MAT102E MIDTERM EXAM 17 July 2013

QUESTION 1 The blanks below will be filled by students. (Except the score)

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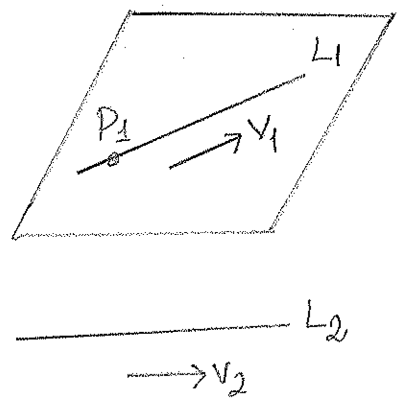
Signature:	Electronic Post(e-mail) address:	Student Number:		

For the solution of this question please use only the front face and if necessary the back face of this page.

[13pt] **a)** Find the equation of the plane containing the line $\mathbf{L_1} : \mathbf{x = 2t+3, y = 4t-1, z = -t+2, -\infty < t < \infty}$ and parallel to the line $\mathbf{L_2} : \mathbf{x = 2s+3, y = s+2, z = 2s-2, -\infty < s < \infty}$.

[12pt] **b)** Find the parametric equations of the tangent line to the curve $\overrightarrow{\mathbf{r}}(\mathbf{t}) = (\mathbf{5}\sin\mathbf{t})\,\overrightarrow{\mathbf{i}} + (\mathbf{5}\cos\mathbf{t})\,\overrightarrow{\mathbf{j}} + (\mathbf{12t})\,\overrightarrow{\mathbf{k}}$ for $\mathbf{t = 3\pi/4}$.

a)



$P_1(3,-1,2)$

$\vec{V_1} = 2i + 4j - k \parallel L_1$

$\vec{V_2} = 2i + j + 2k \parallel L_2$

$$\vec{n} \parallel \vec{V_1} \times \vec{V_2} = \begin{vmatrix} i & j & k \\ 2 & 4 & -1 \\ 2 & 1 & 2 \end{vmatrix} = (8+1)i - (4+2)j + (2-8)k$$

$P_1 \in$ Plane , $\vec{n} = 9i - 6j - 6k \perp$ Plane

$\Rightarrow$ Plane's equation: $9(x-3) - 6(y+1) - 6(z-2) = 0 \Rightarrow 3x - 2y - 2z = 7$

b) $x = 5\sin t$, $y = 5\cos t$, $z = 12t$

$t = 3\pi/4 \Rightarrow x_0 = \frac{5\sqrt{2}}{2}$, $y_0 = -\frac{5\sqrt{2}}{2}$, $z_0 = 9\pi \Rightarrow P_0\left(\frac{5\sqrt{2}}{2}, -\frac{5\sqrt{2}}{2}, 9\pi\right)$

$$\frac{d\vec{r}}{dt} = (5\cos t)\,i + (-5\sin t)\,j + 12\vec{k}$$

$$\left.\frac{d\vec{r}}{dt}\right|_{t=3\pi/4} = -5\frac{\sqrt{2}}{2}i - 5\frac{\sqrt{2}}{2}j + 12k \parallel \text{Tangent Line} \quad , \quad P_0 \in \text{Tang. Line}$$

Tangent Line:

$$x - 5\frac{\sqrt{2}}{2} = -5\frac{\sqrt{2}}{2}t$$

$$y + 5\frac{\sqrt{2}}{2} = -5\frac{\sqrt{2}}{2}t$$

$$z - 9\pi = 12t$$

$$-\infty < t < \infty$$

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QUESTION 2 The blanks below will be filled by students. (Except the score)

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[10pt] **a)** Let $\mathbf{f(x,y)} = \begin{cases} \dfrac{\mathbf{xy-y}}{\mathbf{x^2-2x+y^2+1}} & ,(\mathbf{x,y}) \neq (\mathbf{1,0}) \\ \mathbf{2} & ,(\mathbf{x,y}) = (\mathbf{1,0}) \end{cases}$

Is $\mathbf{f(x,y)}$ continuous at $(\mathbf{1,0})$? Explain your answer.

[15pt] **b)** Find the maximum, minimum and saddle points of $\mathbf{f(x,y) = 2x^4 + xy + y^2}$.

a) $\dfrac{xy-y}{x^2-2x+y^2+1} = \dfrac{(x-1)y}{(x-1)^2+y^2}$, $x-1 = my \Rightarrow \lim\limits_{y\to 0} \dfrac{m\cancel{y^2}}{(1+m^2)\cancel{y^2}} = \dfrac{m}{1+m^2}$

Limit depends on the value of m. Thus, the limit as $(x,y)\to(1,0)$ does not exist and therefore, f is not continuous at (1,0).

b) $f_y = x+2y = 0 \Rightarrow x = -2y$

$f_x = 8x^3 + y = 0 \Rightarrow 8(-8y^3) + y = 0 \Rightarrow y(1-8^2y^2) = 0$

$\to y=0 \Rightarrow x=0$

$\to y=\frac{1}{8} \Rightarrow x=-\frac{1}{4}$

$\to y=-\frac{1}{8} \Rightarrow x=\frac{1}{4}$

$A(0,0)$, $B(-1/4, 1/8)$, $C(1/4, -1/8)$

$\Delta = f_{xx}f_{yy} - f_{xy}^2 = (24x^2).2 - 1 = 48x^2 - 1$

$\Delta\big|_A = -1 < 0 \Rightarrow$ A: sadde point

$\Delta\big|_{B,C} = 48\,\frac{1}{16} - 1 > 0$, $f_{xx}\big|_{B,C} = 24\,\frac{1}{16} > 0 \Rightarrow$ B, C: local min

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[10pt] **a)** Find the equation of the tangent plane of the surface $\cos(\pi \mathbf{x}) - \mathbf{x}^2\mathbf{y} + \mathbf{e}^{\mathbf{yz}} = \mathbf{4} - \mathbf{xz}$ at $\mathbf{P}(\mathbf{0}, \ln \mathbf{3}, \mathbf{1})$.

[15pt] **b)** Find the shortest distance from the origin to the surface $\mathbf{xyz^2} = \mathbf{2}$ by using the method of Lagrange multipliers.

a) $f(x,y,z) = \cos(\pi x) - x^2y + e^{yz} + xz = 4$

$f_x = -\pi \sin(\pi x) - 2xy + z \Rightarrow f_x\big|_P = 1$

$f_y = -x^2 + z e^{yz} \Rightarrow f_y\big|_P = 1 . e^{\ln 3} = 3$

$f_z = y e^{yz} + x \Rightarrow f_z\big|_P = \ln 3\, e^{\ln 3} = 3\ln 3 = \ln 27$

$\nabla f\big|_P = i + 3j + (\ln 27)k \perp$ Tang. Plane , $P \in$ Tang. Plane

Tangent Plane: $1.(x-0) + 3(y-\ln 3) + (\ln 27)(z-1) = 0$

b) $f(x,y,z) = x^2+y^2+z^2$, $g = xyz^2 = 2$ $(x,y,z \neq 0)$

$\vec{\nabla} f = \lambda \vec{\nabla} g \Rightarrow 2xi + 2yj + 2zk = \lambda(yz^2 i + xz^2 j + 2xyz k)$

(*) $2x = \lambda y z^2$

(**) $2y = \lambda x z^2$

$z = \lambda xyz \Rightarrow z(1-\lambda xy) = 0$ $\left\langle \begin{array}{l} z \neq 0 \\ \lambda = \frac{1}{xy} \quad (x,y \neq 0) \end{array} \right.$

