

# NUMERICAL METHODS

Week-1

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## Introduction

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# Why do we need NM?

- **Mathematical models** are an integral part in solving **engineering problems**.
- Many times, these mathematical models are derived from engineering and science principles, while at other times the models may be obtained from experimental data.

# Why do we need NM?

- **Mathematical models** generally result in need of using **mathematical procedures** that include but are not limited to
  - Differentiation,
  - Nonlinear equations,
  - Simultaneous linear equations,
  - Curve fitting by interpolation or regression,
  - Integration,
  - Differential equations.
- These mathematical procedures may be suitable to be solved exactly as you must have experienced in the series of calculus
- **BUT IN MOST CASES, the procedures need to be solved approximately using numerical methods.**

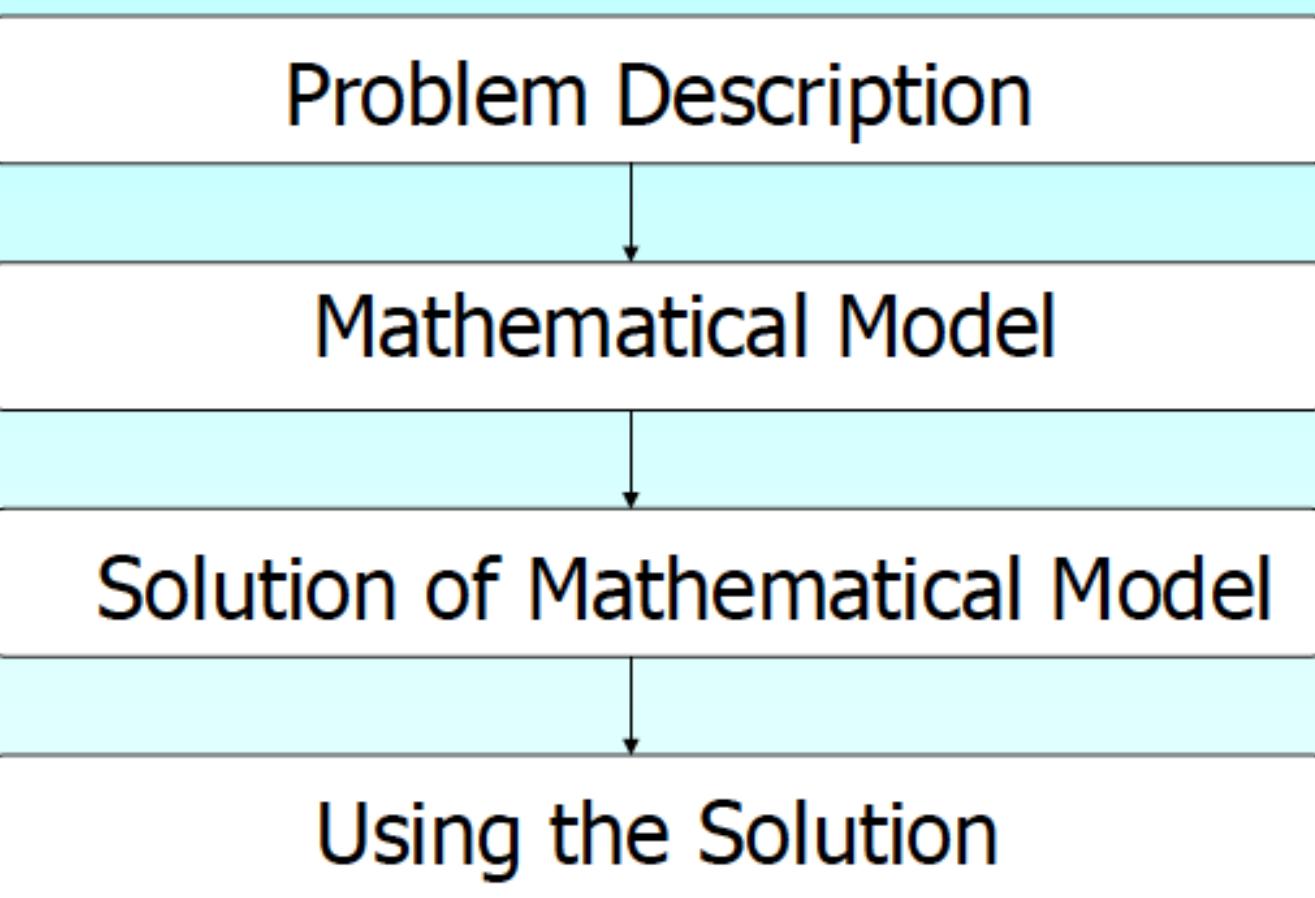
# Why NM in Computer Engineering?

- To analyze and develop **algorithmic methods** for any **function evaluation**.
- To explain the role of and the limitations of the computer in solving mathematical and engineering **problems**.

