MAT202E Midterm Solutions

1) For the conversions given below, perform the operations step by step.

a) $(160,343)_{10} = (?)_2$

<u>Number</u>	<u>Quotient</u>	<u>Remainder</u>	<u>Number</u>	<u>Quotient</u>	<u>Integer part.</u>
$160/2$	80	$0 \rightarrow a_0$	$0.343*2$	0.686	$0 \rightarrow a_{-1}$
$80/2$	40	$0 \rightarrow a_1$	$0.686*2$	1.372	$1 \rightarrow a_{-2}$
$40/2$	20	$0 \rightarrow a_2$	$0.372*2$	0.744	$0 \rightarrow a_{-3}$
$20/2$	10	$0 \rightarrow a_3$	$0.744*2$	1.488	$1 \rightarrow a_{-4}$
$10/2$	5	$0 \rightarrow a_4$	$0.488*2$	0.976	$0 \rightarrow a_{-5}$
$5/2$	2	$0 \rightarrow a_5$	$0.976*2$	1.952	$1 \rightarrow a_{-6}$
$2/2$	1	$0 \rightarrow a_6$	$0.952*2$	1.904	$1 \rightarrow a_{-7}$
$1/2$	0	$0 \rightarrow a_7$	$0.904*2$	1.808	$1 \rightarrow a_{-8}$
			$0.808*2$	1.616	$1 \rightarrow a_{-9}$
			$0.616*2$	1.232	$1 \rightarrow a_{-10}$

$$(160,343)_{10} = (10100000,0101011111)_2$$

b) $(11001,1001)_{10} = (?)_2$

$$\begin{aligned}
(11001)_2 &= (1\cdot 2^0)+(0\cdot 2^1)+(0\cdot 2^2)+(1\cdot 2^3)+(1\cdot 2^4) \\
&= 1+0+0+8+16 \\
&= 25
\end{aligned}$$

$$\begin{aligned}
(\ldots,1001)_2 &= (1\cdot 2^{-1})+(0\cdot 2^{-2})+(0\cdot 2^{-3})+(1\cdot 2^{-4}) \\
&= (0.5)+(0)+(0)+(0.0625) \\
&= 0.5625
\end{aligned}$$

$$(11001,1001)_{10} = (25,5625)_2$$

2) Represent -5,5834x10^{10} as a single precision floating point number using IEEE-754 format

Value = $(-1)^s x(1.m)_2 x2^{e'-127}$

$(-5,5834x10^{10}) \cong (-1)x(1,625)x2^{35}$ $\quad\quad$ $(162)_{10} = (10100010)_2$

$e'-127 = 35$ $\quad\quad$ $e' = 162$ $\quad\quad$ $(0,625)_{10} = (,101)_2$

1	**1**	**0**	**1**	**0**	**0**	**0**	**1**	**0**	**1**	**0**	**1**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
s				e'																m											

3) If B is increased by +2 and C increased by +1, the robot moves 8 step backwards. If A is increased by +1, B is decreased by -2 and C is decreased by -3, the robot does not move. If A is decreased by -1, B is increased by 1 and C is increased by 2, the robot moves 3 steps forward. (A, B, C buttons)

- Write the linear system model of the control panel.
- Solve the linear system by Naïve Gaussian Elimination method.
- Write the pseudo-code of the forward-elimination phase for the algorithm. (*Please refer to lecture notes*)

$$2x_2 + x_3 = -8$$
$$x_1 - 2x_2 - 3x_3 = 0$$
$$-x_1 + x_2 + 2x_3 = 3$$

$$\left[\begin{array}{ccc|c} 0 & 2 & 1 & -8 \\ 1 & -2 & -3 & 0 \\ -1 & 1 & 2 & 3 \end{array}\right]$$
Swap R1 and R2

$$\left[\begin{array}{ccc|c} 1 & -2 & -3 & 0 \\ 0 & 2 & 1 & -8 \\ -1 & 1 & 2 & 3 \end{array}\right]$$
Add R1 to R3

$$\left[\begin{array}{ccc|c} 1 & -2 & -3 & 0 \\ 0 & 2 & 1 & -8 \\ 0 & -1 & -1 & 3 \end{array}\right]$$
Swap R2 and R3

