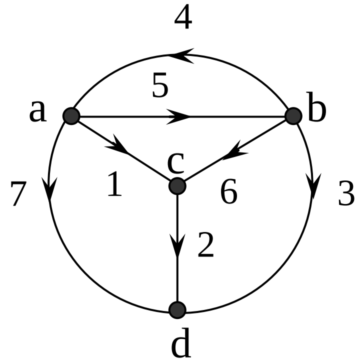
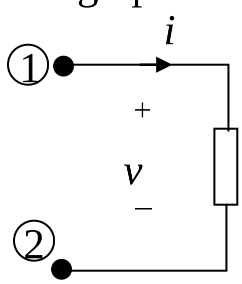
Graph Theory

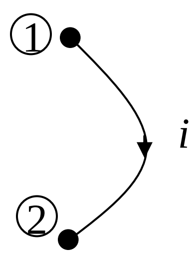
A structure consisting of vertices and edges



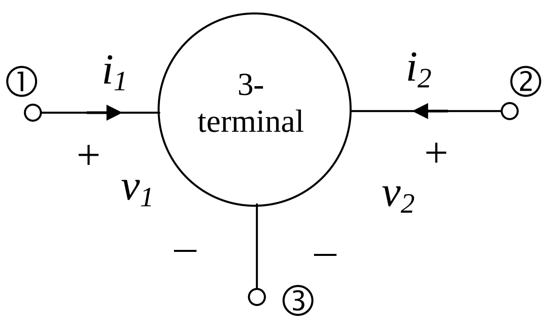
number of vertices=4
number of edges=7

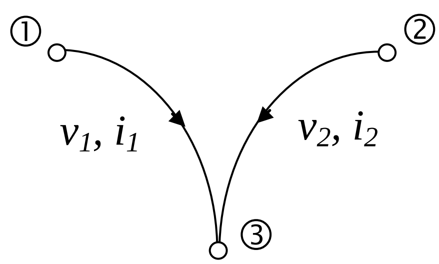
Element graph

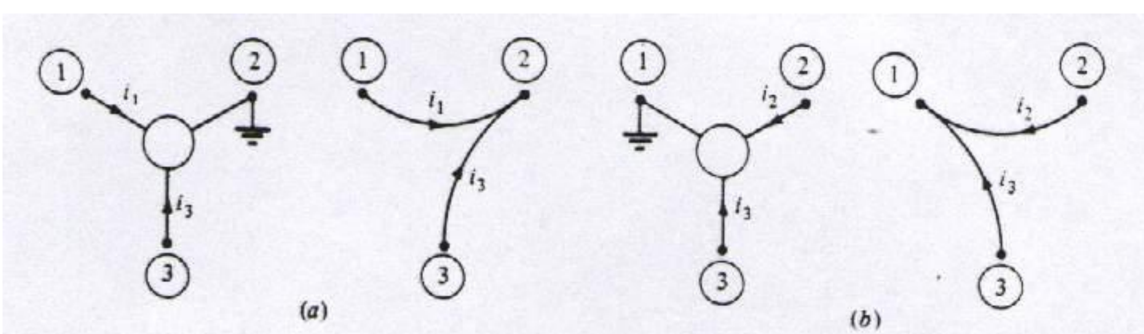




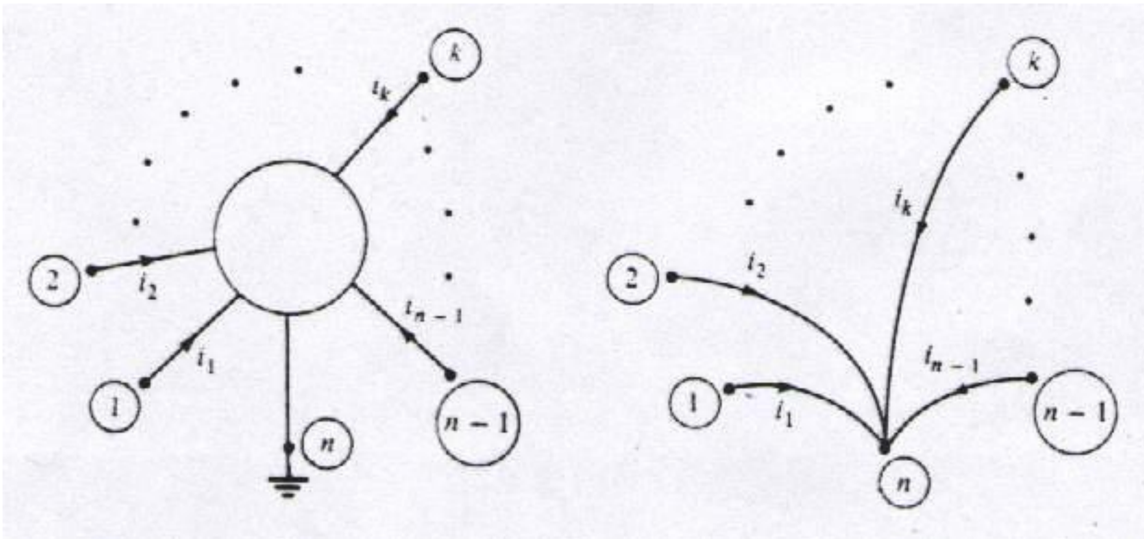
Passive sign convention should be met



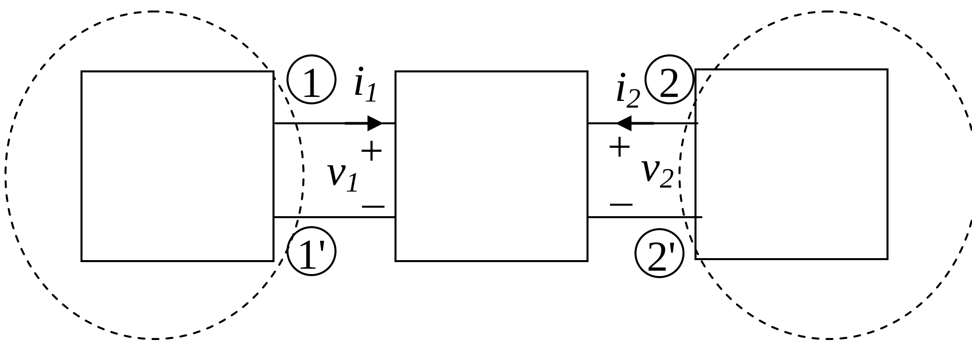


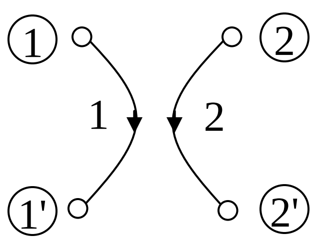


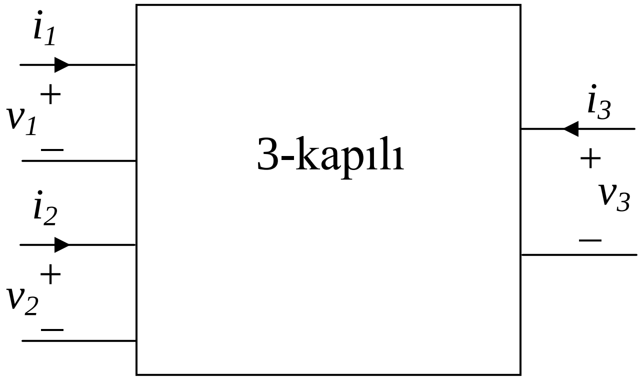
Element graph of an n-terminal

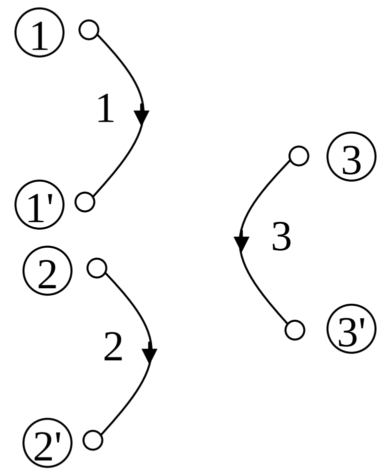


Two-port and multi-port elements

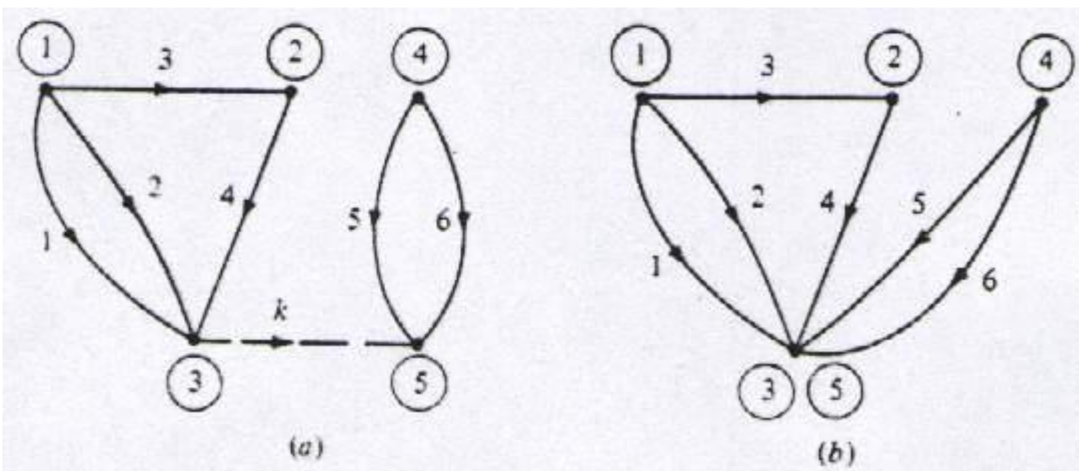




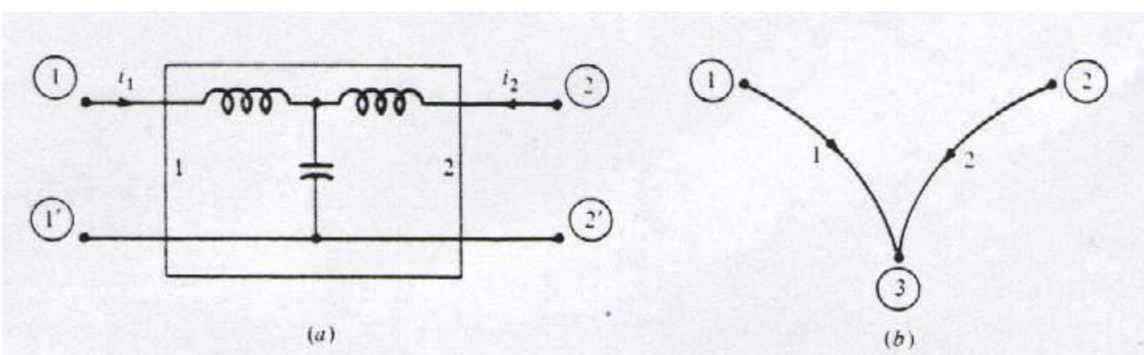




The graph of a 2-port element is disjoint. However any disjoint graph can be converted into a connected graph as shown below.



