1.1 AND 1.2 - PROPOSITIONAL LOGIC (WITH APPLICATIONS)

Definition: A ***proposition*** is a declarative sentence (that is, a sentence that declares a fact) that is either true or false, but not both.

Example 1: Which of the following are propositions?

(***a***) $5 + 7 \leq 19$
(***b***) How many eggs are there in a dozen?
(***c***) The line "Play it again, Sam" occurs in the movie "Casablanca."
(***d***) $x + 3 = 7$
(***e***) If it is raining outside, then the lawn is wet.
(***f***) If I am elected, then I will lower taxes on the middle class.
(***g***) This sentence is false.
(***h***) Every even integer greater than 2 is the sum of two prime numbers.

Example 2: There are three false statements in this example. Can you identify them?

(***a***) $2 + 2 = 4$
(***b***) $3 \cdot 6 = 17$
(***c***) $8 / 4 = 2$
(***d***) $13 – 6 = 5$
(***e***) $5 + 4 = 9$

The ***truth value*** of a proposition is **T** if the proposition is true, and **F** if the proposition is false.

COMPOUND PROPOSITIONS: Let p and q be propositions.

Name	***Negation***	***Conjunction***	***Disjunction* (*inclusive or*)**	***Exclusive or***
Symbol	$\neg p$	$p \wedge q$	$p \vee q$	$p \oplus q$
Pronounced	"not p"	"p and q"	"p or q, or both"	"p or q, but not both"
Is true when…	p is false	both p and q are true	at least one of p and q is true	one of p and q is true and the other is false
Is false when…	p is true	at least one of p and q is false	both p and q are false	both p and q are true or both p and q are false

A ***truth table*** displays the relationships between the truth values of propositions.

A statement of the form "If …, then …" is called an ***implication***.

Example 3: Suppose you make the following statement: "If I go home this weekend, then I will take my parents out to dinner."

Let p and q be propositions.

Name	***Implication***
Symbol	$p \rightarrow q$
Pronounced	"p implies q"
Is true when…	p is false *or* both p and q are true
Is false when…	p is true and q is false

The implication $p \rightarrow q$ can also be expressed as any of the following:

"if p, then q"
"if p, q"
"p implies q"
"p only if q"
"p is sufficient for q"
"a necessary condition for p is q"

"q if p"
"q when p"
"q whenever p"
"q follows from p"
"q is necessary for p"
"a sufficient condition for q is p"

Definition: Given the implication $p \rightarrow q$, then

- $q \rightarrow p$ is called the ***converse*** of $p \rightarrow q$,
- $\neg q \rightarrow \neg p$ is called the ***contrapositive*** of $p \rightarrow q$, and
- $\neg p \rightarrow \neg q$ is called the ***inverse*** of $p \rightarrow q$.

Example 4: Given the proposition

"If it is raining, then the lawn is wet."

What is the truth value of the proposition, its converse, its contrapositive and its inverse?

Example 5: Construct the truth table for the compound proposition $p \rightarrow \neg(p \wedge q)$.

Example 6: Construct the truth table for

$$[(p \vee q) \wedge r] \rightarrow (p \wedge \neg q).$$

Let p and q be propositions.

Name	***Implication***	***Biconditional***
Symbol	$p \rightarrow q$	$p \leftrightarrow q$
Pronounced	"p implies q"	"p if and only if q"
Is true when…	p is false *or* both p and q are true	both p and q are true or both p and q are false
Is false when…	p is true and q is false	one of p and q is true and the other is false

The biconditional $p \leftrightarrow q$ can also be expressed as any of the following:

"p if and only if q"
"p iff q"
"p is necessary and sufficient for q"

PRECEDENCE OF LOGICAL OPERATORS

Question: What does $\neg p \rightarrow q$ mean?

Is it $(\neg p) \rightarrow q$ or $\neg(p \rightarrow q)$?

These have different truth values, so the answer is important.

TRANSLATING ENGLISH SENTENCES

Example 7: Consider the following propositions:

p: You heard the "Flying Pigs" rock concert.
q: You heard the "Y2K" rock concert.
r: You have sore eardrums.

