BLG 202E – Numerical Methods in CE

Spring 2017

Homework 4

Due: 19.05.2017 23:59

Question 1 – The Trapezoidal Rule

$$\int_a^b f(x)dx \approx \frac{\Delta x}{2}(y_0 + 2y_1 + 2y_2 + \cdots + 2y_{n-1} + y_n)$$

(The simplified formula of the trapezoidal rule:

n = segment count, Δx = length of every segment)

$$f(x) = 3x^2 + 25x + 0.2$$

a) Use Trapezoidal Rule to numerically integrate $f(x)$ from $a = 0$ to $b = 2$ and calculate the relative error.

By using integration rules: (*real value*)

$$\int f(x)dx = x^3 + 12.5x^2 + 0.2x + c$$

$$\int_0^2 f(x)dx = 2^3 + 12.5 * 2^2 + 0.2 * 2 - 0^3 + 12.5 * 0^2 + 0.2 * 0 = 58.4$$

By using trapezoidal rule formula, which is given above: (*calculated value*)

$$n = 1, \quad \Delta x = 2, \quad f(2) = 62.2, \quad f(0) = 0.2$$

$$\int_0^2 f(x)dx \approx \frac{2}{2}(62.2 + 0.2) = 62.4$$

$$\%Error = \frac{|58.4 - 62.4|}{58.4} * 100 = 6.8493$$

b) Use the 2-segment Trapezoidal Rule to numerically integrate $f(x)$ from $a = 0$ to $b = 2$ and calculate the relative error.

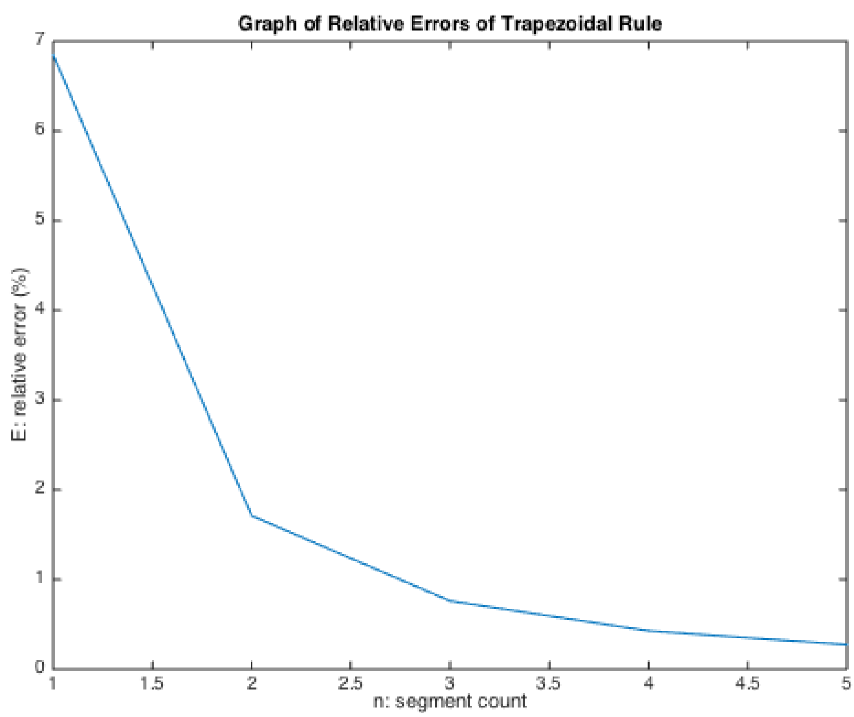
By using trapezoidal rule formula, which is given above: (calculated value)

$$n = 2, \qquad \Delta x = 1, \qquad f(2) = 62.2, \qquad f(1) = 28.2, \qquad f(0) = 0.2$$

$$\int_0^2 f(x)dx \approx \frac{1}{2}(62.2 + 2 * 28.2 + 0.2) = 59.4$$

$$\%Error = \frac{|58.4 - 59.4|}{58.4} * 100 = 1.7123$$

c) Compare calculated relative errors in (a) and (b), and explain the result in your own words. According the results obtained part a. and b., in trapezoidal rule, the relative error *decreases* as the number of segments *increases*. To better observe, I wrote two piece of matlab code, one calculates integral of given function (f) with given interval $[a, b]$ using trapezoidal rule (trapezoidal.m) and the other calculates and plots the relative errors despite the number of segments (n) for given equation in this question (h04q01.m).



