

Discrete Math I – Practice Problems for Exam II

The upcoming exam on Thursday, February 9 will cover the material in Sections 1.7, 1.8, 2.1, 2.2, 2.3, 2.4, and 4.1.

Note that this practice exam is NOT “synchronized” with what you will see on exam day. I won’t purposely present problems here and just give you the same problem with the numbers changed. That is, the following problems do not represent all of the possible types of problems that could appear on the exam. Problems chosen for the exam will be similar to homework problems, the quizzes, and examples done in class. Also note that the number of problems presented in this practice exam may not represent the actual length of the exam you see on the exam day. You should be prepared for a lengthy exam.

IMPORTANT! First try these problems as if it were the real exam; work by yourself without the text or your notes. This is supposed to be a gauge on what you need to work on to prepare for the exam. Answering these problems as you might handle homework problems won’t necessarily give you much of a clue on what you need to work on.

Instructions: Provide all steps necessary to solve the problem. *Unless otherwise stated, your answer must be exact and reasonably simplified.* Additionally, clearly indicate the value or expression that is your final answer. Calculators are NOT allowed.

1. Prove that if x^3 is irrational, then x is irrational. What type of proof did you use?
2. Let $m, n \in \mathbf{N}$. Prove that if $m + n \geq 100$, then either $m \geq 50$ or $n \geq 50$. What type of proof did you use?
3. Suppose $A = \{1, 2, 3, 4, 5\}$. Determine if each statement is true or false.

(a) $\{1\} \in \mathcal{P}(A)$	(b) $\{\{3\}\} \subseteq \mathcal{P}(A)$	(c) $\emptyset \subseteq A$	(d) $\{\emptyset\} \subseteq \mathcal{P}(A)$
(e) $\{\emptyset\} \subseteq \mathcal{P}(A)$	(f) $\{2, 4\} \in A \times A$	(g) $ P(A) = 5$	(h) $(1, 1) \in A \times A$
4. Let $U = \{n \in \mathbf{N} \mid n \leq 15\}$ be the universal set and let $A = \{n \in U \mid n \text{ is even}\}$, $B = \{n \in U \mid n \text{ is prime}\}$, and $C = \{n \in U \mid n < 7\}$. List all of the elements in the following sets.

(a) $A \cap B$	(b) $A - C$	(c) $B \cup \bar{B}$	(d) $\overline{A \cup B \cup C}$
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5. Evaluate each of the following:

(a) $\{1, 2, 3, 4, 5\} \cap \{0, 3, 6\}$;	(b) $\{1, 3, 6, 7\} - \{1, 2, 3, 4, 5\}$;	(c) $ \{1, 3, 4, 5, 6\} $;	(d) $\left\lceil \frac{3}{2} + \left\lfloor \frac{3}{2} \right\rfloor + \left\lfloor -\frac{3}{2} \right\rfloor \right\rceil$
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6. Show that relative complementation is not commutative. *I.e.*, give a counterexample to show that the following equation is not true: $(A - B) - C = A - (B - C)$
7. Let $A = \{n \in \mathbf{N} \mid n \leq 20\}$. List all of the elements of the sets (a) $\{n \in A \mid \lceil n/3 \rceil = 6\}$ and (b) $\{n \in A \mid \lfloor n/2 \rfloor = 6\}$.
8. Let $g : \mathbf{N} \rightarrow \mathbf{N}$ be defined by $g(n) = \max\{0, n - 3\}$ and let $A = \{0, 2, 4, 6\}$.
 - (a) Evaluate $g(A)$.
 - (b) Evaluate $g^{-1}(A)$.
 - (c) Is g a one-to-one function?
 - (d) Is g an onto function?
 - (e) Is g a bijection?
9.
 - (a) How many functions are there from $\{1, 2\}$ to $\{a, b, c\}$?
 - (b) How many of these functions are one-to-one?
 - (c) How many of these functions are onto?
10. With $A = \{x, y, z\}$, let $f, g : A \rightarrow A$ be given by $f = \{(x, y), (y, z), (z, x)\}$ and $g = \{(x, y), (y, x), (z, z)\}$. Determine each of the following functions. Write your answers as a collection of ordered pairs. (a) $f \circ g$; (b) $g \circ f$; (c) f^{-1} ; (d) g^{-1} .
11. Suppose that $\{a_n\}$ is defined recursively by $a_n = a_{n-1}^2 - 1$ and that $a_0 = 2$. Find a_3 and a_4 .
12. Determine each of the following: (a) $18 \bmod 7$; (b) $-88 \bmod 13$; (c) $289 \bmod 17$.
13. Let m be a positive integer, and let a, b , and c be integers. Show that if $a \equiv b \pmod{m}$, then $a - c \equiv b - c \pmod{m}$.

