BLG311E – FORMAL LANGUAGES AND AUTOMATA

2017 SPRING

RECITEMENT 3

1. Let A and B be languages defined over Σ and $\Lambda \notin B$,

a) Propose a solution to the equation $A \cup XB = X$.

b) Show that your solution is correct.

c) Let A={a, b} and B={aa, ab, ba, bb}. Write the solution set and give 5 example words from it.

Solution:

a) Solution is $X=AB^*$

b) When we replace X in the equation:

$A \cup (AB^*)B = A \cup AB^+ = A(\{\Lambda\} \cup B^+) = AB^*$

Solution is correct.

c) $AB^* = \{a, b\}\{aa, ab, ba, bb\}^*$

Examples: {a}, {aaa}, {baa}, {bbbaa}, {aabbabb}

2. Show that following expressions hold. If they do not hold give a counterexample.

a) $A^+A^+ = A^+$

Does not hold.

Let A = {1} :

$A^+ = \{1, 11, 111, 1111, ... 1^n, ...\}$

$A^+A^+ = \{11, 111, 1111, ... 1^n, ...\}$

b) $(A^*B^*)^* = (B^*A^*)^*$

By using Theorem 13 on the slides:

$(A^*B^*)^* = (A \cup B)^* \rightarrow (B^*A^*)^* = (B \cup A)^* \rightarrow (B \cup A)^* = (A \cup B)^*$

Holds

c) $(AB)^* = (BA)^*$

Does not hold

Let A ={0} and B ={1}

$(AB)^* = \{\Lambda, 01, 0101, 010101, ... (01)^n, ...\}$

$(BA)^* = \{\Lambda, 10, 1010, 101010, ... (10)^n, ...\}$

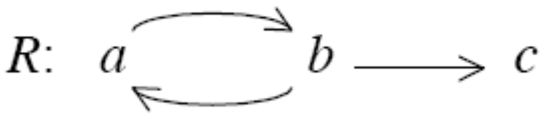
3. Matrix below is a relation defined on the set {a, b, c}. **(a)** Draw the relation graph of the relation itself, **(b)** its powers, **(c)** reflexive, **(d)** symmetric, **(e)** transitive closures as well as **(f)** reflexive closure of its symmetric closure.

	a	**b**	**c**
a	0	1	0
b	1	0	1
c	0	0	0

Solution:

a) $R = \{(a, b), (b, a), (b, c)\}$

Relation graph R:



b) Powers of the relation will be found in **(e)** for finding the transitive closure.

c) Reflexive closure:

$r(R) = R \cup R^0 = R \cup E$, $E = R^0$ (E is the unit relation)

$R = \{(a,b), (b,a), (b,c)\}$

$E = \{(a,a), (b,b), (c,c)\}$

$r(R) = \{(a,b), (b,a), (b,c), (a,a), (b,b), (c,c)\}$

d) Symmetric closure:

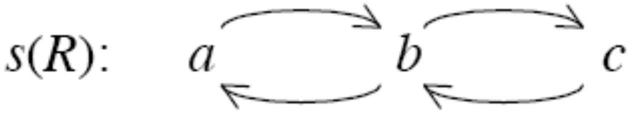
$s(R) = R \cup R^{-1}$

$R = \{(a,b), (b,a), (b,c)\}$

$R^{-1} = \{(b,a) \mid (a,b) \in R\}$

$R^{-1} = \{(a,b), (b,a), (c,b)\}$

$R \cup R^{-1} = s(R) = \{(a,b), (b,a), (b,c), (c,b)\}$

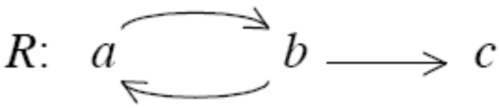


e) Transitive closure:

$$t(R) = \bigcup_{i=1}^{\infty} R^i$$

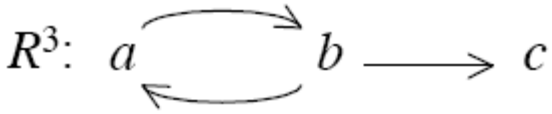
We need to find the powers of the relation for the transitive closure.

$R = \{(a,b), (b,a), (b,c)\}$



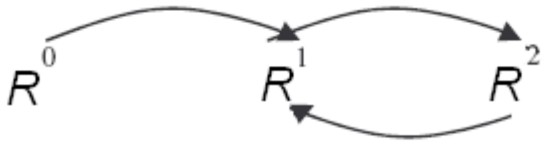
$R^2 = RR = \{(a,b), (b,a), (b,c)\}\{(a,b), (b,a), (b,c)\} = \{(a,a), (b,b), (a,c)\}$

$R^3 = R^2R$ = {(a,a), (b,b), (a,c)} {(a,b), (b,a), (b,c)} = {(a,b), (b,a), (b,c)}



$R^1 = R^3$
$RR = R^3R \rightarrow R^2 = R^4$
$R^{2n+1} = R^1$ and $R^{2n} = R^2$ (n>0)

Powers of the relation graph:



Transitive closure → $t(R) = R \cup R^2$:

$t(R)$ = {(a,b), (b,a), (b,c)} ∪ {(a,a), (b,b), (a,c)}
= {(a,b), (b,a), (b,c), (a,a), (b,b), (a,c)}

f) Reflexive closure of the symmetric closure

rs(*R*) = ?   Let *P* = *s*(*R*)
We know that: *s*(*R*) = {(a,b), (b,a), (b,c), (c,b)}
We need to find *r*(*P*).
r(*P*) = {(a,b), (b,a), (b,c), (c, b), (a,a), (b,b), (c,c)}

4. State the type of each grammar given below with respect to Chomsky hierarchy and explain the reason why that grammar belongs to the type you stated. Write the regular expression for each of these grammars (Give the general form of productions for the grammars for which it is not possible to write a regular expression).

Grammar	**Type and reason**	**Regular expression (or general form)**
S → aS | bS | aba	**Type 3:** A single nonterminal on the left-hand side and a right-hand side consisting of a number of terminals followed by a single nonterminal.	$L(G) = (a \vee b)^{*}aba$
<S> ::= a<S>a | b<S>b | c	**Type 2:** A single nonterminal on the left-hand side and a right-hand side consisting of a string of terminals and nonterminals.	This grammar does not have a regular expression as it is not Type 3. General structure is in the following form: xcx^{R}, $x = (a \vee b)^{*}$.
S → aAbc | abc A → aAbC | abC Cb → bC Cc → cc	**Type 1:** Multiple symbols on the left-hand side and the length of the left-hand side can not exceed the length of the right-hand side.	This grammar does not have a regular expression as it is not Type 3. General structure is in the following form: $a^{n}b^{n}c^{n},\ n > 0$.
<S> ::= a<S>a | b<S>b | c a<S>a ::= ac	**Type 0:** Multiple symbols on the left-hand side and the length of the left-hand side exceeds the length of the right-hand side.	This grammar does not have a regular expression as it is not Type 3. General structure is in the following form: $x(c \vee ac)x^{R},\ x = (a \vee b)^{*}$.