The subcircuit below can be considered either a 2port or 3-terminal.



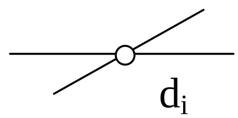
Circuit graph: A graph is a pictorial structure which represents the interconnections of the elements in an electric circuit.

Edge: A directed line which represents a connection between vertices

Vertex (node, point): The edges of an edge

degre (of a vertex): the number of edges connnected to a vertex

$\delta(d_i)=4$



An isolated vertex is a vertex with $\delta=0$

In a graph with n_e elements and n vertices, the sum of degrees of all vertices equals to 2 n_e.

Subgraph is graph formed from a subset of the vertices and edges of a given graph.

Path is a sequence of vertices and edges, with both endpoints of an edge appearing adjacent to it in the sequence

A connected graph is one in which each pair of vertices forms the endpoints of a path. Otherwise, it is disjoint graph.

Loop is a path whose endpoints are the same vertex. Degrees of all vertices in a loop is 2.

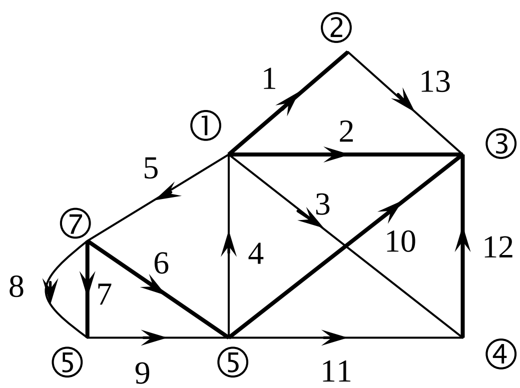
Tree: A tree is a connected and acyclic subgraph which contains all the vertices of the given graph.

Provided that the graph has n_e edges and n_d vertices, in a tree,

1. the number of edges is n_e-1.
2. There is one only one path interconnecting each pair of vertices.

Fundamental Loop set:

Each co-tree of a tree and some branched define a loop. For a given tree, there is n_e-n_d+1 such loops. These loop consitute fundamental loop set.



Fundamental loop sets:
{8, 7}
{9, 7, 6}
{3, 12, 2}
{13, 1, 2}
{11,10, 12}
toplam n_e-n_d+1=13-7+17

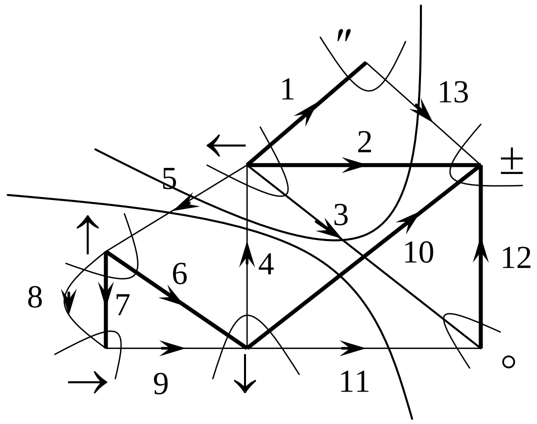
Cut-set: Some elements in graph G satisfying the followins contitute a cut-set.

a) After removal of these elements, graph is split into two parts.

b) Any subset of these elements satisfy a).