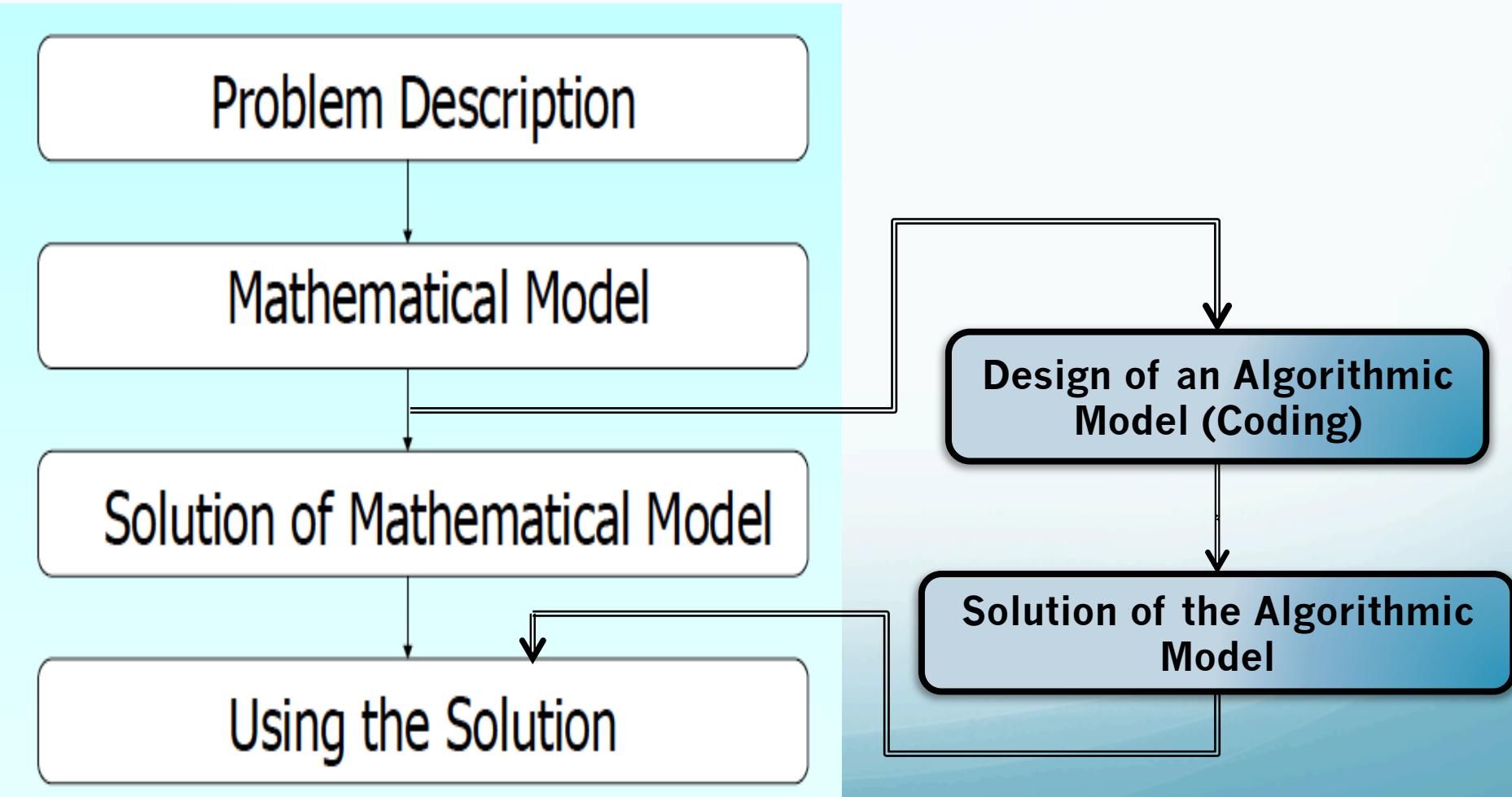
# Why NM in Computer Engineering?

- **Scientific exploration** is often conducted on **computers** rather than laboratory equipment. AT LEAST AT FIRST AS SIMULATION..
- While it is rarely meant to completely replace work in the scientific laboratory, **computer simulation** often complements this work.

# Steps for Solving an Engineering Problem from a generic perspective



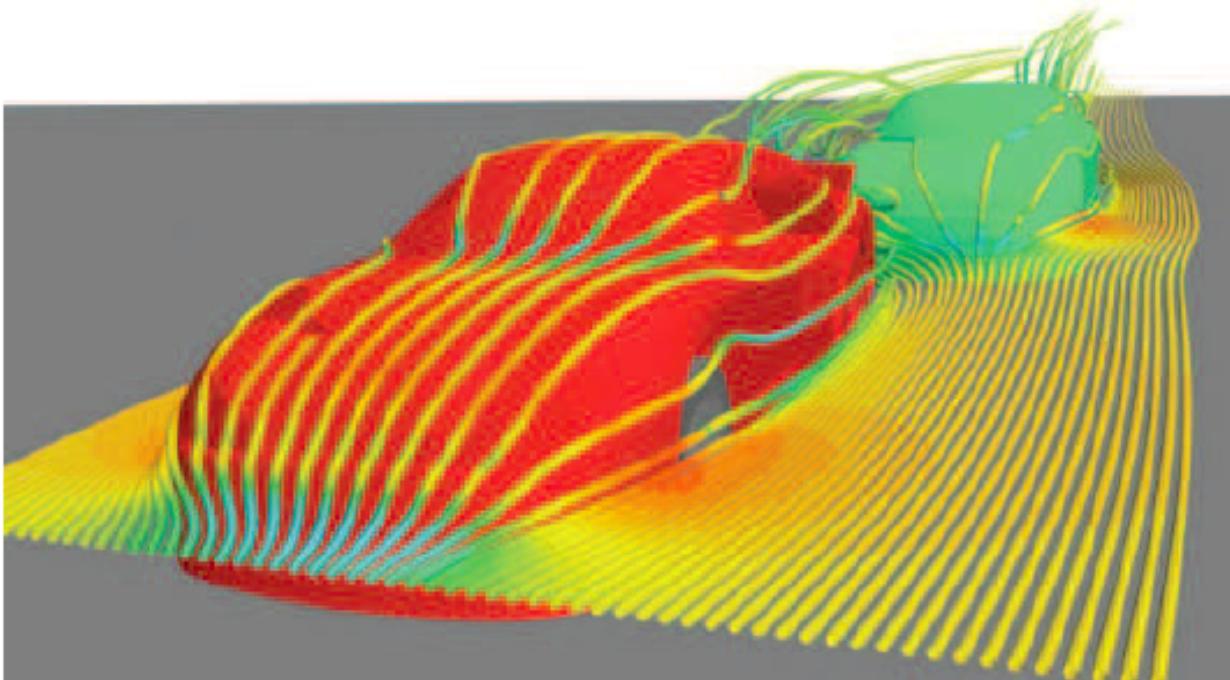
# Steps for Solving an Engineering Problem from CE perspective



# Mathematical Procedures

- Linear Equations
- Nonlinear Equations
- Differentiation
- Curve Fitting
  - Interpolation
  - Regression
- Integration
- Differential Equations
- Other Advanced Mathematical Procedures:
  - Partial Differential Equations
  - Optimization
  - Fast Fourier Transforms

