

Problem Statement 4

Curve: $y = \frac{2x}{(1+x)} \quad \text{--- (1)}$

$A(0,1) \rightarrow m = \frac{y-1}{x-0} = \frac{2}{(1+x)^2} \quad \text{--- (2)}$

In eqⁿ (2) \rightarrow

$$y-1 = \frac{2x}{(1+x)^2}$$

$$y = 1 + \frac{2x}{(1+x)^2}$$

$$y = \frac{(1+x)^2 + 2x}{(1+x)^2} \quad \text{--- (3)}$$

Now, substitute y from (1) in (3)

$$\rightarrow \frac{2x}{1+x} = \frac{1+x^2+4x}{(1+x)^2} \quad \text{--- (4)}$$

$$\therefore 2x(1+x) = x^2 + 4x + 1$$

$$\rightarrow 2x^2 + 2x = x^2 + 4x + 1$$

$$\rightarrow x^2 - 2x - 1 = 0$$

$$\rightarrow x = \frac{2 \pm \sqrt{4+4}}{2}$$

$$x = 1 \pm \sqrt{2}$$

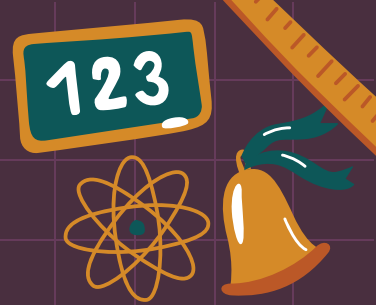
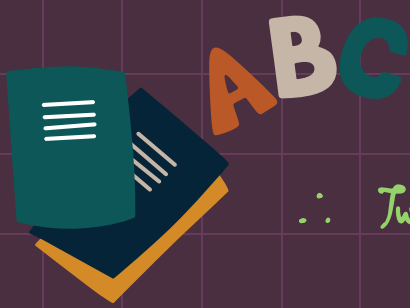
$$+ \quad x = 1 + \sqrt{2}$$

$$y = \frac{2(1+\sqrt{2})}{2+\sqrt{2}}$$

$$- \quad x = 1 - \sqrt{2}$$

$$y = \frac{2(1-\sqrt{2})}{2+\sqrt{2}}$$





\therefore Two points of tangency.

Drawing a rough sketch of curve.

Curve: $y = \frac{2x}{x+1}$

At $x=0$; $y=0$.

Poles: $x=-1$

Also $\frac{dy}{dx} = \frac{2}{(x+1)^2} \rightarrow \frac{dy}{dx} > 0$

[Increasing function]

Now $\lim_{x \rightarrow -\infty} \frac{2x}{x+1} = 2$ and $\lim_{x \rightarrow \infty} \frac{2x}{x+1} = 2$