Fundemantal cut-set:

For a given tree, each of the branches contitutes a cut-set with only some branches of the tree. This cut-set is called fundemantal cut-set.

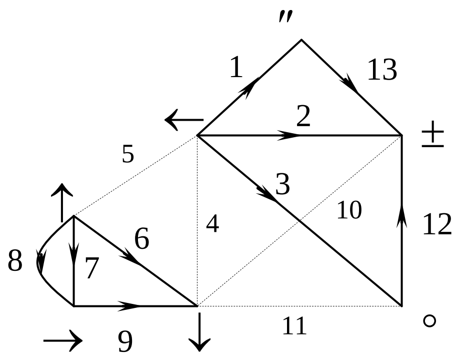


Connected graph

n_e=13 => dal n_d-1=7-1=6

n_d=7=> kiriş n_e-n_d+1=7

- Co-tree elements G_K={3, 4, 5, 8, 9, 11, 13}
- Tree G_T={1, 2, 6, 7, 10, 12}



{5, 4, 10, 11} is a cut-set

- Cut-set G_{DK}={1, 2, 3, 4 , 5}

Graph Matrices

1. Fundamental Loop Matrix

Fill in the entries of the matrix $\mathbf{B_t}=[b_{ij}]$ as follows:

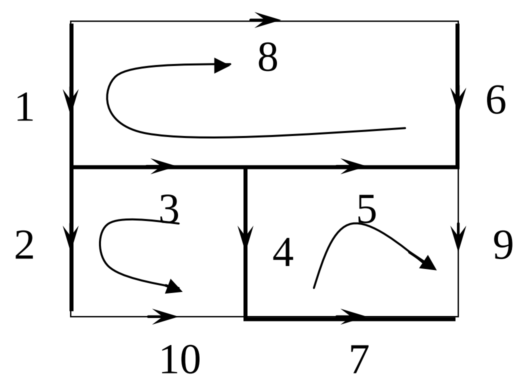
$b_{ij}=0$; jth element is not included in the ith fundamental loop,

$b_{ij}=1$; jth element is included in the ith fundamental loop, and the element is in the same orientation as the loop,

$b_{ij}=-1$; jth element is included in the ith fundamental loop, and the element is in the opposite orientation as the loop.

(the orientation of a loop is the same as that of the co-tree)

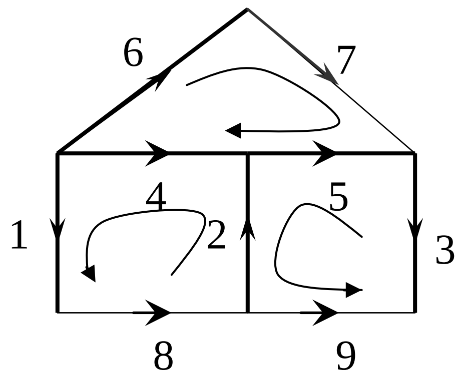
The matrix $\mathbf{B_t}$, thus obtained is called fundamental loop matrix.



Dimensions of matrix B_t is $(n_e-n_d+1) \times (n_e)$

$$\mathbf{B_t} = \begin{array}{c} \\ (8) \\ (9) \\ (10) \end{array} \begin{array}{c} \begin{array}{cccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \end{array} \\ \left| \begin{array}{ccccccc:ccc} -1 & 0 & -1 & 0 & -1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \end{array} \right| \\ \underbrace{\qquad\qquad\qquad\qquad\qquad}_{\text{branches}} \; \underbrace{\qquad\qquad}_{\text{co-tree edges}} \end{array}$$

$$\mathbf{B_t} = [\ \underbrace{\mathbf{B_1}}_{n_d-1} \ \vdots \ \underbrace{\mathbf{U}}_{n_e-n_d+1} \ ] \qquad n_e-n_d+1$$



$$\mathbf{B_t} = \begin{array}{c|cccccc:ccc|}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
(7) & 0 & 0 & 0 & -1 & -1 & 1 & 1 & 0 & 0 \\
(8) & 1 & 1 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\
(9) & 0 & -1 & -1 & 0 & -1 & 0 & 0 & 0 & 1 \\
 & \multicolumn{6}{c}{\underbrace{\hspace{5cm}}_{\text{branches}}} & \multicolumn{3}{c}{\underbrace{\hspace{2.5cm}}_{\text{co-tree edges}}}
\end{array}$$

$$\mathbf{B_t} = [\quad \mathbf{B_1} \quad \vdots \quad \mathbf{U} \quad]$$

Rank $\{B_t\} = n_e - n_d + 1$= number of co-trees= number of fundamental loops

2. Fundamental Cut-Set Matrix

For each fundamental cut-set in graph G, cosntruct the fundamental cut-set matrix $\mathbf{Q_t}$=[q_{ij}] according to the following steps:

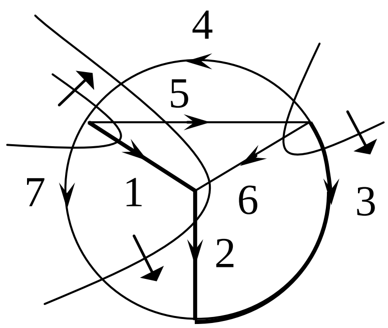
q_{ij}=0; jth element is not included in the set of ith fundamental cut,

q_{ij}=1; jth element is included in the set of ith fundamental cut and it is in the same orientation as the cut-set.

q_{ij}=-1; jth element is included in the set of ith fundamental cut and it is in the opposite orientation as the cut-set.

(the direction of the cut set is chosen as the direction of the branch)

The orientation of cut-set is the same as that of co-tree element and is taken positive.



$$\mathbf{Q_t} = \begin{array}{c} \\ (1) \\ (2) \\ (3) \\ \\ \end{array} \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \left| \begin{array}{ccc:cccc} 1 & 0 & 0 & -1 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 & 1 & -1 & 1 \\ 0 & 0 & 1 & 1 & -1 & 1 & 0 \end{array} \right| \\ \underbrace{\qquad\qquad}_{\text{branches}} \quad \underbrace{\qquad\qquad\qquad}_{\text{co-tree edges}} \end{array}$$

$$\mathbf{Q_t} = [\ \underbrace{\mathbf{U}}_{n_d-1}\ \vdots\ \underbrace{\mathbf{Q_1}}_{n_e-n_d+1}\ ] \qquad n_d-1$$

$$\mathbf{B_t} = \begin{array}{c} \\ (4) \\ (5) \\ (6) \\ (7) \\ \\ \end{array} \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \left| \begin{array}{ccc:cccc} 1 & 1 & -1 & 1 & 0 & 0 & 0 \\ -1 & -1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 & 0 \\ -1 & -1 & 0 & 0 & 0 & 0 & 1 \end{array} \right| \\ \underbrace{\qquad\qquad}_{B_1} \quad \underbrace{\qquad\qquad\qquad}_{U} \end{array}$$

$$\mathbf{Q_1 = -B_1^T} \qquad\qquad \mathbf{B_1 = -Q_1^T}$$

Property: $\mathbf{Q_t B_t^T = 0} \ \Leftrightarrow\ \mathbf{B_t Q_t^T = 0}$

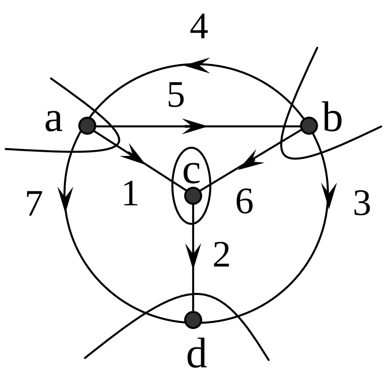
$$\mathbf{Q_t B_t^T = [U\ Q_1][B_1\ U]^T} = \begin{bmatrix} \mathbf{U} & \mathbf{Q_1} \end{bmatrix} \begin{bmatrix} \mathbf{B_1^T} \\ \mathbf{U} \end{bmatrix}$$

$$\mathbf{= B_1^T + Q_1 = B_1^T + (-B_1^T) = 0}$$

Incidence Matrix

Choose all the cut-sets in such a way that one of the parts related to the cut-set consists of a single vertix. The fundamental loop matrix of this specific cut-set is the node incidence matrix, A.

- ✓ Dimensions of A is (n_d) x (n_e)



$$\bar{\mathbf{A}} = \begin{array}{c} \\ (a) \\ (b) \\ (c) \\ (d) \end{array} \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \left| \begin{array}{ccccccc} 1 & 0 & 0 & -1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & -1 & 1 & 0 \\ -1 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & -1 & -1 & 0 & 0 & 0 & -1 \end{array} \right| \end{array}$$

4. Reduced incidence matrix, **A**

The matrix obtained by removing one of the rows of $\bar{A}$ is called reduced incidence matrix

$$\mathbf{A} = \begin{array}{c|ccccccc|} & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \text{(a)} & 1 & 0 & 0 & -1 & 1 & 0 & 1 \\ \text{(b)} & 0 & 0 & 1 & 1 & -1 & 1 & 0 \\ \text{(c)} & -1 & 1 & 0 & 0 & 0 & -1 & 0 \\ \cancel{\text{(d)}} & \cancel{0} & \cancel{-1} & \cancel{-1} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{-1} \end{array} \qquad \text{rank } \mathbf{A} = n_d\text{-}1$$

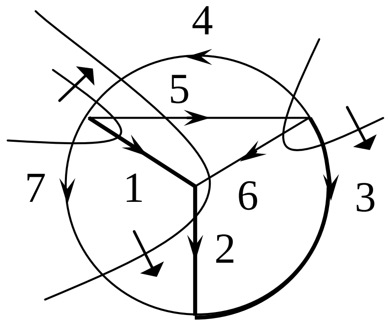
Reduced incidence matrix is a full rank matrix.

