Istanbul Technical University- Fall 2007
Pattern Recognition and Analysis
BBL514E

MIDTERM

Total worth: 25% of your grade.
Date: Wednesday, November 5, 2007.
Time: 120 mins
Closed books and notes. Please write as neat and clear as possible. Good luck!

1. [20 points] What is (use at most three sentences per question):
a) Reject region
b) Bayes' Rule
c) Naïve Bayes Classifier
d) multivariate classification
e) the relationship between bias, variance and mean square error of an estimator:

Given:
Unknown parameter θ
Estimator $d_i = d\,(X_i)$ on sample X_i
Bias: $b_\theta(d) = E\,[d] - \theta$
Variance: $E\,[(d - E\,[d])^2]$
mse= $r\,(d,\theta) = E\,[(d-\theta)^2]$

2. [20 points]
a) What is the difference between PCA (Principal Component Analysis) and Backward Feature Selection. Assume that you are given inputs $X = \{\underline{x}^t\}_{t=1}^N$ where each $\underline{x}^t \in R^d$.

b) Given a multivariate binary classification problem and assumption of normally distributed d dimensional inputs, what are the most complex and least complex classifiers that you could produce? Explain in detail your assumptions to arrive at those classifiers and the number of parameters needed to be estimated from training data.

3) [30 points] Assume a two-class problem with equal a priori class probabilities and Gaussian class-conditional densities as follows:

$$p(x \mid w_1) = N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} a & c \\ c & b \end{bmatrix}\right) \quad \text{and} \quad p(x \mid w_2) = N\left(\begin{bmatrix} d \\ e \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)$$

where $ab - c^2 = 1$.

a) Find the equation of the decision boundary between these two classes in terms of the given parameters, after choosing a logarithmic discriminant function.
b) Determine the constraints on the values of a, b, c, d and e, such that the resulting discriminant function results with a linear decision boundary.

Hint1: If $\underline{\mathbf{x}} \sim \mathrm{N}_d(\underline{\boldsymbol{\mu}}, \Sigma)$, then the pdf for x is given by:

$$p(\underline{\mathbf{x}}) = \frac{1}{(2\pi)^{d/2}|\Sigma|^{1/2}} \exp\left[-\frac{1}{2}(\underline{\mathbf{x}} - \underline{\boldsymbol{\mu}})^T \Sigma^{-1} (\underline{\mathbf{x}} - \underline{\boldsymbol{\mu}})\right]$$

Hint2:

For a 2×2 matrix,

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

the matrix inverse is

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

4) [30 points] For the Bayesian network shown below, compute the following:
a) P(A,B,C,D)
b) P(A|B)
c) P(C|B)