BLG 454E Learning From Data (Spring 2018)
Homework I

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March 9, 2018

1 Question 1

$$P(A) = \frac{1}{4} \text{ (Rains on Saturday)}$$

$$P(B) = \frac{1}{4} \cdot \frac{1}{2} + \frac{3}{4} \cdot \frac{1}{4} = \frac{5}{16} \text{ (Rains on Sunday)}$$

$$P(A \mid B) = \frac{P(A,B)}{P(B)} = \frac{\frac{1}{4} \cdot \frac{1}{2}}{\frac{5}{16}} = \frac{2}{5}$$

2 Question 2

If starting point is in {A} $\rightarrow \frac{1}{7}$

If starting point is in {B, F} $\rightarrow \frac{2}{7} \cdot (\frac{1}{3} + \frac{1}{3} \cdot \frac{1}{6}) = \frac{1}{9}$

If starting point is in {G} $\rightarrow \frac{1}{7} \cdot (\frac{1}{6} + \frac{2}{6} \cdot \frac{1}{3}) = \frac{5}{126}$

If starting point is in {C, E} $\rightarrow \frac{2}{7} \cdot (\frac{1}{3} \cdot \frac{1}{6} + \frac{1}{3} \cdot \frac{1}{3}) = \frac{1}{21}$

If starting point is in {D} $\rightarrow \frac{1}{7} \cdot (\frac{1}{3} \cdot \frac{1}{6}) = \frac{1}{126}$

$$Answer = \frac{1}{7} + \frac{1}{9} + \frac{5}{126} + \frac{1}{21} + \frac{1}{126} = \frac{22}{63}$$

3 Question 3

$$\mu = \frac{\Sigma x_i}{n}, \quad \sigma^2 = \frac{\Sigma (x_i - \mu)^2}{n}$$

$$P(x_1, x_2 ... x_n \mid \mu, \sigma^2) = \frac{1}{\sigma \cdot \sqrt{2 \cdot \pi}} \cdot e^{\dfrac{-(x-\mu)^2}{2 * \sigma^2}}$$

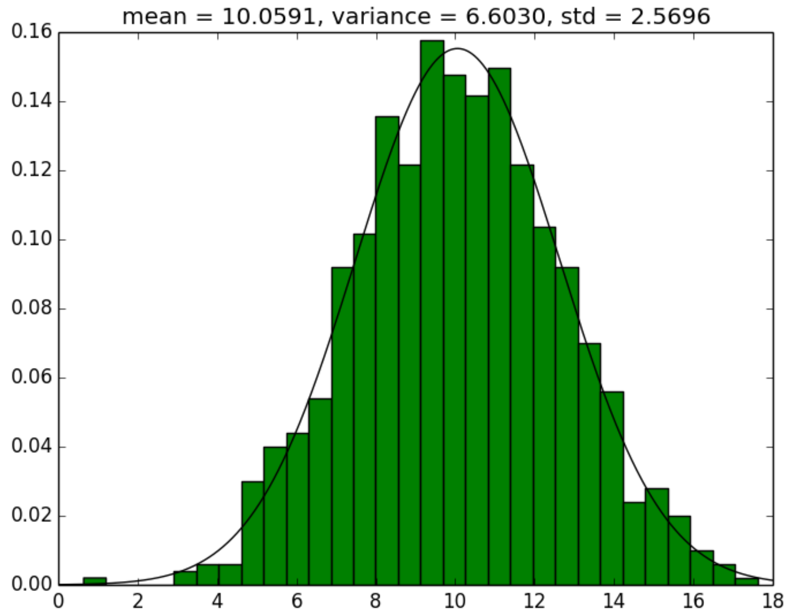

Figure 1: A Data and fixed gaussian distribution with MLE

4 Question 3

a) Naive Bayes classifier:

	X_1		X_2		X_3	
Likelihood	YES	NO	YES	NO	YES	NO
+	$\frac{3}{5}$	$\frac{2}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{1}{5}$
-	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{1}{5}$	$\frac{4}{5}$

b)
probability that the given data is +:

$$P(+ \mid x_1 \cap x_2 \cap x_3) = \frac{P(x_1 \mid +) \cdot P(x_2 \mid +) \cdot P(x_3 \mid +) \cdot P(+)}{P(x_1) \cdot P(x_2) \cdot P(x_3)}$$
$$= \frac{3}{5} \cdot \frac{2}{5} \cdot \frac{4}{5} \cdot \frac{1}{2} = \frac{12}{125}$$

probability that the given data is -:

$$P(- \mid x_1 \cap x_2 \cap x_3) = \frac{P(x_1 \mid -) \cdot P(x_2 \mid -) \cdot P(x_3 \mid -) \cdot P(-)}{P(x_1) \cdot P(x_2) \cdot P(x_3)}$$
$$= \frac{2}{5} \cdot \frac{2}{5} \cdot \frac{1}{5} \cdot \frac{1}{2} = \frac{2}{125}$$

$$\frac{P(+ \mid x_1 \cap x_2 \cap x_3)}{P(+ \mid x_1 \cap x_2 \cap x_3) + P(- \mid x_1 \cap x_2 \cap x_3)} = \frac{6}{7}$$

c)

$$P(x_1) = \frac{1}{2}$$
$$P(x_2) = \frac{2}{5}$$
$$P(x_1 \mid x_2) = \frac{1}{2}$$
$$P(x_1, x_2) = \frac{1}{5}$$
$$P(x_1, x_2) = P(x_1 \mid x_2) \cdot P(x_2) = \frac{1}{2} \cdot \frac{2}{5} = \frac{1}{5}$$

Because of the formulas above give the same result, we can say x_1 and x_2 are dependent.