Property:

✓ $\mathbf{A} = [\ \underbrace{\mathbf{A_1}}_{\text{branches}} \ \vdots\ \underbrace{\mathbf{A_2}}_{\text{co-trees}}\ ] \quad n_d\text{-}1 \Rightarrow \quad \mathbf{A_1^{-1}A = Q_T}$

$$\mathbf{A_1^{-1}A = [\ U \vdots A_1^{-1}A_2\ ] = Q_T = [\ U \vdots Q_1\ ]}$$

$\Rightarrow \quad \mathbf{Q_1 = A_1^{-1}A_2}$



$$\mathbf{A} = \begin{array}{c} \\ (a) \\ (b) \\ (c) \end{array} \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \left| \begin{array}{ccc:cccc} 1 & 0 & 0 & -1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & -1 & 1 & 0 \\ -1 & 1 & 0 & 0 & 0 & -1 & 0 \end{array} \right| \end{array}$$

$$\underbrace{\qquad\qquad}_{\text{dallar } (A_1)} \quad \underbrace{\qquad\qquad}_{\text{kirişler } (A_2)}$$

$$\mathbf{A} = [\quad \mathbf{A_1} \quad \vdots \quad \mathbf{A_2} \quad]$$

$$\mathbf{A_1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix} \qquad A_2 = \begin{bmatrix} -1 & 1 & 0 & 1 \\ 1 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$$\mathbf{Q_t} = \mathbf{A_1^{-1} A} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & -1 & 1 & 0 \\ -1 & 1 & 0 & 0 & 0 & -1 & 0 \end{bmatrix}$$

$$\mathbf{Q_t} = \begin{array}{c} \\ (1) \\ (2) \\ (3) \end{array} \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \left| \begin{array}{ccc:cccc} 1 & 0 & 0 & -1 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 & 1 & -1 & 1 \\ 0 & 0 & 1 & 1 & -1 & 1 & 0 \end{array} \right| \end{array}$$

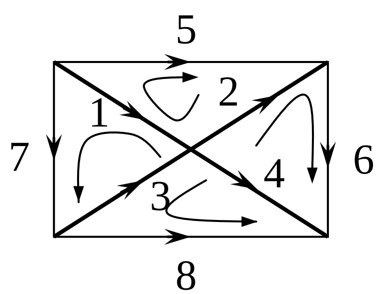
$$\underbrace{\qquad\qquad}_{\text{brances } (U)} \quad \underbrace{\qquad\qquad}_{\text{co-trees } (Q_1)}$$

Second Postulate of Circuit Theory

The fundamental loop matrix of a chosen tree, $\mathbf{B_t}$ and the vector composed of the element voltages $\mathbf{v(t)}$ in a given graph G satisfies:

$$\mathbf{B_t\, v(t) = 0}$$

which is called *fundemantal loop equations*.



n_d=5, n_e=8

$\mathbf{B_t\, v(t)}=0$

$$\mathbf{B_t} = \begin{array}{c} \\ (5) \\ (6) \\ (7) \\ (8) \end{array} \begin{array}{c} \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\ \left| \begin{array}{cccc:cccc} -1 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & -1 & 0 & 0 & 0 & 1 \end{array} \right| \end{array} \qquad \mathbf{v(t)} = \left| \begin{array}{c} v_1(t) \\ v_2(t) \\ v_3(t) \\ v_4(t) \\ v_5(t) \\ v_6(t) \\ v_7(t) \\ v_8(t) \end{array} \right|$$

$$\underbrace{\qquad\qquad\qquad}_{\text{brances } (B_1)} \quad \underbrace{\qquad\qquad\qquad}_{\text{cut-sets } (U)}$$

Fundamental loop equations are given by:

(5) $-v_1(t) - v_2(t) + v_5(t) \quad = \quad 0$

(6) $v_2(t) - v_4(t) + v_6(t) \quad = \quad 0$

(7) $- v_1(t) + v_3(t) + v_7(t) \quad = \quad 0$

(8) $- v_3(t) - v_4(t) + v_8(t) \quad = \quad 0$

Third Postulate of Circuit Theory

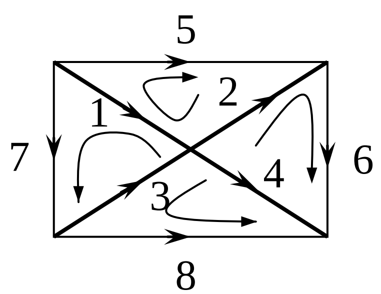
The fundamental cut-set matrix of a chosen tree, $\mathbf{B_t}$ and

the vector composed of the element currents $\mathbf{i(t)}$ in a given graph G

satisfies:

$$\mathbf{Q_t\, i(t)} = \mathbf{0}$$

which is called *fundemantal cut-set equations*.



