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Long/ Short Trading Strategies Design & Backtest

Certificate in Quantitative Finance

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1. Introduction

This report explores cointegration between pairs of front month commodity future price time series using 13 years of data between 2008 and 2021. The pairs were selected from a list of assets from the energy sector, including futures on energy commodities such as crude oil, natural gas, heating oil and an Exchange Trading Fund (ETF) offering exposure to the nuclear power industry. The cointegration analysis was conducted using the Engle-Granger procedure to identify cointegrated pairs with a long-term equilibrium. This report also defines a long/short pair trading strategy based on the cointegration coefficients between variables and on the Ornstein-Uhlenbeck process to determine trading signals. The report also contains strategy backtesting using performance and risk metrics computed on both a training and a test sample as well as rolling cointegrations. Prior to the cointegration analysis a vector autoregression model was implemented.

The code used for the calculations and the visualisations is provided in the attached Jupyter Notebook. It generates three Excel files that are saved in the attached zip file. The proprietary functions used are listed in the Excel file 'Functions.xlsx'.

2. Theoretical Framework and Methodology

a. Vector Autoregression

A vector autoregression model (VAR) are multivariate models that link current observations of a time series with past observations of other time series. VAR models are based on autoregressive models (AR).

An AR calculates the future values of a time series based on its previous time lags. If we consider a time series X_t then for a lag of 2 we have:

$$X_t = \alpha + \beta_1 X_{t-1} + \beta_2 X_{t-2} + \varepsilon_t$$

This an AR(2) model. A VAR model calculates future values of a variable based on its previous lags and the previous lags of other related variables.

If we have two variables $X_{1,t}$ and $X_{2,t}$ a VAR(1) model is described by the following system:

$$X_{1,t} = \alpha_1 + \beta_{1,1} X_{1,t-1} + \beta_{1,2} X_{2,t-1} + \varepsilon_{1,t}$$
$$X_{2,t} = \alpha_2 + \beta_{2,1} X_{1,t-1} + \beta_{2,2} X_{2,t-1} + \varepsilon_{2,t}$$

In a matrix form a VAR(P) model is written (Lütkepohl, Helmut, New Introduction to Multiple Time Series Analysis):

$$\begin{bmatrix} X_{1,t} \\ X_{2,t} \\ \vdots \\ X_{k,t} \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_k \end{bmatrix} + \begin{bmatrix} \beta^1_{1,1} & \beta^1_{1,2} & \cdots & \beta^1_{1,k} \\ \beta^1_{2,1} & \beta^1_{2,2} & \cdots & \beta^1_{2,k} \\ \vdots & \vdots & \ddots & \vdots \\ \beta^1_{k,1} & \beta^1_{k,2} & \cdots & \beta^1_{k,k} \end{bmatrix} \begin{bmatrix} X_{1,t-1} \\ X_{2,t-1} \\ \vdots \\ X_{k,t-1} \end{bmatrix} + \cdots + \begin{bmatrix} \beta^p_{1,1} & \beta^p_{1,2} & \cdots & \beta^p_{1,k} \\ \beta^p_{2,1} & \beta^p_{2,2} & \cdots & \beta^p_{2,k} \\ \vdots & \vdots & \ddots & \vdots \\ \beta^p_{k,1} & \beta^p_{k,2} & \cdots & \beta^p_{k,k} \end{bmatrix} \begin{bmatrix} X_{1,t-p} \\ X_{2,t-p} \\ \vdots \\ X_{k,t-p} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \vdots \\ \varepsilon_{k,t} \end{bmatrix}$$

The expression can be rewritten in a reduced form for a VAR(P) with k variables and time observation from $x_p$to x_T:

$$X = BZ + U$$

Where:

$$X = \begin{bmatrix} x_p & x_{p+1} & \cdots & x_T \end{bmatrix} = \begin{bmatrix} x_{1,p} & x_{1,p+1} & \cdots & x_{1,T} \\ x_{2,p} & x_{2,p+1} & \cdots & x_{2,T} \\ \vdots & \vdots & \ddots & \vdots \\ x_{k,p} & x_{k,p} & \cdots & x_{k,T} \end{bmatrix}$$

$$B = \begin{bmatrix} \alpha & B_1 & \cdots & B_k \end{bmatrix} = \begin{bmatrix} \alpha_1 & \beta^1_{1,1} & \beta^1_{1,2} & \cdots & \beta^1_{1,k} & \cdots & \beta^p_{1,1} & \beta^p_{1,2} & \cdots & \beta^p_{1,k} \\ \alpha_2 & \beta^1_{2,1} & \beta^1_{2,2} & \cdots & \beta^1_{2,k} & \cdots & \beta^p_{2,1} & \beta^p_{2,2} & \cdots & \beta^p_{2,k} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \cdots & \vdots & \vdots & \ddots & \vdots \\ \alpha_k & \beta^1_{k,1} & \beta^1_{k,2} & \cdots & \beta^1_{k,k} & \cdots & \beta^p_{k,1} & \beta^p_{k,2} & \cdots & \beta^p_{k,k} \end{bmatrix}$$

$$Z = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ x_{p-1} & x_p & \cdots & x_{T-1} \\ x_{p-2} & x_{p-1} & \cdots & x_{T-2} \\ \vdots & \vdots & \ddots & \vdots \\ x_0 & x_1 & \cdots & x_{T-p} \end{bmatrix} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ x_{1,p-1} & x_{1,p} & \cdots & x_{1,T-1} \\ x_{2,p-1} & x_{2,p} & \cdots & x_{2,T-1} \\ \vdots & \vdots & \cdots & \vdots \\ x_{k,p-1} & x_{k,p} & \cdots & x_{k,T-1} \\ x_{1,p-2} & x_{1,p-1} & \cdots & x_{1,T-2} \\ x_{2,p-2} & x_{2,p-1} & \cdots & x_{1,T-2} \\ \vdots & \vdots & \ddots & \vdots \\ x_{k,p-2} & x_{k,p-1} & \cdots & x_{k,T-2} \\ \vdots & \vdots & \ddots & \vdots \\ x_{1,0} & x_{1,1} & \cdots & x_{1,T-p} \\ x_{2,0} & x_{2,1} & \cdots & x_{2,T-p} \\ \vdots & \vdots & \ddots & \vdots \\ x_{k,0} & x_{k,1} & \cdots & x_{k,T-p} \end{bmatrix}$$

$$U = \begin{bmatrix} e_{1,p} & e_{1,p+1} & \cdots & e_{1,T} \\ e_{2,p} & e_{2,p+1} & \cdots & e_{2,T} \\ \vdots & \vdots & \ddots & \vdots \\ e_{k,p} & e_{k,p+1} & \cdots & e_{k,T} \end{bmatrix}$$

In order to compute the VAR(P) model parameters, an ordinary least squares estimation is performed:

$$\hat{B} = XZ'(ZZ')^{-1}$$

$$\hat{U} = X - \hat{B}Z$$

The optimal lag can be determined using the Akaike information criterion (AIC) and the Bayesian information criterion (BIC).

The AIC is a measure of the quality of a statistical model. When a statistical model is assessed, it is possible to increase the likelihood by adding a parameter. The AIC penalized the model based on the number of parameters involved. The optimal lag is the one with the lowest AIC.

$$\mathrm{AIC} = 2k - 2\ln(L)$$

Where k is the number of parameters used for the model estimation and L is the maximum likelihood function. This criterion relies on a balance between the adjustment quality of the model and its complexity, which limits overfitting.

BIC is derived from AIC. In addition to the number of parameters, BIC considers the sample size.

$$\mathrm{BIC} = 2\ln(L) + kln(N)$$

Where N is the number of observations in the sample. The optimal model is the one which minimize the BIC:

$$M_{BIC} = \arg\min_{M} BIC(M)$$

b. Stationarity

In order to apprehend cointegration the concept of stationarity of time series needs to be introduced. A stationary process is a stochastic process that keeps the same statistical properties throughout time. If we consider a stationary stochastic process $\{X_t\}$, the probability distribution of X_t would be the same as for $X_{t+\tau}$.

More broadly, with a stationary process, the joint distribution of X_{t_1} and X_{t_2} is similar to the joint distribution of $X_{t_1+\tau}$ and $X_{t_2+\tau}$

Formally a stochastic process $\{X_t\}$ is strictly stationary if

$$F_X\left(x_{t_1}, x_{t_2,} \dots, x_{t_n}\right) = F_X\left(x_{t_1+\tau}, x_{t_2+\tau,} \dots, x_{t_n+\tau}\right)$$

Where F_X is the cumulative density function of unconditional joint distribution of $\{X_t\}$.

Practically, strictly stationary processes have interesting properties as X_{t_1} and X_{t_2} are identically distributed for any integer t_1 and t_2. Hence, their means $\mu = E[X_t]$ and their variances $\sigma^2 = var(X_t)$ are the same. Additionally, any pairs $(X_{t_1}, X_{t_1+\tau})$ and $(X_{t_2}, X_{t_2+\tau})$ have identical bivariate distributions. Thus, the autocovariance only depends on the time interval τ (Taylor, Asset Price Dynamics, Volatility, and Prediction).

$$\lambda_\tau = cov(X_t, X_{t+\tau}) = E[(X_t - \mu)(X_{t+\tau} - \mu)]$$

If the mean and the variance of a stochastic process do not change throughout time, we call the process stationary. Estimation is simpler when we are dealing with stationary processes as their parameters stay constant during the considered period.

c. Dickey-Fuller test

The Dickey-Fuller is a test for unit root where the null hypothesis is the non-stationarity of the considered time series.

We start with an autoregressive process X_t:

$$X_t = \rho X_{t-1} + \varepsilon_t$$
$$X_t - X_{t-1} = (\rho - 1)X_{t-1} + \varepsilon_t$$
$$\Delta X_t = \lambda X_{t-1} + \varepsilon_t$$

Where $\lambda = \rho - 1$

Under the null hypothesis $\lambda = 0$ because in this case we have a non-stationary time series ($X_t \sim I(1)$). The alternative hypothesis is going to be $\lambda \neq 0$ which would mean that the time series is stationary ($X_t \sim I(0)$).

The t-statistic is calculated using the following formula:

$$DF = \frac{\lambda}{standard\ error\ of\ \lambda}$$

The use of traditional critical values for the test may lead to a rejection of the null hypothesis while the stationarity is weak or non-existing.

In 2010, MacKinnon provided tables of critical values for some popular tests of cointegration and unit root, in his paper 'Critical Values for Cointegration Tests'.

The Dickey-Fuller test can be improved by adding a lagged Δ terms if there is serial correlation:

$$\Delta X_t = \lambda X_{t-1} + \sum_{i=1}^{h} \beta_i + \Delta X_{t-i} + \varepsilon_t$$

This is a unit root test for X_t. We will use a lag of 1 for the purpose of the cointegration analysis.

d. Engle-Granger Procedure

Cointegration is a statistic property of time series introduced into the economic analysis by Granger and Newbold in 1974 with their paper on spurious regressions in econometrics and more formally by Engle and Granger in 1987 with their publication on cointegration and error correction. Practically, cointegration allows for the detection of a long-run relationship between two or more time series.

If time series X_t and Y_t are integrated of order 1 and there is a linear combination of these time series integrated of order 0 then, we can say that X_t and Y_t are cointegrated. More formally if we consider X_t and Y_t , 2 time series of order d, and there exist coefficient α and β such that $\alpha X_t + \beta Y_t$ is integrated of order below d, then the two variables are cointegrated.

Cointegration tests are used to identify if the linear combination of two or more $I(1)$ time series are integrated in such manner that they cannot diverge from their long-run equilibrium.

