# Problem Statement 4

Curve: $y = \dfrac{2x}{(1+x)}$ — (1)

A(0,1) :→ $m = \dfrac{y-1}{x-0} = \dfrac{2}{(1+x)^2}$ — (2)

In eqⁿ (2) :→

$$y - 1 = \frac{2x}{(1+x)^2}$$

$$y = 1 + \frac{2x}{(1+x)^2}$$

$$y = \frac{(1+x)^2 + 2x}{(1+x)^2} \quad \text{— (3)}$$

Now, substitute y from (1) in (3)

$$\rightarrow \quad \frac{2x}{1+x} = \frac{1+x^2+4x}{(1+x)^2} \quad \text{— (*)}$$

$$\because \quad 2x(1+x) = x^2+4x+1$$

$$\rightarrow \quad 2x^2 + 2x = x^2+4x+1$$

$$\rightarrow \quad x^2 - 2x - 1 = 0$$

$$\searrow \quad x = \frac{2 \pm \sqrt{4+4}}{2}$$

$$x = 1 \pm \sqrt{2}$$

+

$$x = 1+\sqrt{2}$$

$$y = \frac{2(1+\sqrt{2})}{2+\sqrt{2}}$$

−

$$x = 1-\sqrt{2}$$

$$y = \frac{2(1-\sqrt{2})}{2+\sqrt{2}}$$

$\therefore$ Two points of tangency.

Drawing a rough sketch of curve.

CURVE : $y = \dfrac{2x}{x+1}$

At $x=0$; $y=0$.

Poles : $x=-1$

Also $\dfrac{dy}{dx} = \dfrac{2}{(x+1)^2} \rightarrow \dfrac{dy}{dx} > 0$

[Increasing function]

Now $\lim\limits_{x \to -\infty} \dfrac{2x}{x+1} = 2$ and $\lim\limits_{x \to \infty} \dfrac{2x}{x+1} = 2$