Midterm

1. **There are 2017 computers in the computer network. Any two computers are connected by a cable. Due to congestion in the network, one of the cables periodically burns out in some part of the network, forming a cycle of an even number of computers. Can exactly 2016 working cables remain after some time if none of the burnt cables is repaired?**

2. **2017 scientists arrived at the scientific conference. Each of these scientists knows no more than three other scientists, and their acquaintance is mutual (i.e., if A knows B, then B knows A). At this conference, scientists want to listen to the reports of those scientists with whom they are not yet familiar. Prove that scientists can be divided into 4 sections so that each section is attended by no more than 1007 scientists who do not know each other.**

3. **There are several cities in the country, some pairs of cities are connected by bilateral non-stop airlines belonging to k airlines. It is known that every two lines of one airline have a common end. Prove that all cities can be divided into k + 2 groups so that no two cities from the same group are connected by an airline.**

4. **There are several cities in the country, some pairs of cities are connected by roads, and between each two cities there is a single non-self-intersecting road path. It is known that there are exactly 100 cities in the country, from which there is one road. Prove that it is possible to build 50 new roads so that after that, even if any road is closed, it will be possible to get from each city to any other.**

5. **There are seven lakes in the Ozernaya country, interconnected by ten non-intersecting channels, and from each lake you can swim to any other. How many islands are there in this country?**