$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -6 & -5 \\ 0 & 0 & 0.5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -8 \\ 16 \\ 3 \end{bmatrix}$$

$0.5x_3 = 3$

$x_3 = 6$

$-6x_2 - 5 * 6 = 16$

$x_2 = -\frac{23}{3}$

$x_1 + 2x_2 + x_3 = -8$

$x_1 = \frac{4}{3}$

Details of LU decomposition.

A=LU

LUx=B

Ux=L^{-1}B

x=$U^{-1}L^{-1}$B

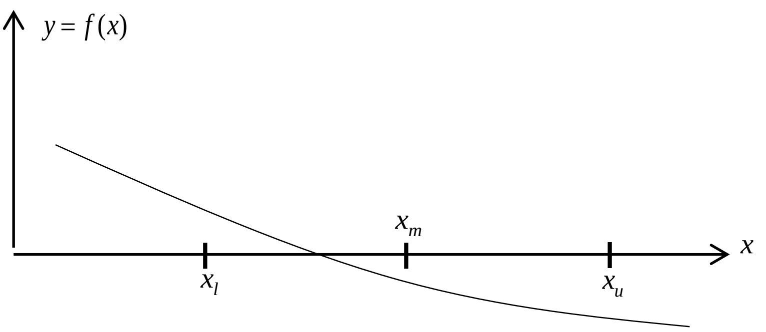
x=L^{-1}Z

Practice Session for week 4

Example of Bisection Method

A function $f(x)$ is defined as $f(x) = x^2 - 4x - 5$. For $y = f(x) = 0$ estimate a root of this function using Bisection method. Use $[0,48]$ as initial estimation range points and use absolute relative approximate error notation for error of estimated root at each iteration.

Recall



$$x_m = \frac{x_l + x_u}{2}$$

$$|E| = \left| \frac{x_m^{new} - x_m^{old}}{x_m^{new}} \right| * 100$$

$f(x_l) * f(x_m) < 0$ ***the root lies between x_l and x_m***

$f(x_l) * f(x_m) > 0$ ***the root lies between x_m and x_u***

$f(x_l) * f(x_m) = 0$ ***x_m is root***

$x_l = 0 \qquad f(x_l) = f(0) = 0 - 0 - 5 = -5$

$x_u = 48 \qquad f(x_u) = f(48) = 2304 - 192 - 5 = 2107 \qquad\qquad x_m = (0 + 48)/2 = 24$

$x_m = 24 \qquad f(x_m) = f(24) = 576 - 96 - 5 = 475$

$f(x_l) * f(x_m) < 0$ *the root lies between* x_l *and* x_m *so new estimation range* $[0,24]$

$x_l = 0 \qquad f(x_l) = f(0) = 0 - 0 - 5 = -5$

$x_u = 24 \qquad f(x_u) = f(24) = 576 - 96 - 5 = 475 \qquad\qquad x_m = (0 + 24)/2 = 12$

$x_m = 12 \qquad f(x_m) = f(12) = 144 - 48 - 5 = 91$

$f(x_l) * f(x_m) < 0$ *the root lies between* x_l *and* x_m *so new estimation range* $[0,12]$

$x_l = 0 \qquad f(x_l) = f(0) = 0 - 0 - 5 = -5$

$x_u = 12 \qquad f(x_u) = f(12) = 144 - 48 - 5 = 91 \qquad\qquad x_m = (0 + 12)/2 = 6$

$x_m = 6 \qquad f(x_m) = f(6) = 36 - 24 - 5 = 7$

$f(x_l) * f(x_m) < 0$ *the root lies between* x_l *and* x_m *so new estimation range* $[0,6]$

$x_l = 0 \qquad f(x_l) = f(0) = 0 - 0 - 5 = -5$

$x_u = 6 \qquad f(x_u) = f(6) = 36 - 24 - 5 = 7 \qquad\qquad x_m = (0 + 6)/2 = 3$

$x_m = 3 \qquad f(x_m) = f(3) = 9 - 12 - 5 = -8$

$f(x_l) * f(x_m) > 0$ *the root lies between* x_m *and* x_u *so new estimation range* $[3,6]$