The two-steps Engle-Granger procedure is a common approach used to identify cointegrated pairs. It was described in the CQF module on cointegration. The first step of the procedure is the test for the stationarity of the spreads between two time series.

We consider two stochastic processes X_t and Y_t both $I(1)$. We begin with the estimation of the static linear regression between the two variables:

$$Y_t = \mu + \beta_1 X_t + e_t$$

We want to determine if the spread time series (residuals) ε_t is stationary.

The first step consists of testing for a unit root in the spread time series:

$$\hat{e}_t = Y_t - \hat{\mu} - \hat{\beta}_1 X_t$$

This is a test for no-cointegration. We usually use the Augmented Dickey-Fuller test which is a unit root test described above.

$$\Delta \hat{e}_t = \hat{\beta}_2 e_{t-1} + \sum_{i=1}^{k} \lambda_i \Delta \hat{e}_{t-i} + \varepsilon_t$$

Test hypothesis:

$\mathcal{H}_0 : \hat{\beta}_2 = 0$ (Unit root/no-cointegration)

$\mathcal{H}_1 : \hat{\beta}_2 \neq 0$

T-statistic:

$$\hat{t}_{\beta_2=0} = \frac{\hat{\beta}_2}{Std(\hat{\beta}_2)}$$

If we conclude that the spreads are non-stationary there is no evidence of a long-run equilibrium and the regression is spurious.

The second step of the Engle-Granger procedure is the estimation of the Error Correction Model.

We now want to integrate the shifted residuals from step 1 in the linear regression of changes in X_t and Y_t:

$$\Delta Y_t = \varphi \Delta X_t - (1-\alpha)\, \hat{e}_{t-1}$$

If we replace $\hat{e}_{t-1}$ by its expression from step 1:

$$\Delta Y_t = \varphi \Delta X_t - (1-\alpha)\, (Y_t - \hat{\beta}_1 X_t - \mu_e)$$

$$E[e_{t-1}] = \mu_e$$

We need to confirm the statistical significance of the coefficient -(1 - α) which is the correction speed. It is small but must be significant for cointegration to happen.

The imbalance $e_{t-1} \neq \mu_e$ is corrected linearly over the long-run. The error correction is linear. If e_{t-1} is below (above) μ_e, the model suggests a small correction upwards (downwards).

$$\Delta Y_t = \varphi_{short-run} \Delta X_t + \varphi_{long-run}(Y_t - \hat{\beta}_1 X_t - \mu_e)$$

e. Ornstein-Ulhenbeck Process

We saw that cointegrated assets relate to each other’~~s~~ due to the stationarity of their spread. It gives a measure of similitude of assets’ risk profiles and describes long-run relationship between asset prices. The β coefficient represents the ratio of the long-leg to the short-leg of the trade.
In order to assess the quality of mean reversion and design trading rules we fit the spread to the Ornstein-Ulhenbeck (OU) process. The OU process is used to model the price of a mean-reverting portfolio. It is described by the following stochastic differential equation (CQF lecture on cointegration):

$$dX_t = \mu(\theta - X_t)dt + \sigma dB_t$$
$$\mu, \sigma > 0$$
$$\theta \in \mathbb{R}$$
$$B \text{ a standard Brownian motion}$$

μ: Long-term mean
θ: Mean-reversion speed
σ: Amplitude of randomness in the system
B: Optimal ratio between two assets
The solution to the stochastic differential equation has a reversion and an autoregression term.

$$e_{t+\tau} = \left(1 - e^{-\theta\tau}\right)\mu_e + e^{-\theta\tau} e_t + \varepsilon_{t,\tau}$$

In order to estimate the speed of mean reversion θ and the equilibrium μ_e and σ_{eq} we run a linear regression:

$$e_t = C + Be_{t-1} + \varepsilon_{t,\tau}$$

We derive the following expressions:

$$e^{-\theta\tau} = B \Rightarrow \theta = -\frac{\ln{(B)}}{\tau}$$

$$\left(1 - e^{-\theta\tau}\right)\mu_e = C \Rightarrow \mu_e = \frac{C}{\left(1 - e^{-\theta\tau}\right)}$$

$$\sigma_{eq} = \sqrt{\frac{SSE \times \tau}{1 - e^{-2\theta\tau}}}$$

Where SSE is the sum of square residuals of the regression.
The magnitude of the mean reversion amount of time of can be measured by its half-life:

$$\tau_{HL} = \frac{\ln{(2)}}{\theta}$$

The optimisation of a trading strategy takes two factors into account. The profit and loss (P&L) per trade and the number of trades that are determined by the entry levels (boundaries).
We consider two time series X_t and Y_t with a cointegrated coefficient β such that the spread $e_t = Y_t - \beta X \sim I(0)$ and U > 0 the upper bound and $L < 0$ the lower bound. We note the equilibrium of the spread μ_e
When $e_t \geq U$, we enter the trade by selling N asset Y and buying $N\beta$ of asset X. We close the trade when $e_t \leq \mu_e$.
Similarly, when $e_t \leq L$, we enter the trade by buying N assets Y and selling $N\beta$ of asset X. We close the trade when $e_t \geq \mu_e$.
A common approach to generate trading signal is to use pair spread Z-score which consists in the normalisation of asset prices spread time series.

$$Z(e_t) = \frac{e_t - \mu_e}{\sigma_{eq}}$$

We enter on bounds $\mu_e \pm Z\,\sigma_{eq}$ and exit around μ_e.
In practice, tighter (wider) boundaries lead to a greater (lower) number of trades and a smaller (bigger) P&L per trade.

3. Cointegration analysis

a. Data Exploration and Analysis

The data used for the cointegration analysis comprises price time series for futures on energy and an ETF replicating the performance of companies involved in the nuclear power production industry. The future contracts are front month, which means that they refer to the nearest expiration date available on the market which make them the most liquid contracts for each underlying commodity. The time series consists of daily prices during a period of 13 years between July 2008 and July 2021. The time series consists of 3274 observations. The prices are quoted in USD.

Table 1.a: Futures characteristics

Future Name	Ticker	Short Name	Contract Unit	Price Quotation	Exchange
Crude Oil Sep 21	CL=F	OIL	1,000 barrels	USD per barrel	NYMEX
Natural Gas Sep 21	NG=F	NGA	10,000 MMBtu	USD per MMBtu	NYMEX
Heating Oil Sep 21	HO=F	HOI	42,000 gallons	U.S. dollars and cents per gallon	NYMEX

Source: CME Group

Table 1.b: ETF characteristic

Future Name	Ticker	Short Name	Underlying Index	Assets under management (18/08/2021)	Exchange
VanEck Vectors Uranium+Nuclear Energy ETF	NLR	NLR	MVIS Global Uranium & Energy Index	27.3M	NYSE Arca

Source: VanEck

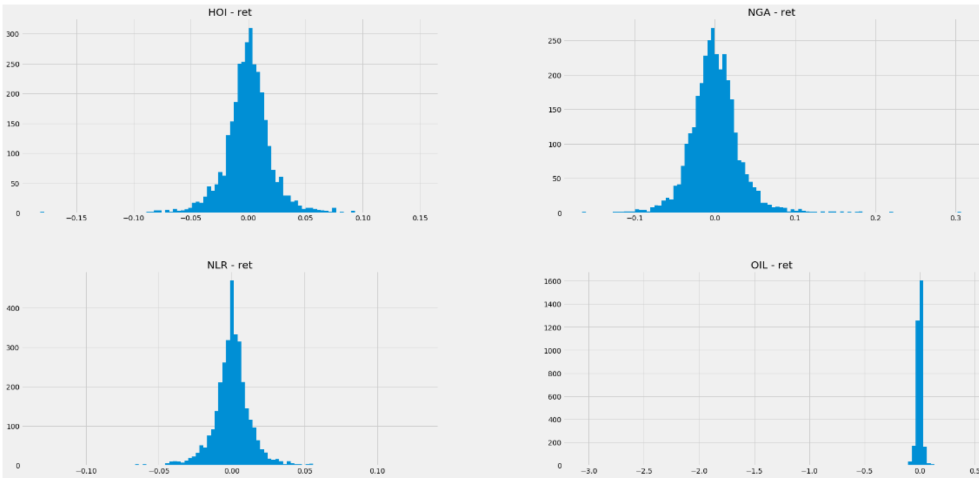
The basic statistical description for daily return data is summarised in the table below:

Table 2: Return series statistical description

	OIL - ret	NGA - ret	HOI - ret	NLR - ret
count	3273.000000	3273.000000	3273.000000	3273.000000
mean	-0.000895	0.000239	0.000089	0.000086
std	0.064473	0.031520	0.021390	0.015087
min	-3.059661	-0.165185	-0.181235	-0.131253
25%	-0.011770	-0.017813	-0.010064	-0.005776
50%	0.000358	-0.000419	0.000376	0.000000
75%	0.012078	0.016155	0.010606	0.006508
max	0.376623	0.306971	0.149301	0.128740
Skew	-34.829772	0.881987	-0.284492	-0.641259
Kurt	1592.152078	6.486798	7.474150	12.498034

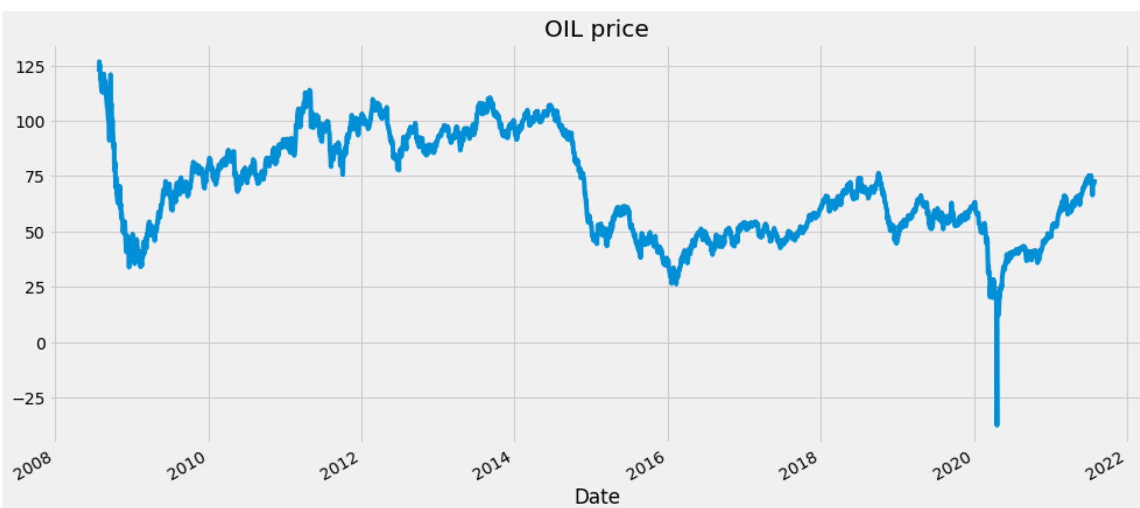
Over the considered period, crude oil future contract returns have a higher standard deviation than the other assets' returns. All the assets' returns are negatively skewed except the natural gas future contract series. Oil futures show a very important negative skewness. Chart 1 displays return distribution for each commodity.

Chart 1: Return distribution



During the first stage of the Covid-19 pandemic future's price on crude oil collapsed early 2020 while the benchmark price for the US crude oil dropped into negative territories.

