

BLG456E

Robotics

Wheeled locomotion & kinematics

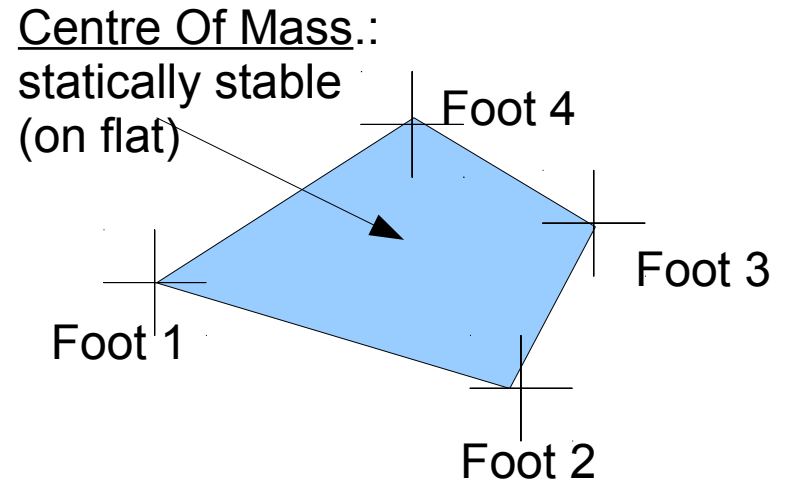
Lecture Contents:

- Locomotion design issues.
- Wheel geometry & kinematics.
- Wheel configurations.
- Car wheels.
- Differential drive.
- Odometry (robot).
- Degrees of freedom & holonomicity.

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Locomotion design issues

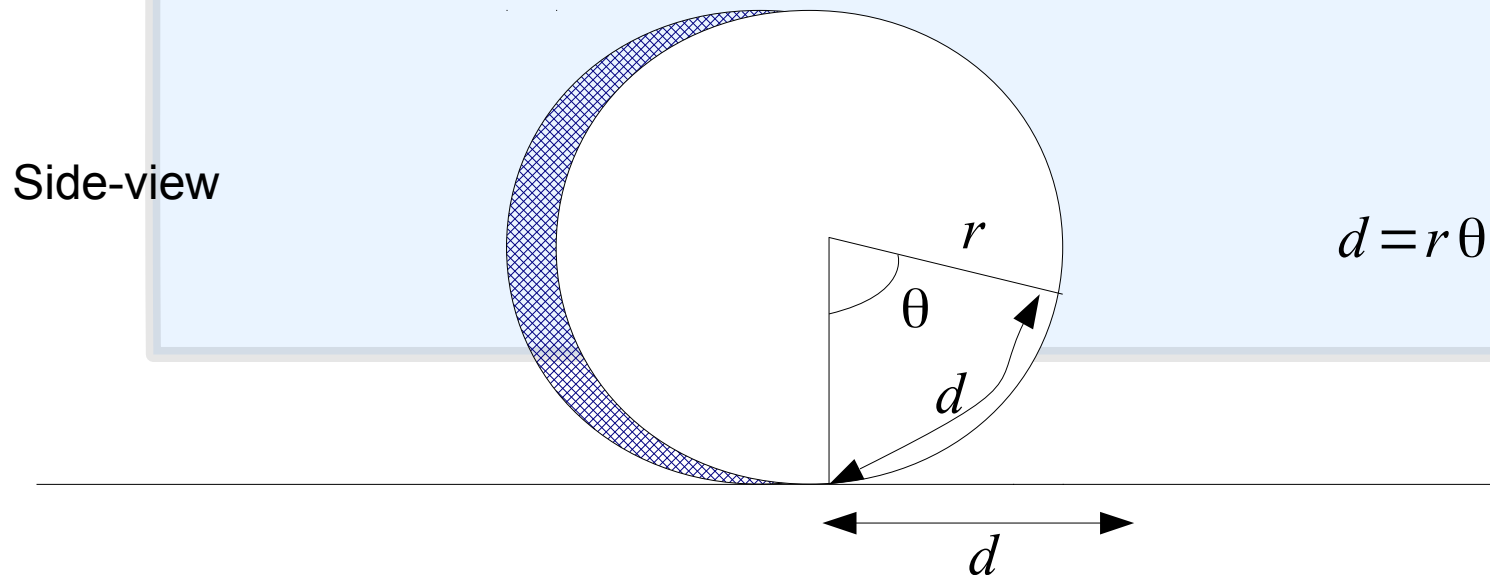
- Required speed, strength.
- Climbing? Flying? Swimming? Tunnelling?
- Terrain – smooth? Rough?
- Power density (never enough).
- Stability:
 - Contact points, centre of gravity (inside concave hull of contact points?).
 - Static stability: hold still, won't fall.
 - Dynamic stability: move right, won't fall.
 - Terrain inclination.
- Contact characteristics:
 - Contact point(s), friction, angle.



Standard wheel constraints

Rolling constraint:

Movement in direction of wheel must have accompanying wheel turning.