Question 2 – The Simpson's Rule

a) Show in detail how to obtain the Simpson rule

$$\int_a^b f(x)dx \approx \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$

Let's consider f is a quadratic polynomial:

$$f(x) = Ax^2 + Bx + C$$

For convenience, let's formulate the area under that parabola passing through those three points: $(-h, y_0), (0, y_1), (h, y_2)$.

$$\int_{-h}^{h} f(x)dx = \left(\frac{Ax^3}{3} + \frac{Bx^2}{2} + Cx\right)\Big|_{-h}^{h}$$

$$\int_{-h}^{h} f(x)dx = \left(\frac{2Ah^3}{3} + 2Ch\right) = \frac{h}{3}(2Ah^2 + 6c)$$

Also the points satisfy the equation:

$$y_0 = f(-h) = Ah^2 - Bh + C$$

$$y_1 = f(0) = C$$

$$y_2 = f(h) = Ah^2 + Bh + C$$

$$y_0 + 4y_1 + y_2 = (Ah^2 - Bh + C) + 4C + (Ah^2 + Bh + C) = (2Ah^2 + 6c)$$

So, the area under parabola turns into:

$$\int_{-h}^{h} f(x)dx = \frac{h}{3}(2Ah^2 + 6c) = \frac{\Delta x}{3}(y_0 + 4y_1 + y_2)$$

Now, we consider the interval $[a, b]$ instead of $[-h, h]$:

$$n = 2, \qquad \Delta x = \left(\frac{b-a}{2}\right), \qquad y_0 = f(a), \qquad y_1 = f\left(\frac{a+b}{2}\right), \qquad y_2 = f(b)$$

$$\int_a^b f(x)dx \approx \frac{\Delta x}{3}(y_0 + 4y_1 + y_2) \approx \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$

b) Use composite Simpson's $^1/_3$ rule to integrate $f(x) = 0.2 + 25x + 3x^2 + 2x^4$ from $a = 0$ to $b = 2$ for $r = 4$ (four equal panels).

The general formula of composite Simpson's rule

$$\int_a^b f(x)dx \approx \frac{b-a}{3n} [y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \cdots + 4y_{n-1} + y_n]$$

$$n = 4, \qquad \Delta x = 0.5,$$

$$f(0) = 0.2, \qquad f(0.5) = 13.575, \qquad f(1) = 30.2, \qquad f(1.5) = 54.575, \qquad f(2) = 94.2$$

$$\int_0^2 f(x)dx \approx \frac{2-0}{12} [0.2 + 4 * 13.575 + 2 * 30.2 + 4 * 54.575 + 94.2] \approx 71.2333$$

Question 3 – The Matlab Application

Write a matlab program that computes an integral numerically, using the composite Trapezoidal, Mid-point and Simpson methods. Input for program consists of integrand, the ends of integration interval, and the number of (uniform) subintervals. Apply your program to the two following integrals.

For this question, I wrote three script file that contains some numerical methods that approximates the value of a definite integral. All script files stored in "Matlab Codes" folder.

- trapezoidal.m → function: trapezoidal (function, a, b, number_of_segment)
- midpoint.m → function: midpoint (function, a, b, number_of_segment)
- simpson.m → function: simpson(function, a, b, number_of_segment) (Note: For this function, the number of segments should be even)
- h04q03 → implemented given functions given in part a. and b. Also calculated the value of given integrals for each implemented method with 10 segments ($n = 10$).

a)

$$\int_0^1 \frac{4}{1+x^2}\,dx = 4 * (\tan^{-1} 1 - \tan^{-1} 0) = 4 * 0.78539 = 3.14156 = \pi$$

Trapezoidal Method with 10 segments = 3.1399

Simpson's Method with 10 segments = 3.1416

Midpoint Method with 10 segments = 3.1424

Trapezoidal Method with 100 segments = 3.1416

Simpson's Method with 100 segments = 3.1416

Midpoint Method with 100 segments = 3.1426

b)

$$\int_0^1 \sqrt{x}\,dx = \left(\frac{2x^{3/2}}{3}\right)\Bigg|_0^1 = \frac{2}{3} = 0.6666$$

Trapezoidal Method with 10 segments = 0.6605

Simpson's Method with 10 segments = 0.6641

Midpoint Method with 10 segments = 0.6684

Trapezoidal Method with 100 segments = 0.6665

Simpson's Method with 100 segments = 0.6666

Midpoint Method with 100 segments = 0.6687

Conclusion: For small segment counts, The Simpson's Method calculates the nearest results, but when the number of segments is increased, all three methods generates very close and accurate results.

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