Discrete Mathematics - Midterm

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1 Instructions

1. Each question is worth 4 points.
2. Attempt as many problems as you can.
3. $F(n)$ denotes the n^{th} number in the Fibonacci sequence defined and discussed in class.

2 Problems

1. *Propositional Logic.*
   Prove the validity of the following arguments:
   (a) $(P \wedge P') \rightarrow Q$.
   (b) $[(A \rightarrow (B \vee C)) \wedge C'] \rightarrow (A \rightarrow B)$.
2. *Predicate Logic.*
   (a) Show that the argument $(\exists x)[P(x) \rightarrow Q(x)] \rightarrow [(\forall x)P(x) \rightarrow (\exists x)Q(x)]$ is valid.
   (b) Consider the verbal argument: *Some plants are flowers. All flowers smell sweet. Therefore, some plants smell sweet.* Is it valid?
3. *Non-Inductive Proof.*
   Prove the following two conjectures.
   (a) A number n is odd if and only if $3n + 5$ is even.
   (b) The square of an odd integer equals $8k + 1$ for some integer k.
4. *Inductive Proof.*
   (a) Show that $5 \mid (7^n - 2^n), \forall n \geq 0$.
   (b) Consider the following inductive proof, which shows that $n = n + 1$, for all integers n. Assume that $P(k)$ is true, i.e., $k = k + 1$. Adding 1 to both sides, we get $k + 1 = (k + 1) + 1$, i.e., $k + 1 = k + 2$ and hence $P(k + 1)$ is true. It therefore, follows that $n = n + 1$, for all integers n, using the first principle of mathematical induction. What is wrong with this proof?

5. *Recurrences.*

(a) Solve the recurrence:

$$\begin{aligned} S(1) &= 4 \\ S(2) &= -2 \\ S(n) &= -S(n-1) + 2 \cdot S(n-2),\ n \geq 3 \end{aligned}$$

(b) Show that $F(n+1) + F(n-2) = 2F(n)$, $n \geq 3$.