n_d=5, n_e=8

$\mathbf{Q_t}\ \boldsymbol{i(t)}=0$

$$\mathbf{Q_t} = \begin{array}{c} \\ (1) \\ (2) \\ (3) \\ (4) \end{array} \begin{array}{c} \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\ \left|\begin{array}{cccc:cccc} 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{array}\right| \\ \underbrace{\qquad\qquad\qquad}_{\text{brances } (B_1)} \quad \underbrace{\qquad\qquad\qquad}_{\text{cut-sets } (U)} \end{array} \qquad \mathbf{i(t)} = \left|\begin{array}{c} i_1(t) \\ i_2(t) \\ i_3(t) \\ i_4(t) \\ i_5(t) \\ i_6(t) \\ i_7(t) \\ i_8(t) \end{array}\right|$$

fundemantal cut-set equations are given by:

(1) $i_1(t)+ i_5(t)+ i_7(t) = 0$

(2) $i_2(t)+ i_5(t) - i_6(t) = 0$

(3) $i_3(t) - i_7(t)+i_8(t) = 0$

(4) $i_4(t)+ i_6(t)+ i_8(t) = 0$

Since $\mathbf{Q_t = A_1^{-1} A}$

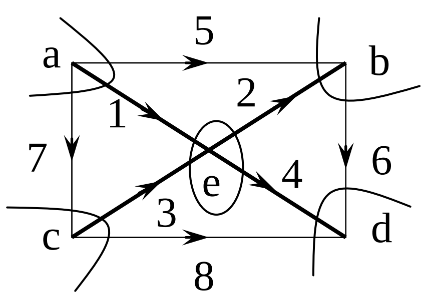
$$\mathbf{Q_t\ i(t)} = \mathbf{0} \qquad \Leftrightarrow \qquad \mathbf{A\ i(t)} = \mathbf{0}$$

$$\underbrace{\left[\quad \mathbf{A_1} \quad\right.}_{\text{brances } (B_1)} \underbrace{\left.\quad \mathbf{A_2} \quad\right]}_{\text{cut-sets } (U)} \begin{bmatrix} \mathbf{i_1(t)} \\ \mathbf{i_2(t)} \end{bmatrix} = \mathbf{0}$$

If we multiply both sides from left with A_1^{-1},

$$\mathbf{A_1^{-1}}\left[\mathbf{A_1} \quad \mathbf{A_2}\right]\begin{bmatrix} \mathbf{i_1(t)} \\ \mathbf{i_2(t)} \end{bmatrix} = \left[\mathbf{U} \quad \mathbf{A_1^{-1}A_2}\right]\begin{bmatrix} \mathbf{i_1(t)} \\ \mathbf{i_2(t)} \end{bmatrix} = \left[\mathbf{U} \quad \mathbf{Q_1}\right]\begin{bmatrix} \mathbf{i_1(t)} \\ \mathbf{i_2(t)} \end{bmatrix} =$$

$$\mathbf{Q_t}\begin{bmatrix} \mathbf{i_1(t)} \\ \mathbf{i_2(t)} \end{bmatrix} = \mathbf{Q_t i(t)} = 0$$



$$\overline{\mathbf{A}} = \begin{array}{c|cccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \hline a & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ b & 0 & -1 & 0 & 0 & -1 & 1 & 0 & 0 \\ c & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 1 \\ d & 0 & 0 & 0 & -1 & 0 & -1 & 0 & -1 \\ e & -1 & 1 & -1 & 1 & 0 & 0 & 0 & 0 \end{array}$$

Reduced incidence matrix, **A**

$$\mathbf{A} = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \left| \underbrace{\begin{array}{cccc} 1 & 2 & 3 & 4 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{array}}_{\mathbf{A_1}} \vdots \underbrace{\begin{array}{cccc} 5 & 6 & 7 & 8 \\ 1 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & -1 & 0 & -1 \end{array}}_{\mathbf{A_2}} \right|$$

$$\mathbf{i(t)} = \left| \begin{array}{c} i_1(t) \\ \\ i_2(t) \end{array} \right| = \left| \begin{array}{c} i_1(t) \\ i_2(t) \\ i_3(t) \\ i_4(t) \\ \hdashline i_5(t) \\ i_6(t) \\ i_7(t) \\ i_8(t) \end{array} \right|$$