# Example: NASCAR AUTO

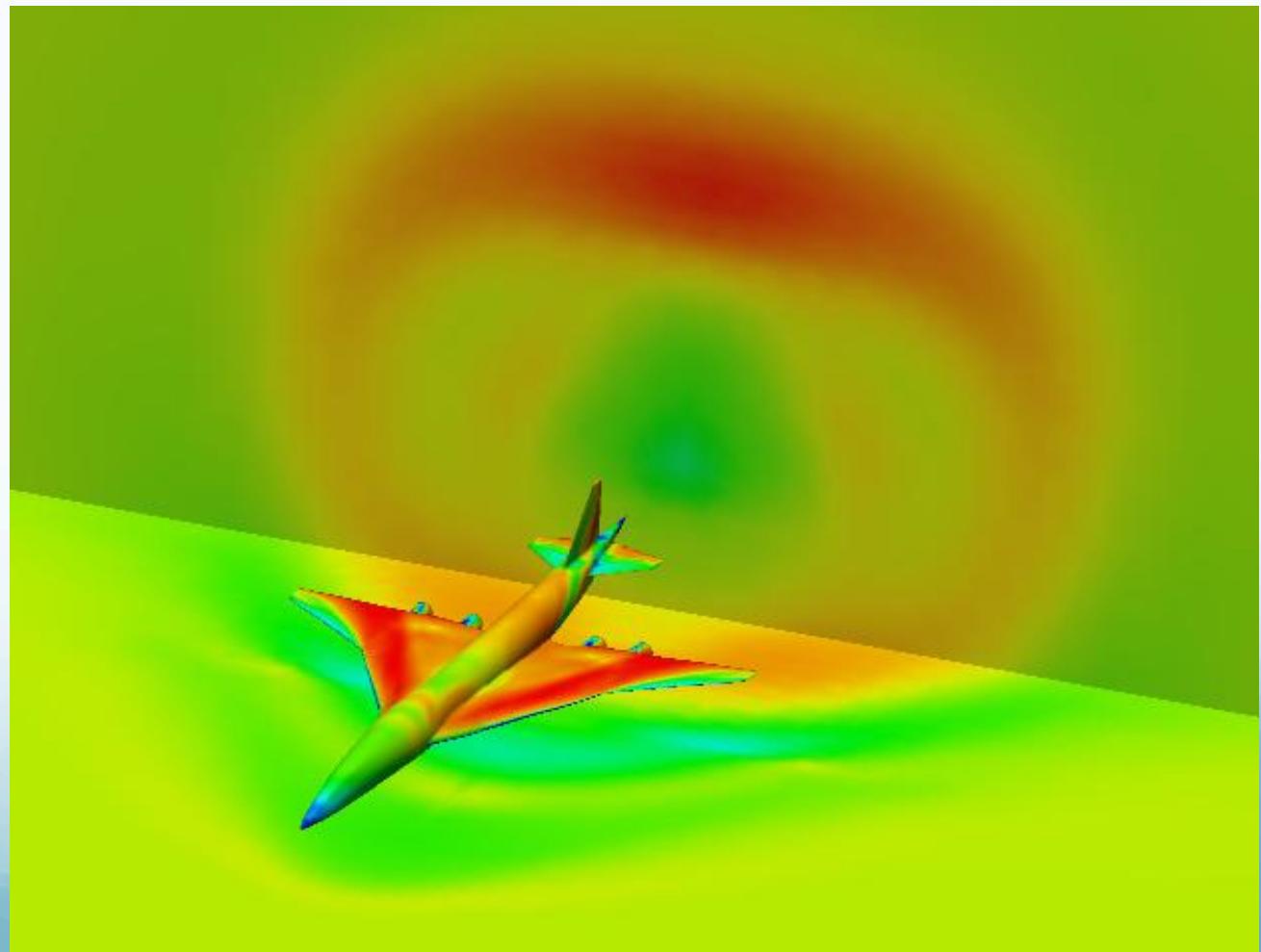


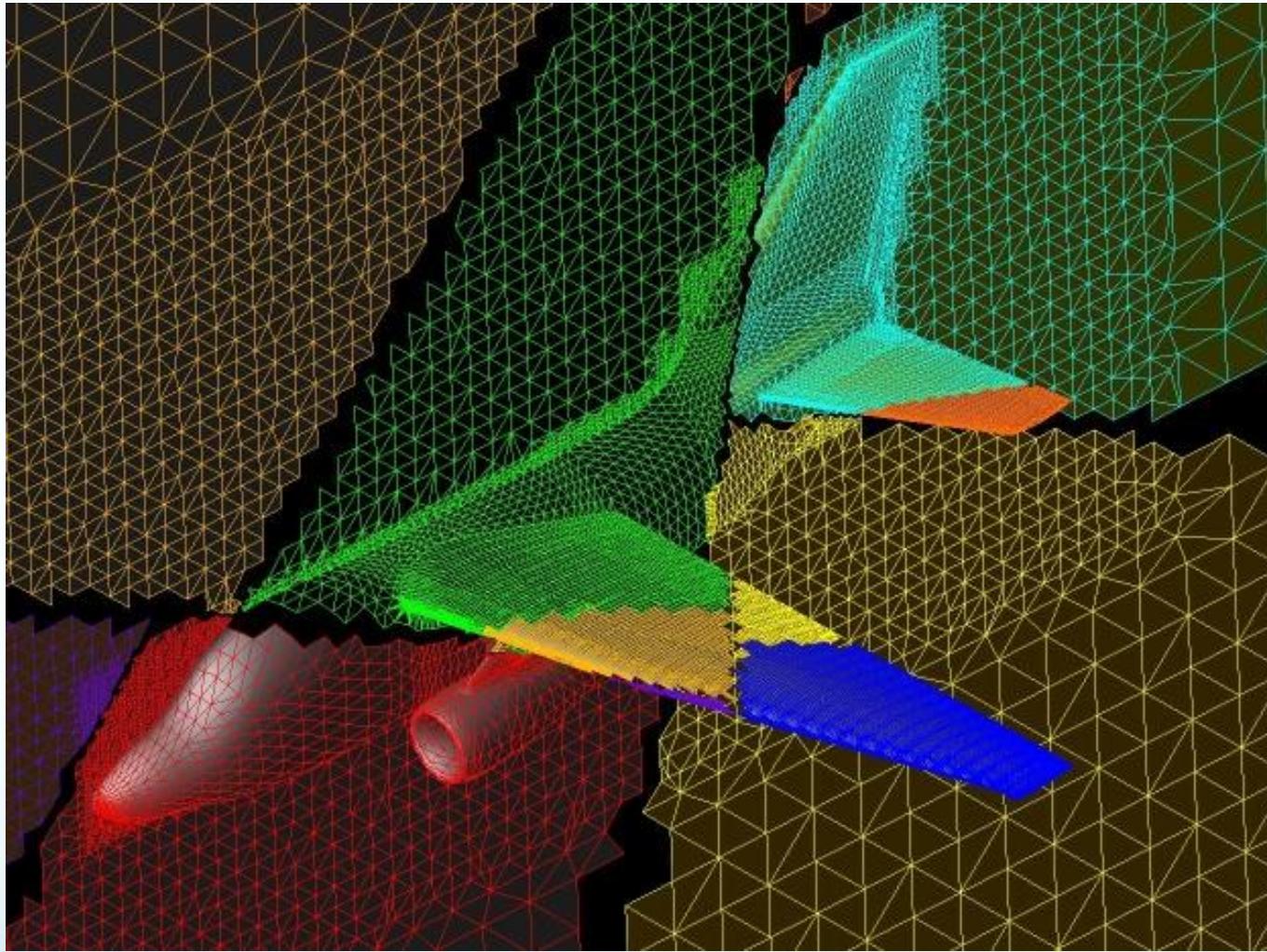
The aerodynamic simulation of two NASCAR autos requires the numerical solution of **partial differential equations (PDEs)** that model the flow of air past the car.

