1.5: NESTED QUANTIFIERS

Nested quantifiers occur when a propositional function of two or more variables has more than one of its variables bound.

Example 1: Translate the statement

$$\forall x\, [(x \neq 0) \rightarrow \exists y\, (xy = 1)]$$

into English, where the universe of discourse for each variable consists of all real numbers.

Example 2: Let $V(x,y)$ be the statement "x has voted for y," where the universe of discourse consists of all people in the United States. Use quantifiers to express each of the following statements.

(*a*) Everybody has voted for Ross.

(*b*) There are at least two people that Linda voted for.

(*c*) There is exactly one person whom everyone has voted for.

Example 3: Express the negation of the proposition

$$\forall x\, [(x \neq 0) \rightarrow \exists y\, (xy = 1)]$$

so that no negation appears in the statement.

The following two equivalences are always true:

$$\forall x\, \forall y\, P(x,y) \equiv \forall y\, \forall x\, P(x,y)$$

and

$$\exists x\, \exists y\, P(x,y) \equiv \exists y\, \exists x\, P(x,y).$$

The following two equivalences are NOT always true:

$$\exists x\, \forall y\, P(x,y) \equiv \forall y\, \exists x\, P(x,y)$$

and

$$\forall x\, \exists y\, P(x,y) \equiv \exists y\, \forall x\, P(x,y).$$

Example 4: Determine the truth values of the following propositions.

(*a*) $\forall x\, \exists y\, [xy = 1]$
(*b*) $\forall x\, \exists y\, [xy = 1 \wedge x \neq 0]$
(*c*) $\forall x\, \exists y\, [x \neq 0 \rightarrow xy = 1]$
(*d*) $\exists y\, \forall x\, [x \neq 0 \rightarrow xy = 1]$

SECTION 1.5 – NESTED QUANTIFIERS

Nested quantifiers occur when a propositional function of two or more variables has more than one of its variables bound.

TRANSLATING STATEMENTS INVOLVING NESTED QUANTIFIERS

Example 1: Translate the statement $\forall x\,[(x \neq 0) \rightarrow \exists y\,(xy = 1)]$ into English, where the universe of discourse for each variable consists of all real numbers.

TRANSLATING SENTENCES INTO LOGICAL EXPRESSIONS

Example 2: Let $V(x,y)$ be the statement "x has voted for y," where the universe of discourse consists of all people in the United States. Use quantifiers to express each of the following statements.

(*a*) Everybody has voted for Ross.
(*b*) There are at least two people that Linda voted for.
(*c*) There is exactly one person whom everyone has voted for.

NEGATING NESTED QUANTIFIERS

Example 3: Express the negation of the proposition $\forall x\ [(x \neq 0) \rightarrow \exists y\ (xy = 1)]$ so that no negation appears in the statement.

THE ORDER OF QUANTIFIERS

The following two equivalences are always true:

$$\forall x\ \forall y\ P(x,y) \equiv \forall y\ \forall x\ P(x,y) \quad \text{and} \quad \exists x\ \exists y\ P(x,y) \equiv \exists y\ \exists x\ P(x,y).$$

The following two equivalences are NOT always true:

$$\exists x\ \forall y\ P(x,y) \equiv \forall y\ \exists x\ P(x,y) \quad \text{and} \quad \forall x\ \exists y\ P(x,y) \equiv \exists y\ \forall x\ P(x,y).$$

Example 4: Determine the truth values of the following propositions.

(*a*) $\forall x\ \exists y\ [xy = 1]$
(*b*) $\forall x\ \exists y\ [xy = 1 \wedge x \neq 0]$
(*c*) $\forall x\ \exists y\ [x \neq 0 \rightarrow xy = 1]$
(*d*) $\exists y\ \forall x\ [x \neq 0 \rightarrow xy = 1]$