Chart 2: Future on crude oil's



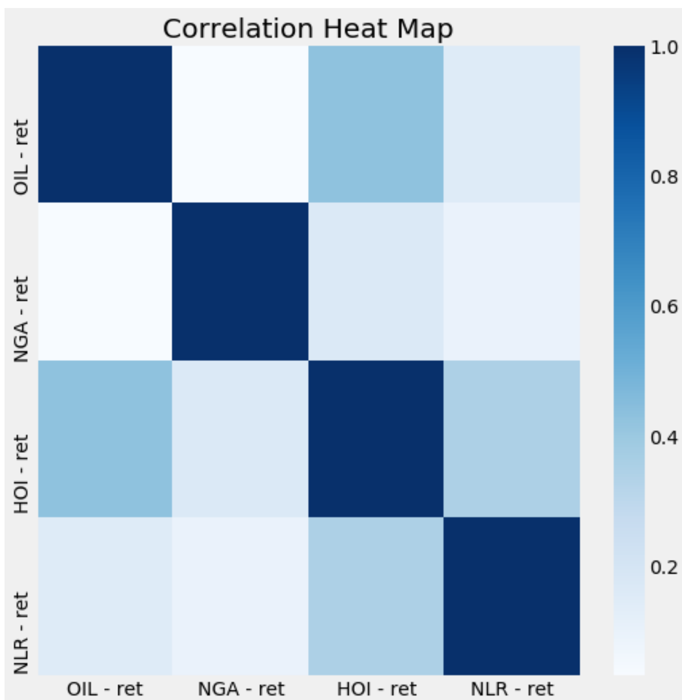
The normality of distributions can be tested using some statistical tests such as the Shapiro-Wilk test and the d'Agostino K-squared test. For both tests the null hypothesis is that the population from which the samples originated are normally distributed. If the test p-value is below $\alpha = 0.05$ we reject the null hypothesis. The table below shows p-values well below 0.05, which means the rejection of the normality of the distributions. Appendix D shows quantile-quantile plots which provide further evidence of non-normality of returns distributions. The charts compare the sample distribution with a normal distribution. If the sample were normally distributed the data points would be on a straight line.

Table 3: Normal distribution test on returns

	OIL - ret	NGA - ret	HOI - ret	NLR - ret
Shapiro - stat	0.224096	9.422314e-01	9.258206e-01	8.628874e-01
Shapiro - pvalue	0.000000	5.590889e-34	1.717026e-37	0.000000e+00
K2 - stat	9094.936103	7.640200e+02	5.293894e+02	8.445922e+02
K2 - pvalue	0.000000	1.244983e-166	1.108053e-115	3.973185e-184

Overall returns are not strongly correlated with a maximum of 0.43 between futures on crude oil and heating oil. It is possible to have weak correlation and cointegrated time series. A strong positive correlation would indicate that prices usually move in the same direction while cointegration reflects a long-term equilibrium between time series. The correlation matrix is displayed in appendix C.

Chart 3: Correlation heat map



The appendix A displays scatterplots regarding pairwise relationships.

b. Vector Autoregressive Model Implementation

The vector Autoregressive Model allows to capture the interdependencies between several stationary time series. Returns time series' stationarity was tested using the advanced Dickey-Fuller (ADF) test with 1 lag. A proprietary code including data transformation and linear regressions was used to perform the test.

Table 4: Returns stationarity test - ADF

	ADF test-statistic	p-value	Lag	Number of observations	Critical value (1%)	Critical value (5%)	Critical value (10%)	Stationary
OIL - ret	-41.739705	1.348566e-305	1	3271	-3.432351	-2.862424	-2.567241	Yes
NGA - ret	-41.914086	1.162270e-307	1	3271	-3.432351	-2.862424	-2.567241	Yes
HOI - ret	-41.129078	2.179889e-298	1	3271	-3.432351	-2.862424	-2.567241	Yes
NLR - ret	-38.896265	2.486591e-272	1	3271	-3.432351	-2.862424	-2.567241	Yes

The VAR function from the statsmodels library can be used to compute the AIC and the BIC easily. Setting a maximum lag at 10, the function generated the following table.

Table 5: VAR(p) - AIC and BIC

	AIC	BIC	FPE	HQIC
0	-28.84	-28.83	2.984e-13	-28.84
1	-28.94	-28.90	2.699e-13	-28.93
2	-28.98	-28.92*	2.583e-13	-28.96*
3	-28.99	-28.89	2.575e-13	-28.95
4	-29.00	-28.87	2.556e-13	-28.95
5	-29.00	-28.84	2.550e-13	-28.94
6	-29.00	-28.82	2.539e-13	-28.93
7	-29.01	-28.79	2.523e-13	-28.93
8	-29.01	-28.76	2.522e-13	-28.92
9	-29.02	-28.74	2.500e-13	-28.92
10	-29.02*	-28.72	2.486e-13*	-28.91

According to the AIC, the optimal model would have 10 lags. However, in order to reduce the risk of overfitting, the VAR model was performed using 3 lags which is consistent with the optimal BIC.

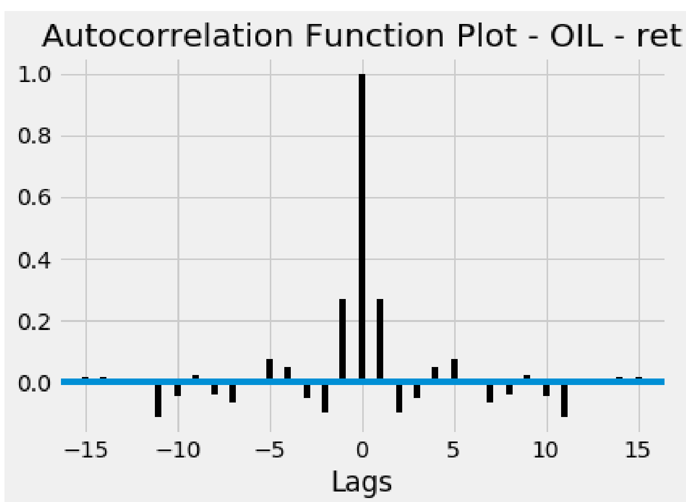
A code was designed to implement the VAR(p) model using linear algebra and matrix calculations.

Table 6: Beta coefficient of the VAR(3) model

	OIL - ret	NGA - ret	HOI - ret	NLR - ret
Const	-0.000665	0.000266	0.000144	0.000095
Lag 1 - OIL - ret	0.405539	0.01461	0.058918	0.008399
Lag 1 - NGA - ret	-0.044295	-0.063349	0.020351	-0.000626
Lag 1 - HOI - ret	-0.479906	-0.031852	-0.11413	-0.010433
Lag 1 - NLR - ret	0.01194	0.032235	0.057677	-0.038643
Lag 2 - OIL - ret	-0.227553	-0.028452	-0.021889	-0.003048
Lag 2 - NGA - ret	-0.038977	-0.012842	0.016036	0.00912
Lag 2 - HOI - ret	0.113207	0.105991	-0.003787	0.002826
Lag 2 - NLR - ret	0.229803	0.00861	0.063211	0.057706
Lag 3 - OIL - ret	0.044674	0.033554	0.028381	0.002145
Lag 3 - NGA - ret	0.041473	0.002769	0.011861	0.005307
Lag 3 - HOI - ret	0.051701	-0.046196	-0.007109	0.005465
Lag 3 - NLR - ret	-0.252321	-0.036732	-0.044441	-0.006948

There is no significant autocorrelation effect except for the future on crude oil which has a correlation of around 0.3 for a lag of 1.

Chart 4: Autocorrelation - Oil



c. Engle-Granger Procedure Implementation

We divided the dataset into two subsets: a training set (70%) and a test set (30%). The following cointegration analysis is performed on the training set.

	Beginning	End	Number of observations
Training set	30/07/2008	01/09/2017	2292
Test set	05/09/2017	29/07/2021	982

Cointegration applies to time series integrated of order 1. The result of the stationarity test for price time series and price difference time series are displayed in appendix F. Price time series are not stationary but the price differences are stationary so they are integrated of order 1.

The first step of the Engle-Granger procedure consists in checking that the spreads between pairs are stationary. The spread are the residuals of ordinary least squares regressions. The advanced Dickey-Fuller test with a lag of 1 is used to determine the stationarity of the spreads. The Engle-Grange – step 1 test was performed using a proprietary code that performs the ordinary least squares regressions and the ADF on the residuals resulting from these regressions. Table 7 presents the results of the spread stationarity test for a significance level of 5%. The detailed static regression outputs are displayed in appendix G.

Table 7: Engle-Granger step 1 results – Training set

x	y	ADF test-statistic	p-value	Critical value (1%)	Critical value (5%)	Critical value (10%)	Stationary
OIL	NGA	-4.407717	0.000011	-3.433209	-2.862803	-2.567442	Yes
OIL	HOI	-3.363484	0.000782	-3.433209	-2.862803	-2.567442	Yes
OIL	NLR	-4.172542	0.000031	-3.433209	-2.862803	-2.567442	Yes
NGA	OIL	-1.666577	0.095735	-3.433209	-2.862803	-2.567442	No
NGA	HOI	-1.435465	0.151291	-3.433209	-2.862803	-2.567442	No
NGA	NLR	-3.611856	0.000311	-3.433209	-2.862803	-2.567442	Yes
HOI	OIL	-3.503078	0.000469	-3.433209	-2.862803	-2.567442	Yes
HOI	NGA	-4.547274	0.000006	-3.433209	-2.862803	-2.567442	Yes
HOI	NLR	-4.409781	0.000011	-3.433209	-2.862803	-2.567442	Yes
NLR	OIL	-2.229643	0.025868	-3.433209	-2.862803	-2.567442	No
NLR	NGA	-4.163551	0.000032	-3.433209	-2.862803	-2.567442	Yes
NLR	HOI	-2.448844	0.014406	-3.433209	-2.862803	-2.567442	No

The pairs (OIL, HOI) and (NLR, NGA) are studied below as they have stationary spreads. The OLS regression coefficients are:

$$NGA \sim 0.047880 \times NLR$$

$$HOI \sim 0.028264 \times OIL$$

The second step of the Engle-Granger test consists in applying the equilibrium correction model in order to test the existence of short-run and long-run relationships between time series. These relationships are significant based on the ordinary least squares regression outlined in the methodology section. The p- values are both below $\alpha = 0.05$ which implies the rejection of the null hypothesis (null coefficients). We can conclude that the two pairs have a long-run equilibrium.

