Numerical Methods in CE

Recitation 5

Linear Programming - Simplex Examples

ÖRNEK 1 (Jensen&Bard, 2003)

enbüyükle $z = 2x_1 + 3x_2$

öyle ki
$$
\begin{aligned}
-x_1 + x_2 &\leq 5 \\
x_1 + 3x_2 &\leq 35 \\
x_1 \qquad &\leq 20 \\
x_1 \geq 0, x_2 &\geq 0
\end{aligned}
$$

ÖRNEK 2 (Winston, 2004)

enbüyükle $z = 60x_1 + 35x_2 + 20x_3$

öyle ki
$$
\begin{aligned}
8x_1 + 6x_2 + x_3 &\leq 48 \\
4x_1 + 2x_2 + 1.5x_3 &\leq 20 \\
2x_1 + 1.5x_2 + 0.5x_3 &\leq 8 \\
x_2 \qquad &\leq 5 \\
x_1, x_2, x_3 &\geq 0
\end{aligned}
$$

ZÜM 1

pivot column

		KATSAYILAR						
Satır	Temel	z	x_1	x_2	s_1	s_2	s_3	ST
0	z	1	-2	-3	0	0	0	0
1	s_1	0	-1	1	1	0	0	5
2	s_2	0	1	3	0	1	0	35
3	s_3	0	1	0	0	0	1	20

$z - 2x_1 +$

		KATSAYILAR							
Satır	Temel	z	x_1	x_2	s_1	s_2	s_3	ST	ORAN TESTİ
0	z	1	-2	-3	0	0	0	0	-
1	s_1	0	-1	1	1	0	0	5	5/1=5*
2	s_2	0	1	3	0	1	0	35	35/3=11.67
3	s_3	0	1	0	0	0	1	20	-

GİREN DEĞİŞKEN ↑ ↓ ÇIKAN DEĞİŞKEN

		KATSAYILAR							
Satır	Temel	z	x_1	x_2	s_1	s_2	s_3	ST	ORAN TESTİ
0	z	1	-5	0	3	0	0	15	-
1	x_2	0	-1	1	1	0	0	5	-
2	s_2	0	4	0	-3	1	0	20	20/4=5*
3	s_3	0	1	0	0	0	1	20	20/1=20

		KATSAYILAR							
Satır	Temel	z	x_1	x_2	s_1	s_2	s_3	ST	ORAN TESTİ
0	z	1	0	0	-0.75	1.25	0	40	-
1	x_2	0	0	1	0.25	0.25	0	10	10/0.25=40
2	x_1	0	1	0	-0.75	0.25	0	5	-
3	s_3	0	0	0	0.75	-0.25	1	15	15/0.75=20*

		KATSAYILAR						
Satır	Temel	z	x_1	x_2	s_1	s_2	s_3	ST
0	z	1	0	0	0	1	1	55
1	x_2	0	0	1	0	0.33	-0.33	5
2	x_1	0	1	0	0	0	1	20
3	s_3	0	0	0	1	-0.33	1.33	20

ÇÖZÜM 2

SATIR	TEMEL	z	x_1	x_2	x_3	s_1	s_2	s_3	s_4	ST	ORAN TESTİ
		KATSAYILAR									
0	z	1	-60	-35	-20	0	0	0	0	0	-
1	s_1	0	8	6	1	1	0	0	0	48	48/8=6
2	s_2	0	4	2	1.5	0	1	0	0	20	20/4=5
3	s_3	0	2	1.5	0.5	0	0	1	0	8	8/2=4*
4	s_4	0	0	1	0	0	0	0	1	5	-

SATIR	TEMEL	z	x_1	x_2	x_3	s_1	s_2	s_3	s_4	ST	ORAN TESTİ
		KATSAYILAR									
0	z	1	0	10	-5	0	0	30	0	240	-
1	s_1	0	0	0	-1	1	0	-4	0	16	-
2	s_2	0	0	-1	0.5	0	1	-2	0	4	4/0.5=8*
3	x_1	0	1	0.75	0.25	0	0	0.5	0	4	4/0.25=16
4	s_4	0	0	1	0	0	0	0	1	5	-

sıfırladım

SATIR	TEMEL	z	x_1	x_2	x_3	s_1	s_2	s_3	s_4	ST	
		KATSAYILAR									
0	z	1	0	0	0	0	10	10	0	280	EN İYİ ÇÖZÜM 1
1	s_1	0	0	-2	0	1	2	-8	0	24	
2	x_3	0	0	-2	1	0	2	-4	0	8	
3	x_1	0	1	1.25	0	0	-0.5	1.5	0	2	
4	s_4	0	0	1	0	0	0	0	1	5	

SATIR	TEMEL	z	x_1	x_2	x_3	s_1	s_2	s_3	s_4	ST	ORAN TESTİ
		KATSAYILAR									
0	z	1	0	0	0	0	10	10	0	280	-
1	s_1	0	0	-2	0	1	2	-8	0	24	-
2	x_3	0	0	-2	1	0	2	-4	0	8	-
3	x_1	0	1	1.25	0	0	-0.5	1.5	0	2	2/1.25=1.6*
4	s_4	0	0	1	0	0	0	0	1	5	5/1=5

