Operations Research - Midterm 1

Inst. Yakup Görkem Gökmen

2019 Fall
October 25
Duration: 120 mins.

Name and Surname: ____________________

Signature: ____________________

Question No.	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

Read the **Instructions** below carefully!

- The duration of the exam is 120 mins.
- You are not permitted to leave the exam in first 30 minutes.
- Asking questions to the observers is not allowed.
- Write your answers on the provided blank A4 papers.
- If you have a question, raise your hand and wait for instructor to come. Be careful not to disturb your friends.
- Any electronic devices will be turned off and put away. Calculators are NOT allowed.

Good luck to you all!

MIDTERM 1

1. (20 points) Consider a computer network consisting of n computers labeled 1 through n. Suppose that you wish to send K MBytes of information from computer 1 to computer n. For each computer i, $i = 1, \ldots, n$, let $L(i)$ denote the subset of computers with which computer i has a direct connection. Let c_{ij} denote the capacity of the (directed) link in MBytes from computer i, $i = 1, \ldots, n$ to computer $j \in L(i)$. Your goal is to send this information from computer 1 to computer n so that the maximum congestion on any link is as small as possible. Congestion on a link is defined to be the ratio of the amount of the information sent on that link per period to the capacity of the link. Formulate an LP problem to achieve this goal.

2. (20 points) Suppose that you would like to devise an investment plan. You have n investment options labeled $1, \ldots, n$. The annual expected rate of return on investment option j is given by c_j, $j = 1, \ldots, n$. You wish to find out the ratio of your assets to be allocated to each one of the n investment options. In order to ensure a diversified portfolio, you wish to ensure that the difference between the ratios of your assets allocated among any two different investment options should not be over a threshold value δ, where $\delta \in (0, 1)$. Furthermore, based on risk assessments, financial experts suggest a portfolio allocation vector given by $(r_1, \ldots, r_n)$, where r_j denotes the ratio allocated to investment option j, $j = 1, \ldots, n$, and $\sum_{j=1}^{n} r_j = 1$. You wish to design your portfolio in such a way that your actual allocation vector will not significantly deviate from the suggested allocation vector. The deviation is measured using the sum of the absolute values of the differences of the ratios corresponding to each investment option. You wish to ensure that the deviation does not exceed a threshold value β, where $\beta > 0$. Formulate an LP problem so as to maximize your total annual expected rate of return.

3. Consider the fixed cost transportation problem discussed in class. We are given a set V_1 of potential suppliers and a set V_2 of demand points. The capacity of potential supplier $i \in V_1$ is q_i units and the demand of customer $j \in V_2$ is d_j units. The unit cost of shipping from potential supplier $i \in V_1$ to demand point $j \in V_2$ is c_{ij}. The fixed cost of using a potential supplier $i \in V_1$ is f_i. Demand of each customer should be satisfied. However, you need to fulfill the following additional requirements:

   (a) (6 points) Facilities 1 and 2 cannot be used together.

   (b) (7 points) At least one of the following two requirements should be satisfied: (i) At least two of the facilities 3, 4, and 5 should be used; (ii) At most two of the facilities 6, 7, and 8 can be used.

   (c) (7 points) If the facilities 9 and 10 are used together, then neither one of the facilities 11 and 12 can be used.

   Formulate this version of the problem as an IP problem to minimize the total cost.

4. (20 points) The university wishes to schedule the final exams for the classes offered in the fall semester. Suppose that there are n courses labeled $1, \ldots, n$. Each course should be assigned to a 3-hour time slot during the final exam period. However, the registrar should ensure that no two classes taken by the same student are scheduled at the same time slot. For course i, let C_i denote the set of other courses that have at least one common enrolled student (so that course i cannot be scheduled at the same time slot with *any* one of the courses in C_i), $i = 1, \ldots, n$. The university wishes to schedule all the final exams using the smallest number of time slots. Formulate this problem as an IP problem.

5. (20 points) Suppose that you are trying to plan your course work. In your degree program, there are K specialization areas labeled $1, \ldots, K$. Let C denote the set of all offered courses. For each specialization area k, there is a set of designated courses $C_k \subseteq C$, $k = 1, \ldots, K$. In order to receive a certificate in specialization area k, you need to take at least l_k courses from the set C_k, $k = 1, \ldots, K$. Some courses are listed in more than one specialization area, i.e., the sets C_k may not be disjoint. However, a course $c \in C$ can be counted to fulfill the requirements of at most u_c specialization areas. You can take at most L courses in your degree program. You wish to select your courses in such a way that you fulfill the requirements of the largest number of specialization areas. Formulate this problem as an IP problem.