

3) Over a computer sub-network, three end-users connected to a wireless media via smart-phones. These users download data using the internet connection of the wireless router. In a time interval, internet data download speed statistics (KB/sn) of these three users recorded and modelled with functions for each user. Modelled functions for data download speed statistics of these users are given in the following lines.

$$F_{USER1}(t) = -0.0189t^2 + 1.3406t + 80.7743$$

$$F_{USER2}(t) = -0.0043t^2 + 0.4259t + 105.2688$$

$$F_{USER3}(t) = 0.0181t^2 - 0.8689t + 105.9157$$

Data download speed functions are available for a time interval is 100 seconds. Considering the given functions answer the following questions;

- Using single Trapezoidal rule calculate the downloaded data size (kilobytes) in [10,70] time range for each smart-phone user.
- Calculate the true errors and absolute relative true errors considering the values you find in section (a).
- Using multiple Trapezoidal rule calculate data sizes (kilobytes) in [10,70] time range for each smart-phone user by getting Trapezoidal segment counts as 2,4,6,10,20.
- Calculate the true errors and absolute relative true errors considering the values you find in section (c).
- Write your segments, approximate values, true values, true errors, absolute true errors as tabulated form. (Segments : 1,2,4,6,10. Note that you should use previous section results, do not calculate necessary elements again.)
- Plot the data download speeds of users for time range [0, 100] and compare your calculations with graphics.
- Using Simpson's method for integration solve section (a), (b), (c), (d), (e).

## Answers

3)

$$a) \int_a^b f(x)dx \approx \text{Area of trapezoid} \qquad \int_a^b f(x)dx \approx (b-a) \left[ \frac{f(a) + f(b)}{2} \right]$$

$$\begin{aligned} F_1(t) &= -0.0189t^2 + 1.3406t + 80.7743 \quad [10, 70] \\ \int_{10}^{70} F_1(t)dt &\approx (70-10) \left[ \frac{82.0063 + 92.2903}{2} \right] = 5228.8980 \end{aligned}$$

$$F_2(t) = -0.0043t^2 + 0.4259t + 105.2688 \quad [10, 70]$$

$$\int_{10}^{70} F_1(t) dt \approx (70 - 10) \left[ \frac{114.0118 + 109.0978}{2} \right] = 6693.2880$$

$$F_3(t) = 0.0181t^2 - 0.8689t + 105.9157 \quad [10, 70]$$

$$\int_{10}^{70} F_1(t) dt \approx (70 - 10) \left[ \frac{133.7827 + 99.0367}{2} \right] = 6984.5820$$

True integration results;

$$F(t) = -0.0063t^3 + 0.6703t^2 + 80.7743t \quad [10, 70]$$

$$\int_{10}^{70} F_1(t) dt = F(70) - F(10) = 6777.7710 - 868.4730 = 5909.2980$$

$$F(t) = -0.00143t^3 + 0.21295t^2 + 105.2688t \quad [10, 70]$$

$$\int_{10}^{70} F_1(t) dt = F(70) - F(10) = 7921.7810 - 1072.5530 = 6849.2280$$

$$F(t) = 0.00603t^3 - 0.43445t^2 + 105.9157t \quad [10, 70]$$

$$\int_{10}^{70} F_1(t) dt = F(70) - F(10) = 7.3535840 - 1.0217420 = 6331.8420$$

b)

	True Error	Relative Abs. True Error
User - 1	5909.2980 - 5228.8980 = 680.4	(680.4 / 5909.2980) x 100 = %11.5140
User - 2	6849.2280 - 6693.2880 = 155.94	(155.94 / 6849.2280) x 100 = %2.2767
User - 3	6331.8420 - 6984.5820 = -652.74	(652.74 / 6331.8420) x 100 = %10.3088

c) 
$$\int_a^b f(x) dx \approx \frac{(b-a)}{2n} \left[ f(a) + 2 \left\{ \sum_{i=1}^{n-1} f(a+ih) \right\} + f(b) \right]$$

$$F_1(t) = -0.0189t^2 + 1.3406t + 80.7743 \quad [10, 70]$$

$$\int_a^b f(x) dx \approx \frac{(70-10)}{2(2)} [f(10) + 2\{f(40)\} + f(70)]$$

$$\int_a^b f(x)dx \approx \frac{(70-10)}{2(4)} \left[ f(10) + 2\{f(25) + f(40) + f(55)\} + f(70) \right]$$

$$\int_a^b f(x)dx \approx \frac{(70-10)}{2(6)} \left[ f(10) + 2\{f(20) + f(30) + f(40) + f(50) + f(60)\} + f(70) \right]$$