Table 8.a: Equilibrium model – NLR-NGA - Training set

	x	y	coef Δx	tstat Δx	p-value Δx	coef e_t-1	tstat e_t-1	pvalue e_t-1	Test
0	NLR	NGA	0.025022	6.309144	3.357843e-10	-0.010211	-4.526313	0.000006	Reject H0
1	NGA	NLR	0.705394	6.561282	6.575668e-11	-0.009738	-3.562024	0.000376	Reject H0

Table 8.b: Equilibrium model – OIL-HOI - Training set

	x	y	coef Δx	tstat Δx	p-value Δx	coef e_t-1	tstat e_t-1	pvalue e_t-1	Test
0	OIL	HOI	0.020020	61.273418	0.0	-0.010308	-3.698348	0.000222	Reject H0
1	HOI	OIL	31.019839	61.298393	0.0	-0.011539	-3.570269	0.000364	Reject H0

4. Pairs Trading strategy

a. Trading Signal Generation

The definition of a trading strategy is required to compute the following parameters:

- The weights $\beta_{Coint} = [1, -\beta]$ to obtain the spread.
- The speed of mean-reversion of the spread. The speed can be reflected by the half-life which measures the time between the equilibrium states (i.e. $e_t = \mu_e$)
- The upper and the lower bound as entry and exit levels defined by σ_{eq}

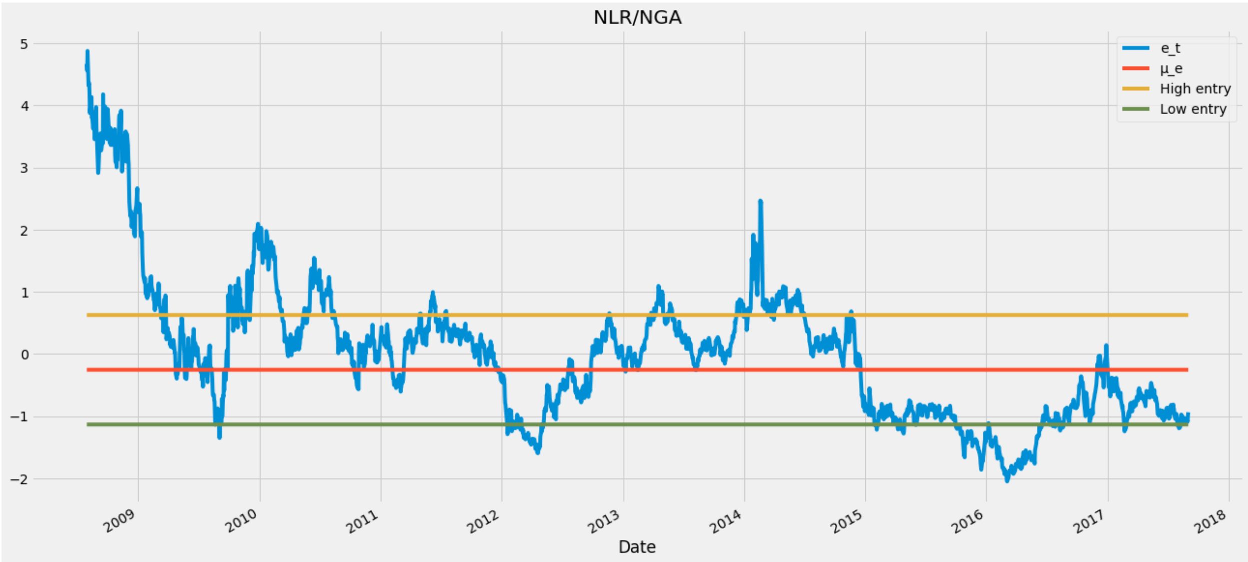
The parameters needed for the strategy implementation can be calculated using the Ornstein-Ulhenbeck process described in the methodology section. The following results were computed using a proprietary function. Heating oil future contract and the Vaneck ETF were used as dependent variables for the OLS regression.

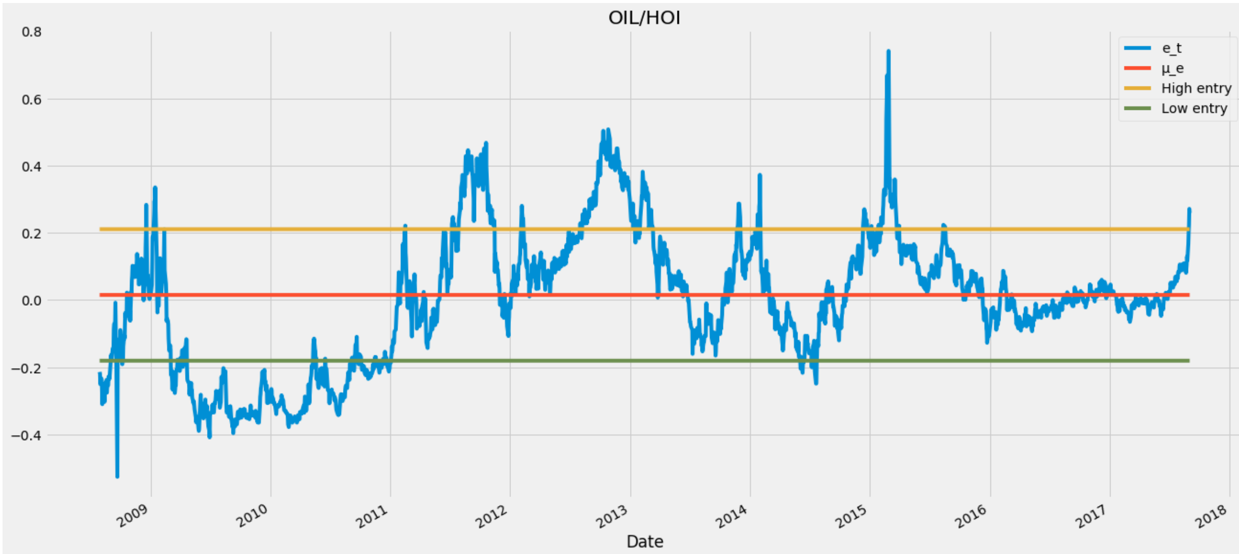
Table 9: Ornstein-Ulhenbeck process

	NLR/NGA	OIL/HOI
μ_e	-0.254972	0.015236
T	0.003968	0.003968
θ	2.421100	2.821670
σ_OU	1.937531	0.464673
Half Life	0.286294	0.245651
σ_eq	0.880496	0.195605
High entry	0.625524	0.210841
Low entry	-1.135468	-0.180369
β	0.047880	0.028264

The high entry and low entry where computed using the formula $\mu_e \pm Z\sigma_{eq}$. With Z = 1.

Chart 5: Spread vs μ_e and $\pm\sigma_{eq}$ – Training set





A common strategy is to enter the trade when the spread is above $\mu_e + Z\sigma_{eq}$ (short position) or below $\mu_e - Z\sigma_{eq}$ (long position) and to close it when the spread reaches μ_e.

If we have two cointegrated time series such as $Y_t \sim \beta X_t$ the long and short positions are defined below:

- Long position: Buy N of asset Y and sell βN of asset X
- Short position: Buy $N\beta$ of asset X and sell N asset Y

Where $N > 0$.

In order to hedge the strategy against the negative effect of extreme spread widenings and potential break of the cointegration we also close the trade when the spread is above $\mu_e + 3 \times Z\sigma_{eq}$ and when it is below $\mu_e - 3 \times Z\sigma_{eq}$.

As a first step, the implementation of this trading strategy was implemented using the parameters obtained by fitting the OU process summarised in table 11. For the purpose of this exercise, the cost of trading was assumed to be at 0.5% based on the trading fees of Boursorama bank, a French online bank. The pairs trading strategy P&Ls are summarized in table 11 below for both pairs crude oil-heating oil and Vaneck ETF-natural gas. For both pairs, the strategy outperformed the buy and hold strategy (50% invested in each asset of the pair) by a significant margin. Both buy and hold strategies had negative cumulative returns while both pairs trading strategies had positive cumulative returns.

Table 11: Trading strategy P&L – Z=1 – fees = 0.5% – Training set

```
NLR - NGA
----------
                   Strategy Buy & hold
P&L (incl fees)    5.926278  -0.291932
Number of trades  26.000000         NA
Trading fees       0.136224         NA

OIL - HOI
----------
                   Strategy Buy & hold
P&L (incl fees)    0.244298  -0.528276
Number of trades  28.000000         NA
Trading fees       0.143957         NA
```

This trading strategy could be refined using an appropriate Z in order to optimize the P&L. In practice, tighter (wider) bounds lead to a greater (lower) number of trades and a smaller (bigger) P&L per trade.

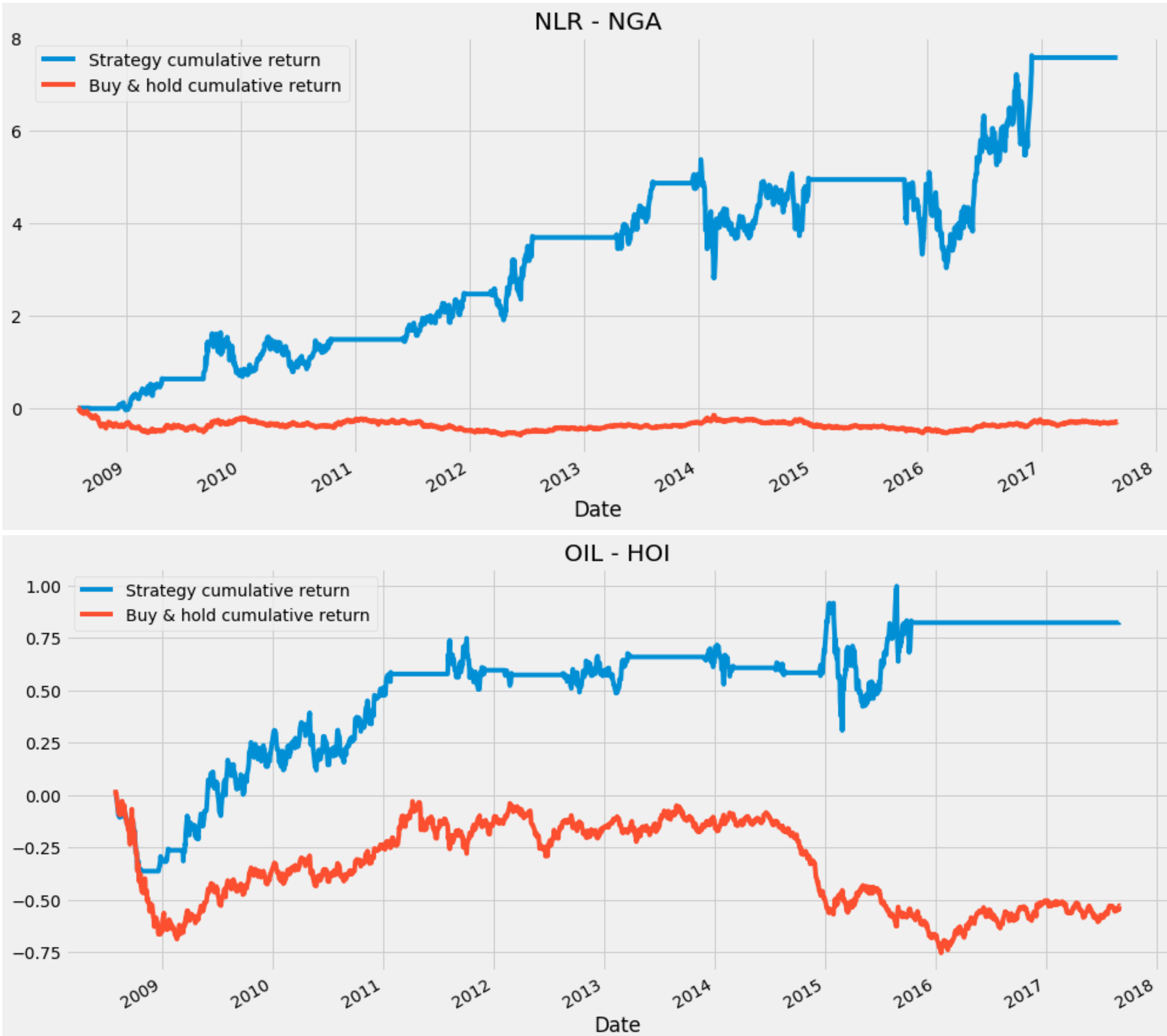
In order to confirm the optimised Z, the P&L was recalculated for several Zs between in the [0.7, 1.3] range. The results are summarised in the table below.

Table 12: Optimisation of Z – Training set

Z	NLR/NGA - P&L	NLR/NGA - N_trades	NLR/NGA - Total fees	OIL/HOI - P&L	OIL/HOI - N_trades	OIL/HOI - Total fees
0.7	3.063648	46.0	0.241012	0.177625	61.0	0.313620
0.8	4.080424	38.0	0.199097	0.171177	40.0	0.205653
0.9	5.738281	30.0	0.157182	0.120479	34.0	0.174805
1.0	5.926278	26.0	0.136224	0.244298	28.0	0.143957
1.1	7.285664	20.0	0.104788	0.298499	26.0	0.133674
1.2	7.570468	19.0	0.099549	0.526090	24.0	0.123392
1.3	4.334729	27.0	0.141464	0.823785	22.0	0.113109

The optimal Z is 1.2 for the pair NLR-NGA and 1.3 for the pair OIL-HOI based on the cumulative returns and the number of trades which we try to minimise. The P&Ls reported in the table above take into accounts the trading fees.

Chart 6: Cumulative returns – Z in (1.2, 1.3) - Training set



b. Backtesting

The strategies need to be tested on the test sample. A proprietary code was built to compute performance and risk metrics.

Table 13: Performance metrics

Compound annual growth rate (CAGR)	$(1 + cumulative\ return)^{\frac{1}{N}} - 1$
Annualised Volatility	$\sqrt{\frac{\sum_{i=1}^{N}(r_i-\bar{r})}{N-1}} \times \sqrt{252} = \sigma\sqrt{252}$
Annualised Sharpe Ratio	$\frac{\bar{r}}{\sigma} \times \sqrt{252}$

Maximum Drawdown (6 months)	$\frac{(P-L)}{L}$

Where

- N is the number of years (i.e. number of days / 252).
- r_i are the daily returns
- $\bar{r}$ is the average daily return
- σ is the daily standard deviation
- P is the peak high
- L is the through low

On the training set, both strategies had positive performance. Also, they outperformed their respective buy and hold strategy in terms of P&L and Sharpe ratio. They both have a better Sharpe ratio and 6-months maximum drawdown than their buy and hold counterparts.

On the test set, both strategies have a positive P&L with a CAGR of 5% for the pair NLR-NGA and a CAGR of 24% for the pair OIL/HOI.

The pair OIL/HOI pairs trading strategy has a Sharpe ratio of 1.24 on the test set which is significantly better than its buy and hold counterpart's Sharpe ratio of -0.30. Its maximum drawdown is at -18% which is low compared to the buy and hold strategy (-142%).

The pairs strategy on the NLR-NGA underperformed the buy and hold strategy with a Sharpe ratio of 0.31 against 0.47 and a maximum drawdown of -49% against -35%.

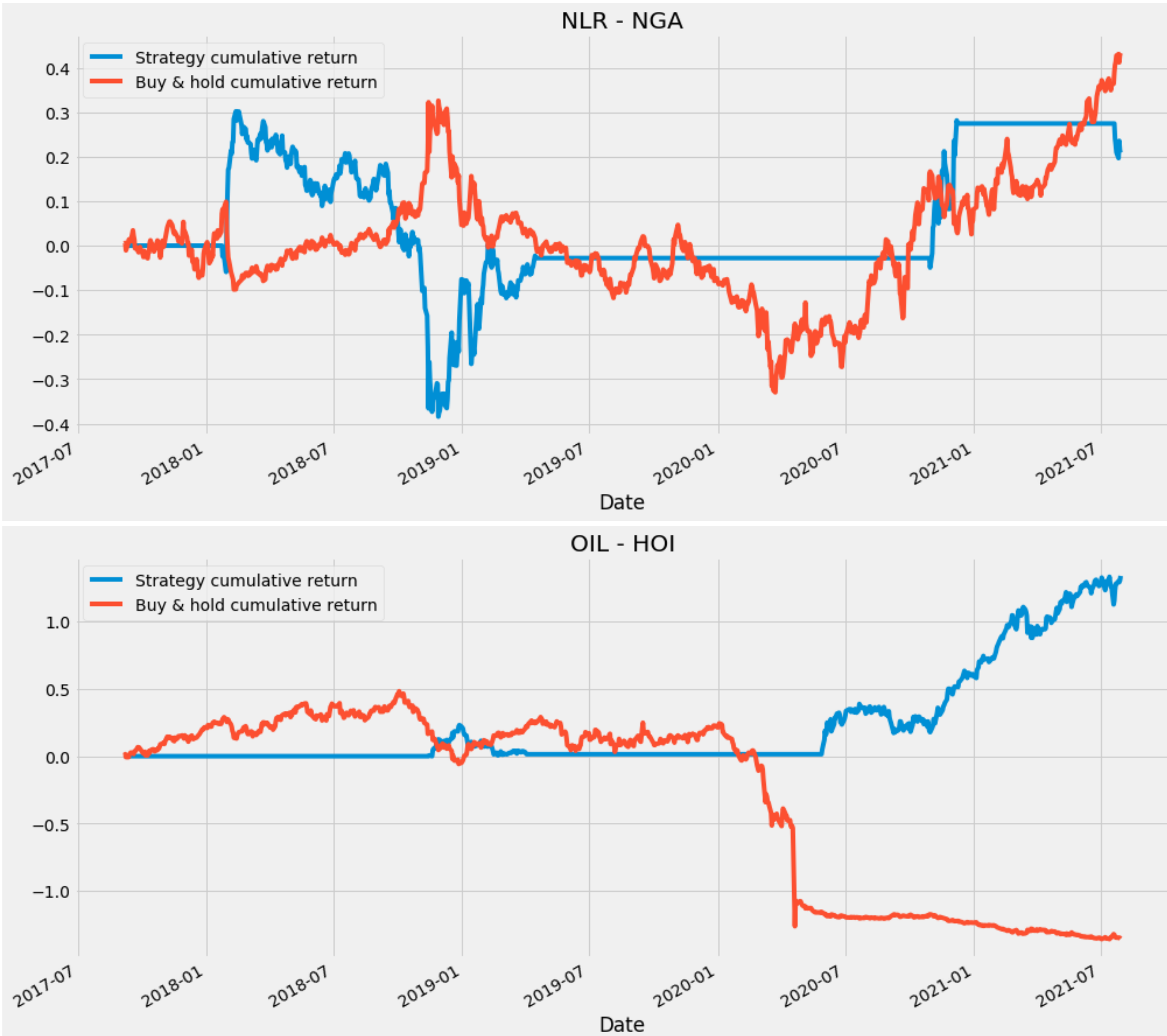
Both pairs trading strategies show rolling Sharpe ratios on 6-month windows moving between approximately 0.3 and -0.2 (appendix J).

The cumulative returns plots for the pairs trading strategies and the buy and hold strategies on the test period are displayed on chart 7 below. The spreads and the trading positions are displayed on charts in appendix I. The maximum drawdowns are plotted in appendix K.

Table 14: Pairs trading strategy performance against buy and hold strategy (Z=(1.2, 1.3))

	P&L	CAGR	Annualised Volatility	Annualised Sharpe Ratio	Max Drawdown (6m)
NLR/NGA train strat	7.570468	0.266562	0.361156	0.835480	-0.401401
OIL/HOI train strat	0.823785	0.068331	0.227834	0.403144	-0.385625
NLR/NGA train buy & hold	-0.291932	-0.037260	0.290278	0.013491	-0.464409
OIL/HOI train buy & hold	-0.528276	-0.079323	0.346636	-0.065626	-0.677537
NLR/NGA test strat	0.208948	0.049951	0.300701	0.312996	-0.490149
OIL/HOI test strat	1.339671	0.244024	0.190352	1.242468	-0.183527
NLR/NGA test buy & hold	0.432916	0.096807	0.281426	0.468449	-0.359632
OIL/HOI test buy & hold	-1.356190	NaN	0.969452	-0.298672	-1.421009

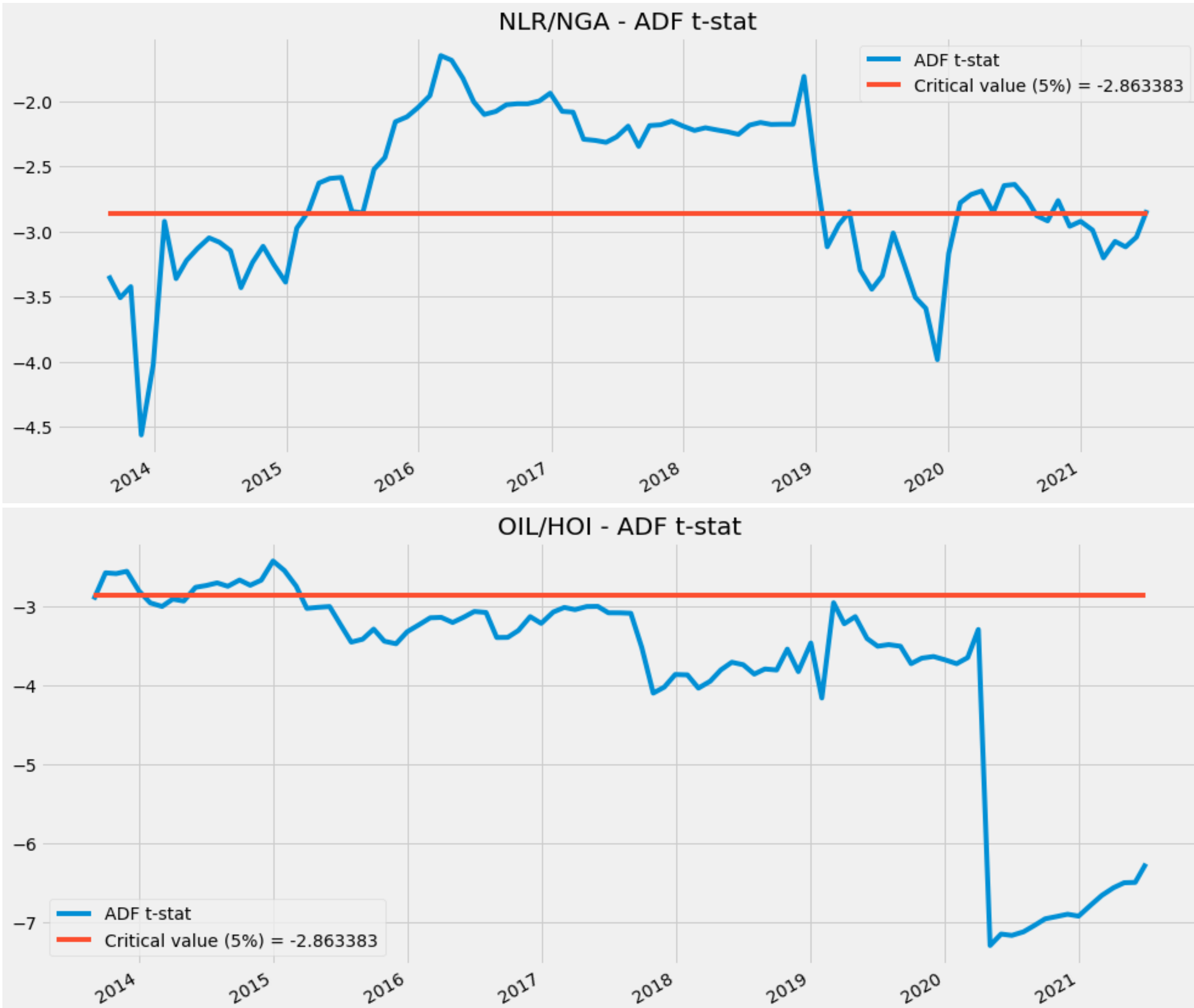
Chart 7: Cumulative return – Z in (1.2, 1.3) – test set



The Engle-Granger procedure showed some evidence of cointegration between the two pairs over the training set period.

