MAT 271E Probability and Statistics, Spring 2018
Homework #2
Due Nov 11, 2018 11pm

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Section 3.1: Random Variables

(3.20) Determine the mean and variance for the following continuous random variable X with probability density function f(x) given by

$$f(x) = \begin{cases} \frac{1}{\lambda} \cdot e^{-x/\lambda} & ; x >= 0 \text{ and } \lambda > 0 \\ 0 & ; \text{otherwise} \end{cases}$$

$$\begin{aligned}
\mu_x &= \int_{-\infty}^{\infty} x \cdot f(x) dx \\
&= \int_{-\infty}^{0} x \cdot 0 \cdot dx + \int_{0}^{\infty} x \cdot \frac{1}{\lambda} \cdot e^{-x/\lambda} dx \\
&= \frac{1}{\lambda} \cdot \int_{0}^{\infty} x \cdot e^{-x/\lambda} dx \\
&= \frac{1}{\lambda} \cdot (x \cdot (-\lambda) \cdot e^{-x/\lambda} \Big|_0^{\infty} - \int_0^{\infty} \lambda \cdot e^{-x/\lambda} dx) \\
&= \frac{1}{\lambda} \cdot (0 + \lambda * \lambda \cdot e^{-x/\lambda} \Big|_0^{\infty}) \\
&= \lambda \\
\sigma_x^2 &= \int_{-\infty}^{\infty} (x - \mu_x)^2 f(x) dx \\
&= \int_{-\infty}^{0} (x - \lambda)^2 \cdot 0 \cdot dx + \int_{0}^{\infty} (x - \lambda)^2 \cdot \frac{1}{\lambda} \cdot e^{-x/\lambda} dx \\
&= \frac{1}{\lambda} \cdot \int_{0}^{\infty} (x - \lambda)^2 * e^{-x/\lambda} * dx \\
&= \frac{1}{\lambda} \cdot ((x - \lambda)^2 \cdot (-\lambda \cdot e^{-x/\lambda}) \Big|_0^{\infty} - \int_0^{\infty} -\lambda * e^{-x/\lambda} \cdot 2 \cdot (x - \lambda) dx) \\
&= \frac{1}{\lambda} \cdot (0 + 2\lambda \cdot (\int_0^{\infty} x * e^{-x/\lambda} * dx - \lambda \int_0^{\infty} e^{-x/\lambda} dx)) \\
&= \frac{1}{\lambda} \cdot (0 + 2\lambda * (\lambda \cdot \lambda + \lambda \cdot \lambda)) \\
&= 4\lambda^2
\end{aligned}$$

Section 3.2: Permutations and Combinations

(3.20) A company wants to purchase 4 electronic systems. After all the system models were reviewed, 8 foreign made and 10 U.S. made systems were considered to satisfy all the security requirements for the company.

(a) if the systems are chosen at random, find the probability that 2 of the systems selected are foreign made.

$$P(x) = \frac{\binom{8}{2} * \binom{10}{2}}{\binom{18}{4}} = \frac{28 * 45}{3060} = 0.41176$$

(b) what is the probability that all the four systems selected are U.S. made?

$$P(x) = \frac{\binom{10}{4}}{\binom{18}{4}} = \frac{210}{3060} = 0.06862$$

(c) what is the probability that all of the 4 systems selected are foreign made?

$$P(x) = \frac{\binom{8}{4}}{\binom{18}{4}} = \frac{70}{3060} = 0.022875$$

(d) what is the probability that at least 2 of the systems are U.S. made?

$$P(x) = 1 - \left(\frac{\binom{8}{4}}{\binom{18}{4}} + \frac{\binom{8}{3} * \binom{10}{1}}{\binom{18}{4}} \right) = 0.794117$$

(3.40) If 3 persons are to be selected randomly from 5 persons for a committee, determine the different possible combinations.

$$\binom{5}{3} = 10$$

Section 3.3: Discrete Distributions

(3.45) Batches of 50 shock absorbers from a production process are tested for conformance to quality requirements. The mean number of non-conforming absorbers in a batch is 5. Assume that the number of non-conforming shock absorbers in a batch, denoted as x, is a binomial random variable.

(a) find n and p

(b) find p(x 2)

(c) find p(x 49).

(3.49) Five per cent of a large batch of high-strength steel components purchased for a mechanical system are defective.

(a) if seven components are randomly selected, find the probability that exactly three will be defective

(b) find the probability that two or more components will be defective.

(3.57) The probability that a person undergoes a heart operation will recover is 0.6. Determine the probability that of the six patients who undergo similar heart operation:

(a) none will recover

(b) all will recover

(c) half will recover

(d) at least half will recover.

(3.59) Given that the probability of an individual patient suffers a bad reaction from injection of a particular serum is 0.001. Determine the probability that out of 2000 individual patients.

(a) exactly 3 individuals will suffer a bad reaction

(b) more than 2 individuals will suffer a bad reaction. Use Poisson distribution.

(3.60) Use binomial distribution and repeat Problem 3.59.

Section 3.4: Continuous Probability Distributions

(3.72) The mass, , of a particular electronic component is normally distributed with a mean of 66 g and a standard deviation of 5 g. Determine

(a) the per cent of components that will have a mass less than 72 g

(b) the per cent of components that will have a mass in excess of 72 kg

(c) the per cent of components that will have a mass between 61 and 72 g.

Section 3.4: Approximating Probability Distributions

(3.80) Determine the probability that in a sample of 10 machine components chosen at random, exactly two will be defective by using

(a) the binomial distribution

(b) the Poisson approximate to the binomial distribution.

Given that 10% of the machine components produced in that manufacturing process are defective.