(*) $2x = \frac{1}{x\cancel{y}} \cancel{y} z^2 \Rightarrow 2x^2 = z^2$

(**) $2y = \frac{1}{\cancel{x}y} \cancel{x} z^2 \Rightarrow 2y^2 = z^2$

$\Big\}$ $\underbrace{x = y}$ $= \pm z/\sqrt{2}$

x and y must have the same sign since $g > 0$

$g = \frac{z}{\sqrt{2}} \cdot \frac{z}{\sqrt{2}} z^2 = \frac{z^4}{2} = 2 \Rightarrow z^4 = 4$, $x^2 = y^2 = 1$

$f = 1 + 1 + 2 = 4 \Rightarrow$ distance $= 2$ //

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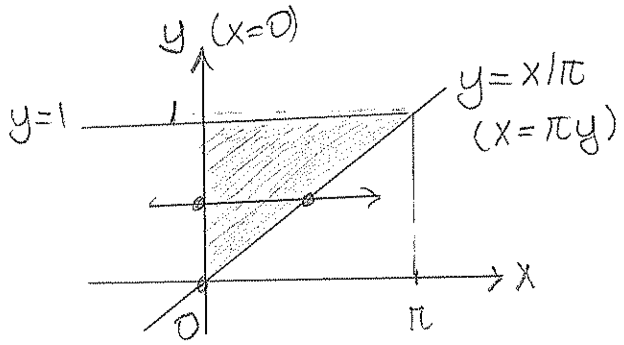
[13pt] **a)** Consider the integral $\int_0^{\pi}\int_{x/\pi}^{1} \mathbf{y^4 \sin(xy^2)\ dy\ dx}$. Sketch the region of integration and evaluate the integral.

[12pt] **b)** **D** is the solid right cylinder whose base is in the region between $\mathbf{r} = \sin\theta$ and $\mathbf{r} = \mathbf{2}\sin\theta$ in the **xy**-plane and whose top lies in the plane $\mathbf{z} = \mathbf{2} - \mathbf{x}$. Sketch the region **D**. Write a triple integral to calculate the volume of **D**. Do not evaluate the integral.

a) $\frac{x}{\pi} \le y \le 1$

$0 \le x \le \pi$



$$I = \int_0^{\pi}\int_{x/\pi}^{1} y^4 \sin(xy^2)\, dy\, dx$$

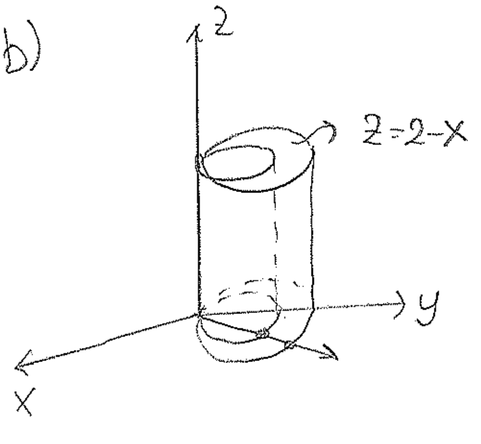
$$= \int_0^1 \left(\int_0^{\pi y} y^2 \sin(xy^2)\, dx\right) y^2 dy$$

$$= -\int_0^1 \left(\cos(xy^2)\Big|_{x=0}^{\pi y}\right) y^2\, dy$$

$$I = -\int_0^1 y^2(\cos(\pi y^3) - 1)\, dy = \int_0^1 y^2 dy - \int_0^1 \frac{3\pi}{3\pi} y^2 \cos(\pi y^3)\, dy$$

$$= \frac{y^3}{3} - \frac{1}{3\pi}\sin(\pi y^3)\Big|_0^1 = \frac{1}{3} - \frac{1}{3\pi}(0-0) = \frac{1}{3}$$

b)



$$V = \int_{\theta=0}^{\pi}\int_{r=\sin\theta}^{2\sin\theta}\int_{z=0}^{2-r\cos\theta} dz\, r\, dr\, d\theta$$