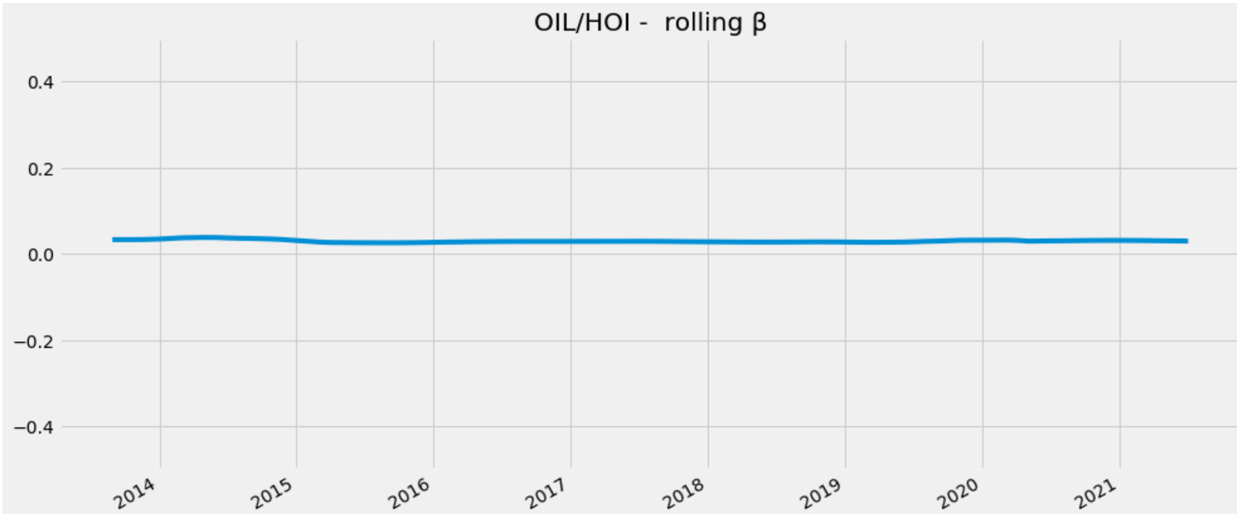
We now want to test if cointegration is persistent throughout time. We performed the Engle procedure every 21 days on 1260 days (5 years) rolling windows which makes 95 tests. The results for the Engle-Granger step 1 test are presented on charts below. These charts display the ADF test statistic against the 5%-critical value.

Chart 8: Rolling cointegration – Engle-Granger step 1



The chart above shows some stationarity of the spreads for the pair OIL-HOI starting in 2015. The statistical significance increased significantly in 2020. Chart 8 below shows that the static regression coefficient β is stable. The spreads of the pair NLR-NGA do not show consistent stationarity except in 2014 and 2019.

Chart 8: Rolling cointegration – Static Regression – Coefficient β



In order to complete the Engle-Granger procedure, the error correction model was also implemented for each data point. The pair OIL-HOI passes the cointegration tests (step 1 and step 2) 81 times out of 95 data points (85%) which confirms the stability of the cointegration throughout time.

A summary of the results is displayed in the table below. The 'coint test' columns reflect the combination of both the Engle-Grange step tests (i.e. it is a fail if one of the two tests fails). The full results are available in the attached excel files 'rolling cointegration NLR-NGA.xlsx' and 'rolling cointegration HOI-OIL.xlsx'.

Table 15: Rolling cointegration – Engle-Granger procedure

	Step 1 - NLR-NGA	Coint Test - NLR-NGA	Step 1 - OIL-HOI	Coint Test - OIL-HOI
Pass	38.0	38.0	81.000000	81.000000
Fail	57.0	57.0	14.000000	14.000000
Pass Rate	0.4	0.4	0.852632	0.852632

5. Conclusion

The study conducted in this report revealed cointegration for two pairs: the crude oil and the heating oil front month future contracts, and the front month future contract on natural gas and the VanEck Vectors Uranium+Nuclear Energy ETF.

For both pairs, the spread stationarity was confirmed through the augmented Dickey-Fuller test and a long-term equilibrium has been established using the error correction model between 30 July 2008 and 01 September 2017 (in-sample period).

Fitting the Ornstein-Ulhenbeck process, we developed a trading strategy designed to take advantage of the mean-reversion of the spreads. If the spreads were below (above) a specified lower (upper) bound we entered a long (short) position. The trade was liquidated when the spreads reached their expected level. This strategy produced positive results in terms of performance on the in-sample period and out-of-sample periods (between 05 September 2017 and 29 July 2021).

In order to confirm the stability of the cointegration between pairs, we also performed cointegration tests on 5-year rolling windows between 2013 and 2021. The tests showed a persisting cointegration for the pair crude oil-natural gas, however we did not find evidence of continuous cointegration for the pair natural gas and the VanEck Vectors Uranium+Nuclear Energy ETF.

The Engle-Granger procedure presents some limitations. For example, the choice of the dependent variable is arbitrary and may have an impact on the statistical estimations and, as a result, on the trading strategy performance. Additionally, the procedure relies on the estimation of spreads that will feed a stationarity test which consists of two ordinary least squares regressions. Therefore, the error of the first regression would be passed on to the second. Another limitation is that the Engle-Granger procedure can only be applied to pairs. Further analysis could be conducted using the Johansen procedure which can be applied for multivariate cointegrations.

6. References

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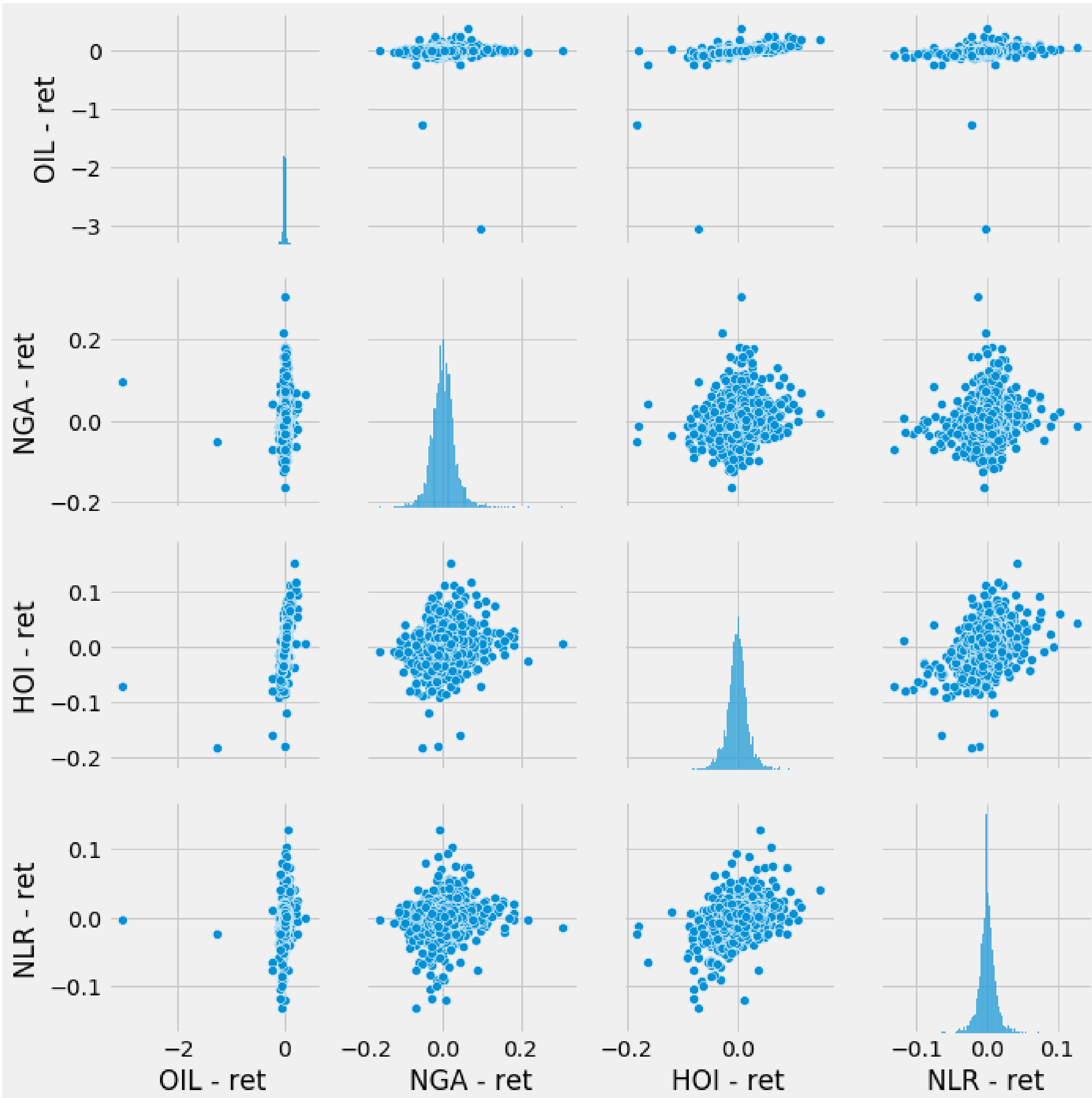
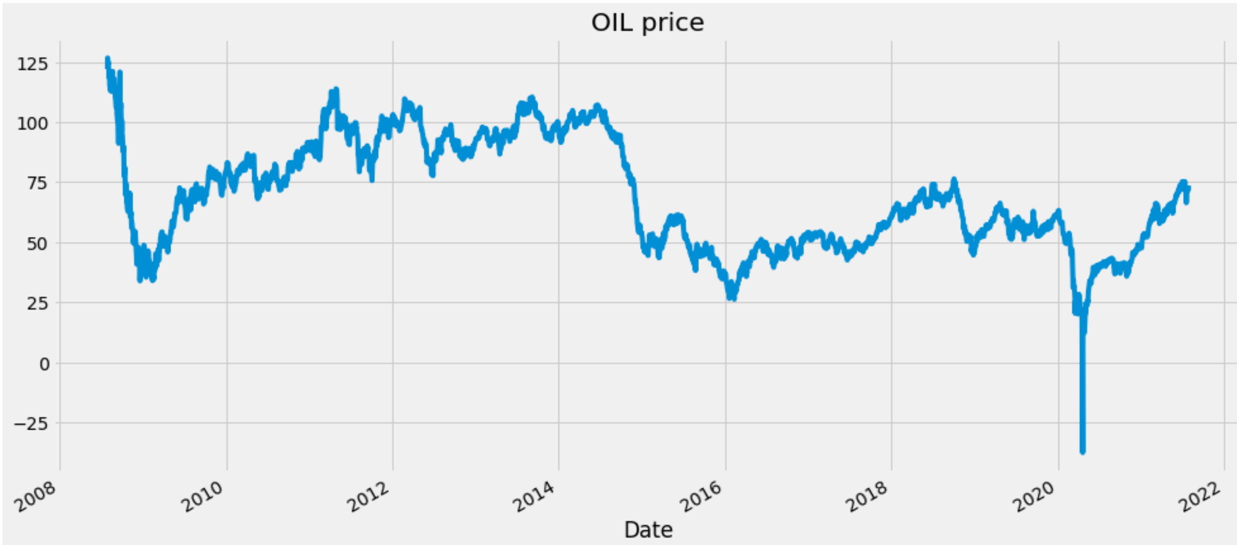
CQF lecture on cointegration

