40AR015 – Exercises 2

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E1 - 1 Given the images in the Figure 1.1 and 1.2, find co-occurrence matrices in gray levels for d= (1,1), (1,0), (0,1)

0	2	0	1	0
1	0	1	1	0
0	0	2	0	0
1	1	0	0	2
0	0	1	1	0

Figure 1.1

0	1	0	1	0
0	1	0	1	0
0	1	0	1	0
0	1	0	1	0
0	1	0	1	0

Figure 1.2

Let the image in the Figure 1.1 be declared A and the image in the Figure 1.2 be declared B. In the following table are shown co-occurrence matrices for corresponding pictures and displacements vectors.

Figure \ disp. vec.	d(1,1)	d(1,0)	d(0,1)
A	$\frac{1}{16}\begin{pmatrix}3 & 3 & 2\\ 5 & 1 & 0\\ 1 & 1 & 0\end{pmatrix}$	$\frac{1}{20}\begin{pmatrix}4 & 3 & 3\\ 5 & 3 & 0\\ 2 & 0 & 0\end{pmatrix}$	$\frac{1}{20}\begin{pmatrix}4 & 6 & 1\\ 4 & 1 & 1\\ 3 & 0 & 0\end{pmatrix}$
B	$\frac{1}{16}\begin{pmatrix}0 & 8\\ 8 & 0\end{pmatrix}$	$\frac{1}{20}\begin{pmatrix}0 & 10\\ 10 & 0\end{pmatrix}$	$\frac{1}{20}\begin{pmatrix}12 & 0\\ 0 & 8\end{pmatrix}$

Table 1 Co- occurrence matrices of given figures and displacemenet vectors

E1 - 2 Compute the entropy, energy and homogeneity

The entropy, energy and homogeneity of co-occurrence matrices are computed using following equations.

$$\text{Entropy} = -\sum_i \sum_j P[i,j] \log P[i,j]$$

$$\text{Energy} = \sum_i \sum_j P^2[i,j]$$

$$\text{Homogeneity} = \sum_i \sum_j \frac{P[i,j]}{1+|i-j|}$$

The desired features for each combination of figures and displacement vectors are computed in the following table.

Figure	Disp. vec.	Entropy	Energy	Homogeneity
A	d(1,1)	2.555	0.195	0.594
A	d(1,0)	2.528	0.180	0.633
A	d(0,1)	2.509	0.200	0.592
B	d(1,1)	1	0.500	0.500
B	d(1,0)	1	0.500	0.500
B	d(0,1)	0.971	0.520	1.000

Table 2 Features of computed co-occurences matrices from Table 1

E2 - 1 Segment the image in Figure 2.1 and obtain regions of more than 5px

0	0	0	0	0	0	0	0	90	90	90	0
0	90	90	90	0	0	0	0	0	90	0	0
0	90	90	90	0	0	0	0	0	0	0	0
0	90	90	90	0	0	0	90	90	0	0	0
0	90	90	90	0	0	0	90	90	0	0	0
0	90	90	90	0	0	0	90	90	0	0	0
0	0	90	90	90	90	90	90	90	0	0	0
0	0	0	90	90	90	90	90	90	0	0	0
0	0	0	0	90	90	90	0	0	0	90	90
0	0	0	0	0	90	0	0	0	90	90	0
0	90	0	0	0	0	0	0	90	90	0	0
0	0	0	0	0	0	0	0	0	0	0	0

Figure 2 Image to be segmented

The first operation to be done is binarization of the image in Figure 1. This operation can be seen in the following Figure.

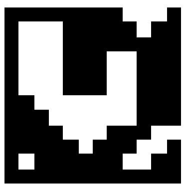


Figure 3 Binarized image from Figure 2

The next step is obtaining regions with more than five pixels and segmentation of image from Figure 3. This process is shown in the Figure 4.