Translate each of the following English sentences into compound propositions.

(*a*) You heard the "Flying Pigs" rock concert, and you have sore eardrums.

(*b*) You have sore eardrums only if you heard the "Flying Pigs" rock concert but not the "Y2K" rock concert.

LOGIC AND BIT OPERATORS

A **bit** is a *b*inary dig*it*, *i.e.* 0 or 1. To do logical analysis using bits, let T (true) be represented by a 1 and F (false) represented by a 0.

Logical Operations	$\wedge$	$\vee$	$\oplus$
Bit Operations	bitwise AND	bitwise OR	bitwise XOR

A **bit string** is a sequence of zero or more bits. The **length** of this string is the number of bits in the string.

Example 8: Apply the bit operations to the bit strings 1100 and 0101.

Example 9: In 1978, Raymond Smullyan authored the book *What is the Name of This Book?* in which he wrote about an island that had only two types of inhabitants: knights and knaves. Knights only said statements that were true and knaves only said statements that were false. An explorer encounters two inhabitants *A* and *B*.

(*a*) *A* says "I am a knave and *B* is a knight." What type of inhabitants are *A* and *B*?

(*b*) If instead, suppose *A* says "Either I am a knave or *B* is a knight." Now can you say what *A* and *B* are?

SECTION 1.1 AND 1.2 – PROPOSITIONAL LOGIC (WITH APPLICATIONS)

PROPOSITIONS

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(***f***) If I am elected, then I will lower taxes on the middle class.
(***g***) This sentence is false.
(***h***) Every even integer greater than 2 is the sum of two prime numbers.
(A prime number is a positive integer whose only divisors are 1 and itself – 1 is not considered a prime number.)

Example 2: There are three false statements in this example. Can you identify them?

(***a***) $2 + 2 = 4$
(***b***) $3 \cdot 6 = 17$
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The ***truth value*** of a proposition is **T** if the proposition is true, and **F** if the proposition is false.

Let p and q be propositions.

Name	Symbol	Pronounced	Is true when…	Is false when…
Negation	$\neg p$	"not p"	p is false	p is true
Conjunction	$p \wedge q$	"p and q"	both p and q are true	at least one of p and q is false
Disjunction (inclusive or)	$p \vee q$	"p or q, or both"	at least one of p and q is true	both p and q are false
Exclusive or	$p \oplus q$	"p or q, but not both"	one of p and q is true and the other is false	both p and q are true or both p and q are false

A ***truth table*** displays the relationships between the truth values of propositions.

IMPLICATIONS

A statement of the form "If …, then …" is called an ***implication***.

Example 3:

Suppose you make the following statement: "If I go home this weekend, then I will take my parents out to dinner."

Name	Symbol	Pronounced	Is true when…	Is false when…
Implication	$p \rightarrow q$	"p implies q"	p is false *or* both p and q are true	p is true and q is false

The implication $p \rightarrow q$ can also be expressed as any of the following:

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Given the implication $p \rightarrow q$, then

- $q \rightarrow p$ is called the ***converse*** of $p \rightarrow q$,
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- $\neg p \rightarrow \neg q$ is called the ***inverse*** of $p \rightarrow q$.

Example 4: Given the proposition "If it is raining, then the lawn is wet." What is the truth value of the proposition, its converse, its contrapositive and its inverse?

Example 5: Construct the truth table for the compound proposition $p \to \neg(p \wedge q)$.

Example 6: Construct the truth table for $[(p \vee q) \wedge r] \to (p \wedge \neg q)$.

To summarize, let p and q be propositions.

Name	Symbol	Pronounced	Is true when…	Is false when…
Implication	$p \to q$	"p implies q"	p is false *or* both p and q are true	p is true and q is false
Biconditional	$p \leftrightarrow q$	"p if and only if q"	both p and q are true or both p and q are false	one of p and q is true and the other is false

The biconditional $p \leftrightarrow q$ can also be expressed as any of the following:

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LOGIC PUZZLES

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