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BLG 202E – Numerical Methods
Assignment 4

Solutions

Solution 1

The composite trapezoidal method is

$$\int_a^b f(x)dx \approx \frac{h}{2}\sum_{i=1}^{r}\Big[f(t_{i-1})+f(t_i)\Big]$$
$$= \frac{h}{2}[f(t_0)+2f(t_1)+2f(t_2)+\cdots+2f(t_{r-1})+f(t_r)]$$
$$= \frac{h}{2}[f(a)+2f(t_1)+2f(t_2)+\cdots+2f(t_{r-1})+f(b)].$$

So that in this question, trapezoidal equation is for interval [0, 3] with N = 10

(In this case step size h must be (b-a)/N = 0.3)

$$Icomp,trap = \frac{h}{2}[f(0)+2f(0.3)+2f(0.6)+2f(0.9)+2f(1.2)+2f(1.5)+2f(1.8)$$
$$+2f(2.1)+2f(2.4)+2f(2.7)+f(3)]$$

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```
syms x

f = inline((2.718182.^-x)*sin(x), 'x');
h = 0.3
xi = 0;
yn = 0; %% new approximate
yp = f(xi); %% previous approximate
sum = 0;

while xi < 3
xi = xi + h;
sum = sum + 2*f(xi)
yp = yn;
end

sum = sum + f(3);
result = sum * h/2
```

$result =$ 0.5136, where number of interval is 10. For small number of interval, error is nor smaller than $\in_{step}$ (0.001). For just ten interval, absolute error is 0.0018(small than $\in_{step}$)

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Solution 2

The composite Simpson's rule is

$$\int_a^b f(x)dx \approx \frac{h}{3}\left[f(a) + 2\sum_{k=1}^{r/2-1} f(t_{2k}) + 4\sum_{k=1}^{r/2} f(t_{2k-1}) + f(b) \right].$$

And exact integration is:

$\frac{(2x-1)e^{2x}}{4} + C$ interval 0 and 4, exact value is

5216.426477

$Isimp = 5256.754$

Absolute relative true error = 0.0075

```
syms x
h = 1;
xi = 0;
xs = 4;
yn = 0; %% new approximate
yp = f(xi); %% previous approximate
sum = 0;


f = inline((2.718182.^2*x)*x, 'x');
%%Isimp = h/3*(f(xi) + 2*f(xi+2*h) + 4*f(xi + h) + 4*f(xi+3*h)+f(xs))
Isimp = h/3*(f(0) + 2*f(2) + 4*f(1) + 4*f(3)+f(4))