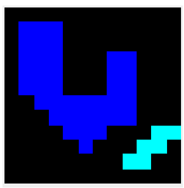


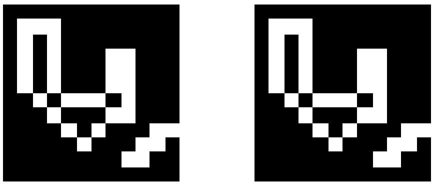
Figure 4 Segmented image with regions with more than 5 pixels

E2 - 2 Extract the contour and obtain the perimeter, area, geometric center, orientation, magnitude and compacity

Firstly, the contour was extracted. The gradient techniques for obtaining the contour were not suitable for this problem. That is why different approaches were chosen. For this problem, two approaches were found to be optimal – the morphological operations and contour extraction based on connectivity analysis. Both are retuning the same result when set correctly. The result when the connectivity is set not only on horizontal and vertical directions, but also on the diagonal connections (connectivity=8) and when the morphological operation

$$\beta(A) = A - (A \otimes B)$$

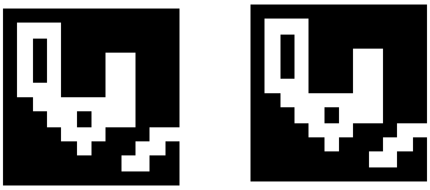
is performed using filter B = [0 1 0; 1 1 1;0 1 0]. Also called boundary extraction.



a) b)

Figure 5 a) result obtained by means of morph. operation beta(A). b) result obtained by means of algorithm with connectivity set to 8

When the same operations are performed but with connectivity equal to 4 (when connectivity is set in horizontal and vertical directions) and when the morphological operation is performed using filter B = [1 1 1; 1 1 1;1 1 1], the result is following.



a) b)

Figure 6 a) result obtained by means of morph. operation beta(A). b) result obtained by means of algorithm with connectivity set to 4

It is obvious that when the connectivity is set to 8, the resulting contours of segmented objects is clearer.

Obtained features from the segmented objects (perimeter, area, geometric center, orientation, magnitude and compacity) are shown in the following table. The colours of objects correspond to the Figure 4.

	Object 1 (bigger)	Object 2 (smaller)
Perimeter	28	6
Area	38	6
Geom. center	[5.29 5.89]	[10.50 10.00]
Orientation	-37.78	39.69
Magnitude*	11.6675; -1.4729	11.1881; -9.6047
Compacity	20.632	6.00

Table 3 Features of segmented obejcts

*The magnitude is computed by the formula that returns 2 values. It is not clear how to operate with these results. However, if the norm of is computed, the results are 11.6675 for Object 1 and 14.7453 for Object 2.

E3 Distinguish between two types of ship sails using different classifiers

The goal of the third exercise is to train 3 classifiers (K-nearest neighbor, Gaussian Bayes and Mahalanobis) on samples of two classes of data.

Class 1: (6,6)(6,8)(7,6)(7,7)(8,6)(8,7)(8,9)(9,5)(9,8)(10,4)(10,2)(11,3)
Class 2: (4,5)(4,6)(5,4)(5,5)(6,4)(7,5)(9,7)(10,5)(10,6)(10,7)(11,8)

Figure 7 Samples of classes to be distinguished

All the sample data must be used to build the classifiers and all the data must be used for validation.

E3 - 1 Obtain discriminant function for each classifier (best K in case of k-nn)

The Gaussian Bayes classifier was obtained first. This classifier is based on the following inequality.

$$\ln p(W_i) - \frac{1}{2}\ln|C_i| - \frac{1}{2}[X - m_i]^T C_i^{-1}[X - m_i] > \ln p(W_j) - \frac{1}{2}\ln|C_j| - \frac{1}{2}[X - m_j]^T C_j^{-1}[X - m_j]$$

To obtain the surface that divides both classes (discriminant function), the d(1) and d(2) must be computed first (those are left and right sides of the inequality) and equality must be found between them. In other words, once the equation d(1)-d(2)=0 is satisfied, that is the found discriminant function. To obtain d(1) and d(2), firstly the means of classes of the data must be computed

```
mean1 = 1×2
        8.2500    5.9167
```

```
mean2 = 1×2
        7.3636    5.6364
```
,

the covariances of the classes must be computed

```
Cov1 = 2×2
         2.5682   -2.2500
        -2.2500    4.4470
```

```
Cov2 = 2×2
         7.2545    2.2455
         2.2455    1.6545
```
,

and prior probabilities for each class

```
P1 = 0.5217          P2 = 0.4783
```
.

With these, the discriminant function of the Gaussian Bayes classifier is:

$$\mathrm{d}(1) - \mathrm{d}(2) = -0.231x^2 - 0.677xy + 0.3191y^2 + 7.9319x + 1.811y - 38.4.$$

When the discriminant function is shown along with the original (true) data, this is the obtained graph. It is clear from the graph that some of the original data were misclassified. Three data points, to be exact.

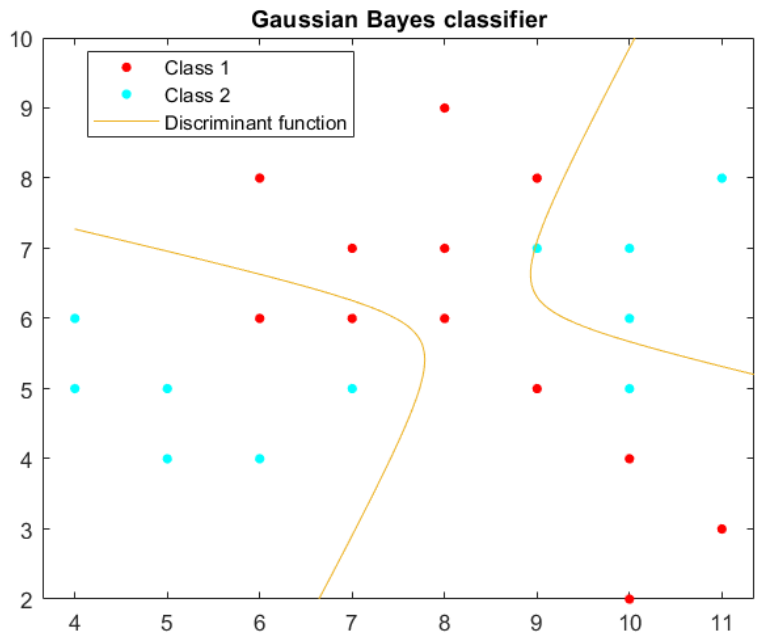


Figure 8 Discriminant function of the Gaussian Bayes classifier along with the true data

Next classifier to be obtained is the Mahalanobis classifier. The Mahalanobis distance is defined as:

$$D_M(X) = \sqrt{(X-M)^T C^{-1}(X-M)}$$

where X is data, M is mean and C is covariance matrix. Now after obtaining the square distances for each class and subtracting them, the discriminant function of the Mahalanobis classifier is:

$$d(1) - d(2) = 0.462x^2 + 1.353xy - 0.638y^2 - 15.864x - 3.621y + 77.07,$$

where d(1) and d(2) are $D_m^2(X)$.

Let's take a look on how the discriminant function of the Mahalanobis classifier look like.

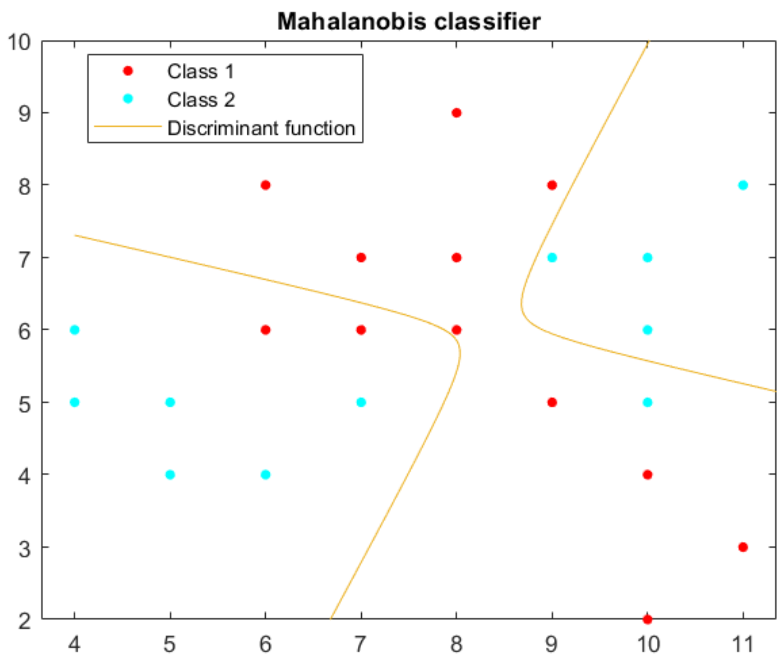


Figure 9 Discriminant function of the Mahalanobis classifier along with the true data

It can be seen in the previous Figure that some of the data were again misclassified. Same data as the last time. However, the classifiers are not the same at all. It can be seen that the space between two hyperboles in the Gaussian Bayes classifier is significantly larger than in the case of Mahalanobis classifier.

For the k-nearest neighbour classifier, the best K must be obtained. Best K was found in two ways. First way was to try all possible values of K in the algorithm. The number of correctly classified data using K = 1:11 is:

```
17    17    20    18    18    16    18    13    14    12    10
```

The K with best result (most well classified data) is the best K- that is **K=3**. The other way of finding out the best K was using built-in Matlab function for k-nearest neighbour classification and evaluating the loss, so some kind of metric is used when evaluating the best K parameter. The smallest loss was obtained also for K=3. The K = 3 is the best obtained K and verified. The resulting classification of training data is shown in the following Figure.

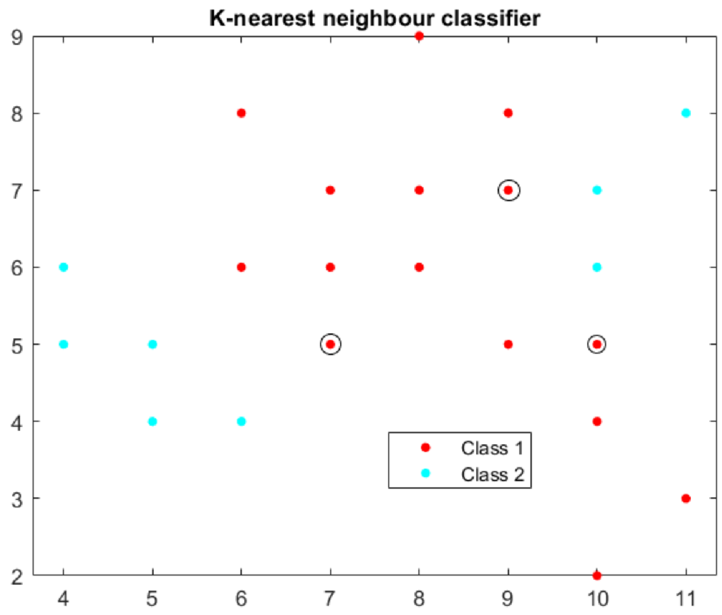


Figure 9 Data classified using K-nearest neighbour classifier

In the figure there are three data points from the original set of data that are wrongly classified, those are encircled in the image.

E3 - 2 Which is the best classifier with respect to the number of samples well classified?

The best classifier cannot be chosen by this criterion only, because all of the classifiers have the same number of well classified samples. Every classifier classified well 20 out of 23 samples, while having 3 out of 23 misclassified.

E3 - 3 Obtain the classification of new samples of data

The samples of data to be classified using found classifiers are following.

(8,3)(8,5)(8,8)(10,8)

Figure 10 Test data

When using Gaussian Bayes classifier to classify the test data, the way the samples are classified is shown in the following figure, where encircled data points are the test data newly classified. In other words, the samples (8,3), (8,5) and (8,8) are classified as class number 1 and the sample (10,8) is classified as class number 2.

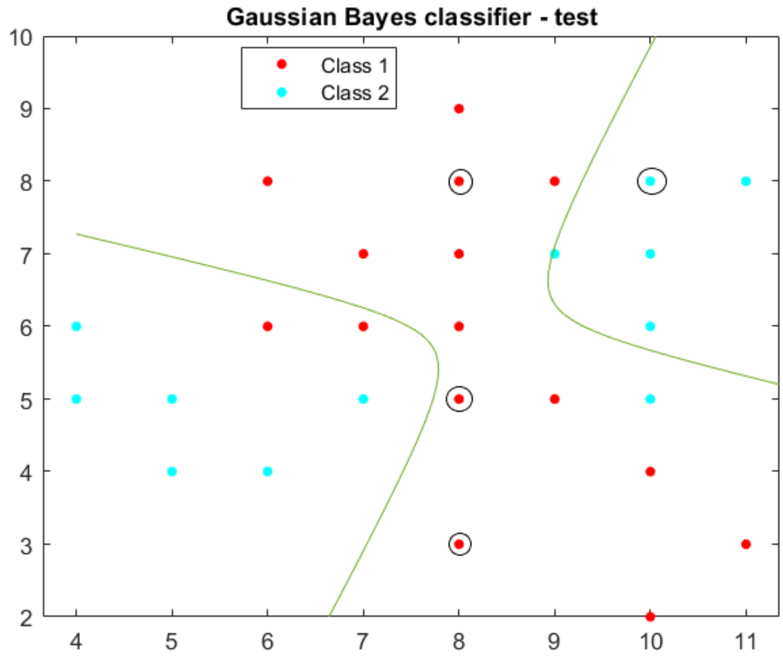


Figure 11 Classification of test data using Gaussian Bayes classifier

Mahalanobis classifier classified the test data in exactly same way as the Gaussian Bayes classifier. That means that first three data samples are classified as class 1 and the last sample as class two. The classification of test data by Mahalanobis classifier is shown in the following figure, where, again, the encircled data points are the test samples newly classified.

