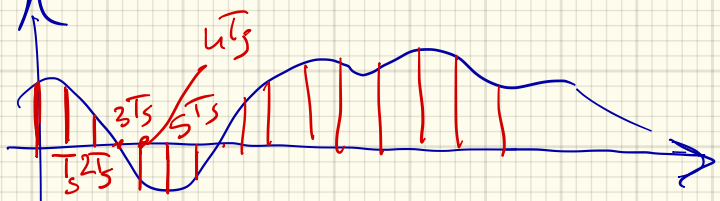
Signals & Systems

06.02.2018 Week 1

Q. What is a signal? 1D. $-\infty < t < +\infty$.

$x(t)$: CT : Continuous-time signal



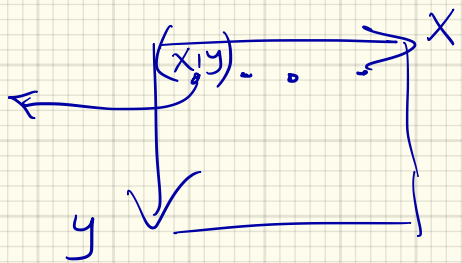
$x(t) : ( )$

$x[n] : [ ]$

DT signal $x(n)$

$n \in \mathbb{Z}$ integer.

2D: $I(x,y)$



$x(\theta) = \sin(\theta)$ ← radians ; unit of angle

$x(t) = \sin(\omega t)$; ω : rad/s

Angular frequency :

$x(t) = A \sin(\omega t + \phi)$ ← phase : radians

$\omega = 2\pi f$

$f = \frac{\omega}{2\pi}$

$\omega = 2\pi f = \frac{2\pi}{T}$

$\sin(\omega t + \phi) = 0$

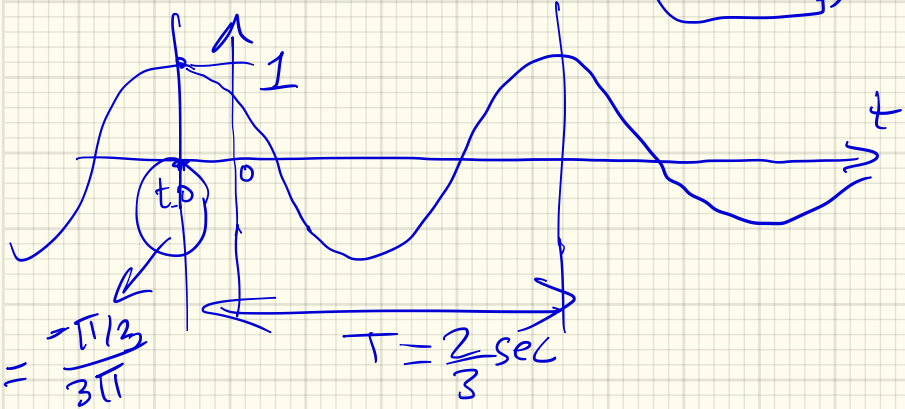
$t_0 = -\frac{\phi}{\omega}$ sec

ex: $x(t) = \cos\left(3\pi t + \frac{\pi}{3}\right)$; $\omega = 3\pi$ rad/s

$f = \frac{3\pi}{2\pi} = 1.5\,Hz$

$T = \frac{2}{3}\,sec$

freq. Unit $\boxed{Hz = \frac{1}{sec}}$



$t_o = \frac{-\pi/3}{3\pi}$

$t_o = -\frac{1}{9}\,sec$

Periodic Signal (Def) If $x(t) = x(t+T)$, $\forall t$

then $x(t)$ is a periodic signal, & T is a period of it.

ex: $x(t) = 3\sin(4\pi t + \pi/8)$

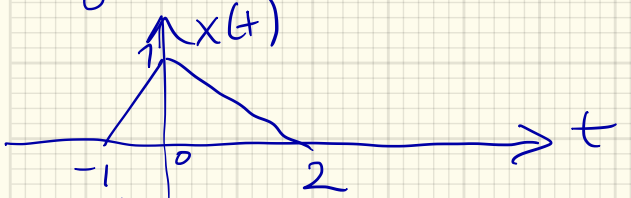
$= 3\cos\left(4\pi t + \frac{\pi}{8} - \frac{\pi}{2}\right)$

