MATEMATİK 102 ARA SINAV 10 TEMMUZ 2012

SORU 2 Aşağıdaki boşluklar öğrenci tarafından doldurulacaktır. (Puan Hariç)

Soyadı:	Adı:	Grup No:	Sıra No:	Puan
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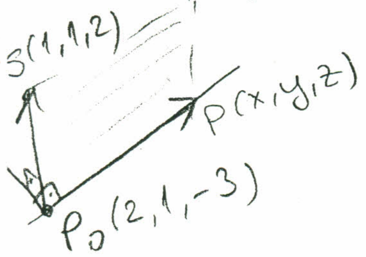
Lütfen bu soruyu bu kağıdın ön yüzünü ve gerekirse arka yüzünü kullanarak cevaplayınız.

[13puan] a) $(\mathbf{1}, \mathbf{1}, \mathbf{2})$ noktasından geçen ve $\mathbf{x = 5t + 2}$, $\mathbf{y = 1 + t}$, $\mathbf{z = 4t - 3}$ doğrusunu içeren düzlemin denklemini yazınız.

[12puan] b) $\vec{\mathbf{r}}(\mathbf{t}) = \mathbf{t}\cos\mathbf{t}\,\vec{\mathbf{i}} + \mathbf{t}\sin\mathbf{t}\,\vec{\mathbf{j}} + \dfrac{\mathbf{2\sqrt{2}}}{\mathbf{3}}\mathbf{t}^{3/2}\,\vec{\mathbf{k}}$ vektörel denklemi ile verilen eğrinin $\mathbf{0 \le t \le \pi}$ arasındaki uzunluğunu hesaplayın.

Çözüm:

a) $S(1,1,2)$, $x = 2+5t$, $y = 1+t$, $z = -3+4t$, $\vec{v}(5,1,4)$



$\overrightarrow{P_0P} = \vec{v} = 5\vec{i} + \vec{j} + 4\vec{k}$

$\overrightarrow{P_0S} = -\vec{i} + 5\vec{k}$

$$\vec{N} = \vec{v} \times \overrightarrow{P_0S} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 5 & 1 & 4 \\ -1 & 0 & 5 \end{vmatrix} = 5\vec{i} - 29\vec{j} + \vec{k}$$

9 Normali bulana.

$\vec{N}$ vektörüne dik, $(1,1,2)$'den geçen düzlemin denklemi:

$A(x-x_0) + B(y-y_0) + C(z-z_0) = 0$

$5(x-1) - 29(y-1) + (z-2) = 0$ 4 denklemi yazmak

$\boxed{5x - 29y + z = -22}$ //

b) $\vec{r}(t) = t\cos t\,\vec{i} + t\sin t\,\vec{j} + \dfrac{2\sqrt{2}}{3} t^{3/2}\,\vec{k}$, $0 \le t \le \pi$

$L = \displaystyle\int_{t_0}^{t_1} \left| \frac{d\vec{r}}{dt} \right| dt$ 2, $\dfrac{d\vec{r}}{dt} = (\cos t - t\sin t)\vec{i} + (\sin t + t\cos t)\vec{j} + \sqrt{2}\, t^{1/2}\,\vec{k}$ 2

$\left|\dfrac{d\vec{r}}{dt}\right| = \sqrt{\cos^2 t + t^2\sin^2 t - 2t\cancel{\cos t\sin t} + \sin^2 t + t^2\cos^2 t + 2t\cancel{\sin t\cos t} + 2t}$

$= \sqrt{1 + t^2 + 2t} = \sqrt{(t+1)^2} = |t+1| = t+1$ 3

$0 \le t \le \pi$

$L = \displaystyle\int_0^{\pi} (t+1)\,dt = \frac{t^2}{2} + t \Big|_0^{\pi} \Rightarrow \boxed{L = \frac{\pi^2}{2} + \pi}$ //

3 2

QUESTION 2 The blanks below will be filled by students. (Except the score)

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Signature:	Electronic Post(e-mail) address:	Student Number:		

[10 pts] a) Let $\mathbf{f(x,y)}$ be a function given by

$$f(x,y)=\begin{cases}\dfrac{\mathbf{x-2}}{\sqrt{\mathbf{(x-2)^2+y^2}}} & ; \quad \mathbf{(x,y)\neq(2,0)} \\ \mathbf{1} & ; \quad \mathbf{(x,y)=(2,0)}\end{cases}.$$

Is $\mathbf{f(x,y)}$ continuous at the point $\mathbf{(2,0)}$? Give reasons for your answer.