Discrete Math I – Answers to Practice Problems for Exam II

1. Use a proof of the contrapositive.

Suppose x is rational.

Then $x = p/q$ where p and q are integers and $q \neq 0$.

Then $x^3 = p^3/q^3$ and since p^3 and q^3 are integers with $q^3 \neq 0$, then x^3 is also of the form of a rational number.

Thus, if x^3 is irrational, then x is irrational.

QED.

2. Use a proof by contradiction.

So assume that $m+n \geq 100$ and that it is not true that either $m \geq 50$ or $n \geq 50$ (or both).

This implies that both $m < 50$ and $n < 50$ must be true.

Adding together the two inequalities preserves the inequality so $m < 50$ and $n < 50$ imply that $m+n < 50 + 50 = 100$.

This contradicts our assumption that $m+n \geq 100$.

Thus, we must have that either $m \geq 50$ or $n \geq 50$ (or both).

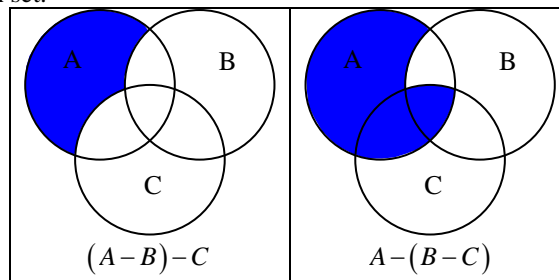
QED.

3. TTTTFFFT

4. (a) $\{2\}$; (b) $\{8,10,12,14\}$; (c) U ; (d) $\{9,15\}$

5. (a) $\{3\}$; (b) $\{6,7\}$; (c) 5; (d) 1

6. Draw the Venn diagram of each set.



Any choice of sets A , B , and C can be made provided $A \cap C \neq \emptyset$, that is, the intersection of A and C is non-empty.

The simplest example would be $A = C = \{1\}$ and $B = \emptyset$; this would imply that $(A - B) - C = \emptyset$ and $A - (B - C) = \{1\}$.

7. (a) $\{16,17,18\}$; (b) $\{12,13\}$

8. (a) $\{0,1,3\}$; (b) $\{0,1,2,3,5,7,9\}$; (c) No, because $g(0) \neq g(1)$; (d) Yes; (e) No.

- 9.

(a) There are 9 functions from $\{1,2\}$ to $\{a,b,c\}$.

(b) Six of the functions are one-to-one ($g_2, g_3, g_4, g_6, g_7, g_8$).

(c) None of them are onto.

Function	Mapping
g_1	$\{(1,a), (2,a)\}$
g_2	$\{(1,a), (2,b)\}$
g_3	$\{(1,a), (2,c)\}$
g_4	$\{(1,b), (2,a)\}$
g_5	$\{(1,b), (2,b)\}$
g_6	$\{(1,b), (2,c)\}$
g_7	$\{(1,c), (2,a)\}$
g_8	$\{(1,c), (2,b)\}$
g_9	$\{(1,c), (2,c)\}$

- 10.

(a) Since

- $(f \circ g)(x) = f(g(x)) = f(y) = z,$
- $(f \circ g)(y) = f(g(y)) = f(x) = y,$ and
- $(f \circ g)(z) = f(g(z)) = f(z) = x,$

then $f \circ g = \{(x, z), (y, y), (z, x)\}.$

(b) Since

- $(g \circ f)(x) = g(f(x)) = g(y) = x,$
- $(g \circ f)(y) = g(f(y)) = g(z) = z,$ and
- $(g \circ f)(z) = g(f(z)) = g(x) = y,$

then $g \circ f = \{(x, x), (y, z), (z, y)\}.$

(c) $f^{-1} = \{(x, z), (y, x), (z, y)\}$

(d) $g^{-1} = \{(x, y), (y, x), (z, z)\} = g$

11. $a_3 = 63; a_4 = 3968$

12. (a) 4; (b) 3; (c) 0

13. Since $a \equiv b \pmod{m}$ we have $m \mid a - b$. Hence there is an integer k such that $a - b = mk$. It follows that $(a - c) - (b - c) = a - b = mk$. This implies that $(a - c) - (b - c) \mid a - b = mk$ so $a - c \equiv b - c \pmod{m}.$