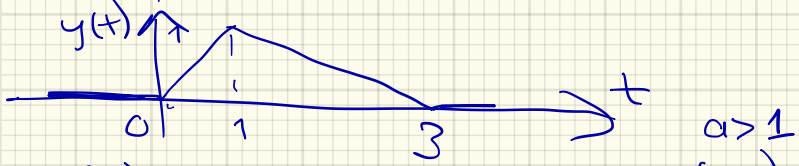
↳ phase shift btw sine & cosine

Shifting & Scaling Signals :

$y(t) = x(t-1)$ $\longrightarrow$ $x(t-t_0)$ $t_0 > 0$ → shift to the right by t_0

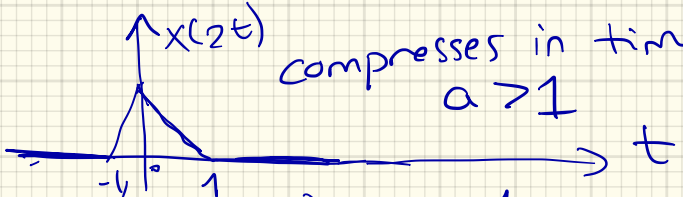
$x(t+t_0)$ (left)

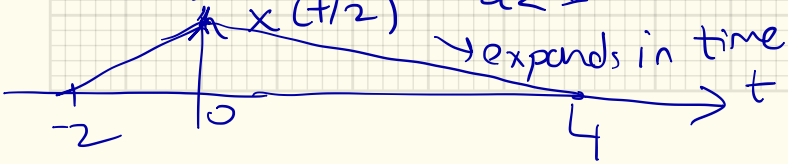




$a > 1$

$y(t) = x(2t)$; $y(t) = x(at)$





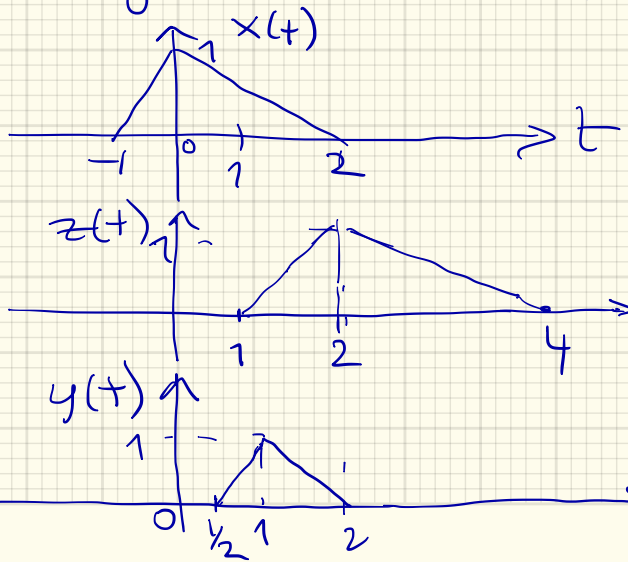
ex: Both time shifting & scale: First Shift then Scale ★

$y(t) = x(2t-2)$

$z(t) = x(t-2)$: first shift then scale

$y(t) = z(2t)$

$y(t) = x(2t-2)$

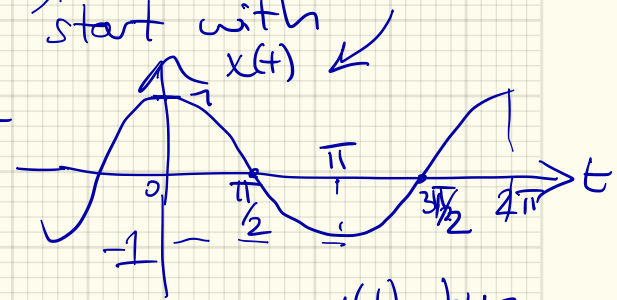


exercise:

