BLG 202E
Recitation 3

1. LU Decomposition

$$\begin{bmatrix} 1 & 2 & 4 \\ 3 & 8 & 14 \\ 2 & 6 & 13 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 13 \\ 4 \end{bmatrix} \qquad \begin{gathered} Ax=b\,, A=LU \\ MA=U \\ M^{(2)}M^{(1)}A=U \end{gathered}$$

Finding M(1) :

$$M^{(1)}A=A^{(1)}$$

$$\begin{gathered} l_{21}=A_{21}/A_{11} \\ l_{31}=A_{31}/A_{11} \end{gathered} \qquad \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 4 \\ 3 & 8 & 14 \\ 2 & 6 & 13 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 2 & 2 \\ 0 & 2 & 5 \end{bmatrix}$$

Finding M(2) :

$$M^{(2)}A^{(1)}=A^{(2)}=U$$

$$l_{32}=A_{32}/A_{22} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 4 \\ 0 & 2 & 2 \\ 0 & 2 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

L(lower triangular) and U(upper triangular) has been found

$$L=\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix} \qquad U=\begin{bmatrix} 1 & 2 & 4 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix} \qquad \begin{gathered} Ax=b\,, A=LU \\ LUx=b\,, Ux=y \\ Ly=b \end{gathered}$$

Ly = b forward substitution

$$\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 13 \\ 4 \end{bmatrix} \qquad y=\begin{bmatrix} 3 \\ 4 \\ -6 \end{bmatrix}$$

Ux = y backward substitution

$$\begin{bmatrix} 1 & 2 & 4 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ -6 \end{bmatrix} \qquad x=\begin{bmatrix} -2 \\ 4 \\ 3 \end{bmatrix}$$

2. SVD

http://www.d.umn.edu/~mhampton/m4326svd_example.pdf