$$\int_a^b f(x)dx \approx \frac{(70-10)}{2(10)} \left[ f(10) + 2\left\{ \begin{array}{l} f(16) + f(22) + f(28) + f(34) + f(40) + \\ f(46) + f(52) + f(58) + f(64) \end{array} \right\} + f(70) \right]$$

$$\int_a^b f(x)dx \approx \frac{(70-10)}{2(20)} \left[ f(10) + 2\left\{ \begin{array}{l} f(13) + f(16) + f(19) + f(22) + f(25) + f(28) + f(31) + \\ f(34) + f(37) + f(40) + f(43) + f(46) + f(49) + f(52) + \\ f(55) + f(58) + f(61) + f(64) + f(67) \end{array} \right\} + f(70) \right]$$

	Approximate values		
	User-1	User-2	User-3
2-segments	<b>5739.1980</b>	<b>6809.3880</b>	<b>6495.8820</b>
4-segments	<b>5866.7730</b>	<b>6838.4130</b>	<b>6373.7069</b>
6-segments	<b>5890.3980</b>	<b>6843.7880</b>	<b>6351.0820</b>
10-segments	<b>5902.4940</b>	<b>6846.5400</b>	<b>6339.4980</b>
20-segments	<b>5907.5970</b>	<b>6847.7009</b>	<b>6334.6109</b>

True values		
User-1	User-2	User-3
5909.2980	6849.2280	6331.8420

d) For user-1;

2-segment 5909.2980 - 5739.1980 = 170,01

(170,0100 / 5909.2980) \* 100 = 2.8769

4-segment 5909.2980 - 5866.7730 = 42.525

(42.5250 / 5909.2980) \* 100 = 0.7196

6-segment 5909.2980 - 5890.3980 = 18,9

(18,9000 / 5909.2980) \* 100 = 0.3198

10-segment  $5909.2980 - 5902.4940 = 6.804$   
 $(6.8040 / 5909.2980) * 100 = 0.1151$   
 20-segment  $5909.2980 - 5907.5970 = 1.701$   
 $(1.7010 / 5909.2980) * 100 = 0.0287$

2-segment  $6849.2280 - 6809.3880 = 39.84$   
 $(39.8400 / 6849.2280) * 100 = 0.5816$   
 4-segment  $6849.2280 - 6838.4130 = 10.815$   
 $(10.8150 / 6849.2280) * 100 = 0.1579$   
 6-segment  $6849.2280 - 6843.7880 = 5.44$   
 $(5.4400 / 6849.2280) * 100 = 0.0794$   
 10-segment  $6849.2280 - 6846.5400 = 2.688$   
 $(2.6880 / 6849.2280) * 100 = 0.0392$   
 20-segment  $6849.2280 - 6847.7009 = 1.5271$   
 $(1.5271 / 6849.2280) * 100 = 0.0222$

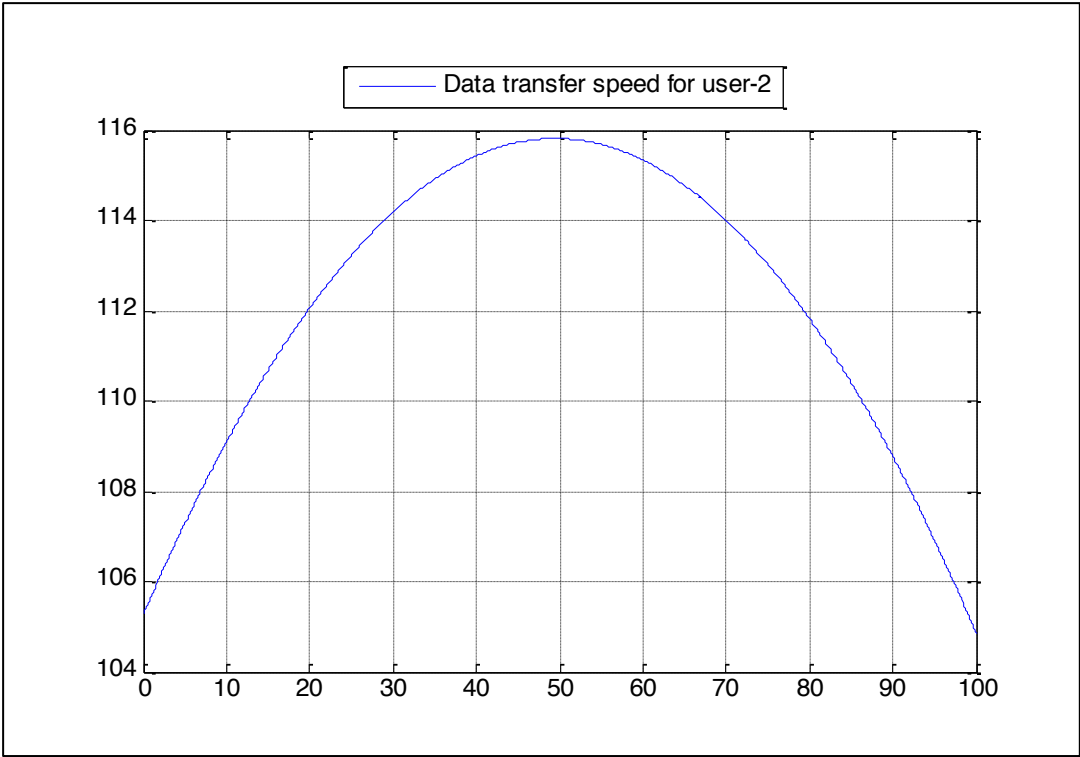
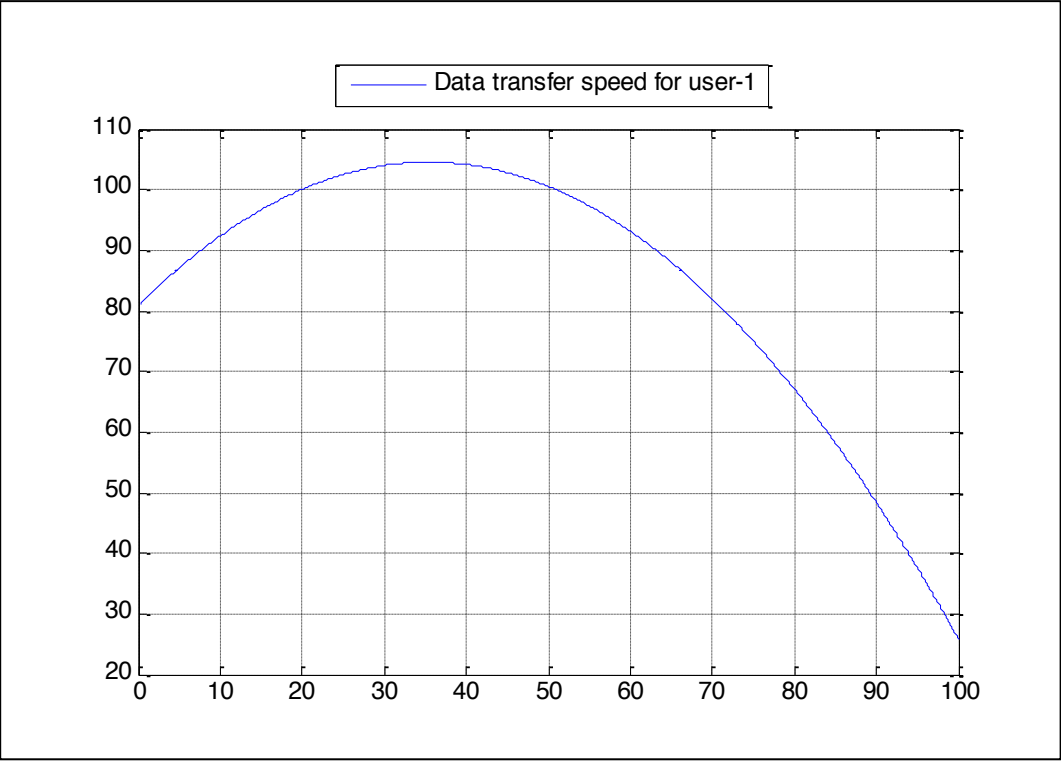
2-segment  $6331.8420 - 6495.8820 = -164.04$   
 $(164.0400 / 6331.8420) * 100 = 2.5907$   
 4-segment  $6331.8420 - 6373.7069 = -41.8649$   
 $(41.8649 / 6331.8420) * 100 = 0.6611$   
 6-segment  $6331.8420 - 6351.0820 = -19.24$   
 $(19.2400 / 6331.8420) * 100 = 0.3038$   
 10-segment  $6331.8420 - 6339.4980 = -7.656$   
 $(7.6560 / 6331.8420) * 100 = 0.1209$   
 20-segment  $6331.8420 - 6334.6109 = -2.7689$   
 $(2.7689 / 6331.8420) * 100 = 0.0437$