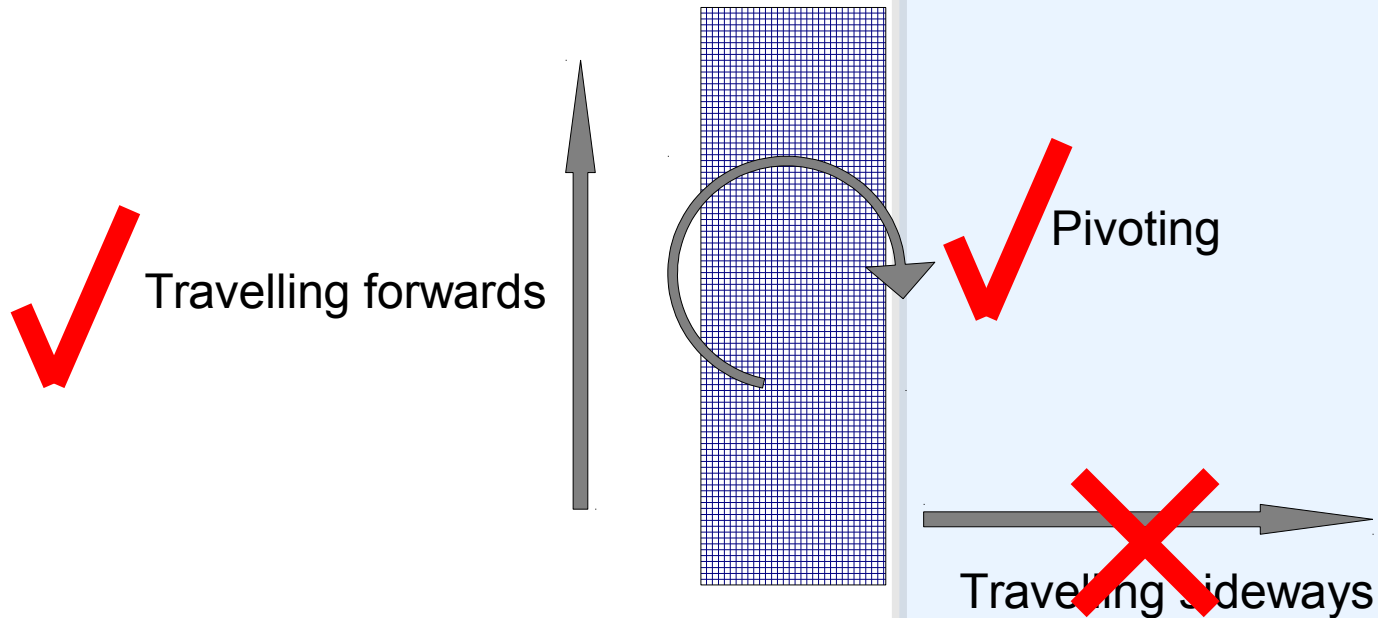


Exercise: (3 minutes) A wheel of radius 0.5m turns through half a full rotation (π radians) as it travels in a straight line. How far did it travel?

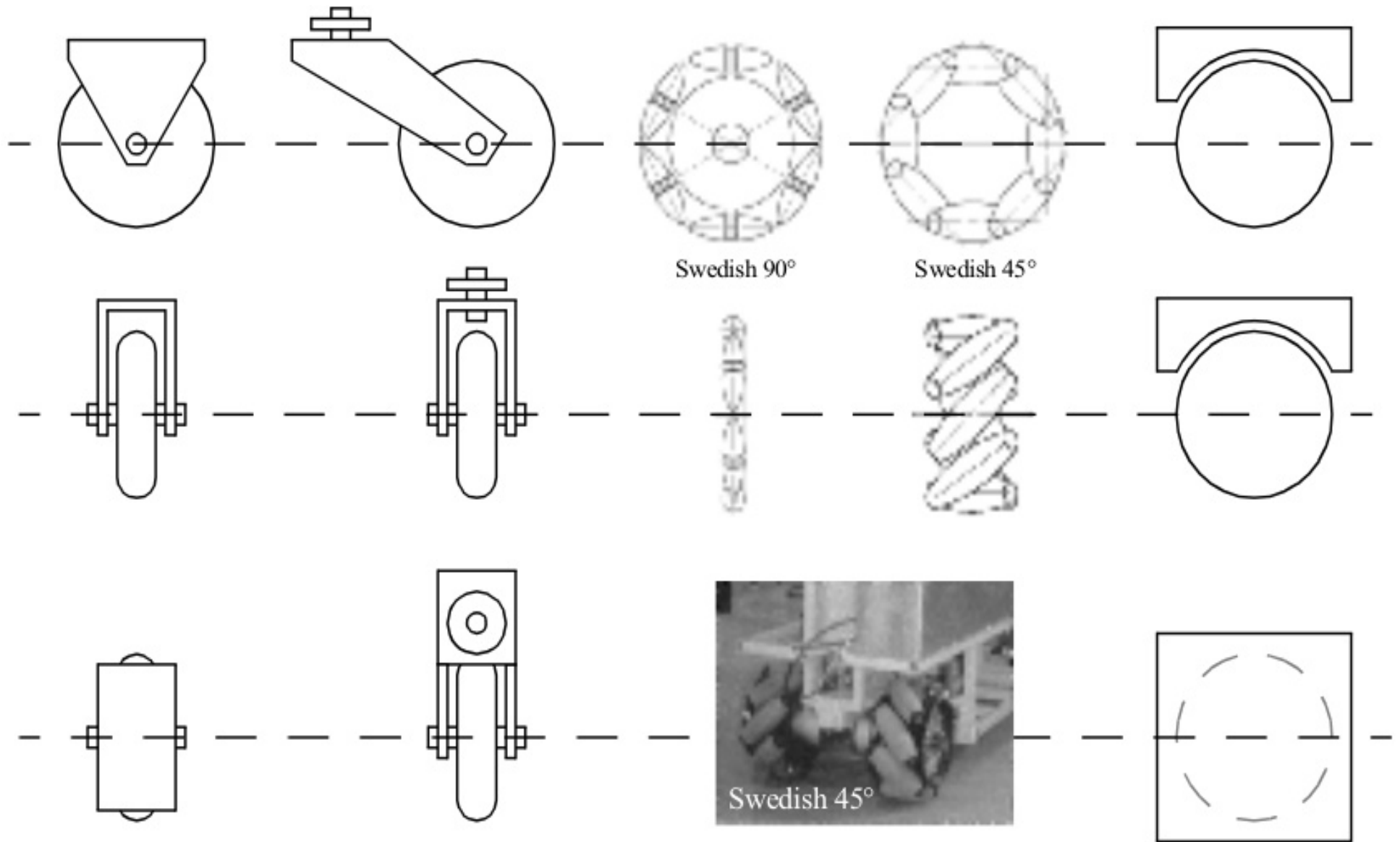
Standard wheel constraints

Sliding constraint:

Wheel can't go sideways.



Other kinds of wheel



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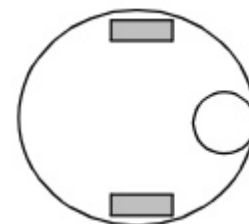
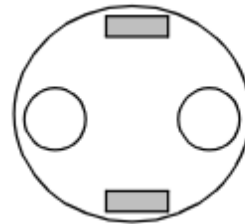
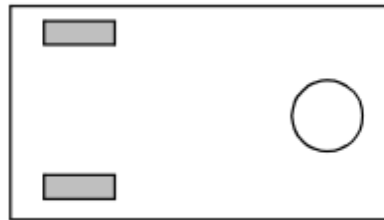
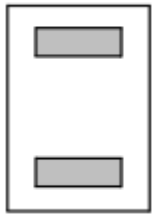
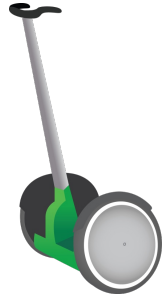
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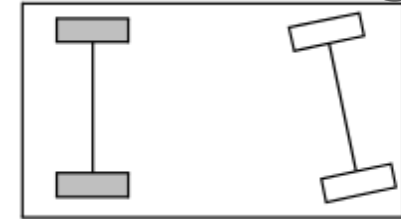
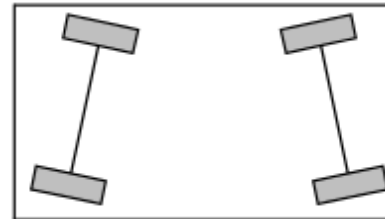
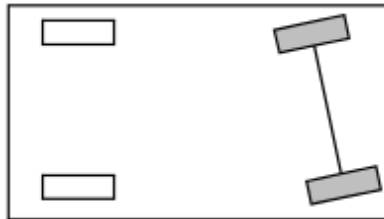
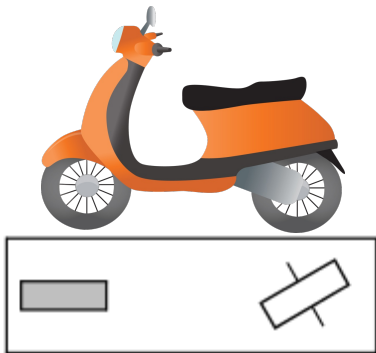
Robot wheel configurations



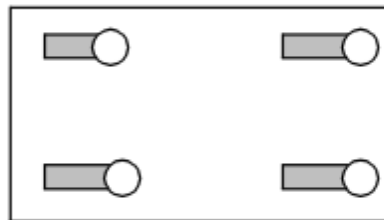
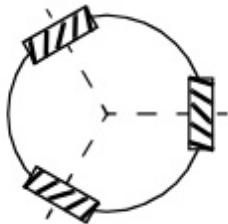
Differential drive



Steered wheels



With powered omnidirectional or castor wheels

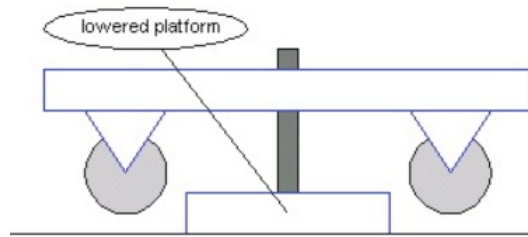


More kinds of wheeled locomotion

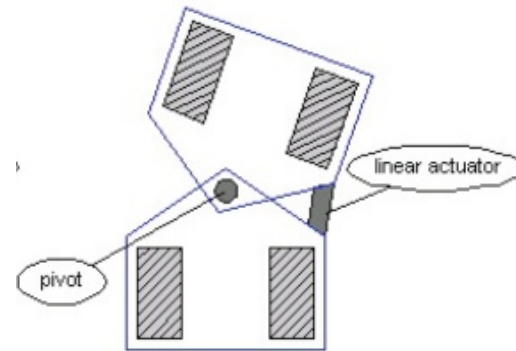
Skid-steer
(tanks).



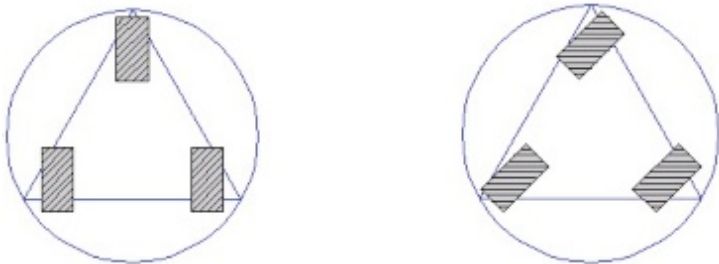
Pivot drive.



Articulated drive.



Synchronous drive.



Hybrid walkers.

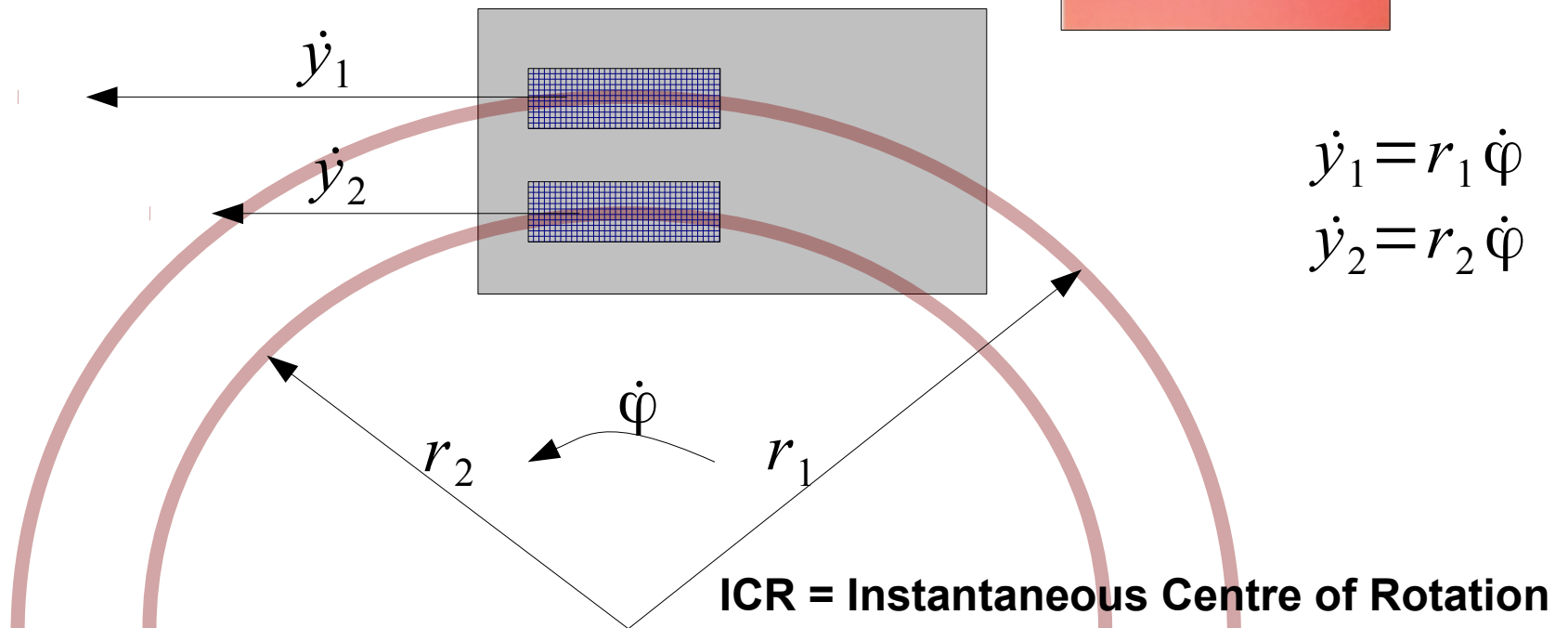


Cars are more complicated

1: Differentials

Problem: Powered wheels while going through a turn must either skid or turn at different rates.

