BLG435E
Artificial Intelligence

Practice Session 3: Logic

Logical Representation

- Write down logical representations for the following sentences.
  - Some students took both Communications and AI courses.
  - There is a student who does not take any art courses.
  - No person likes an expensive and low quality appliance.
  - Only one student registered in AI took Project course.
  - All students take the same exam.

Logical Representation

- Some students took both Communications and AI courses.

Logical Representation

- Some students took both Communications and AI courses.

$$\exists x \;\; Student(x) \land Take\ (x, Communications) \land Take\ (x, AI)$$

Logical Representation

- There is a student who does not take any art courses.

Logical Representation

- There is a student who does not take any art courses.

$$\exists x \forall y \ \ Student(x) \land ArtCourse(y) \Rightarrow \neg Take\,(x, y)$$

Logical Representation

- No person likes an expensive and low quality appliance.

Logical Representation

- No person likes an expensive and low quality appliance.

$$\forall x, y \;\; Person(x) \wedge Appliance(y) \wedge Expensive(y) \\ \wedge LowQuality(y) \Rightarrow \neg Like\,(x, y)$$

Logical Representation

- Only one student registered in AI took Project course.

Logical Representation

- Only one student registered in AI took Project course.

$$\exists x \forall y \;\; Student(x) \wedge Take(x, AI) \wedge Take(x, Project) \wedge Student(y) \wedge Take(y, AI) \wedge Take(y, Project) \Rightarrow x = y$$

Logical Representation

- All students take the same exam.

Logical Representation

- All students take the same exam.

$$\forall x, y, z \;\; Student(x) \land Student(y) \land Exam(z) \land Take(x, z) \Rightarrow Take(y, z)$$

Resolution

- Write down logical representations for the following sentences, suitable for use with Generalized Modus Ponens.
  - Horses, cows, and pigs are mammals.
  - An offspring of a horse is a horse.
  - Bluebeard is a horse.
  - Bluebeard is Charlie's parent.
  - Offspring and parent are inverse relations.
  - Every mammal has a parent.

Resolution

- Horses, cows, and pigs are mammals.

Resolution

- Horses, cows, and pigs are mammals.

$$\forall x \;\; Horse(x) \Rightarrow Mammal(x)$$
$$\forall x \;\; Cow(x) \Rightarrow Mammal(x)$$
$$\forall x \;\; Pig(x) \Rightarrow Mammal(x)$$

Resolution

- An offspring of a horse is a horse.

Resolution

- An offspring of a horse is a horse.

$$\forall x, y \;\; Offspring(x, y) \wedge Horse(y) \Rightarrow Horse(x)$$

Resolution

- Bluebeard is a horse.

Resolution

- Bluebeard is a horse.

$$Horse(Bluebeard)$$

Resolution

- Bluebeard is Charlie’s parent.

Resolution

- Bluebeard is Charlie's parent.

$$Parent(Bluebeard, Charlie)$$

Resolution

- Offspring and parent are inverse relations.

Resolution

- Offspring and parent are inverse relations.

$$\forall x, y \;\; Offspring(x, y) \Rightarrow Parent(y, x)$$
$$\forall x, y \;\; Parent(x, y) \Rightarrow Offspring(y, x)$$

Resolution

- Every mammal has a parent.

Resolution

- Every mammal has a parent.

$$\forall x \exists y \;\; Mammal(x) \Rightarrow Parent(y, x)$$

Resolution

- Use resolution to prove the query: $Horse(Charlie)$ by using the KB built in the first part of the question.
- First, we need to convert the KB into CNF.
- Then, we will apply resolution by showing $KB \wedge \neg\alpha$ is unsatisfiable.

Resolution

$$\forall x \;\; Horse(x) \Rightarrow Mammal(x)$$
$$\forall x \;\; Cow(x) \Rightarrow Mammal(x)$$
$$\forall x \;\; Pig(x) \Rightarrow Mammal(x)$$

Resolution

$$\forall x \;\; Horse(x) \Rightarrow Mammal(x)$$
$$\forall x \;\; Cow(x) \Rightarrow Mammal(x)$$
$$\forall x \;\; Pig(x) \Rightarrow Mammal(x)$$

- **In CNF:**

$$\neg Horse(x) \lor Mammal(x)$$
$$\neg Cow(x) \lor Mammal(x)$$
$$\neg Pig(x) \lor Mammal(x)$$

Resolution

$$\forall x, y \ \ Offspring(x, y) \land Horse(y) \Rightarrow Horse(x)$$

Resolution

$$\forall x, y \;\; Offspring(x, y) \land Horse(y) \Rightarrow Horse(x)$$

- **In CNF:**

$$\neg(Offspring(x, y) \land Horse(y)) \lor Horse(x)$$
$$\neg Offspring(x, y) \lor \neg Horse(y) \lor Horse(x)$$

Resolution

$Horse(Bluebeard)$
$Parent(Bluebeard, Charlie)$

Resolution

- **Already in CNF:**

$$Horse(Bluebeard)$$
$$Parent(Bluebeard, Charlie)$$

Resolution

$$\forall x, y \;\; Offspring(x, y) \Rightarrow Parent(y, x)$$
$$\forall x, y \;\; Parent(x, y) \Rightarrow Offspring(y, x)$$

Resolution

$$\forall x, y \;\; Offspring(x, y) \Rightarrow Parent(y, x)$$
$$\forall x, y \;\; Parent(x, y) \Rightarrow Offspring(y, x)$$

- **In CNF:**

$$\neg Offspring(x, y) \vee Parent(y, x)$$
$$\neg Parent(x, y) \vee Offspring(y, x)$$