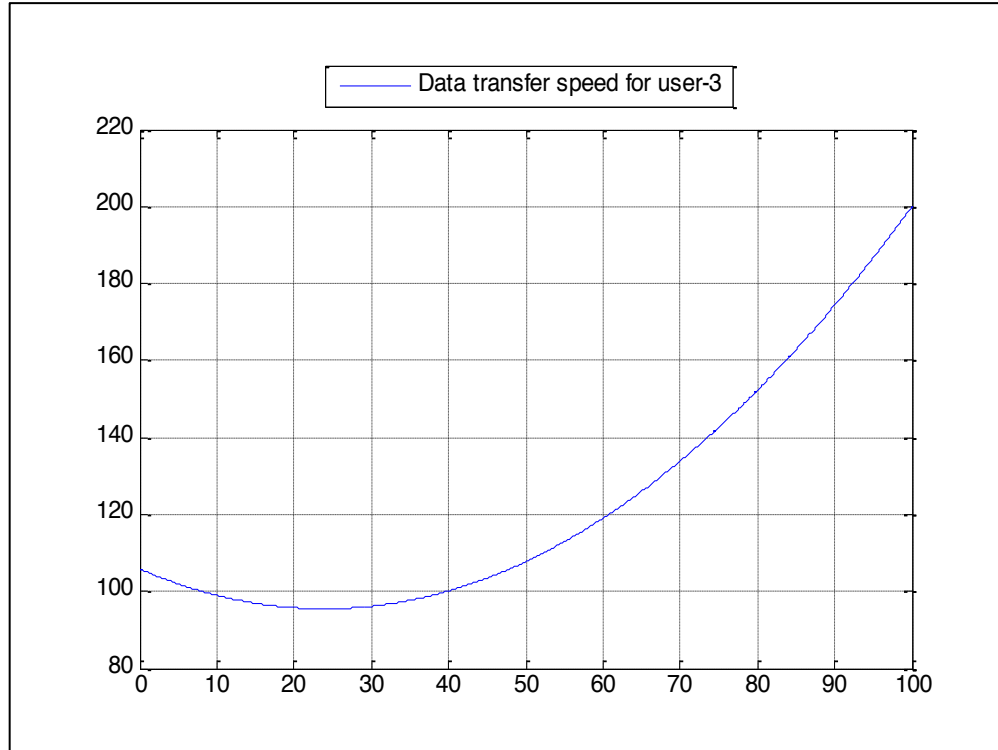
e)

	Approximate values		
Segments	User-1	User-2	User-3
2	5739.1980	6809.3880	6495.8820
4	5866.7730	6838.4130	6373.7069

6	5890.3980	6843.7880	6351.0820
10	5902.4940	6846.5400	6339.4980
20	5907.5970	6847.7009	6334.6109
	<b>True Errors</b>		
2	170.01	39.84	0.5816
4	42.525	10.815	0.1579
6	18.9	5.44	0.0794
10	6.804	2.688	0.0392
20	1.701	1.5271	0.0222
	<b>Abs. Rel. True Errors</b>		
2	2.8769	-164.04	2.5907
4	0.7196	-41.8649	0.6611
6	0.3198	-19.24	0.3038
10	0.1151	-7.656	0.1209
20	0.0287	-2.7689	0.0437

f)





$$\text{g) } \int_a^b f_2(x) dx \approx \frac{h}{3} \left[ f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right] \approx \frac{(b-a)}{6} \left[ f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right]$$

$$F_1(t) = -0.0189t^2 + 1.3406t + 80.7743 \quad [10, 70]$$

$$\int_{10}^{70} F_1(t) dt \approx \frac{70-10}{6} [82.0063 + 4f(40) + 92.2903] = 10 [82.0063 + 416.6332 + 92.2903] = 5909.2980$$

$$F_2(t) = -0.0043t^2 + 0.4259t + 105.2688 \quad [10, 70]$$

$$\int_{10}^{70} F_1(t) dt \approx (70-10) \left[ \frac{+}{2} \right] = 6693.2880$$

$$\int_{10}^{70} F_1(t) dt \approx \frac{70-10}{6} [114.0118 + 4f(40) + 109.0978] = 10[114.0118 + 461.6992 + 109.0978] = 6848.0880$$

$$F_3(t) = 0.0181t^2 - 0.8689t + 105.9157 \quad [10, 70]$$

$$\int_{10}^{70} F_1(t) dt \approx \frac{70-10}{6} [133.7827 + 4f(40) + 99.0367] = 10[133.7827 + 400.4788 + 99.0367] = 6332.9820$$