Solution: A "differential".

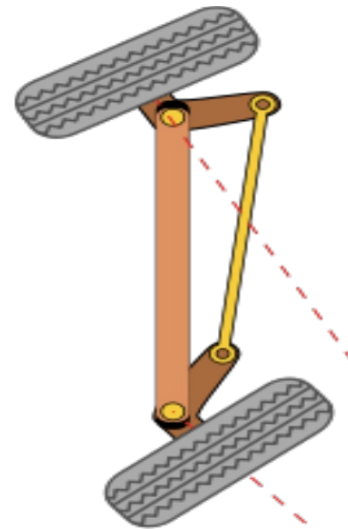
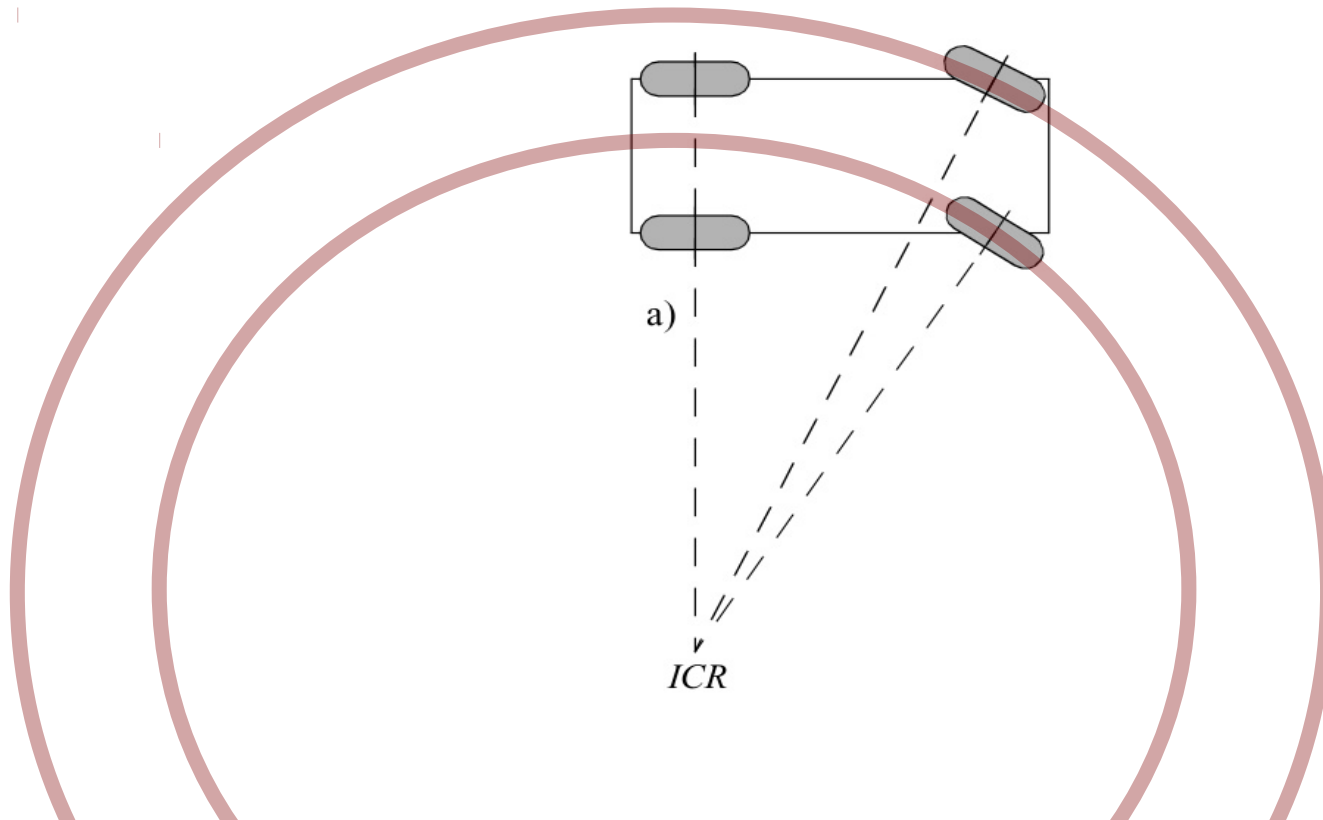