fun = @(x) exp(2*x).*x;
ExactIntegral = integral(fun, 0, 4)
error = abs(ExactIntegral - Isimp)/ExactIntegral
```

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Solution 3

The basic midpoint rule as like following

$$I_f \approx I_{mid} = (b-a) f\left(\frac{a+b}{2}\right).$$

So we need to find all midpoint which given number of midpoints. For example m=2, number of midpoint is 2 and

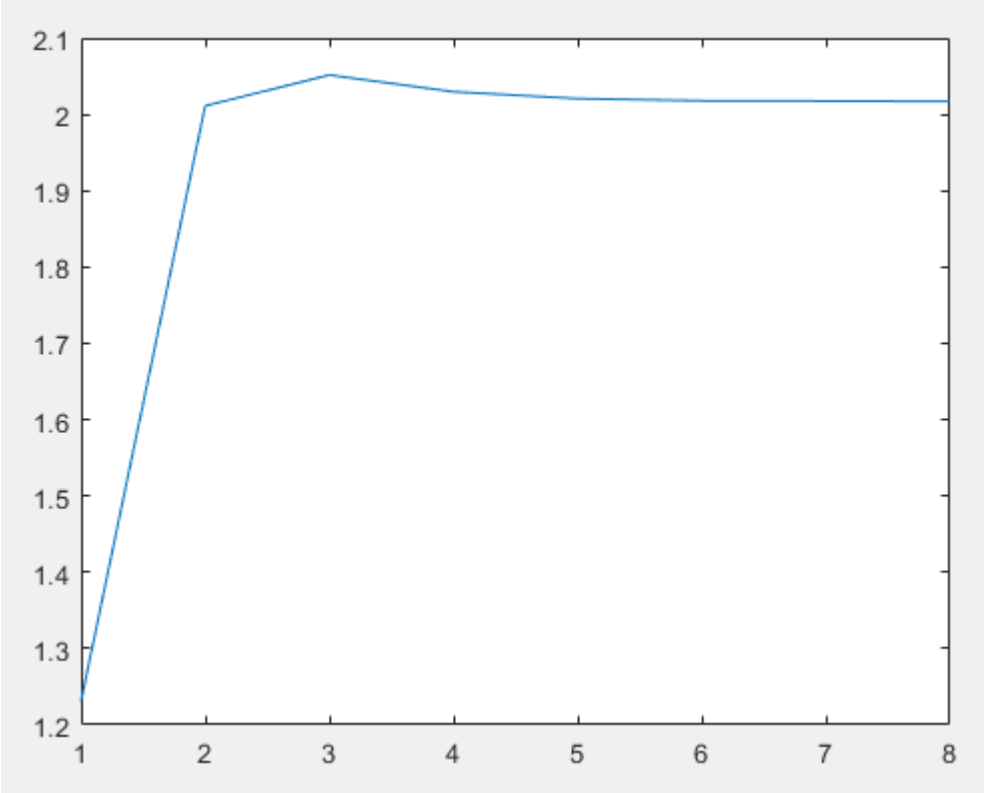
m0 = 0.5

m1 = 1.5

General formula of MATLAB code for every midpoint is

```
syms x

f = inline(1+(2.718182.^-x)*sin(8*x^(2/3)), 'x');
m = [2,4,8,16,32,60,70,100];
h = 2./m;
i = 1;
first = 0;
second = 0 + h;
mid_first = (first + second) / 2;
result = 0;
mid = 0;
while i < 9
    mid = mid_first(i);
        while mid < 2
        result = result + f(mid);
        mid = mid + h(i);
        end
    result = result * h(i);
    results(i) = result;
    mid = 0;
    result = 0;
    i = i + 1
end
results
plot(results)
```



approximation results according to the number of sample values

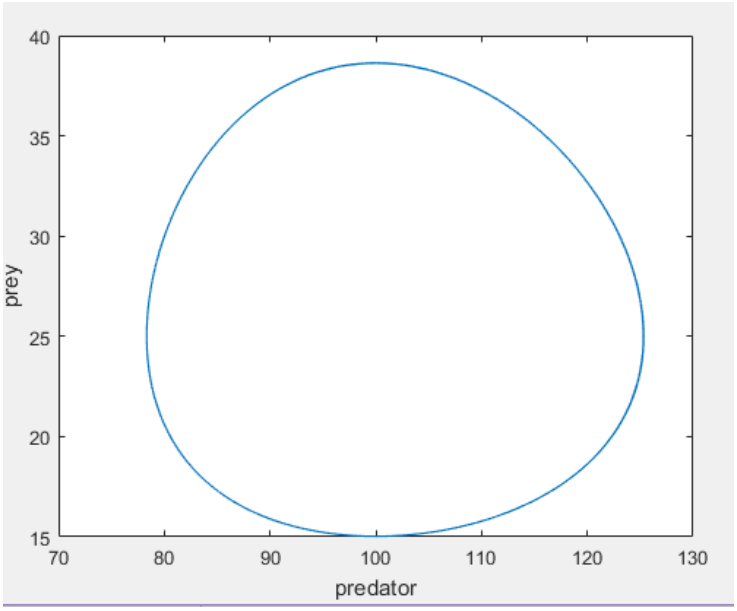
Solution 4

Runge Kutta 2 stage MATLAB code as following:

```
clear;
to = 0; %%integrated from 0
tf = 100; %%integrated to 100
yo = [80 30]; %%initial value
options = odeset('RelTol', 1e-5);
[t y] = ode45('y1y2',[to tf],yo,options);
plot(y(:,1),y(:,2))
ylabel('prey')
xlabel('predator')



function y1y2 =y1y2(t,y)
 y1y2(1) = .25*y(1) - .01*y(1)*y(2);
 y1y2(2) = -y(2) +.01*y(1)*y(2);
 y1y2 = [y1y2(1) y1y2(2)]';
end
```



predator-prey solution in phase plane: the curve of y1(t) vs. y2(t) yields a limit cycle

Solution 5

Calculation of error in each step size value for Forward Euler Method.

```
s = [.00005,.0001,.0005,.001];
h = [.00005,.0001,.0005,.001]*pi;
N = 1./[.00005,.0001,.0005,.001];
N = N/2;
y0 = 1;

result_forward = [0,0,0,0];
i = 1;
while i < 5
    y = y0; t = 0;
    for k=1:N(i)
    y = y + h(i)*(-1000*(y - cos (t)) - sin (t));
    t = t + h(i);
    end
    result_forward(i) = y;
    i
    i = i + 1;
end

plot(result_forward)
i = 1;
while i < 5
    fprintf('Approximation Result for h = %i: %i\n', s(i),
result_forward(i))
    i = i + 1;
end
```

After the calculation, error results will be as following:

Approximation Result for h = 5.000000e-05: 7.421676e-11
Approximation Result for h = 1.000000e-04: 1.405548e-10
Approximation Result for h = 5.000000e-04: 3.741929e-10
Approximation Result for h = 1.000000e-03: -3.669057e+159

It can be seen that after the value h = .0005π, we get the error as -3.6e+159 for h = .001π which means that a blowup has occurred. This result from stability difference between forward and backward Euler method.

In Forward Euler Method,

$$|1 + h\lambda| \leq 1 \Rightarrow h \leq \frac{2}{|\lambda|}.$$

the constant step size h must not exceed a certain bound that depends on the problem which is being approximately solved.

However; in Backward Euler Method, there is no restriction about step size. Because,

$$y_{i+1} = \frac{1}{1 - h\lambda} y_i.$$

For Backward Euler Method for any h > 0 and λ < 0, there is no absolute stability restriction.

For h = 0.01π,

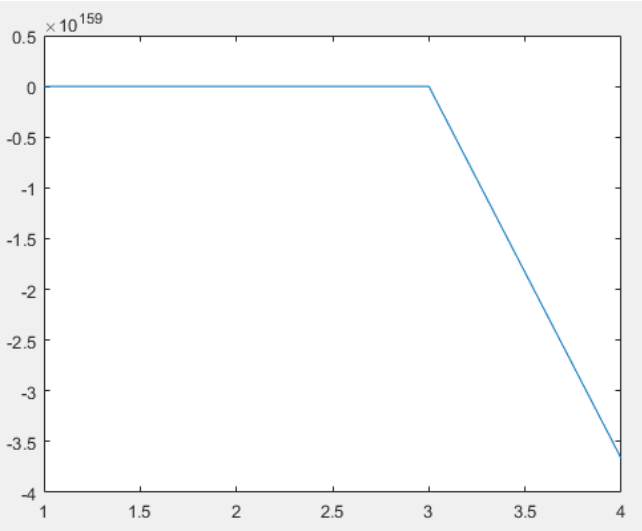
Error in Forward Euler	= -3.6e+159
Error in Backward Euler	= -3.2e-9

Where for this problem $\lambda = 1000$, then

Conclusion, h ≥0.01π, the result blows up

h ≤ 0.002, there is no problem for Forward Euler

For Backward Euler, there is no any problem for any h or λ.



In this figure, we can see blowup point.