[15 pts] b) Find the directional derivative of the function $\mathbf{f(x,y,z)=e^x\cos(yz)}$ at the point $\mathbf{P_0(2,1,0)}$, in the direction of the unit tangent vector of $\overrightarrow{\mathbf{r}}(\mathbf{t})=\mathbf{2t^2}\overrightarrow{\mathbf{i}}+\mathbf{t^3}\overrightarrow{\mathbf{j}}+(\mathbf{2-2t})\overrightarrow{\mathbf{k}}$ at $\mathbf{t=1}$.

let $x-2=my$ (3)

$$\lim_{(x,y)\to(2,0)} f(x,y)=\lim_{\substack{y\to 0\\(x-2=my)}}\frac{my}{\sqrt{m^2y^2+y^2}}=\underbrace{\lim_{y\to 0}\frac{my}{\sqrt{1+m^2}\,|y|}}_{(2)}$$

$$=\lim_{y\to 0}\frac{\mp my}{\sqrt{1+m^2}\,y}=\frac{\mp m}{\sqrt{1+m^2}} \quad (2)$$

The limit varies for all m. Thus, the limit does not exist. (1)

f(x,y) is not cont. at the point (2,0) (2)

or $x-2=r\cos\theta$ (2) $\sqrt{(x-2)^2+y^2}=r$ $\quad \lim_{r\to 0}\frac{r\cos\theta}{r}=\cos\theta$ (1) (2)

$y=r\sin\theta$ (2)

$r\to 0$

$$\vec\nabla f=f_x\vec i+f_y\vec j+f_z\vec k$$

b) $\left(\frac{df}{ds}\right)\Big|_{P_0,\vec u}=\vec\nabla f\Big|_{P_0}\cdot\vec u$ (1)

$$=\underbrace{e^x\cos(yz)\vec i}_{(1)}-\underbrace{ze^x\sin(yz)\vec j}_{(1)}-\underbrace{ye^x\sin(yz)\vec k}_{(1)}$$

$\vec\nabla f\Big|_{P_0}=e^2\vec i$ (2)

(1) $\vec u=\dfrac{d\vec r/dt}{|d\vec r/dt|}$, $\dfrac{d\vec r}{dt}=4t\vec i+3t^2\vec j-2\vec k$ (2) $\dfrac{d\vec r}{dt}\Big|_{t=1}=4\vec i+3\vec j-2\vec k$ (1)

$\left|\dfrac{d\vec r}{dt}\right|\Big|_{t=1}=\sqrt{29}$ (1) $\quad \vec u=\dfrac{4}{\sqrt{29}}\vec i+\dfrac{3}{\sqrt{29}}\vec j-\dfrac{2}{\sqrt{29}}\vec k$ (2)

$$\frac{df}{ds}\Big|_{P_0,\vec u}=e^2\vec i\cdot\left(\frac{4}{\sqrt{29}}\vec i+\frac{3}{\sqrt{29}}\vec j-\frac{2}{\sqrt{29}}\vec k\right)=\frac{4e^2}{\sqrt{29}} \quad (2)$$

SORU 3 Aşağıdaki boşluklar öğrenci tarafından doldurulacaktır. (Puan Hariç)

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Lütfen bu soruyu bu kağıdın ön yüzünü ve gerekirse arka yüzünü kullanarak cevaplayınız.

[15 puan] a) $\mathbf{x^3 + 3x^2y^2 + y^3 + 4xy - z^2 = 0}$ ve $\mathbf{x^2 + y^2 + z^2 = 11}$ yüzeylerinin arakesit eğrisine $(\mathbf{1, 1, 3})$ noktasında teğet olan doğrunun parametrik denklemlerini yazızın.

[10 puan] b) $\mathbf{f(x, y) = x^3 + xy^2 - x}$ fonksiyonunun yerel maksimum, yerel minumum ve semer(eyer) noktalarını bulunuz.

ÇÖZÜM

a) $f(x,y,z) = x^3 + 3x^2y^2 + y^3 + 4xy - z^2 = 0$; $g(x,y,z) = x^2+y^2+z^2 = 11$ } Düzey yüzeyleridir.