SATIR	TEMEL	z	x_1	x_2	x_3	s_1	s_2	s_3	s_4	ST	
		KATSAYILAR									
0	z	1	0	0	0	0	10	10	0	280	
1	s_1	0	1.6	0	0	1	1.2	-5.6	0	27.2	
2	x_3	0	1.6	0	1	0	1.2	-1.6	0	11.2	EN İYİ ÇÖZÜM 2
3	x_2	0	0.8	1	0	0	-0.4	1.2	0	1.6	
4	s_4	0	-0.8	0	0	0	0.4	-1.2	1	3.4	

ÇÖZÜM 3

Birthday Problem

An interesting problem that can be solved by using simulation is the famous **birthday problem**. Suppose that in a room of n people, each of the 365 days of the year is equally likely to be someone's birthday. From probability theory, it can be shown that, contrary to intuition, only 23 people need be present for the chances to be better than fifty-fifty that at least two of them will have the same birthday! (It is always fun to try this experiment at a large party or in class to see it work in practice.)

Many people are curious about the theoretical reasoning behind this result, so we discuss it briefly before solving the simulation problem. After someone is asked his or her birthday, the chances that the next person asked will not have the same birthday are 364/365. The chances that the third person's birthday will not match those of the first two people are 363/365. The chances of two successive independent events occurring is the product of the probability of the separate events. (The sequential nature of the explanation does not imply that the events are dependent.) In general, the probability that the nth person asked will have a birthday different from that of anyone who has already been asked is

$$\left(\frac{365}{365}\right)\left(\frac{364}{365}\right)\left(\frac{363}{365}\right)\cdots\left(\frac{365-(n-1)}{365}\right)$$

The probability that the nth person asked will provide a match is 1 minus this value. A table of the quantity $1 - (365)(364)\cdots[365-(n-1)]/365^n$ shows that with 23 people, the chances are 50.7%; with 55 or more people, the chances are 98.6% or almost theoretically certain that at least two out of 55 people will have the same birthday. (See Table 13.1.)

Without using probability theory, we can write a routine that uses the random-number generator to compute the approximate chances for groups of n people. Clearly, all that is

TABLE 13.1 Birthday Problem

n	Theoretical	Simulation
5	0.027	0.028
10	0.117	0.110
15	0.253	0.255
20	0.411	0.412
22	0.476	0.462
23	0.507	0.520
25	0.569	0.553
30	0.706	0.692
35	0.814	0.819
40	0.891	0.885
45	0.941	0.936
50	0.970	0.977
55	0.986	0.987

needed is to select n random integers from the set $\{1, 2, 3, \ldots, 365\}$ and to examine them in some way to determine whether there is a match. By repeating this experiment a large number of times, we can compute the probability of at least one match in any gathering of n people.

One way of writing a routine for simulating the birthday problem follows. In it we use the approach of checking off days on a calendar to find out whether there is a match. Of course, there are many other ways of approaching this problem.

Function procedure *Probably* calculates the probability of repeated birthdays:

```
real function Probably(n, npts)
integer i, npts;    logical Birthday;    real sum ← 0
for i = 1 to npts do
    if Birthday (n) then sum ← sum + 1
end for
Probably ← sum/real(npts)
end function Probably
```

Logical function *Birthday* generates n random numbers and compares them. It returns a value of `true` if these numbers contain at least one repetition and `false` if all n numbers are different.

```
logical function Birthday(n)
integer i, n, number;   logical array (days_i)_1:365
real array (r_i)_1:n
call Random((r_i))
for i = 1 to 365 do
    days(i) ← false
end for
```

```
Birthday ← false
for i = 1 to n do
    number ← integer (365r_i + 1)
    if days(number) then
        Birthday ← true
        exit loop i
    end if
    days(number) ← true
end for
end function Birthday
```

The results of the theoretical calculations and the simulation are given in Table 13.1.

Linear Programming- Graphical Solution Examples

Q1) A farmer can plant up to 8 acres of land with wheat and barley. He can earn \$5,000 for every acre he plants with wheat, \$3000 for every acre he plants with barley.

His use of a necessary pesticide is limited by federal regulations to 10 gallons for entire 8 acres. Wheat requires 2 gallons of pesticide for every acre and barley requires 1 gallon per acre.