$y(t) = \cos(2\pi t + \frac{\pi}{4})$

$x(t) = \cos(t)$

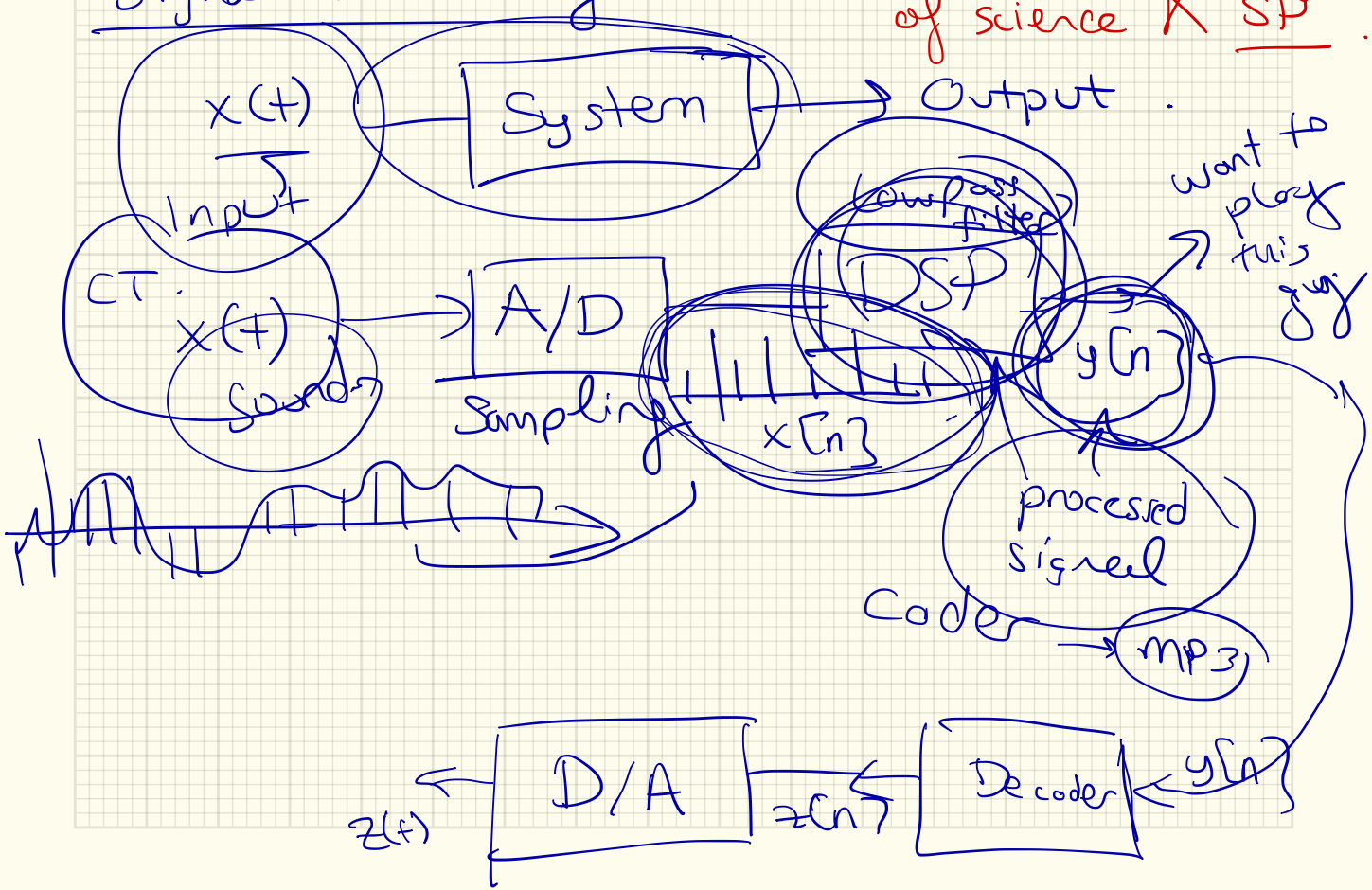
start with



obtain y(t) by first shift then scale

Signal Processing :

Mathematics is the language of science X SP .



$x(t) = A\cos(2\pi f_0 t + \phi)$ → Know 3 parameters of the wave.

$\omega_0 = 2\pi f_0$

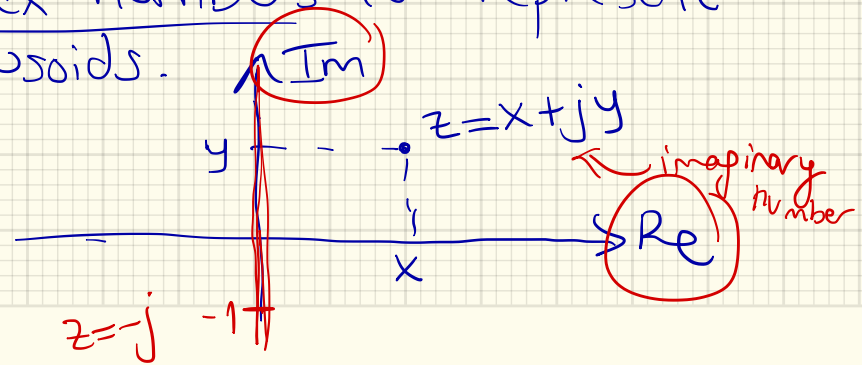
Know how to plot a sinusoid.