An auto body must be smooth and sleek, so it is often modeled using cubic (or higher order) **splines**.

# Example: AIRCRAFT

Sampled **Euler** Calculation





Parallel computation on an **unstructured mesh** showing the domain decomposition of 16 processors of a distributed memory computer.

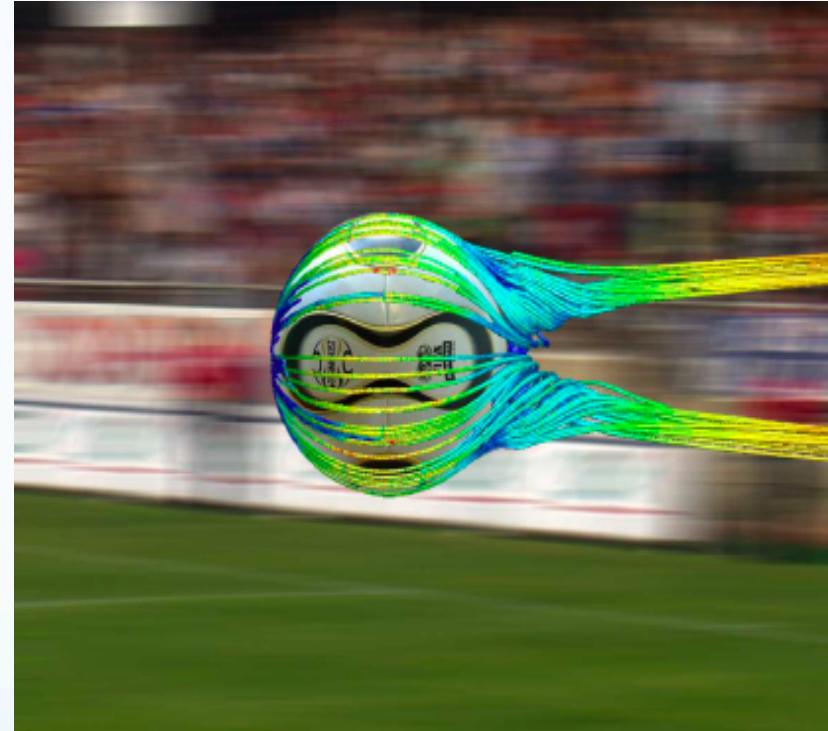
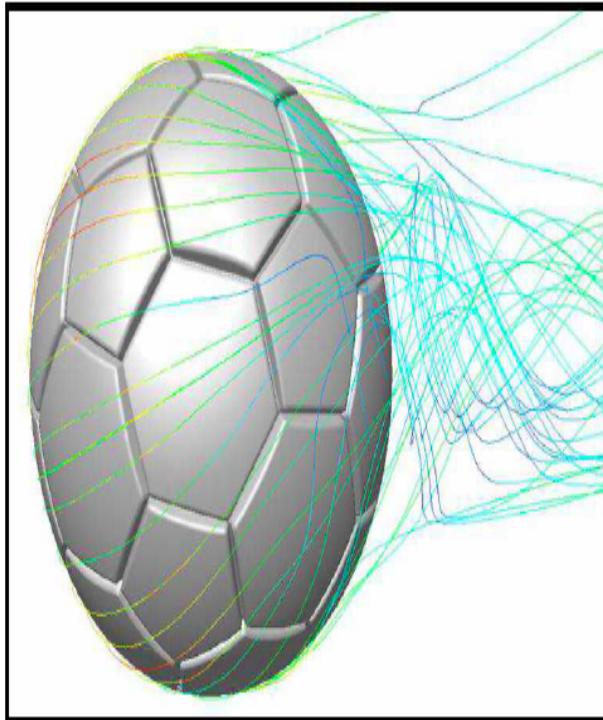
# Example: YODA



Yoda's fight scene between Yoda with Count Dooku in Star Wars Episode II.

Two layers of Yoda's clothing, seen in (b) and (c), were computed separately. A process known as **collision detection** ensured that the inner layer of clothing did not intersect the outer robe and become visible.

# Example: BALL



The aerodynamic behavior of a soccer ball can be modeled by NM. High speed airflow path lines colored by local velocity over the soccer ball. → Designed and solved by **Differential Equations**.