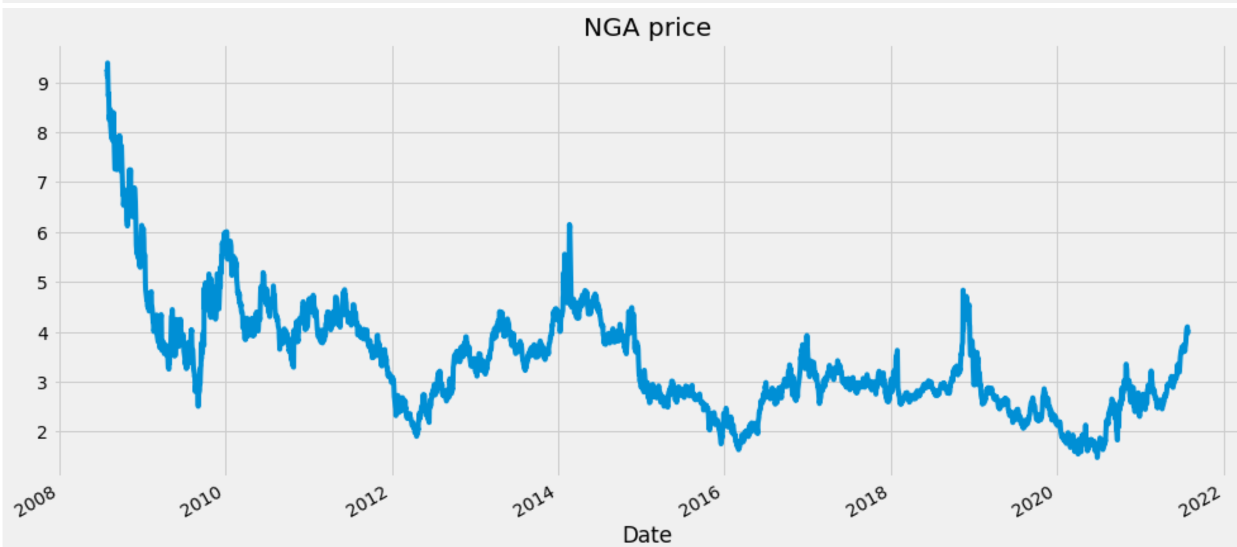
7. Appendix

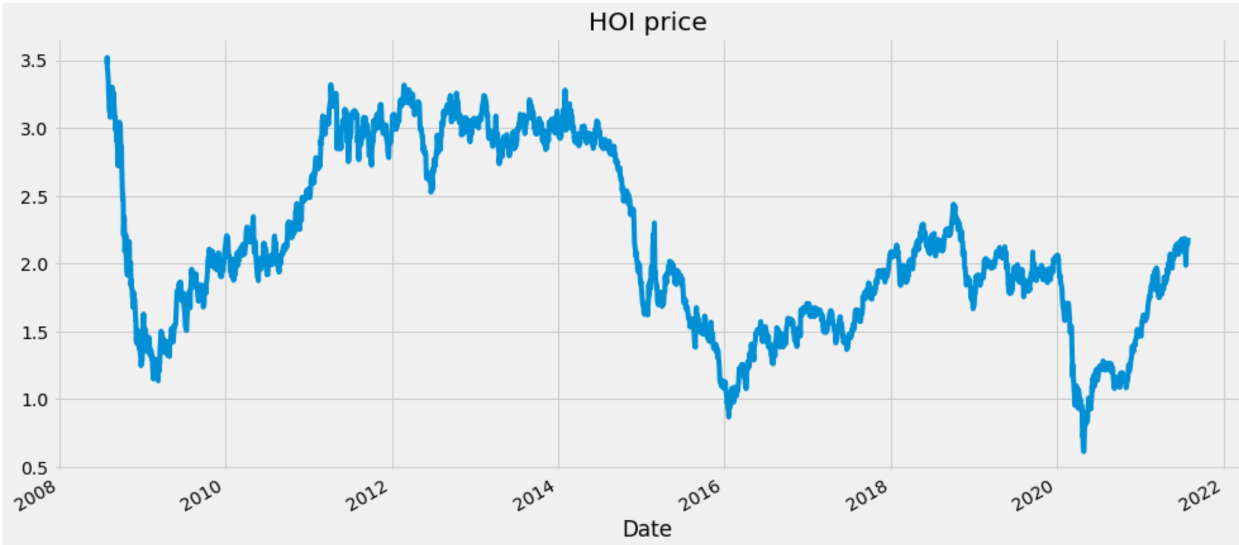
Appendix A: Returns pairwise relationships

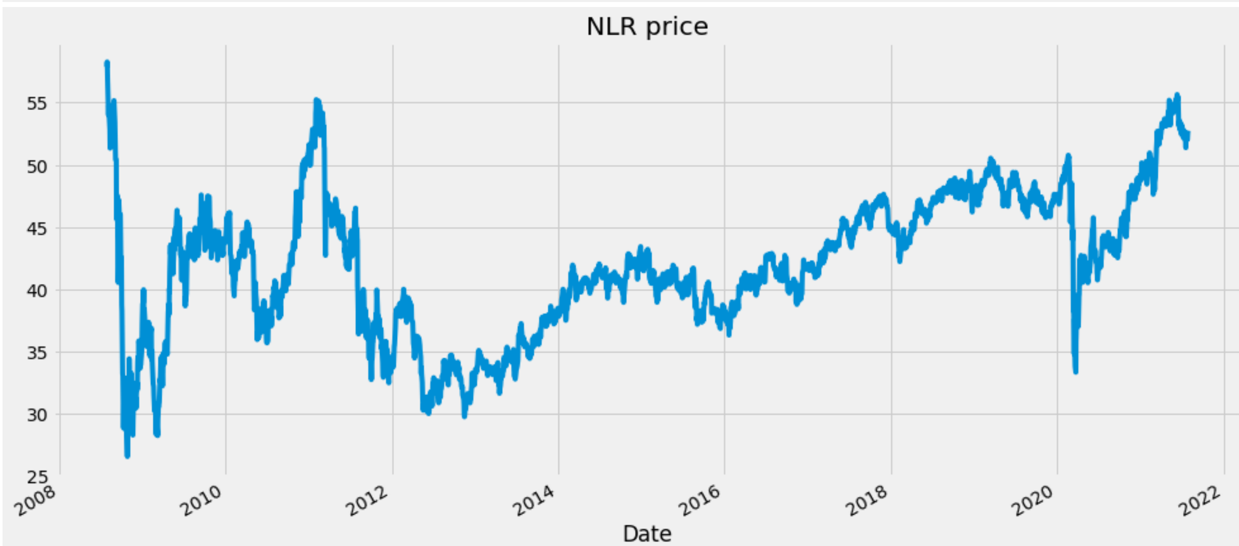


Appendix B: Price charts





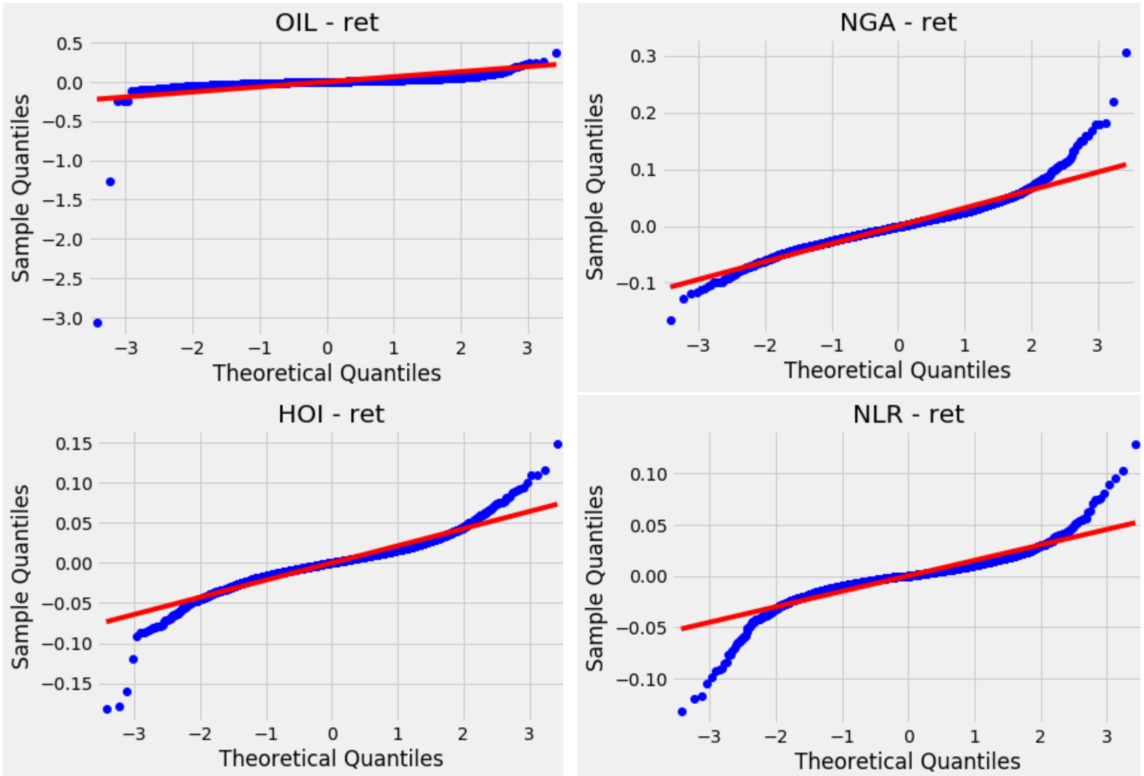




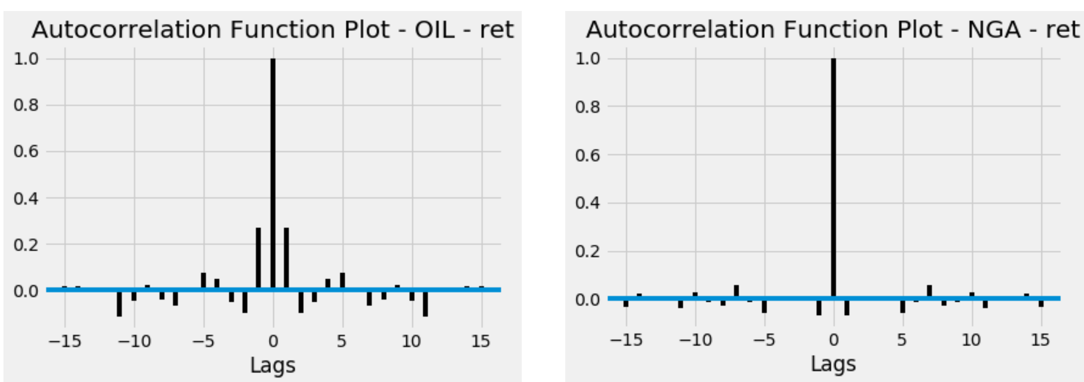
Appendix C: Correlation matrix

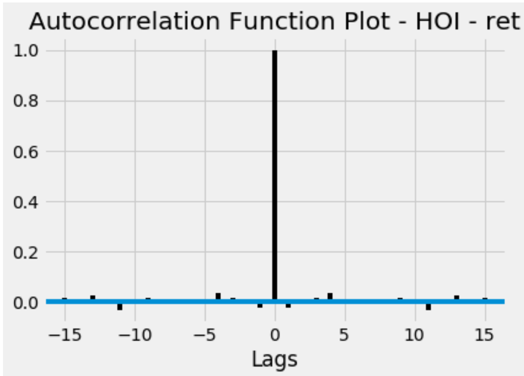
	OIL - ret	NGA - ret	HOI - ret	NLR - ret
OIL - ret	1.000000	0.031407	0.431391	0.153123
NGA - ret	0.031407	1.000000	0.170210	0.096777
HOI - ret	0.431391	0.170210	1.000000	0.350346
NLR - ret	0.153123	0.096777	0.350346	1.000000