Given a sinusoid plot → extract the parameters of the sine.

Sinusoids → $x(t) = A e^{jwt}$

$e^{jwt} = \exp(jwt)$

Use Complex numbers to represent sinusoids.



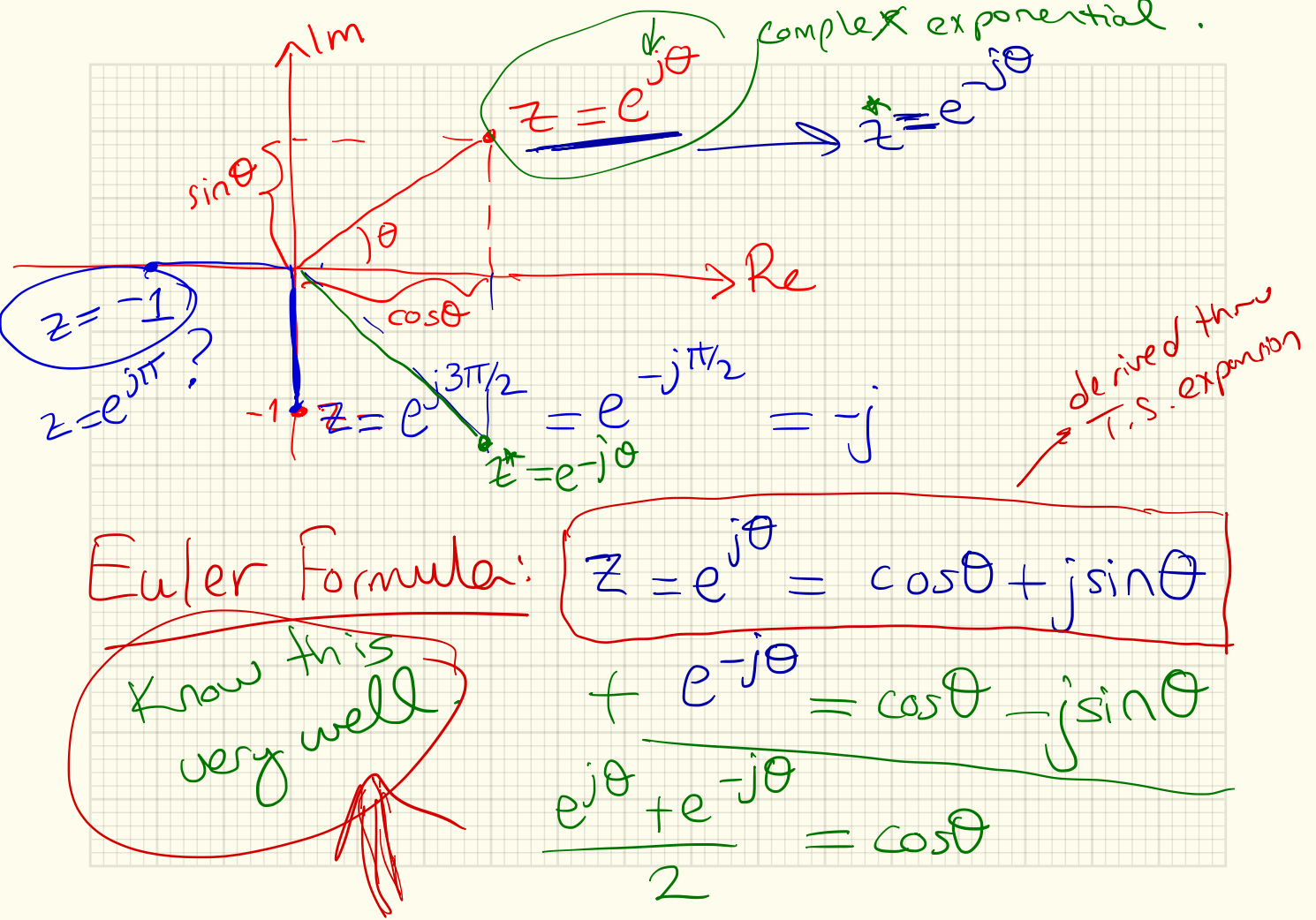
$z^2 = -1 \Rightarrow z = \mp j$

$j^2 = -1$

$j^3 = -j$

$j^4 = 1$

$j^5 = j$


complex exponential.
Im
Re
$z = e^{j\theta}$
$\sin\theta$
θ
$\cos\theta$
$z^* = e^{-j\theta}$
$z = -1$
$z = e^{j\pi}$?
-1
$z = e^{j3\pi/2} = e^{-j\pi/2} = -j$
$z^* = e^{-j\theta}$
derived thru T.S. expansion
Euler Formula:
$z = e^{j\theta} = \cos\theta + j\sin\theta$
