1.6: RULES OF INFERENCE

Example 1: Consider the following argument.

Using Words	Using Symbols
If it works, then use it.	$W \to U$
It works.	W

Therefore, use it.	$\therefore U$

Name	Rule of Inference	Name	Rule of Inference
Addition	p ------ $\therefore p \vee q$	*Modus tollens*	$\neg q$ $p \to q$ -------- $\therefore \neg p$
Simplification	$p \wedge q$ ------ $\therefore p$	*Hypothetical syllogism*	$p \to q$ $q \to r$ -------- $\therefore p \to r$
Conjunction	p q ------ $\therefore p \wedge q$	*Disjunctive syllogism*	$p \vee q$ $\neg p$ ------ $\therefore q$
Modus ponens	p $p \to q$ -------- $\therefore q$	*Resolution*	$p \vee q$ $\neg p \vee r$ ------ $\therefore q \vee r$

Example 2: Prove the following argument is valid. Justify each step by citing the rule of inference needed.

<u>Using Words</u>	<u>Using Symbols</u>
If the car is not full, then it is not sunny.	$\neg F \to \neg S$
It is sunny or lunchtime.	$S \vee L$
It is not lunchtime.	$\neg L$
Therefore, the car is full.	----------
	$\therefore\ F$

The solution written for Example 2 is an example of a ***valid proof*** (*a.k.a.* a ***formal proof***).

Given some hypotheses and some ***conclusion*** q, form the chain

$$p_1, p_2, \ldots, p_n, q$$

of propositions where each p_i is

- a hypothesis,
- a logical equivalence, or
- a consequence using a rule of inference.

Example 4: Determine if the following is a valid argument.
Penguins are birds.
All birds are able to fly.
Therefore, penguins are able to fly.

Example 5: Determine if the following is a valid argument.
Giraffes have four legs.
Cows have four legs.
Therefore, giraffes are taller than cows.

Example 6: Determine if the following is a valid argument.
If Joe wins the state lottery, he can afford a new car.
Joe did not win the state lottery.
Therefore, Joe cannot afford a new car.

Example 7: Prove that $-1 = 1$.

SECTION 1.6 – RULES OF INFERENCE

Example 1: Consider the following argument.

Using Words	Using Symbols
If it works, then use it.	$W \to U$
It works.	W
Therefore, use it.	----------
	$\therefore U$

RULES OF INFERENCE

Name	Rule of Inference
Addition	p ------ $\therefore\ p \vee q$
Simplification	$p \wedge q$ ------ $\therefore\ p$
Conjunction	p q ------ $\therefore\ p \wedge q$
Modus ponens	p $p \to q$ -------- $\therefore\ q$

Name	Rule of Inference
Modus tollens	$\neg q$ $p \to q$ -------- $\therefore\ \neg p$
Hypothetical syllogism	$p \to q$ $q \to r$ -------- $\therefore\ p \to r$
Disjunctive syllogism	$p \vee q$ $\neg p$ ------ $\therefore\ q$
Resolution	$p \vee q$ $\neg p \vee r$ ------ $\therefore\ q \vee r$

VALID ARGUMENTS

Example 2: Prove the following argument is valid. Justify each step by citing the rule of inference needed.

Using Words	Using Symbols
If the car is not empty, then it is not sunny.	$\neg F \to \neg S$
It is sunny or lunchtime.	$S \vee L$
It is not lunchtime.	$\neg L$
Therefore, the car is empty.	----------
	$\therefore F$

The solution written for Example 2 is an example of a This is an example of a ***valid proof*** (*a.k.a.* a ***formal proof***).

Given some hypotheses and some ***conclusion*** q, form the chain $p_1, p_2, \ldots, p_n, q$ of propositions where each p_i is

- a hypothesis,
- a logical equivalence, or
- a consequence using a rule of inference.

Example 3: From hypotheses $(p \rightarrow q) \wedge r$, $\neg(r \wedge q)$ and $s \rightarrow p$, conclude $\neg s$.

FALLACIES

Example 4:
Penguins are birds.
All birds are able to fly.
Therefore, penguins are able to fly.

Example 5:
Giraffes have four legs.
Cows have four legs.
Therefore, giraffes are taller than cows.

Example 6:
If Joe wins the state lottery, he can afford a new car.
Joe did not win the state lottery.
Therefore, Joe cannot afford a new car.

Example 7: Prove that $-1 = 1$.

EQUIVALENCES AND IMPLICATION EQUIVALENCES

Double negation law: $\neg(\neg p) \equiv p$

Identity laws: $p \vee \mathbf{F} \equiv p$, $p \wedge \mathbf{T} \equiv p$

Domination laws: $p \vee \mathbf{T} \equiv \mathbf{T}$, $p \wedge \mathbf{F} \equiv \mathbf{F}$

Negation laws: $p \vee \neg p \equiv \mathbf{T}$, $p \wedge \neg p \equiv \mathbf{F}$

Idempotent laws: $p \vee p \equiv p$, $p \wedge p \equiv p$

Commutative laws: $p \vee q \equiv q \vee p$, $p \wedge q \equiv q \wedge p$

Associative laws: $p \vee (q \vee r) \equiv (p \vee q) \vee r$,
$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$

Distributive laws: $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$,
$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

Absorption laws: $p \vee (p \wedge q) \equiv p$, $p \wedge (p \vee q) \equiv p$

DeMorgan's laws: $\neg(p \vee q) \equiv \neg p \wedge \neg q$,
$\neg(p \wedge q) \equiv \neg p \vee \neg q$

1. $p \to q \equiv \neg p \vee q$
2. $p \to q \equiv \neg q \to \neg p$
3. $p \vee q \equiv \neg p \to q$
4. $p \wedge q \equiv \neg(p \to \neg q)$
5. $\neg(p \to q) \equiv p \wedge \neg q$
6. $(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$
7. $(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$
8. $(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$
9. $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$
10. $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$
11. $p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
12. $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
13. $\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

RULES OF INFERENCE

p

$\therefore\ p \vee q$ (*Addition*)

$p \wedge q$

$\therefore\ p$ (*Simplification*)

p
q

$\therefore\ p \wedge q$ (*Conjunction*)

p
$p \to q$

$\therefore\ q$ (*Modus ponens*)

$\neg q$
$p \to q$

$\therefore\ \neg p$ (*Modus tollens*)

$p \to q$
$q \to r$

$\therefore\ p \to r$ (*Hypothetical syllogism*)

$p \vee q$
$\neg p$

$\therefore\ q$ (*Disjunctive syllogism*)

$p \vee q$
$\neg p \vee r$

$\therefore\ q \vee r$ (*Resolution*)