Cars are more complicated

2: Ackermann Steering

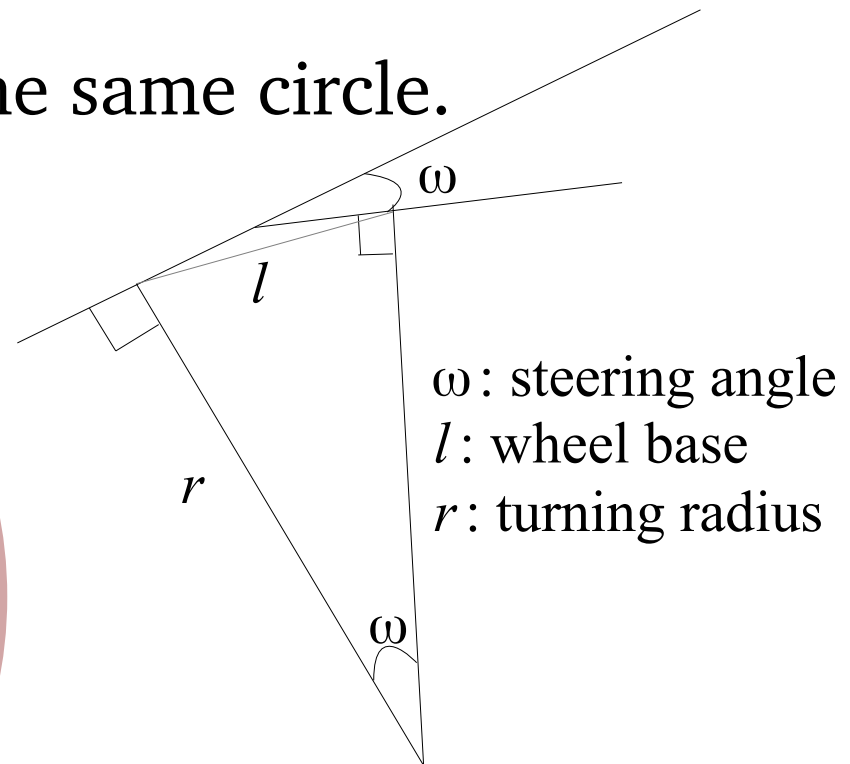
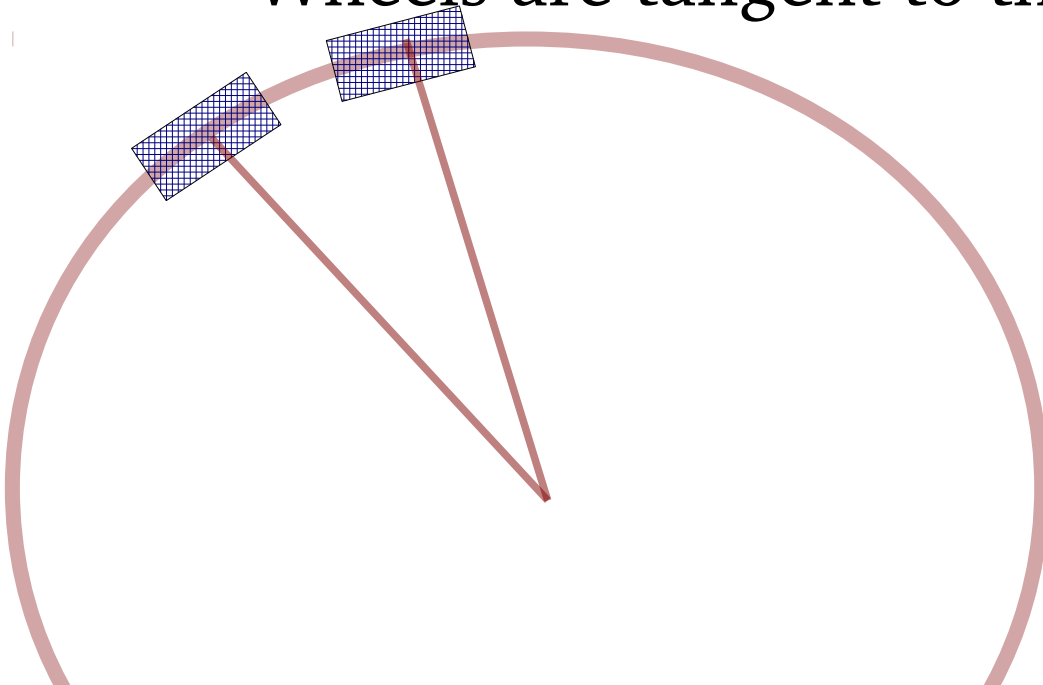
Problem: Parallel wheels during steering are on circles of different radii and so skid.

Solution: Ackermann steering.

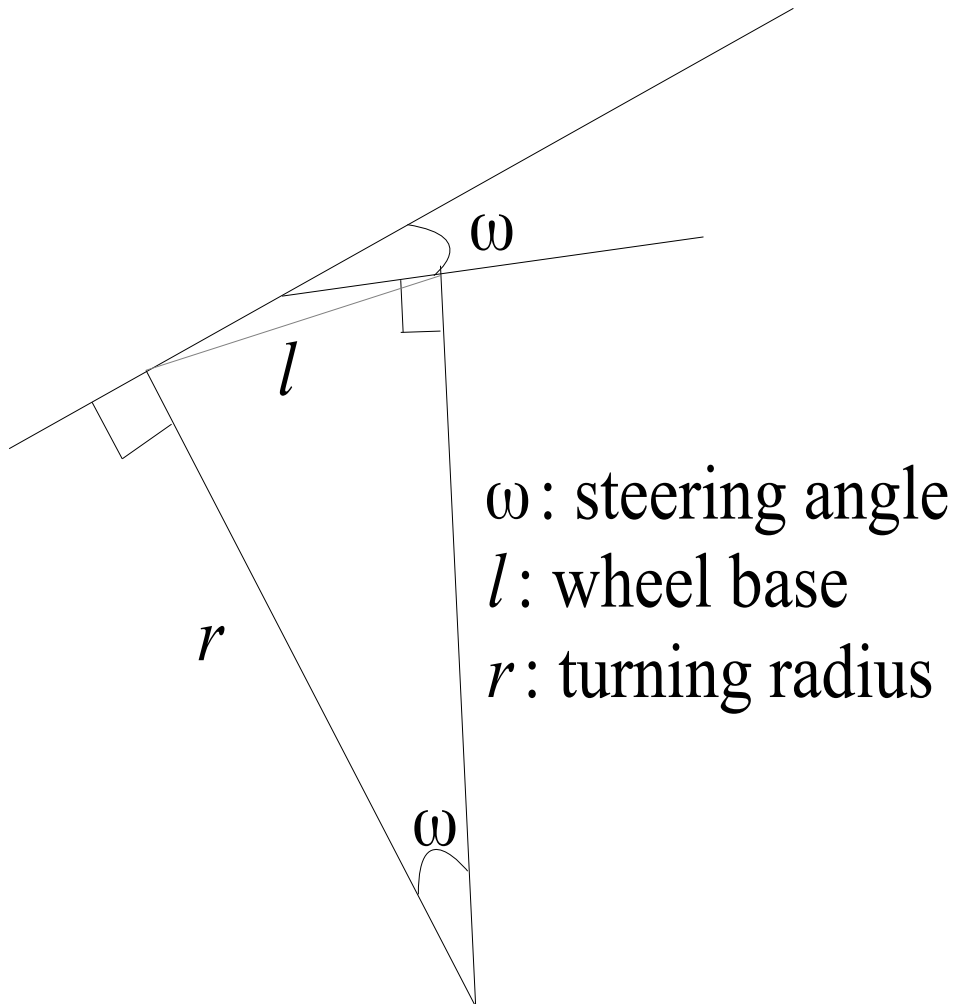


Common simplification with cars motion planning:

- Assumption 1:
 - 1 wheel in front, 1 wheel in back.
- Assumption 2:
 - Wheels are tangent to the same circle.