Incidence equations, **A i(t) = 0**

(a) $i_1(t) + i_5(t) + i_7(t) = 0$

(b) $-\,i_2(t) - i_5(t) + i_6(t) = 0$

(c) $i_3(t) - i_7(t) + i_8(t) = 0$

(d) $-\,i_4(t) - i_6(t) - i_8(t) = 0$

$$\mathbf{A}_1^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$\mathbf{A_1^{-1}A_2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & -1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} = \mathbf{Q_1}$$

$$\mathbf{A_1^{-1}}\begin{bmatrix} \mathbf{A_1} & \mathbf{A_2} \end{bmatrix} \begin{bmatrix} \mathbf{i_1(t)} \\ \mathbf{i_2(t)} \end{bmatrix} = \begin{bmatrix} \mathbf{U} & \mathbf{A_1^{-1}A_2} \end{bmatrix} \begin{bmatrix} \mathbf{i_1(t)} \\ \mathbf{i_2(t)} \end{bmatrix} = \mathbf{i_1(t)} + \mathbf{Q_1 i_2(t)} = \mathbf{0}$$

Tellegen's Theorem

Assume that two different circuits have the same graph G.

Circuit D

$\mathbf{B_t\, v(t) = 0}$

$\mathbf{Q_t\, i(t) = 0}$

Circuit D'

$\mathbf{B_t\, v'(t) = 0}$

$\mathbf{Q_t\, i'(t) = 0}$

(**v(t), i(t)** ile **v'(t), i'(t)** farklı)

Explicitly,

$$\begin{bmatrix}\mathbf{B_1} & \mathbf{U}\end{bmatrix}\begin{bmatrix}\mathbf{v_1(t)}\\ \mathbf{v_2(t)}\end{bmatrix}=\mathbf{0} \qquad \begin{bmatrix}\mathbf{B_1} & \mathbf{U}\end{bmatrix}\begin{bmatrix}\mathbf{v'_1(t)}\\ \mathbf{v'_2(t)}\end{bmatrix}=\mathbf{0}$$

$$\begin{bmatrix}\mathbf{U} & \mathbf{Q_1}\end{bmatrix}\begin{bmatrix}\mathbf{i_1(t)}\\ \mathbf{i_2(t)}\end{bmatrix}=\mathbf{0} \qquad \begin{bmatrix}\mathbf{U} & \mathbf{Q_1}\end{bmatrix}\begin{bmatrix}\mathbf{i'_1(t)}\\ \mathbf{i'_2(t)}\end{bmatrix}=\mathbf{0}$$

$\mathbf{v_2(t) + B_1\, v_1(t) = 0}$ $\qquad$ $\mathbf{v'_2(t) + B_1\, v'_1(t) = 0}$

$\mathbf{i_1(t) + Q_1\, i_2(t) = 0}$ $\qquad$ $\mathbf{i'_1(t) + Q_1\, i'_2(t) = 0}$

$\mathbf{v_2(t) = -B_1\, v_1(t)}$ $\qquad$ $\mathbf{v'_2(t) = -B_1\, v'_1(t)}$

$\mathbf{i_1(t) = -Q_1\, i_2(t)}$ $\qquad$ $\mathbf{i'_1(t) = -Q_1\, i'_2(t)}$

$$\mathbf{v^T(t) i(t)} = \begin{bmatrix}\mathbf{v_1^T(t)} & \mathbf{v_2^T(t)}\end{bmatrix}\begin{bmatrix}\mathbf{i_1(t)}\\ \mathbf{i_2(t)}\end{bmatrix} = \mathbf{v_1^T(t)\, i_1(t)} + \mathbf{v_2^T(t)\, i_2(t)} =$$

$\mathbf{v_1^T(t)\ [\ -Q_1 i_2(t)\ ] + [\ -v_1^T(t) B_1^T\ ]\ i_2(t) =}$

$\mathbf{v_1^T(t)\ B_1^T\ i_2(t) - v_1^T(t)\ B_1^T\ i_2^T(t) = 0} \quad \mathbf{(\ Q_1 = -B_1^T\ )}$

Thus $\mathbf{v^T(t) i(t) \equiv 0}$

Similarly, we can deduce the followings for the circuit D'

$\mathbf{v'^T(t)\ i'(t) \equiv 0}$

$\mathbf{v^T(t)\ i'(t) \equiv 0}$

$\mathbf{v'^T(t)\ i(t) \equiv 0}$

From these expressions, we can obtain the following property:

In an electrical circuit, the sum of instantanous power equals zero,

i.e. $p(t) = \mathbf{v(t)^T\, i(t)} = \sum_{k=1}^{n_e} \mathbf{v_k\, (t) i_k\, (t) \equiv 0}$

Energy generated = energy dissipated

(the law of conservation of energy)