1: Casuality and Stability

(a)

Not casual: The function is dependent on future
Stable: x and y can be bounded.

(b)

Casual: The function is independent on future
Not stable: x and y cannot be bounded.

(c)

Casual: The function is independent on future
Stable: The function cannot exceed 1.

(d)

Casual: The function is independent on future
Stable: The function cannot exceed 1.

2: Convolution

$$
\begin{aligned}
h(t) &= 5e^{-0.5(t-3)}[u(t-3) - u(t-11)] \\
x(t) &= u(t-2) \\
y[n] &= \sum_{k=-\infty}^{\infty} h[k][n-k] \\
&= \sum_{k=-\infty}^{\infty} 5e^{-0.5(k-3)}[u(k-3) - u(k-11)]u(n-2-k)
\end{aligned}
$$

3: FFT Implementation

```
Emre:Homework3 KEO$ python3 fft.py 10
Array size = 10
Built-in fft algorithm tooks 8.988380432128906e-05 seconds
My fft algorithm tooks 9.918212890625e-05 seconds
Results are not the same
Emre:Homework3 KEO$ python3 fft.py 100
Array size = 100
Built-in fft algorithm tooks 0.00011181831359863281 seconds
My fft algorithm tooks 0.0002200603485107422 seconds
Results are the same
Emre:Homework3 KEO$ python3 fft.py 1000
Array size = 1000
Built-in fft algorithm tooks 0.0003399848937988281 seconds
My fft algorithm tooks 0.001371145248413086 seconds
Results are the same
Emre:Homework3 KEO$ python3 fft.py 10000
Array size = 10000
Built-in fft algorithm tooks 0.002541780471801758 seconds
My fft algorithm tooks 0.012373924255371094 seconds
Results are the same
Emre:Homework3 KEO$ python3 fft.py 100000
Array size = 100000
Built-in fft algorithm tooks 0.02759099006652832 seconds
My fft algorithm tooks 0.13215303421020508 seconds
Results are the same
Emre:Homework3 KEO$ python3 fft.py 1000000
Array size = 1000000
Built-in fft algorithm tooks 0.3025031089782715 seconds
My fft algorithm tooks 1.2809691429138184 seconds
Results are the same
Emre:Homework3 KEO$
```

Figure 1: The compare results of fft algorithms

4: Convolution

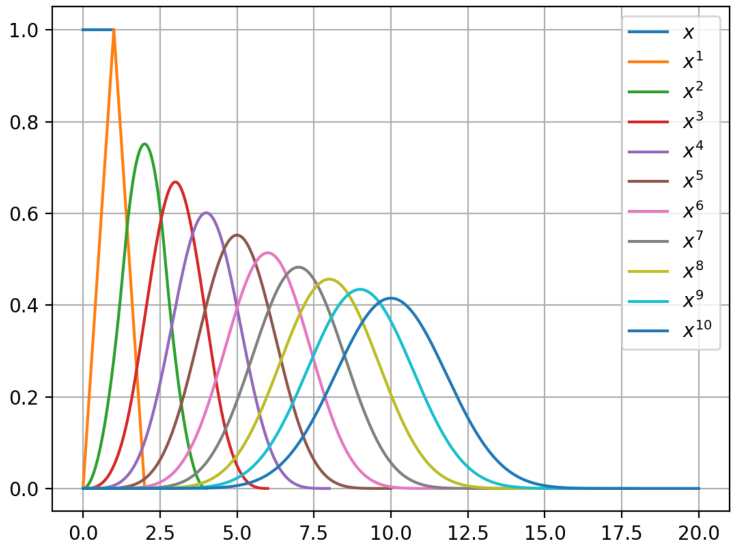

Figure 2: The graph of convolution result

5: Frequency Response and Superposition

(a)

$$\begin{aligned}
H(jw) &= \int_{-\infty}^{\infty} h(t)e^{-jwt}dt \\
H(jw) &= \int_{-\infty}^{\infty} (\delta(t-2) - 0.2e^{-0.2(t-2)}[u(t-2)])e^{-jwt}dt \\
&= \int_{-\infty}^{\infty} \delta(t-2)e^{-jwt}dt + \int_{-\infty}^{\infty} -0.2e^{-0.2(t-2)}[u(t-2)]e^{-jwt}dt \\
&= e^{-2jw} - 0.2e^{0.4}\int_{2}^{\infty} e^{-(0.2+jw)t}dt \\
&= e^{-2jw} + 0.2e^{0.4}\frac{e^{-(0.4+2jw)}}{-(0.2+jw)} \\
&= e^{-2jw} - \frac{0.2e^{-2jw}}{0.2+jw} \\
&= \frac{jwe^{-2jw}}{0.2+jw}
\end{aligned}$$

(b)

(c)

$$\begin{aligned}
x_1(t) &= 5, \qquad x_2(t) = 10cos(0.2t), \qquad x_3(t) = u(t) \\
y_1(t) &= H(j0) = 0 \\
y_2(t) &= 10\frac{e^{-0.4j}}{\sqrt{2}}cos(0.2t + \frac{\pi}{4}) \\
y_3(t) &= u(t) \cdot h(t) \\
&= u(t-2) + \int_{2}^{\infty} -0.2e^{-0.2(t-2)}u(t-2)u(\tau - t)dt \\
&= u(t-2)(1 - 0.2e^{0.4}(t-2)) \\
y(t) &= 10\frac{e^{-0.4j}}{\sqrt{2}}cos(0.2t + \frac{\pi}{4}) + u(t-2)(1 - 0.2e^{0.4}(t-2))
\end{aligned}$$

6: Fourier Transforms

(a)

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{sin^2(10w)}{2w^2}e^{jwt}dw$$

(b)

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{1}{25+w^2}e^{jwt}dw$$

(c)

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{-a(t-2)}u(t-2)cos(w_0t)e^{-jwt}dt$$
$$=\frac{1}{2\pi}\int_{2}^{\infty}e^{-a(t-2)}cos(w_0t)e^{-jwt}dt$$

7: