

Istanbul Technical University

Department of Computer Engineering

BLG 202E – Numerical Methods

Assignment 3

Solutions

**Solution 1**

**Solution 1.a**

Linear interpolant through the points with abscissae 7 and 14 can be obtained like following:

, ,

are the most accurate ones because 12 is more near to interval of 0-14 data points; not 21. So that to observe the most accurate function, interpolant data point must be selected properly.

**Solution 1.b**

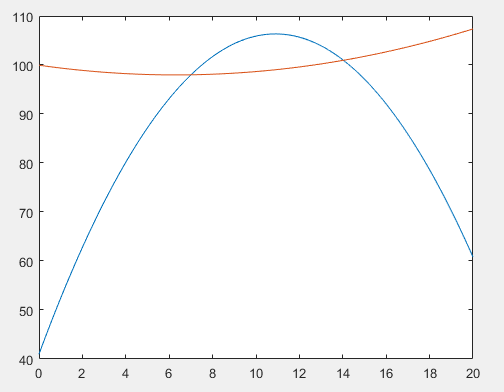
x = linspace(0,20);

y1 = -0.55\*x.^2+11.99\*x+41

y2 = 0.05\*x.^2-0.63\*x+99.96

figure

plot(x,y1,x,y2)



The interval of 6 and 14, result is close each other so that more accurate. On the other hand, lower and upper bound is equal. These two quadratic interpolant just give an approximate result in short area.

**Solution 2**

**Solution 2.a**

To find the upper bound of interval we need to interpolating polynomial found in solution b.

is expression for the error in polynomial interpolation.

Maximum value of third degree polynomial can be found by derivation .

Maximum value of can be obtained while . So that is max value.

**Solution 2.b**

Using Lagrangian Interpolation, can be found as following:

**Solution 2.c**

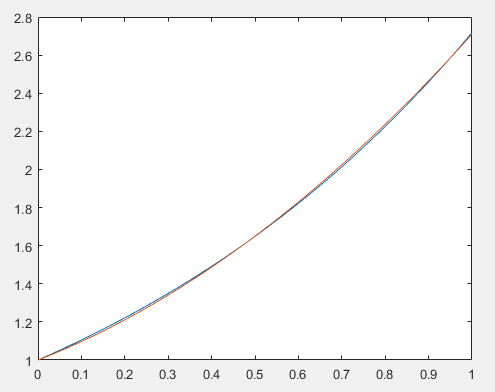
x = linspace(0,1);

y1 = 2.718182.^x;

y2 = 0.82\*x.^2+0.89\*x+1;

figure

plot(x,y1,x,y2)



It can be seen easily that two function is same (first one is second derivative which is , and second one is interpolating polynomial found by Lagrange Method).

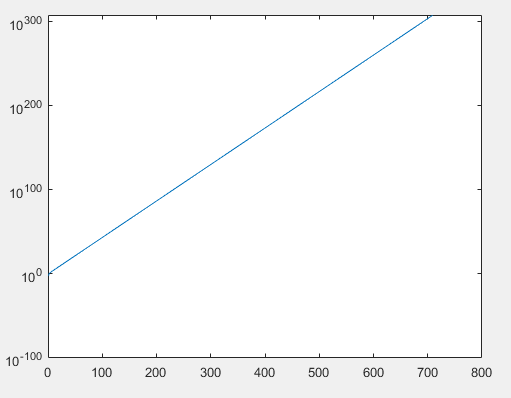
**Solution 2.d**

x = 0:1000;

y = (2.718182.^x)-(0.82\*x.^2+0.89\*x+1);

figure

semilogy(x,y)



**Solution 3**

**Solution 3.a**

can be derived by using Taylor approximation 5-pt formula. A centered formula (at ):

Expand about at , ,

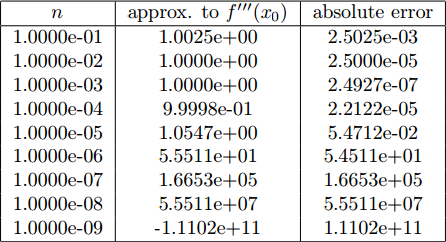
Then by adding and multiplying, it can be like following equation:

Third derivative is determined as following:

Truncation Error

**Solution 3.b**

As can bee seen for the first three values of n, when we reduce h by a factor of 10, the error goes down by afactor of 100, so the method is second order.For example, apply the formula with one value of h and then h/2. It can be seen easily the error should be reduced by a factor of 4.



syms x

syms h

f = inline(2.718182.^x, 'x');

fpp = inline(1/(2\*h.^3 )\*(f(2\*h)-f(-2\*h)-2\*(f(h)- f(-h))),'h'); %% approximate derivation

k = [1:1:9]

h = 10.^-k

result = fpp(h)

err = abs(1-result) %% absolute error

plot(k,err)

For gives the closest approximation to

**Solution 3.c**

When decreases, absolute error increase rapidly doe to round of error. The round off error is inversely proportional to as h approach to 0.

**Solution 3.d**

By using 7-pt central difference formula, it can be obtained in fourth order.

at , , , ,

**Solution 4**

syms x

syms h

fpp = inline((sin(1.2+h)-2\*sin(1.2)+sin(1.2-h))/h^2, 'h'); %% approximate derivation

true\_value = -sin(1.2) %% absolute derivative of sin(x)

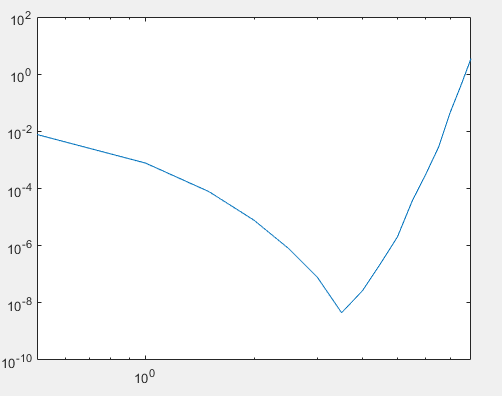
k = [0:.5:8]

h = 10.^-k

result = fpp(h)

err = abs(true\_value-result) %% absolute error

loglog(k,err)



Increasing values corresponding to decreasing values, absolute error increase rapidly in and maximum error is obtained in . It arise from the floating point error of . So that approximating the socond derivative gives closest values while which means absolute error is zero.