$x_l = 3 \qquad f(x_l) = f(3) = 9 - 12 - 5 = -8$

$x_u = 6 \qquad f(x_u) = f(6) = 36 - 24 - 5 = 7 \qquad\qquad x_m = (3 + 6)/2 = 4{,}5$

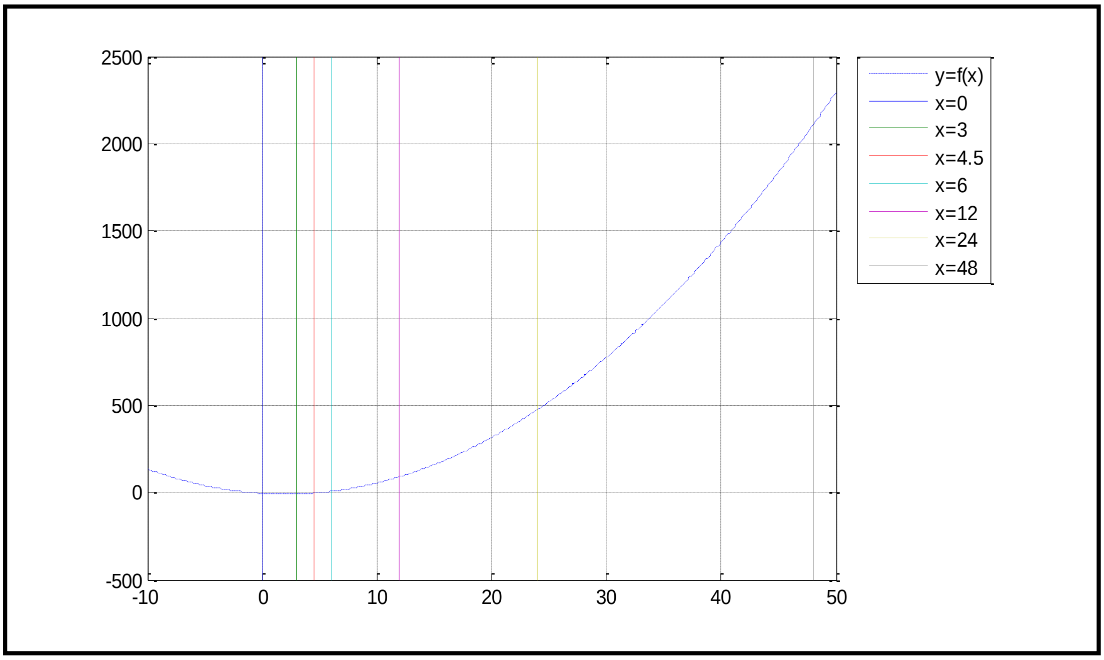
$x_m = 4{,}5 \qquad f(x_m) = f(4{,}5) = 20{,}25 - 18 - 5 = -2{,}75$

$f(x_l) * f(x_m) > 0$ *the root lies between* x_m *and* x_u *so new estimation range* $[4.5\ ,\ 6]$

Table - Estimation of root for initial range given at each iteration. Iteration will be continued until root is estimated with an acceptable error.

Iteration	x_l	x_u	x_m	$f(x_l)$	$f(x_u)$	$f(x_m)$	E
1	**0**	**48**	**24**	**-5**	**2107**	**475**	**-**
2	**0**	**24**	**12**	**-5**	**475**	**91**	**%100**
3	**0**	**12**	**6**	**-5**	**91**	**7**	**%100**
4	**0**	**6**	**3**	**-5**	**7**	**-8**	**%100**
5	**3**	**6**	**4,5**	**-8**	**7**	**-2,75**	**%33,33**

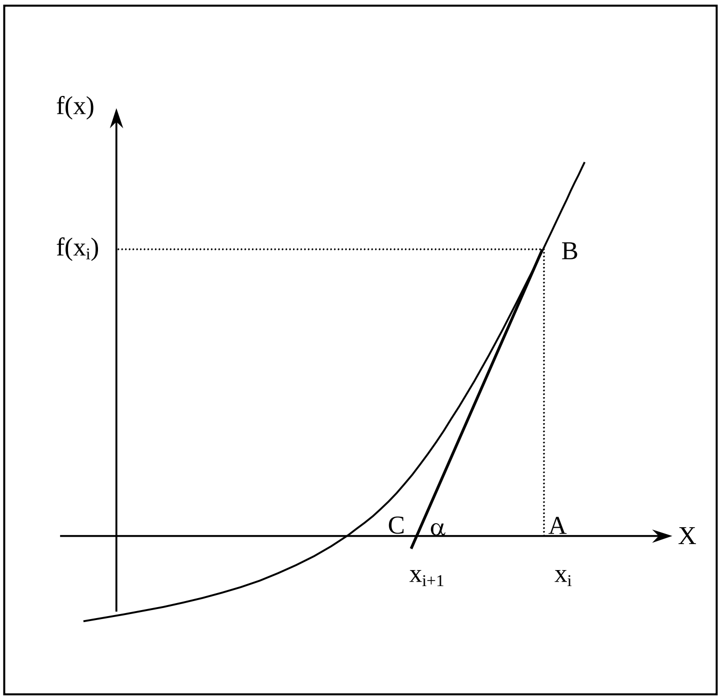
Figure - Bisection lines for given non-linear equation.



Example of Newton Raphson Method

A function $f(x)$ is defined as $f(x) = x^3 - 10x^2 + 100$. For $y = f(x) = 0$ estimate a root of this function using Newton-Raphson method. Initial guess value of root x_0 is 15 and use absolute relative approximate error notation for error of estimated root.

Recall



$$\boxed{\tan(\alpha) = \frac{AB}{AC}}$$

$$\boxed{f'(x_i) = \frac{f(x_i)}{x_i - x_{i+1}}}$$

$$\boxed{x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}}$$

Gist of prediction of x_{i+1}

$$\boxed{|E| = \left| \frac{x_{i+1} - x_i}{x_{i+1}} \right| * 100}$$

Function : $f(x) = x^3 - 10x^2 + 100$

Derivative of function : $f^{'}(x) = 3x^2 - 20x$

$$x_1 = 15 - \frac{f(15)}{f^{'}(15)} = 15 - \frac{3375 - 2250 + 100}{675 - 300} = 15 - \frac{1225}{375} = 15 - 3,26 = 11,74$$

$$x_2 = 11,74 - \frac{f(11,74)}{f^{'}(11,74)} = 11,74 - \frac{1618,09 - 1378,27 + 100}{413,48 - 234,8} = 11,74 - \frac{339,82}{178,68} = 11,74 - 1,89 = 9,85$$

$$x_3 = 9,85 - \frac{f(9,85)}{f^{'}(9,85)} = 9,85 - \frac{955,67 - 970,22 + 100}{291,06 - 197} = 9,85 - \frac{85,45}{94,06} = 9,85 - 1,89 = 8,94$$