Car motion exercises



(1)

I set my steering angle ω to $\frac{\pi}{8}$.

My car has a wheel base of 0.25 m .

--- what is my turning radius r ?

(2)

With my steering angle fixed,

I move forward 0.5 m .

(car is moving on the arc of the circle)

--- What is the angle $\Delta\theta$ of the arc that the car is moving through?

(3)

My car's initial orientation is $\theta_i = \frac{\pi}{2}$.

--- What is my car's new orientation θ_f ?

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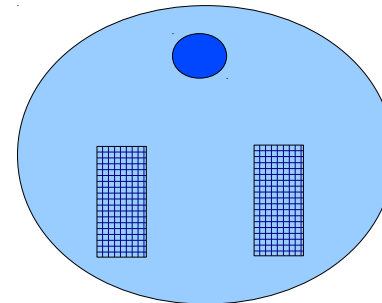
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Differential Drive

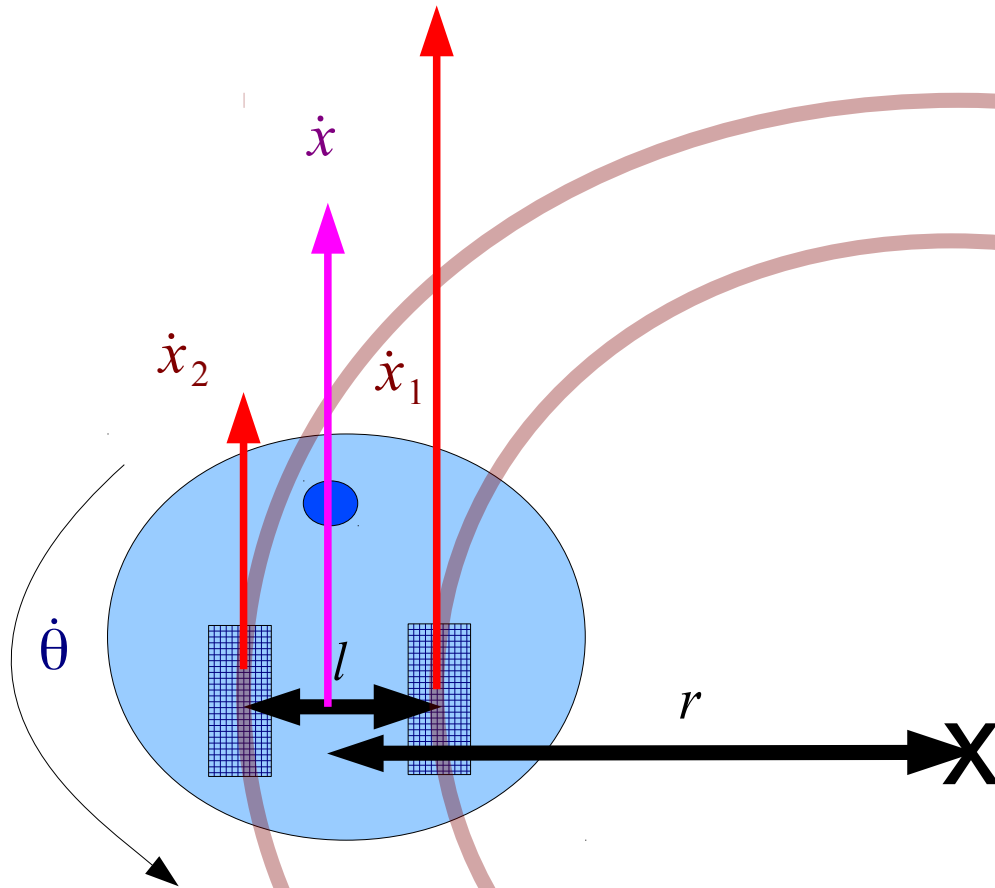


- 2 wheels on common axis.
- Wheels rotate different speeds – differentially.
- Usually 3rd passive wheel (caster/omnidirectional) for balance.
- Can:
 - Rotate in-place.
 - Move forward or back.
 - Move on curve.





Differential Drive is simpler



$$\dot{\theta}\left(r - \frac{l}{2}\right) = \dot{x}_1$$

$$\dot{\theta}\left(r + \frac{l}{2}\right) = \dot{x}_2$$

↓

$$\dot{\theta} = \frac{\dot{x}_1 - \dot{x}_2}{l}$$

$$r = \frac{l}{2} \frac{\dot{x}_1 + \dot{x}_2}{\dot{x}_2 - \dot{x}_1}$$

$$\dot{x} = \frac{\dot{x}_1 + \dot{x}_2}{2}$$

Odometry exercises (differential drive)



A large differential drive robot has wheels of 0.4m diameter (0.2m radius). The wheel encoder has 100 holes. The wheel base is 0.5m.

- (1) The encoder in the left wheel measures 250 pulses, and the one in the right measures 500 pulses. How far has each wheel travelled?
- (2) Assuming it took 0.5 seconds to travel this distance, what is the forward velocity of each wheel over this period?
- (3) Using the equations on the right, determine the angular velocity and forward velocity of the robot.
- (4) Draw the circle through which the robot is turning.

$$\dot{\theta}\left(r - \frac{l}{2}\right) = \dot{x}_1$$

$$\dot{\theta}\left(r + \frac{l}{2}\right) = \dot{x}_2$$



$$\dot{\theta} = \frac{\dot{x}_1 - \dot{x}_2}{l}$$

$$r = \frac{l}{2} \frac{\dot{x}_1 + \dot{x}_2}{\dot{x}_2 - \dot{x}_1}$$

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Configuration space vs phase space

Mobile robot

4-joint arm

Configuration:

Location of all parts.

$$\xi = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

$$\xi = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix}$$

Phase state:

Location & velocity of parts.

$$\Xi = \begin{bmatrix} x \\ y \\ \theta \\ \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}$$

$$\Xi = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{\theta}_4 \end{bmatrix}$$

Workspace state:

Location (sometimes pose) of effector.

$$\chi = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\chi = \begin{bmatrix} x \\ y \end{bmatrix}$$

Introduction to physical Degrees Of Freedom: poses and velocities

Dim	Quantity	DOF
2	Linear Displacement	2
2	Linear Velocity	2
2	Orientation	1
2	Rotational velocity	1
2	Pose	3
2	Rigid motion	3
2	Non-rigid pose/motion	?