Resolution

$$\forall x \exists y \;\; Mammal(x) \Rightarrow Parent(y, x)$$

Resolution

$$\forall x \exists y \ \ Mammal(x) \Rightarrow Parent(y, x)$$

- **In CNF:**

  $\neg Mammal(x) \vee Parent(G(x), x)$ where $G(x)$ is a skolem constant

Resolution

- **KB in CNF:**
  1) $\neg Horse(x) \vee Mammal(x)$
  2) $\neg Cow(x) \vee Mammal(x)$
  3) $\neg Pig(x) \vee Mammal$
  4) $\neg Offspring(x, y) \vee \neg Horse(y) \vee Horse(x)$
  5) $Horse(Bluebeard)$
  6) $Parent(Bluebeard, Charlie)$
  7) $\neg Offspring(x, y) \vee Parent(y, x)$
  8) $\neg Parent(x, y) \vee Offspring(y, x)$
  9) $\neg Mammal(x) \vee Parent(G(x), x)$

Resolution

1) $\neg Horse(x) \lor Mammal(x)$
2) $\neg Cow(x) \lor Mammal(x)$
3) $\neg Pig(x) \lor Mammal$
4) $\neg Offspring(x, y) \lor \neg Horse(y) \lor Horse(x)$
5) $Horse(Bluebeard)$
6) $Parent(Bluebeard, Charlie)$
7) $\neg Offspring(x, y) \lor Parent(y, x)$
8) $\neg Parent(x, y) \lor Offspring(y, x)$
9) $\neg Mammal(x) \lor Parent(G(x), x)$

10) $\neg \boldsymbol{\alpha}$: $\neg \boldsymbol{Horse(Charlie)}$

Resolution

1) $\neg Horse(x) \vee Mammal(x)$
2) $\neg Cow(x) \vee Mammal(x)$
3) $\neg Pig(x) \vee Mammal$
4) $\neg Offspring(x, y) \vee \neg Horse(y) \vee Horse(x)$
5) $Horse(Bluebeard)$
6) $Parent(Bluebeard, Charlie)$
7) $\neg Offspring(x, y) \vee Parent(y, x)$
8) $\neg Parent(x, y) \vee Offspring(y, x)$
9) $\neg Mammal(x) \vee Parent(G(x), x)$
10) **$\neg \boldsymbol{\alpha}$: $\neg \boldsymbol{Horse(Charlie)}$**
11) **$\boldsymbol{Offspring(Charlie, Bluebird)}$ by using 6 and 8 (x:Bluebeard, y:Charlie)**

Resolution

1) $\neg Horse(x) \vee Mammal(x)$
2) $\neg Cow(x) \vee Mammal(x)$
3) $\neg Pig(x) \vee Mammal$
4) $\neg Offspring(x, y) \vee \neg Horse(y) \vee Horse(x)$
5) $Horse(Bluebeard)$
6) $Parent(Bluebeard, Charlie)$
7) $\neg Offspring(x, y) \vee Parent(y, x)$
8) $\neg Parent(x, y) \vee Offspring(y, x)$
9) $\neg Mammal(x) \vee Parent(G(x), x)$
10) **$\neg\boldsymbol{\alpha}$: $\neg \boldsymbol{Horse(Charlie)}$**
11) **$\boldsymbol{Offspring(Charlie, Bluebird)}$ by using 6 and 8 (x:Bluebeard, y:Charlie)**
12) **$\boldsymbol{\neg Horse(Bluebeard) \vee Horse(Charlie)}$ by using 4 and 11 (x:Charlie, y:Bluebeard)**

Resolution

1) $\neg Horse(x) \vee Mammal(x)$
2) $\neg Cow(x) \vee Mammal(x)$
3) $\neg Pig(x) \vee Mammal$
4) $\neg Offspring(x, y) \vee \neg Horse(y) \vee Horse(x)$
5) $Horse(Bluebeard)$
6) $Parent(Bluebeard, Charlie)$
7) $\neg Offspring(x, y) \vee Parent(y, x)$
8) $\neg Parent(x, y) \vee Offspring(y, x)$
9) $\neg Mammal(x) \vee Parent(G(x), x)$
10) $\neg \boldsymbol{\alpha}$: $\neg \boldsymbol{Horse(Charlie)}$
11) $\boldsymbol{Offspring(Charlie, Bluebird)}$ **by using 6 and 8 (x:Bluebeard, y:Charlie)**
12) $\neg \boldsymbol{Horse(Bluebeard)} \vee \boldsymbol{Horse(Charlie)}$ **by using 4 and 11 (x:Charlie, y:Bluebeard)**
13) $\boldsymbol{Horse(Charlie)}$ **by using 5 and 12**

Resolution

1) $\neg Horse(x) \lor Mammal(x)$
2) $\neg Cow(x) \lor Mammal(x)$
3) $\neg Pig(x) \lor Mammal$
4) $\neg Offspring(x, y) \lor \neg Horse(y) \lor Horse(x)$
5) $Horse(Bluebeard)$
6) $Parent(Bluebeard, Charlie)$
7) $\neg Offspring(x, y) \lor Parent(y, x)$
8) $\neg Parent(x, y) \lor Offspring(y, x)$
9) $\neg Mammal(x) \lor Parent(G(x), x)$
10) $\boldsymbol{\neg\alpha}$: $\boldsymbol{\neg Horse(Charlie)}$
11) $\boldsymbol{Offspring(Charlie, Bluebird)}$ **by using 6 and 8 (x:Bluebeard, y:Charlie)**
12) $\boldsymbol{\neg Horse(Bluebeard) \lor Horse(Charlie)}$ **by using 4 and 11 (x:Charlie, y:Bluebeard)**
13) $\boldsymbol{Horse(Charlie)}$ **by using 5 and 12**
14) **{} empty clause by using 10 and 13 →** $KB \land \neg\alpha$ is unsatisfiable