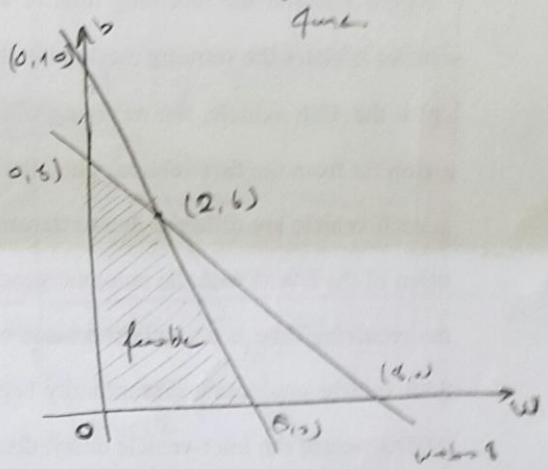
What is the maximum profit the farmer can make?

w: wheat
b: barley

$p = 5000w + 3000b$ ← objective function
profit

Constraints

$w \geq 0$ $\quad w + b \leq 8$
$b \geq 0$ $\quad 2w + b \leq 10$



Intersection points

(0,8); (5,0); (2,6); (0,0)

$w + b = 8$
$2w + b = 10$
$w = 2$
$b = 6$

$p(0,8) = 18000\$$
$p(5,0) = 25000\$$
$\boxed{p(2,6) = 28000\$}$
$p(0,0) = 0\ \$$

Q2) A painter has exactly 32 units of yellow dye, 54 units of green dye. He plans to mix as many gallons as possible of color A and B.

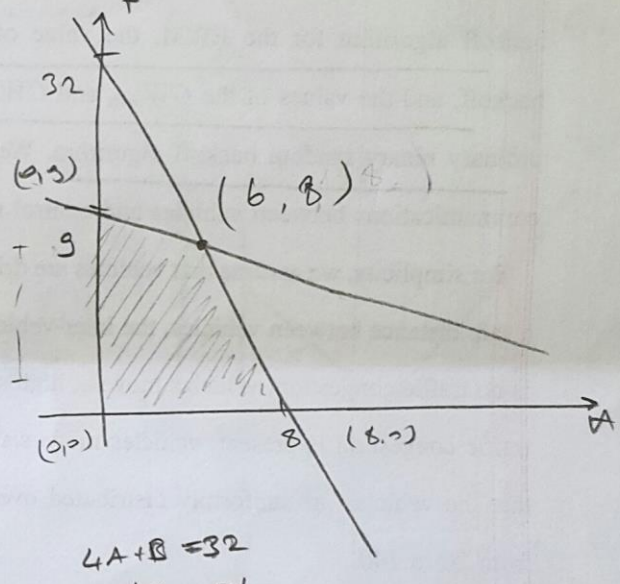
- Each gallon of color A requires 4 units of yellow dye and 1 units of green dye
- Each gallon of color B req. 1 unit of yellow dye and 6 units of green dye

Goal = A + B

$A \geq 0 \quad B \geq 0$

$4A + B \leq 32$

$A + 6B \leq 54$



Extreme points

(0,9) (6,8) (8,0) (0,0)

Goal (0,9) = 9

$\boxed{\text{Goal } (6,8) = 14}$ ✓

$4A + B = 32$

$A + 6B = 54$

$23B = 54 \cdot 4 - 32$

$= 216 - 32$

$B = 184/23 = 8$

Q4) A garden shop wishes to prepare a supply of special fertilizer at a minimal cost by mixing two fertilizers A and B.

The mixture contains

at least 45 units phosphate
at least 36 units nitrate
at least 40 units ammonium

Fertilizer A costs the shop \$.97 per pound.
Fertilizer B costs the shop \$1.89 per pound

Fert. A contains 5 units phosphate
2 units nitrate
2 units ammonium

Fer. B 3 p
3 n
5 a.

How many pounds of each fertilizer should shop use in order to minimize their costs?

minimize

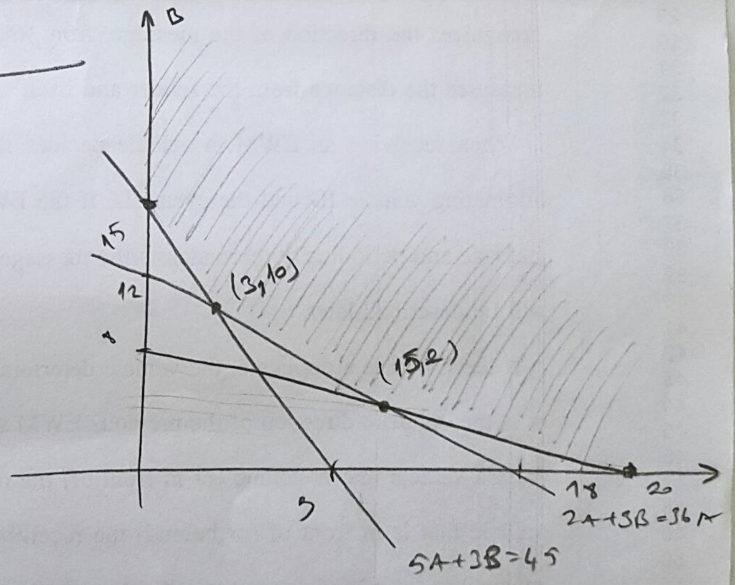
$$Cost = 0.97A + 1.89B$$

$A \geq 0,\ B \geq 0$

$5A + 3B \geq 45$

$2A + 3B \geq 36$

$2A + 5B \geq 40$



Intersection points

(0,15) 28.35\$
(3,10) 21,81\$
(15,2) 18,33\$ minimum
(20,0) 19,40\$