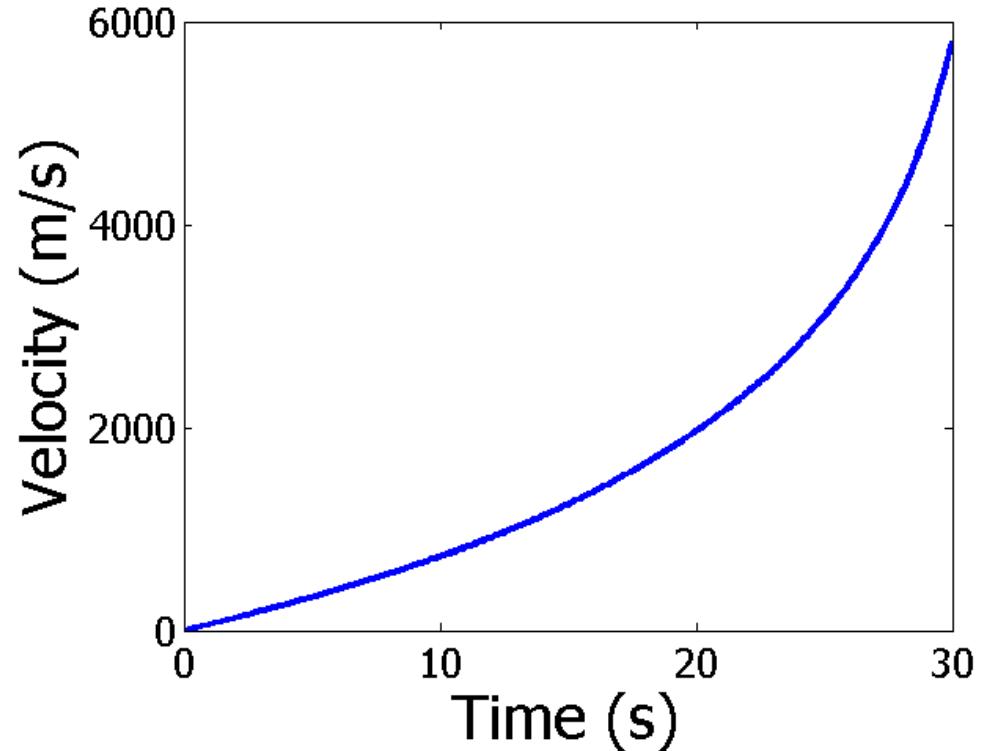
# Example: WWW



A representation  
of the graph of  
the **World Wide  
Web** created by  
David Gleich

# Example: Differentiation

What is the acceleration  
at t=7 seconds?



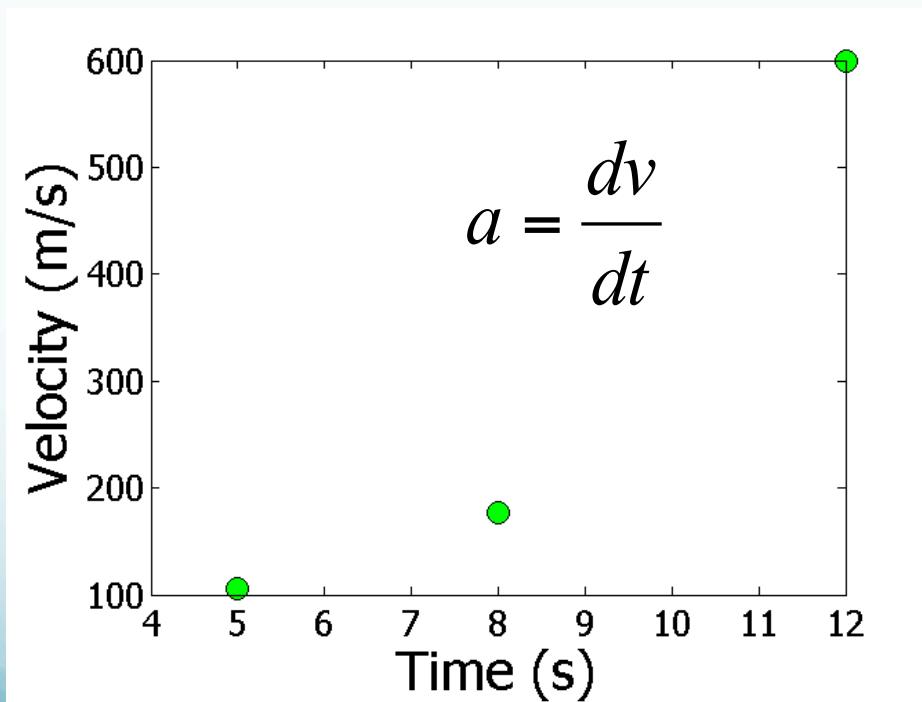
$$v(t) = 2200 \ln\left(\frac{16 \times 10^4}{16 \times 10^4 - 5000t}\right) - 9.8t$$

$$a = \frac{dv}{dt}$$

# Differentiation

What is the acceleration at t=7 seconds?