$$\left[\begin{array}{ccc|c} 1 & -2 & -3 & 0 \\ 0 & -1 & -1 & 3 \\ 0 & 2 & 1 & -8 \end{array}\right]$$
Add twice R2 to R3

$$\left[\begin{array}{ccc|c} 1 & -2 & -3 & 0 \\ 0 & -1 & -1 & 3 \\ 0 & 0 & -1 & -2 \end{array}\right]$$
Add -1 times R3 to R2
Add -3 times R3 to R1

$$\left[\begin{array}{ccc|c} 1 & -2 & 0 & 6 \\ 0 & -1 & 0 & 5 \\ 0 & 0 & -1 & -2 \end{array}\right]$$
Add -2 times R2 to R1

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -4 \\ 0 & -1 & 0 & 5 \\ 0 & 0 & -1 & -2 \end{array}\right]$$
Multiply R2 by -1
Multiply R3 by -1

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 2 \end{array}\right] \qquad \begin{array}{l} x_1 = -4 \\ x_2 = -5 \\ x_3 = 2 \end{array}$$

4) Given the experimental data below, answer the following questions, by using the Newton's Divided Difference Method:
   - Draw the divided difference table for degree 3.
   - Estimate f(0,3).
   - It is discovered that f(0,4) is underestimated by 10 and f(0,6) is overestimated by 5. Under these new circumstances, by what amount (in percentage) the estimation of f(0,3) found in question-b is changed?

x	0.0	0.2	0.4	0.6
f(x)	15.0	21.0	30.0	51.0

x	**f(x)**	**[1]**	**[2]**	**[3]**
0.0	15.0	30	37.5	187.5
0.2	21.0	45	150	
0.4	30.0	105		
0.6	51.0			

Corresponding polynomial degree 3 obtained by using formula

$f(x) = f_0^{[0]} + (x - x_0) f_0^{[1]} + (x - x_0)(x - x_1) f_0^{[2]} + (x - x_0)(x - x_1)(x - x_2) f_0^{[3]}$ Therefore;

$$f(x) = 15.0 + (x)30 + (x)(x-0.2)37.5 + (x)(x-0.2)(x-0.4)187.5$$
$$f(x) = 187.5x^3 - 75x^2 + 37.5x + 15$$

And the value of $f(0.3)$ is;
$$f(0.3) = 24.5625$$

Updating overestimated and underestimated numbers.

x	**f(x)**	**[1]**	**[2]**	**[3]**
0.0	15.0	30	162.5	-541.6667
0.2	21.0	95	-162.5	
0.4	40.0	30		
0.6	46.0			

$$f(x) = 15.0 + (x)30 + (x)(x-0.2)162.5 + (x)(x-0.2)(x-0.4)\left(-\frac{1625}{3}\right)$$

$$f(x) = \frac{-1625x^3}{3} + 487.5x^2 - \frac{275x}{6} + 15$$

And the value of $f(0.3)$ is;
$$f(0.3) = 30.5$$

5) A loan of ***A*** Turkish Lira (TL) is repaid by making n equal monthly payments of ***M*** TL, starting a month after the loan is made. It can be shown that if the monthly interest rate is ***r***, then

$$A.r = M\left(1 - \frac{1}{(1+r)^n}\right)$$

- A car loan of 10000 TL was repaid in 60 monthly payments of 250 TL. Use the Newton-Raphson Method to find the monthly interest rate with the absolute relative approximate error smaller than %0,0019.
- Write the pseudo-code for the algorithm. (*Please refer to lecture notes*)

$$Ar = M\left(1 - \frac{1}{(1+r)^n}\right) \qquad 10000r = 250\left(1 - \frac{1}{(1+r)^{60}}\right)$$

$$f(r) = 40r + \frac{1}{(1+r)^{60}} - 1 \qquad f'(r) = 40 - \frac{60}{(1+r)^{61}}$$

And Newton's iteration;

$$r_{n+1} = r_n - \frac{40r_n + 1/(1+r_n)^{60} - 1}{40 - 60/(1+r_n)^{61}}$$