Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 1

Wednesday 2 May 2018 (afternoon)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

11 pages

12EP01

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

Let $f(x) = \sqrt{x+2}$ for $x \geq -2$ and $g(x) = 3x - 7$ for $x \in \mathbb{R}$.

(a) Write down $f(14)$. [1]

(b) Find $(g \circ f)(14)$. [2]

(c) Find $g^{-1}(x)$. [3]

2.   [Maximum mark: 6]

The following box-and-whisker plot shows the number of text messages sent by students in a school on a particular day.
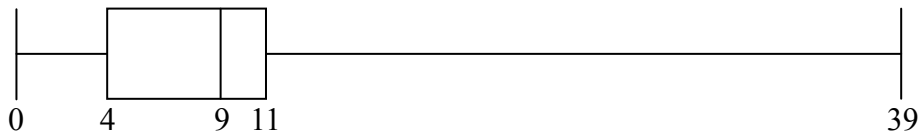
(a)   Find the value of the interquartile range.   [2]

(b)   One student sent k text messages, where $k > 11$. Given that k is an outlier, find the least value of k.   [4]

3.　[Maximum mark: 7]

Consider a function $f(x)$, for $-2 \leq x \leq 2$. The following diagram shows the graph of f.

(a)　Write down the value of

　　(i)　$f(0)$;

　　(ii)　$f^{-1}(1)$.　　　　　　　　　　　　　　　　　　　　　　　　　　　[2]

(b)　Write down the range of f^{-1}.　　　　　　　　　　　　　　　　　　[1]

(c)　On the grid above, sketch the graph of f^{-1}.　　　　　　　　　　　[4]

(This question continues on the following page)

(Question 3 continued)

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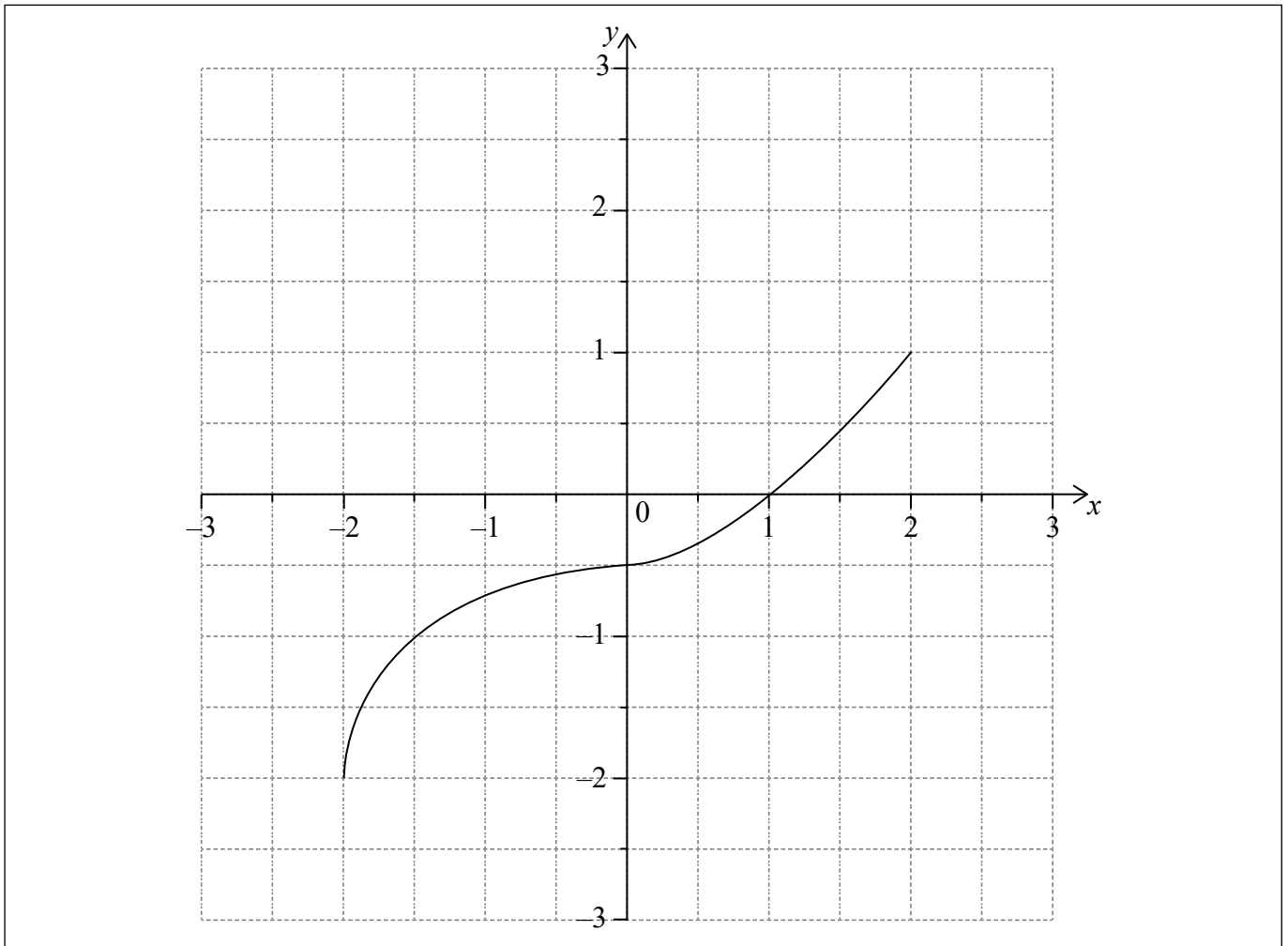. . . . .
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4. [Maximum mark: 7]

Let $f(x) = ax^2 - 4x - c$. A horizontal line, L, intersects the graph of f at $x = -1$ and $x = 3$.

(a) (i) The equation of the axis of symmetry is $x = p$. Find p.

(ii) Hence, show that $a = 2$. [4]

(b) The equation of L is $y = 5$. Find the value of c. [3]

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5.   [Maximum mark: 7]

Let $f(x) = \dfrac{1}{\sqrt{2x-1}}$, for $x > \dfrac{1}{2}$.

(a)   Find $\displaystyle\int \left(f(x) \right)^2 \mathrm{d}x$.   [3]

(b)   Part of the graph of f is shown in the following diagram.
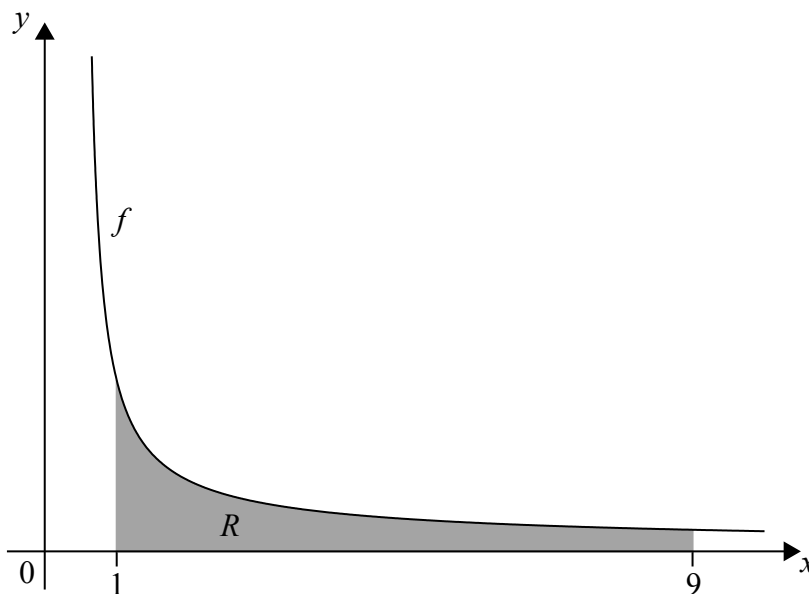
The shaded region R is enclosed by the graph of f, the x-axis, and the lines $x = 1$ and $x = 9$. Find the volume of the solid formed when R is revolved $360°$ about the x-axis.   [4]
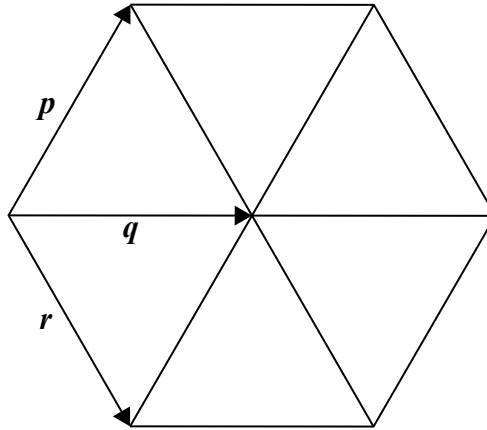
Turn over

6. [Maximum mark: 6]

Six equilateral triangles, each with side length $3 \, \text{cm}$, are arranged to form a hexagon. This is shown in the following diagram.

diagram not to scale

The vectors p, q and r are shown on the diagram.

Find $p \cdot (p + q + r)$.

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7.   [Maximum mark: 7]

Consider $f(x)$, $g(x)$ and $h(x)$, for $x \in \mathbb{R}$ where $h(x) = (f \circ g)(x)$.

Given that $g(3) = 7$, $g'(3) = 4$ and $f'(7) = -5$, find the gradient of the normal to the curve of h at $x = 3$.

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 13]

A function $f(x)$ has derivative $f'(x) = 3x^2 + 18x$. The graph of f has an x-intercept at $x = -1$.

(a) Find $f(x)$. [6]

(b) The graph of f has a point of inflexion at $x = p$. Find p. [4]

(c) Find the values of x for which the graph of f is concave-down. [3]

9. [Maximum mark: 16]

Point A has coordinates $(-4, -12, 1)$ and point B has coordinates $(2, -4, -4)$.

(a) Show that $\overrightarrow{AB} = \begin{pmatrix} 6 \\ 8 \\ -5 \end{pmatrix}$. [1]

(b) The line L passes through A and B.

   (i) Find a vector equation for L.

   (ii) Point $C(k, 12, -k)$ is on L. Show that $k = 14$. [6]

(c) (i) Find $\overrightarrow{OB} \bullet \overrightarrow{AB}$

   (ii) Write down the value of angle OBA. [3]

Point D is also on L and has coordinates $(8, 4, -9)$.

(d) Find the area of triangle OCD. [6]

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10. [Maximum mark: 15]

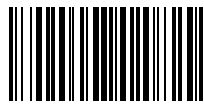The first two terms of an infinite geometric sequence are $u_1 = 18$ and $u_2 = 12\sin^2 \theta$, where $0 < \theta < 2\pi$, and $\theta \neq \pi$.

(a) (i) Find an expression for r in terms of θ.

(ii) Find the possible values of r. [5]

(b) Show that the sum of the infinite sequence is $\dfrac{54}{2 + \cos(2\theta)}$. [4]

(c) Find the values of θ which give the greatest value of the sum. [6]

Please **do not** write on this page.

Answers written on this page
will not be marked.