Time (s)	5	8	12
Vel (m/s)	106	177	600



# Example:

## Simultaneous Linear Equations

Find the velocity profile, given

Time (s)	5	8	12
Vel (m/s)	106	177	600

$$v(t) = at^2 + bt + c, 5 \leq t \leq 12$$

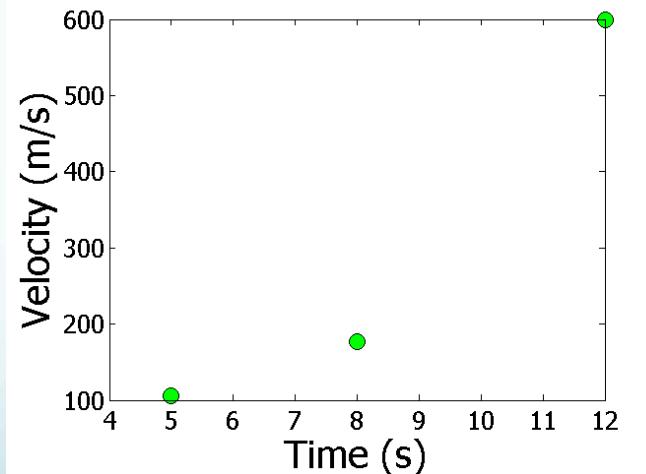


Three simultaneous linear equations

$$25a + 5b + c = 106$$

$$64a + 8b + c = 177$$

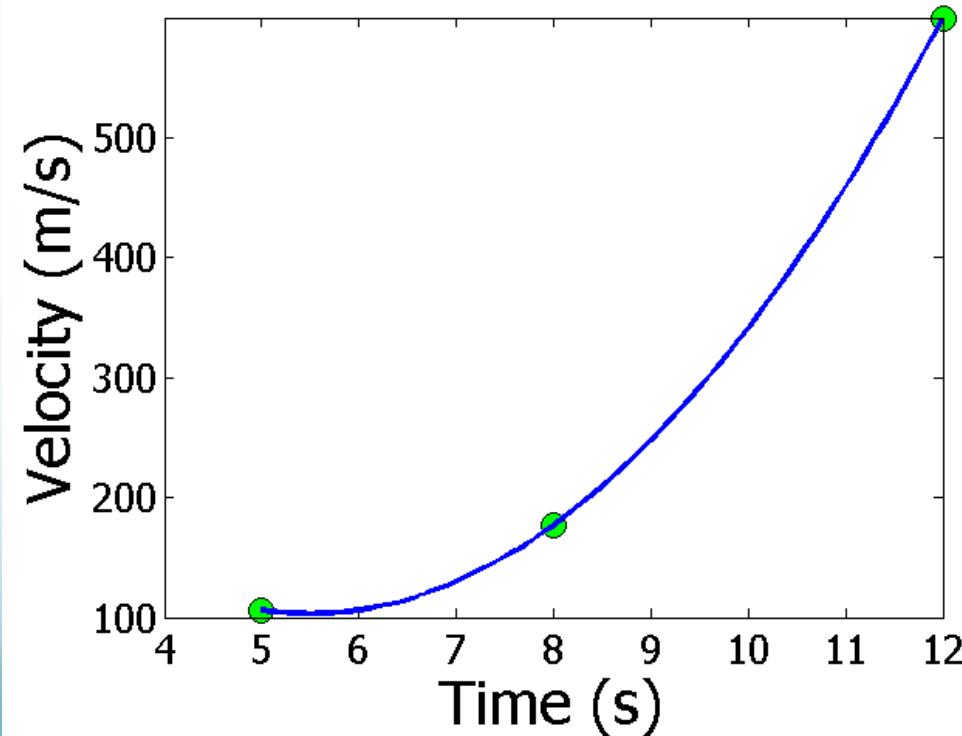
$$144a + 12b + c = 600$$



# Example: Interpolation

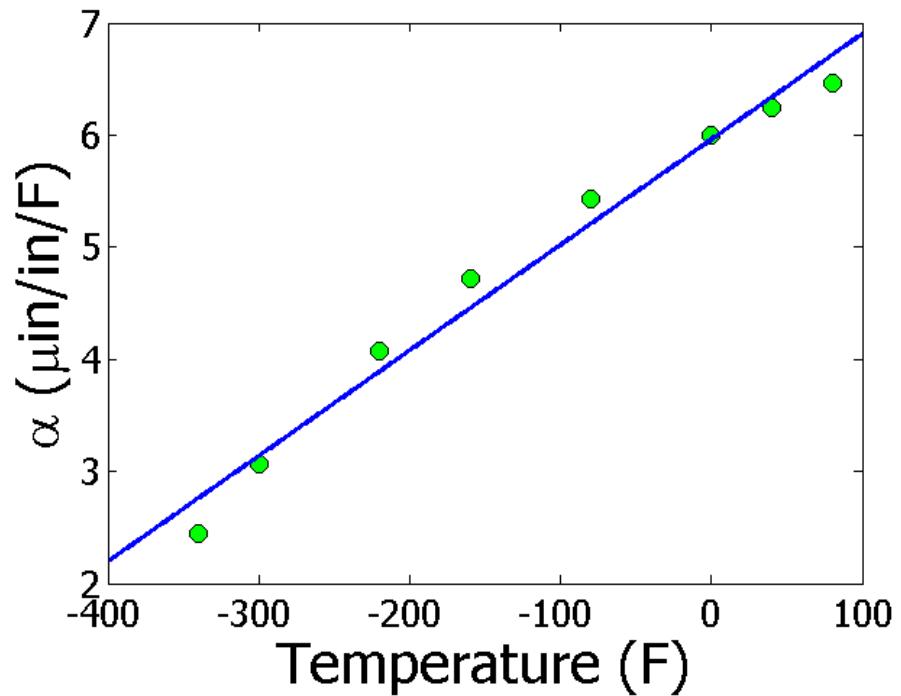
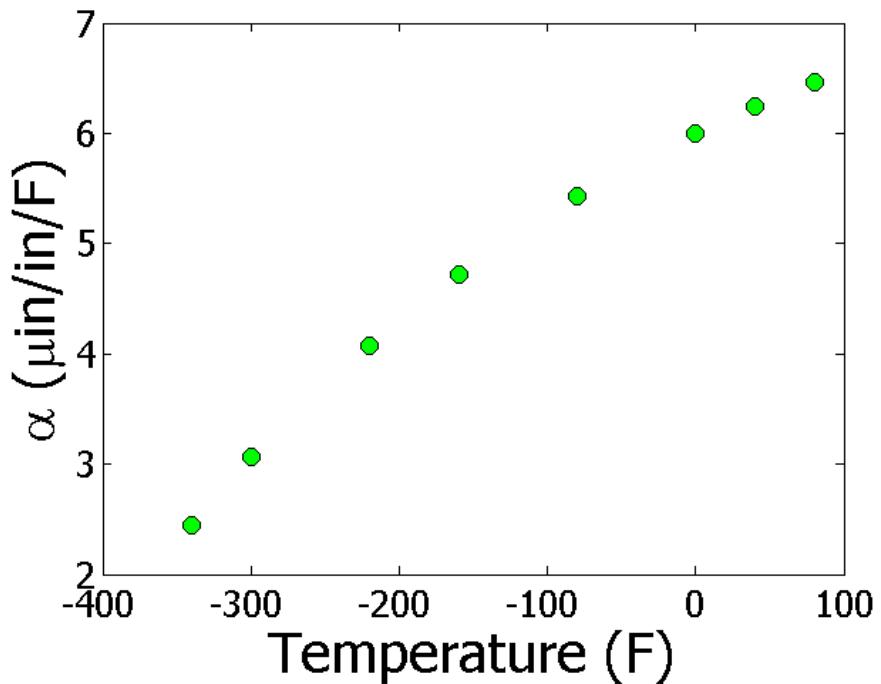
What is the velocity of the rocket at  $t=7$  seconds?

Time (s)	5	8	12
Vel (m/s)	106	177	600



# Example: Regression

Thermal expansion coefficient data for cast steel



# NM and Algorithms..

# Evaluation of a Polynomial Function

$$p(x) = a_0 + a_1x + a_2x^2 + \cdots + a_{n-1}x^{n-1} + a_nx^n$$

we group the terms in a **nested multiplication**:

$$p(x) = a_0 + x(a_1 + x(a_2 + \cdots + x(a_{n-1} + x(a_n)) \cdots))$$

```
integer i, n;  real p, x;  real array (ai)0:n
p ← an
for i = n - 1 to 0 do
    p ← ai + xp
end for
```

# Taking the derivative of a function

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

A computer has the capacity of imitating the limit operation by using a sequence of numbers  $h$  such as

$$h = 4^{-1}, 4^{-2}, 4^{-3}, \dots, 4^{-n}, \dots$$

The sequence  $1/4^n$  consists of machine numbers in a binary computer and, for this experiment on a 32-bit computer.

The following is pseudocode to compute  $f(x)$  at the point  $x = 0.5$ , with  $f(x) = \sin x$ :

```
program First
integer i, imax, n  $\leftarrow$  30
real error, y, x  $\leftarrow$  0.5, h  $\leftarrow$  1, emax  $\leftarrow$  0
for i = 1 to n do
    h  $\leftarrow$  0.25h
    y  $\leftarrow$  [sin(x + h) – sin(x)]/h
    error  $\leftarrow$  |cos(x) – y|; output i, h, y, error
    if error > emax then emax  $\leftarrow$  error; imax  $\leftarrow$  i end if
end for
    output imax, emax
end program First
```