| \le x \le |x|$.

Recall that the absolute value is a piecewise function defined by

$$|x| = x \text{ if } x \ge 0 \text{ and } |x| = -x \text{ if } x < 0.$$

Example 2: Prove that n is even if and only if $3n+2$ is even.

Recall that $P \leftrightarrow Q \equiv (P \rightarrow Q) \wedge (Q \rightarrow P)$.

Question: Suppose you wanted to prove that 4 propositions, labeled P, Q, R, and S, are equivalent. What is the minimum number of implications that must be proven?

Example 3(a): Prove or disprove that there is a prime number between any consecutive multiples of 10.

All prime numbers less than 2000

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199, 211, 223, 227, 229, 233, 239, 241, 251, 257, 263, 269, 271, 277, 281, 283, 293, 307, 311, 313, 317, 331, 337, 347, 349, 353, 359, 367, 373, 379, 383, 389, 397, 401, 409, 419, 421, 431, 433, 439, 443, 449, 457, 461, 463, 467, 479, 487, 491, 499, 503, 509, 521, 523, 541, 547, 557, 563, 569, 571, 577, 587, 593, 599, 601, 607, 613, 617, 619, 631, 641, 643, 647, 653, 659, 661, 673, 677, 683, 691, 701, 709, 719, 727, 733, 739, 743, 751, 757, 761, 769, 773, 787, 797, 809, 811, 821, 823, 827, 829, 839, 853, 857, 859, 863, 877, 881, 883, 887, 907, 911, 919, 929, 937, 941, 947, 953, 967, 971, 977, 983, 991, 997, 1009, 1013, 1019, 1021, 1031, 1033, 1039, 1049, 1051, 1061, 1063, 1069, 1087, 1091, 1093, 1097, 1103, 1109, 1117, 1123, 1129, 1151, 1153, 1163, 1171, 1181, 1187, 1193, 1201, 1213, 1217, 1223, 1229, 1231, 1237, 1249, 1259, 1277, 1279, 1283, 1289, 1291, 1297, 1301, 1303, 1307, 1319, 1321, 1327, 1361, 1367, 1373, 1381, 1399, 1409, 1423, 1427, 1429, 1433, 1439, 1447, 1451, 1453, 1459, 1471, 1481, 1483, 1487, 1489, 1493, 1499, 1511, 1523, 1531, 1543, 1549, 1553, 1559, 1567, 1571, 1579, 1583, 1597, 1601, 1607, 1609, 1613, 1619, 1621, 1627, 1637, 1657, 1663, 1667, 1669, 1693, 1697, 1699, 1709, 1721, 1723, 1733, 1741, 1747, 1753, 1759, 1777, 1783, 1787, 1789, 1801, 1811, 1823, 1831, 1847, 1861, 1867, 1871, 1873, 1877, 1879, 1889, 1901, 1907, 1913, 1931, 1933, 1949, 1951, 1973, 1979, 1987

Example 3(b): Prove or disprove that there is a prime number between any consecutive multiples of 100.

Example 3(c): Prove or disprove that there is a prime number between any consecutive multiples of 1000.

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PROOFS OF EQUIVALENCE (SPECIAL PROOFS)

Example 2: Prove that n is even if and only if $3n+2$ is even.

Recall that $P \leftrightarrow Q \equiv (P \rightarrow Q) \wedge (Q \rightarrow P)$.

Question: Suppose you wanted to prove that 4 propositions, labeled *P*, *Q*, *R*, and *S*, are equivalent. What is the minimum number of implications that must be proven?

PROOFS BY COUNTEREXAMPLE (SPECIAL PROOFS)

Example 3(a): Prove or disprove that there is a prime number between any consecutive multiples of 10.

Take a look at the list of all primes less than 2000.

Example 3(b): Prove or disprove that there is a prime number between any consecutive multiples of 100.

Example 3(c): Prove or disprove that there is a prime number between any consecutive multiples of 1000.