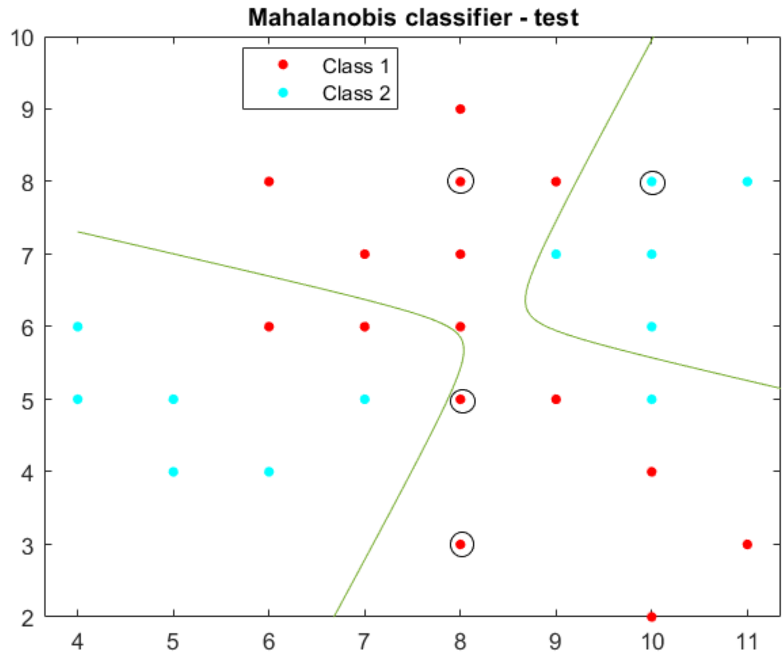


Figure 12 Classification of test data using Mahalanobis classifier

Representation of test data classified by K-nearest neighbour is not as clear as the previous two approaches, since the class of one sample is defined by its neighbours. Nonetheless, the data samples are classified to the same classes as with the two previous classifiers. That means that the samples (8,3), (8,5) and (8,8) are classified as class 1 and the sample (10,8) is classified as class 2. This result is also shown in the following figure, where encircled data samples are the ones newly classified from test dataset.

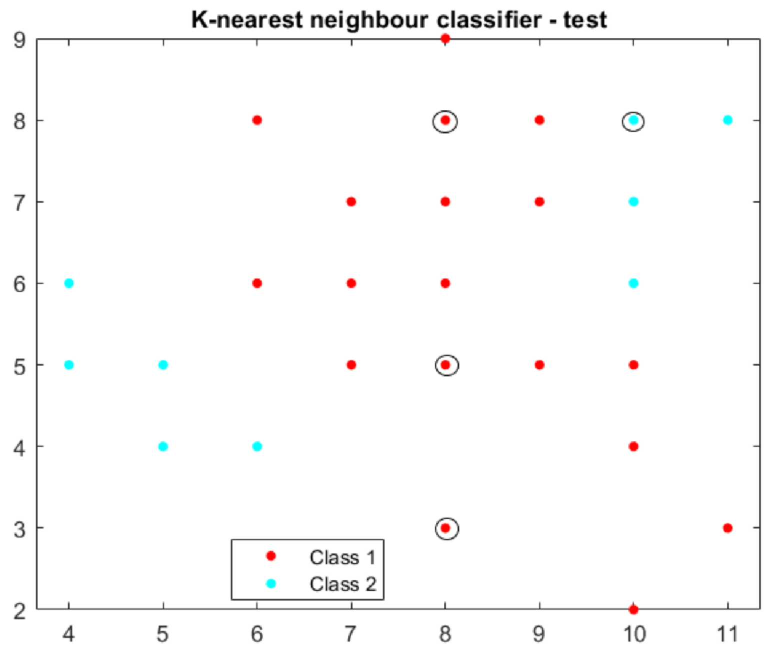


Figure 13 Classification of test data using K-nearest neighbour classifier