$+ \; e^{-j\theta} = \cos\theta - j\sin\theta$
$\frac{e^{j\theta} + e^{-j\theta}}{2} = \cos\theta$
know this very well

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2} \quad ; \sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

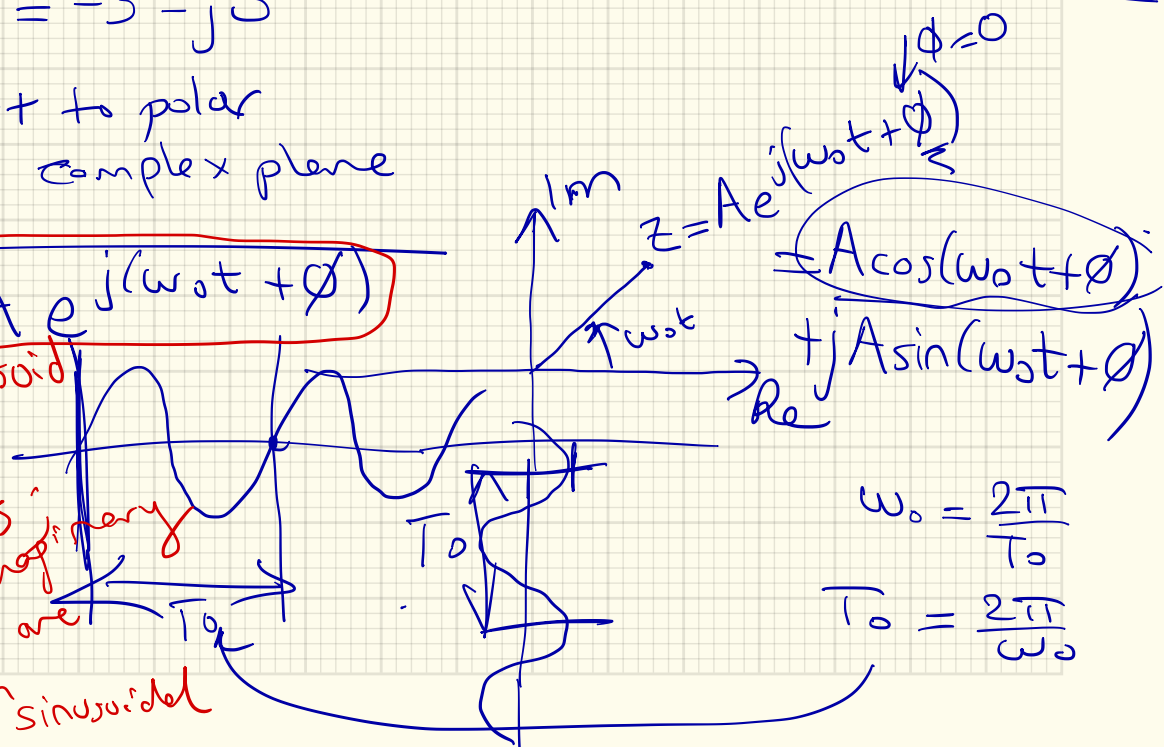
ex: $z = -3 - j3$

convert to polar

plot in complex plane

$$x(t) = A e^{j(\omega_0 t + \phi)}$$

complex sinusoid has 2 components: a real & imaginary which are both sinusoidal



$$\omega_0 = \frac{2\pi}{T_0}$$

$$T_0 = \frac{2\pi}{\omega_0}$$