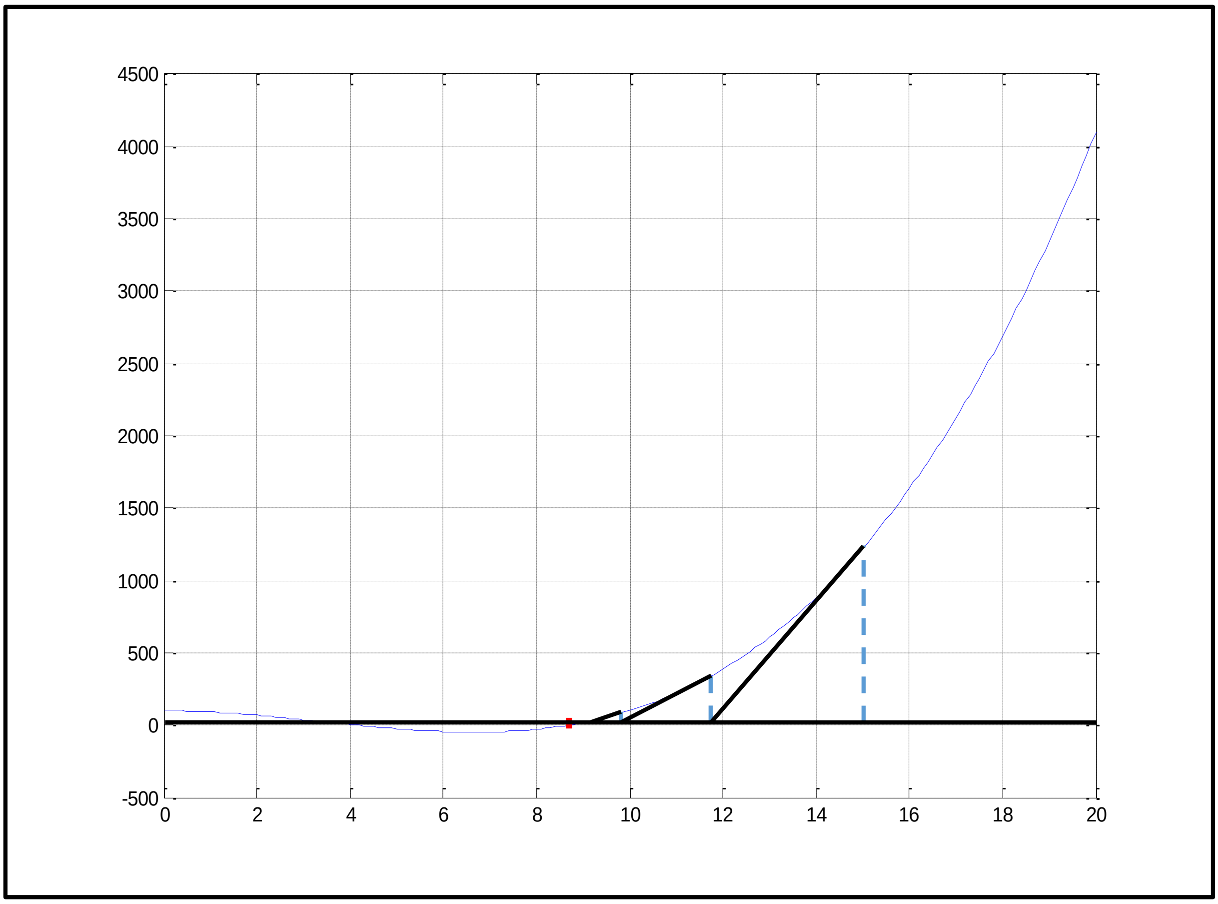
$$x_4 = 8,94 - \frac{f(8,94)}{f^{'}(8,94)} = 8,94 - \frac{714,51 - 799,23 + 100}{239,77 - 178,8} = 8,94 - \frac{15,28}{60,97} = 8,94 - 0,25 = 8,69$$

$$x_5 = 8,69 - \frac{f(8,69)}{f^{'}(8,69)} = 8,69 - \frac{656,23 - 755,16 + 100}{226,54 - 173,8} = 8,69 - \frac{1,07}{52,74} = 8,69 - 0,02 = 8,67$$

Table - Estimation of root for the function described in Q2 at each iteration. Iteration will be continued until root is estimated with an acceptable error.

Iteration	x	$f(x)$	$f^{'}(x)$	E
0	$x_0 = 15$	**1225**	**375**	**-**
1	$x_1 = 11,74$	**339,82**	**178,68**	**%27,76**
2	$x_2 = 9,85$	**85,45**	**94,06**	**%19,18**
3	$x_3 = 8,94$	**15,28**	**60,97**	**%10,17**
4	$x_4 = 8,69$	**1,07**	**52,74**	**%2,87**
5	$x_5 = 8,67$	**-**	**-**	**%0,23**

Figure – Graphical analysisof Newton-Raphson method for given non-linear equation.


4500
4000
3500
3000
2500
2000
1500
1000
500
0
-500
0
2
4
6
8
10
12
14
16
18
20