Dim	Quantity	DOF
3	Linear Displacement	3
3	Linear Velocity	3
3	Orientation	3
3	Rotational velocity	3
3	Pose	6
3	Rigid motion	6
3	Non-rigid pose/motion	?

**Constraints can reduce the degrees of freedom
(e.g. 3D body constrained to lie in plane, limited DOF robot arm).**

Non-holonomicity and degrees of freedom

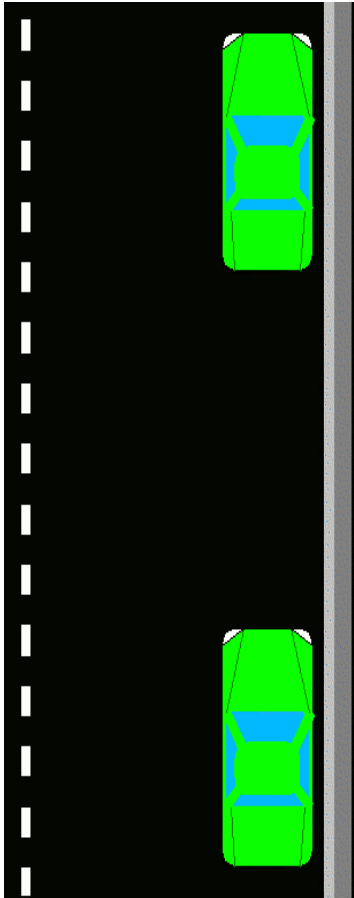
Degrees of Freedom (DOF):

Minimum number of parameters to describe body's configuration (e.g. x, y, θ).

Differentiable Degrees of Freedom (DOF):

Minimum number of parameters that can be instantaneously changed (e.g. x, θ).

Non-holonomicity and degrees of freedom



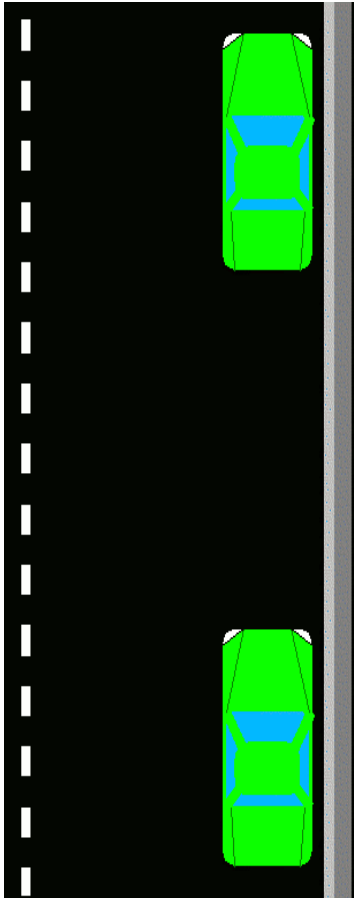
If Differentiable DOF (DDOF) < Total DOF:

- *non-holonomic robot*. e.g. Differential-drive robot, Car.

Else:

- *holonomic robot*. e.g. Swedish-wheel based robot.

Non-holonomicity and degrees of freedom



Non-holonomic constraint:

- Equality constraint.
- Applies to time-derivatives.
- Is non-integrable.

Holonomic constraint:

- Equality constraint.
- Applies to configurations.
- Derivative is integrable.

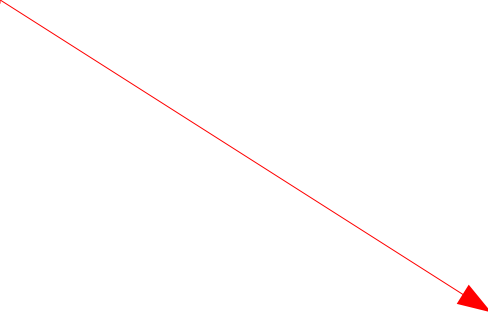
Manoeuvrability

Degree of manoeuvrability

=

DDOF  Steerability.

- Differential drive robot:
 - DDOF = 2.
 - Degree of steerability = 0.
- Basic car-type robot:
 - DDOF = 1.
 - Degree of steerability = 1.
- Omnidirectional (e.g. 3 Swedish wheels):
 - DDOF = 3.
 - Degree of steerability = 0.



Only additive if degrees of freedom are mutually independent.

$$\text{Manoeuvrability} = \text{DDOF} + \text{Steerability}$$

Robot	DOF	DDOF	Steerability	Manoeuvrability
Abstract car	3	1	1	2
Differential drive	3	2	0	2
Omnidirectional	3	3	0	3

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Differential-drive pose control with goal-centred coordinates