True integration results;

$$F(t) = -0.0063t^3 + 0.6703t^2 + 80.7743t \quad [10, 70]$$

$$\int_{10}^{70} F_1(t) dt = F(70) - F(10) = 6777.7710 - 868.4730 = 5909.2980$$

$$F(t) = -0.00143t^3 + 0.21295t^2 + 105.2688t \quad [10, 70]$$

$$\int_{10}^{70} F_1(t) dt = F(70) - F(10) = 7921.7810 - 1072.5530 = 6849.2280$$

$$F(t) = 0.00603t^3 - 0.43445t^2 + 105.9157t \quad [10, 70]$$

$$\int_{10}^{70} F_1(t) dt = F(70) - F(10) = 7.3535840 - 1.0217420 = 6331.8420$$

	True Error	Relative Abs. True Error
User - 1	5909.2980 - 5909.2980 = 0	(0 / 5909.2980) x 100 = %0.00
User - 2	6849.2280 - 6848.0880 = 1.14	(1.14 / 6849.2280) x 100 = %0.0166
User - 3	6331.8420 - 6332.9820 = -1.14	(1.14 / 6849.2280) x 100 = %0.0166



$$\int_a^b f(x)dx \approx \frac{(b-a)}{3n} \left[ f(x_0) + 4 \sum_{\substack{i=1 \\ i=odd}}^{n-1} f(x_i) + 2 \sum_{\substack{i=2 \\ i=even}}^{n-2} f(x_i) + f(x_n) \right]$$

$$F_1(t) = -0.0189t^2 + 1.3406t + 80.7743 \quad [10, 70]$$

	Approximate values		
	User-1	User-2	User-3
2-segments	5909.2980	6848.0880	6332.9820
4-segments	5909.2980	6848.0880	6332.9820
6-segments	5909.2980	6848.0880	6332.9820
10-segments	5909.2980	6848.0880	6332.9820
20-segments	5909.2980	6848.0880	6332.9820

True values		
User-1	User-2	User-3
5909.2980	6849.2280	6331.8420

For user-1;

2-segment  $5909.2980 - 5909.2980 = 0$   $(0 / 5909.2980) * 100 = \%0$   
4-segment  $5909.2980 - 5909.2980 = 0$   $(0 / 5909.2980) * 100 = \%0$   
6-segment  $5909.2980 - 5909.2980 = 0$   $(0 / 5909.2980) * 100 = \%0$   
10-segment  $5909.2980 - 5909.2980 = 0$   $(0 / 5909.2980) * 100 = \%0$   
20-segment  $5909.2980 - 5909.2980 = 0$   $(0 / 5909.2980) * 100 = \%0$

For user-2;

2-segment  $6849.2280 - 6849.2280 = 0$   $(0 / 6849.2280) * 100 = \%0$   
4-segment  $6849.2280 - 6849.2280 = 0$   $(0 / 6849.2280) * 100 = \%0$

6-segment       $6849.2280 - 6849.2280 = 0$        $(0 / 6849.2280) * 100 = \%0$   
10-segment     $6849.2280 - 6849.2280 = 0$        $(0 / 6849.2280) * 100 = \%0$   
20-segment     $6849.2280 - 6849.2280 = 0$        $(0 / 6849.2280) * 100 = \%0$

For user-3;

2-segment       $6331.8420 - 6331.8420 = 0$        $(0 / 6331.8420) * 100 = \%0$   
4-segment       $6331.8420 - 6331.8420 = 0$        $(0 / 6331.8420) * 100 = \%0$   
6-segment       $6331.8420 - 6331.8420 = 0$        $(0 / 6331.8420) * 100 = \%0$   
10-segment     $6331.8420 - 6331.8420 = 0$        $(0 / 6331.8420) * 100 = \%0$   
20-segment     $6331.8420 - 6331.8420 = 0$        $(0 / 6331.8420) * 100 = \%0$

	Approximate values		
Segments	User-1	User-2	User-3
2	5909.2980	6849.2280	6331.8420
4	5909.2980	6849.2280	6331.8420
6	5909.2980	6849.2280	6331.8420
10	5909.2980	6849.2280	6331.8420
20	5909.2980	6849.2280	6331.8420
	True Errors		
2	0.00	0.00	0.00
4	0.00	0.00	0.00
6	0.00	0.00	0.00
10	0.00	0.00	0.00
20	0.00	0.00	0.00
	Abs. Rel. True Errors		
2	0.00	0.00	0.00
4	0.00	0.00	0.00
6	0.00	0.00	0.00

10	0.00	0.00	0.00
20	0.00	0.00	0.00