$\vec{\nabla f} = (3x^2 + 6xy^2 + 4y)\vec{i} + (6x^2y + 3y^2 + 4x)\vec{j} - 2z\vec{k}$ 2

$\vec{\nabla f}\Big|_{(1,1,3)} = 13\vec{i} + 13\vec{j} - 6\vec{k}$ 2

$\vec{\nabla g} = 2x\vec{i} + 2y\vec{j} + 2z\vec{k}$ 2 $\Rightarrow \vec{\nabla g}\Big|_{(1,1,3)} = 2\vec{i} + 2\vec{j} + 6\vec{k}$ 2

$\vec{v} = \vec{\nabla f} \times \vec{\nabla g} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 13 & 13 & -6 \\ 2 & 2 & 6 \end{vmatrix} = 90\vec{i} - 90\vec{j}$ 4

$x = x_0 + at$ 1, $y = y_0 + bt$ 1, $z = z_0 + ct$ 1 $\Rightarrow \boxed{x = 1 + 90t, \ y = 1 - 90t, \ z = 3}$

b) $f(x,y) = x^3 + xy^2 - x$

$f_x = 3x^2 + y^2 - 1 = 0$

$f_y = 2xy = 0$ → $x = 0 \Rightarrow y^2 - 1 = 0 \Rightarrow y = \mp 1 \Rightarrow A(0,1)$ 1, $B(0,-1)$ 1

→ $y = 0 \Rightarrow 3x^2 - 1 = 0 \Rightarrow x = \mp\frac{1}{\sqrt{3}} \Rightarrow C(\frac{1}{\sqrt{3}}, 0)$ 1, $D(-\frac{1}{\sqrt{3}}, 0)$ 1

$f_{xx} = 6x$, $f_{yy} = 2x$, $f_{xy} = 2y$ 1

$\Delta = f_{xx} \cdot f_{yy} - (f_{xy})^2$ 1

$\Delta\Big|_A = -4 < 0 \Rightarrow (0,1)$ eyer noktası 1

$\Delta\Big|_B = -4 < 0 \Rightarrow (0,-1)$ eyer noktası 1

$\Delta\Big|_C = 4 > 0$ ve $f_{xx}\Big|_C = 2\sqrt{3} > 0 \Rightarrow (\frac{1}{\sqrt{3}}, 0)$ yerel min. noktası 1

$\Delta\Big|_D = 4 > 0$ ve $f_{xx}\Big|_D = -2\sqrt{3} < 0 \Rightarrow (-\frac{1}{\sqrt{3}}, 0)$ yerel mak. noktası 1

QUESTION 4 The blanks below will be filled by students. (Except the score)

Surname:	Name:	Group Number:	List Number:	**Score**
Signature:	Electronic Post(e-mail) address:	Student Number:		

For the solution of this question please use only the front face and if necessary the back face of this page.

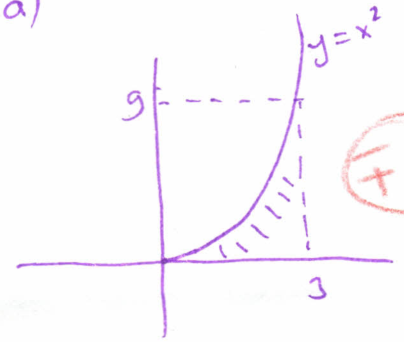
[10 pts] a) Evaluate the integral $\int_0^9 \int_{\sqrt{y}}^3 \cos(\mathbf{x^3})\, \mathbf{dx\, dy}$.

[15 pts] b) Let **D** be the region bounded below by the cone $\mathbf{z} = \sqrt{\mathbf{x^2+y^2}}$ and above by the paraboloid $\mathbf{z = 2 - x^2 - y^2}$.

i) Write the triple integral that calculates the volume of the region **D** in Cartesian coordinates in the order **dzdxdy** . (Do not evaluate the integral)

ii) Write the triple integral that calculates the volume of the region **D** in cylindirical coordinates in the order **dzdrdθ**. (Do not evaluate the integral)

a)



+2

$$\int_0^3 \int_0^{x^2} \cos x^3 \, dy\, dx = \int_0^3 x^2 \cos x^3 \, dx$$

5 1

$u = x^3$

$du = 3x^2 dx$

2

$$= \int_0^{27} \frac{\cos u}{3}\, du$$

1

$$= \frac{\sin 27}{3}$$

1

b)

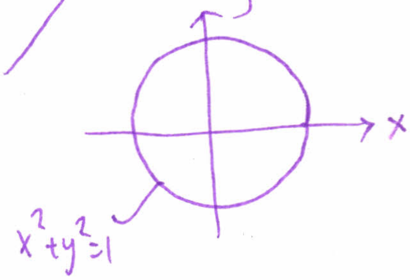
$x^2+y^2 = z^2 \Rightarrow z = 2 - z^2 \Rightarrow z^2 + z - 2 = 0$

$\Rightarrow (z+2)(z-1) = 0$

$\Rightarrow z = 1$

5

$x^2+y^2=1$ is the curve of intersection



(i) $$\int_{-1}^{1} \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \int_{\sqrt{x^2+y^2}}^{2-x^2-y^2} dz\, dx\, dy$$

5

(ii) $$\int_0^{2\pi} \int_0^1 \int_r^{2-r^2} r\, dr\, d\theta$$

5