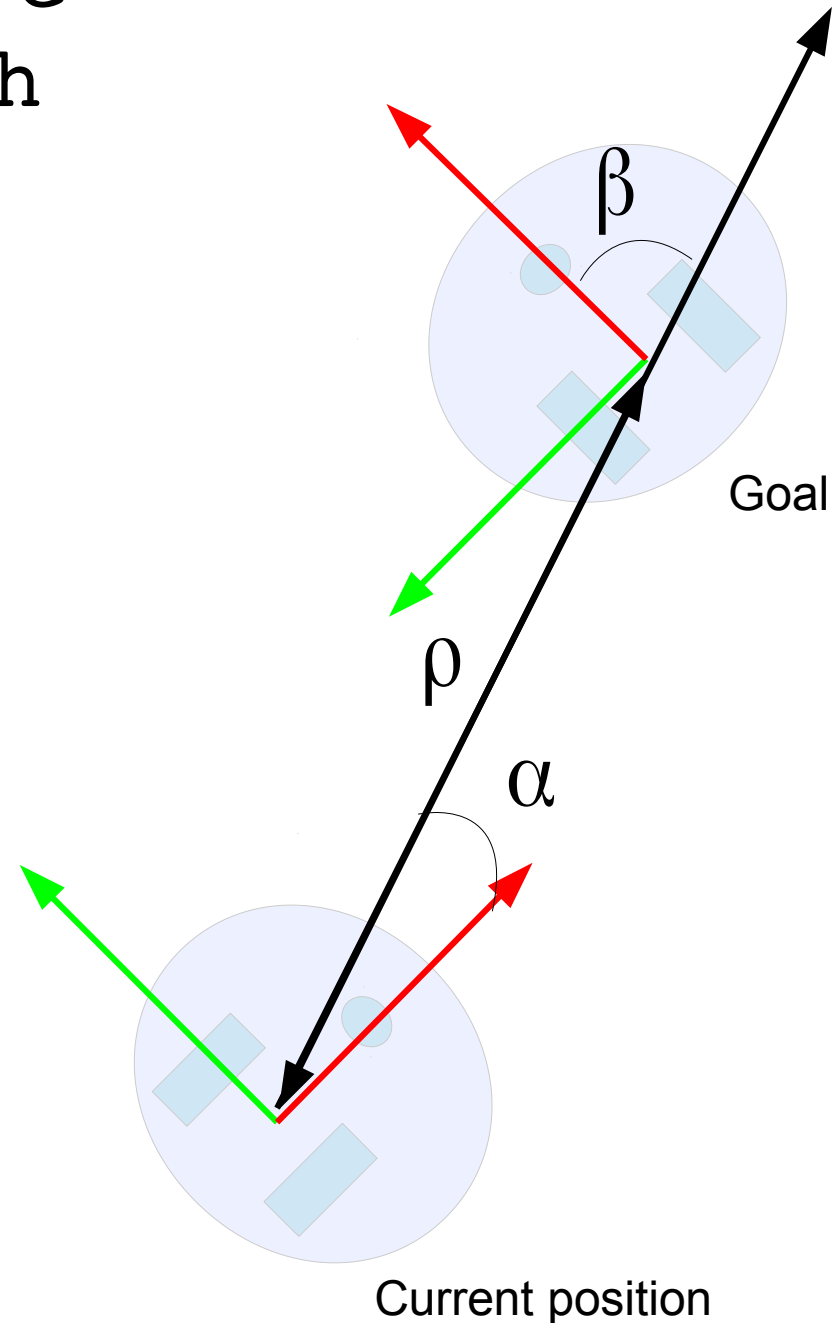
A simple control law:

$$\dot{x}_r = k_\rho \rho$$

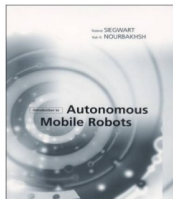
$$\dot{\theta}_r = k_\alpha \alpha + k_\beta \beta$$

Question: How to calculate
 ρ, α, β ?

Need $\Delta x, \Delta y, \Delta \theta$



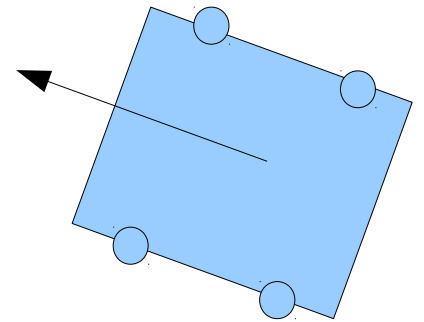
See Siegwart & Nourbakhsh section 3.6 for more details.



Review exercise

- A robot is at the following pose:

$$\begin{bmatrix} x_w \\ y_w \\ \theta_w \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ \pi/4 \end{bmatrix}$$



1) Draw a set of axes and a picture of the robot at the pose. Any picture is okay, but do indicate direction.

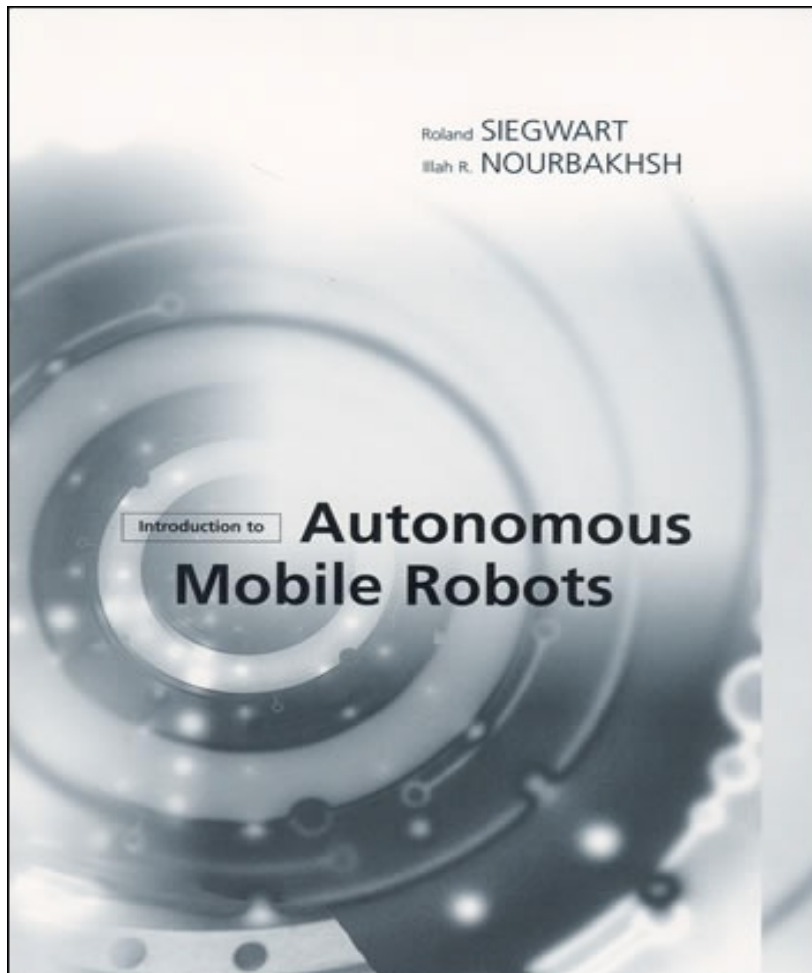
Questions: Holonomicity

- Are these robots holonomic or non-holonomic?
 - A differential drive robot?
 - A robot on Swedish / mecanum wheels?
 - A car-like robot?
 - A quadcopter robot?
- Are these constraints holonomic or non-holonomic?
 - A Swedish-wheeled robot blocked by wall?
 - A robot running on railway tracks?
 - A Swedish-wheeled robot blocked by a moving wall?
 - A Swedish-wheeled robot with an upper speed limit?

Review

- What is the difference between:
 - Differential drive/steering.
 - A differential.

Wheeled locomotion reading



Siegwart &
Nourbakhsh.

Chapters 2 & 3.