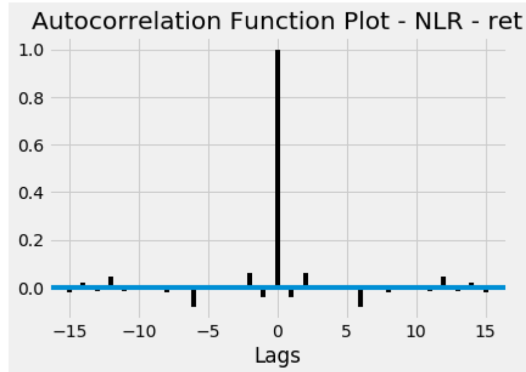
Appendix D: Quantile-Quantile plots



Appendix E: Autocorrelation







Appendix F: Stationary tests on prices and price differences

Appendix F.1: Stationary tests on prices

	ADF test-statistic	p-value	Lag	Number of observations	Critical value (1%)	Critical value (5%)	Critical value (10%)	Stationary
OIL	-1.658268	0.097400	1	2290	-3.433209	-2.862803	-2.567442	No
NGA	-2.528240	0.011530	1	2290	-3.433209	-2.862803	-2.567442	No
HOI	-1.428974	0.153148	1	2290	-3.433209	-2.862803	-2.567442	No
NLR	-0.890192	0.373456	1	2290	-3.433209	-2.862803	-2.567442	No

Appendix F.2: Stationary tests on price differences

	ADF test-statistic	p-value	Lag	Number of observations	Critical value (1%)	Critical value (5%)	Critical value (10%)	Stationary
OIL	-35.701862	2.860810e-222	1	2289	-3.43321	-2.862804	-2.567443	Yes
NGA	-34.675534	4.361765e-212	1	2289	-3.43321	-2.862804	-2.567443	Yes
HOI	-34.782980	3.782398e-213	1	2289	-3.43321	-2.862804	-2.567443	Yes
NLR	-32.419377	4.837728e-190	1	2289	-3.43321	-2.862804	-2.567443	Yes

Appendix G: Engle-Granger step 1 – OLS regressions result

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    NGA   R-squared:                       0.163
Model:                            OLS   Adj. R-squared:                  0.163
Method:                 Least Squares   F-statistic:                     447.0
Date:                Thu, 19 Aug 2021   Prob (F-statistic):           8.98e-91
Time:                        18:19:27   Log-Likelihood:                -3355.0
No. Observations:                2292   AIC:                             6714.
Df Residuals:                    2290   BIC:                             6726.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
```

```
==============================================================================
OIL            0.0198      0.001     21.141      0.000       0.018       0.022
const          2.2146      0.074     30.119      0.000       2.070       2.359
==============================================================================
Omnibus:                      609.264   Durbin-Watson:                   0.013
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             1692.098
Skew:                           1.385   Prob(JB):                         0.00
Kurtosis:                       6.170   Cond. No.                         264.
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specifie
d.
                            OLS Regression Results
==============================================================================
Dep. Variable:                    HOI   R-squared:                       0.921
Model:                            OLS   Adj. R-squared:                  0.921
Method:                 Least Squares   F-statistic:                 2.668e+04
Date:                Thu, 19 Aug 2021   Prob (F-statistic):               0.00
Time:                        18:19:27   Log-Likelihood:                 516.77
No. Observations:                2292   AIC:                            -1030.
Df Residuals:                    2290   BIC:                            -1018.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
OIL            0.0283      0.000    163.333      0.000       0.028       0.029
const          0.1512      0.014     11.139      0.000       0.125       0.178
==============================================================================
Omnibus:                        8.193   Durbin-Watson:                   0.023
Prob(Omnibus):                  0.017   Jarque-Bera (JB):                7.605
Skew:                           0.101   Prob(JB):                       0.0223
Kurtosis:                       2.803   Cond. No.                         264.
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specifie
d.
                            OLS Regression Results
==============================================================================
Dep. Variable:                    NLR   R-squared:                       0.000
Model:                            OLS   Adj. R-squared:                  0.000
Method:                 Least Squares   F-statistic:                     1.035
```

```
Date:                Thu, 19 Aug 2021   Prob (F-statistic):              0.309
Time:                        18:19:27   Log-Likelihood:                -6901.2
No. Observations:                2292   AIC:                         1.381e+04
Df Residuals:                    2290   BIC:                         1.382e+04
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
OIL           -0.0045      0.004     -1.017      0.309      -0.013       0.004
const         40.1723      0.345    116.291      0.000      39.495      40.850
==============================================================================
Omnibus:                       75.777   Durbin-Watson:                   0.017
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               95.357
Skew:                           0.375   Prob(JB):                     1.97e-21
Kurtosis:                       3.661   Cond. No.                         264.
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specifie
d.
                            OLS Regression Results
==============================================================================
Dep. Variable:                    OIL   R-squared:                       0.163
Model:                            OLS   Adj. R-squared:                  0.163
Method:                 Least Squares   F-statistic:                     447.0
Date:                Thu, 19 Aug 2021   Prob (F-statistic):           8.98e-91
Time:                        18:19:27   Log-Likelihood:                -10266.
No. Observations:                2292   AIC:                         2.054e+04
Df Residuals:                    2290   BIC:                         2.055e+04
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
NGA            8.2428      0.390     21.141      0.000       7.478       9.007
const         44.4267      1.509     29.432      0.000      41.467      47.387
==============================================================================
Omnibus:                      327.959   Durbin-Watson:                   0.007
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               80.338
Skew:                          -0.046   Prob(JB):                     3.59e-18
Kurtosis:                       2.087   Cond. No.                         13.9
==============================================================================
```

```
Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specifie
d.
                            OLS Regression Results
==============================================================================
Dep. Variable:                    HOI   R-squared:                       0.089
Model:                            OLS   Adj. R-squared:                  0.089
Method:                 Least Squares   F-statistic:                     225.1
Date:                Thu, 19 Aug 2021   Prob (F-statistic):           1.32e-48
Time:                        18:19:27   Log-Likelihood:                -2283.9
No. Observations:                2292   AIC:                             4572.
Df Residuals:                    2290   BIC:                             4583.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
NGA            0.1797      0.012     15.003      0.000       0.156       0.203
const          1.6039      0.046     34.585      0.000       1.513       1.695
==============================================================================
Omnibus:                    59342.465   Durbin-Watson:                   0.004
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              180.300
Skew:                           0.106   Prob(JB):                     7.05e-40
Kurtosis:                       1.642   Cond. No.                         13.9
==============================================================================


Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specifie
d.
                            OLS Regression Results
==============================================================================
Dep. Variable:                    NLR   R-squared:                       0.042
Model:                            OLS   Adj. R-squared:                  0.042
Method:                 Least Squares   F-statistic:                     101.3
Date:                Thu, 19 Aug 2021   Prob (F-statistic):           2.41e-23
Time:                        18:19:27   Log-Likelihood:                -6852.1
No. Observations:                2292   AIC:                         1.371e+04
Df Residuals:                    2290   BIC:                         1.372e+04
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
NGA            0.8846      0.088     10.064      0.000       0.712       1.057
```

```
const          36.5645      0.340    107.445      0.000      35.897      37.232
==============================================================================
Omnibus:                        7.041   Durbin-Watson:                   0.017
Prob(Omnibus):                  0.030   Jarque-Bera (JB):                8.636
Skew:                          -0.003   Prob(JB):                       0.0133
Kurtosis:                       3.301   Cond. No.                         13.9
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specifie
d.
                            OLS Regression Results
==============================================================================
Dep. Variable:                    OIL   R-squared:                       0.921
Model:                            OLS   Adj. R-squared:                  0.921
Method:                 Least Squares   F-statistic:                 2.668e+04
Date:                Thu, 19 Aug 2021   Prob (F-statistic):               0.00
Time:                        18:19:28   Log-Likelihood:                -7562.5
No. Observations:                2292   AIC:                         1.513e+04
Df Residuals:                    2290   BIC:                         1.514e+04
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
HOI           32.5843      0.199    163.333      0.000      32.193      32.975
const          0.9942      0.473      2.103      0.036       0.067       1.922
==============================================================================
Omnibus:                       38.451   Durbin-Watson:                   0.024
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               22.127
Skew:                           0.022   Prob(JB):                     1.57e-05
Kurtosis:                       2.521   Cond. No.                         9.53
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specifie
d.
                            OLS Regression Results
==============================================================================
Dep. Variable:                    NGA   R-squared:                       0.089
Model:                            OLS   Adj. R-squared:                  0.089
Method:                 Least Squares   F-statistic:                     225.1
Date:                Thu, 19 Aug 2021   Prob (F-statistic):           1.32e-48
Time:                        18:19:28   Log-Likelihood:                -3451.9
```

```
No. Observations:                2292   AIC:                             6908.
Df Residuals:                    2290   BIC:                             6919.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
HOI            0.4980      0.033     15.003      0.000       0.433       0.563
const          2.5691      0.079     32.653      0.000       2.415       2.723
==============================================================================
Omnibus:                      605.685   Durbin-Watson:                   0.012
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             1614.312
Skew:                           1.395   Prob(JB):                         0.00
Kurtosis:                       6.020   Cond. No.                         9.53
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specifie
d.
                            OLS Regression Results
==============================================================================
Dep. Variable:                    NLR   R-squared:                       0.019
Model:                            OLS   Adj. R-squared:                  0.018
Method:                 Least Squares   F-statistic:                     43.31
Date:                Thu, 19 Aug 2021   Prob (F-statistic):           5.76e-11
Time:                        18:19:28   Log-Likelihood:                -6880.2
No. Observations:                2292   AIC:                         1.376e+04
Df Residuals:                    2290   BIC:                         1.378e+04
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
HOI           -0.9749      0.148     -6.581      0.000      -1.265      -0.684
const         42.0483      0.351    119.757      0.000      41.360      42.737
==============================================================================
Omnibus:                      149.344   Durbin-Watson:                   0.018
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              229.623
Skew:                           0.527   Prob(JB):                     1.37e-50
Kurtosis:                       4.138   Cond. No.                         9.53
==============================================================================

Warnings:
```

```
[1] Standard Errors assume that the covariance matrix of the errors is correctly specifie
d.
                            OLS Regression Results
==============================================================================
Dep. Variable:                    OIL   R-squared:                       0.000
Model:                            OLS   Adj. R-squared:                  0.000
Method:                 Least Squares   F-statistic:                     1.035
Date:                Thu, 19 Aug 2021   Prob (F-statistic):              0.309
Time:                        18:19:28   Log-Likelihood:                -10470.
No. Observations:                2292   AIC:                         2.094e+04
Df Residuals:                    2290   BIC:                         2.096e+04
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
NLR           -0.1009      0.099     -1.017      0.309      -0.295       0.094
const         78.9334      3.979     19.836      0.000      71.130      86.737
==============================================================================
Omnibus:                    11287.632   Durbin-Watson:                   0.005
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              181.168
Skew:                          -0.196   Prob(JB):                     4.57e-40
Kurtosis:                       1.679   Cond. No.                         328.
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specifie
d.
                            OLS Regression Results
==============================================================================
Dep. Variable:                    NGA   R-squared:                       0.042
Model:                            OLS   Adj. R-squared:                  0.042
Method:                 Least Squares   F-statistic:                     101.3
Date:                Thu, 19 Aug 2021   Prob (F-statistic):           2.41e-23
Time:                        18:19:28   Log-Likelihood:                -3509.8
No. Observations:                2292   AIC:                              7024.
Df Residuals:                    2290   BIC:                              7035.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
NLR            0.0479      0.005     10.064      0.000       0.039       0.057
const          1.7915      0.191      9.381      0.000       1.417       2.166
```

```
==============================================================================
Omnibus:                      496.539   Durbin-Watson:                   0.012
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             1124.582
Skew:                           1.212   Prob(JB):                    6.31e-245
Kurtosis:                       5.429   Cond. No.                         328.
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specifie
d.
                            OLS Regression Results
==============================================================================
Dep. Variable:                    HOI   R-squared:                       0.019
Model:                            OLS   Adj. R-squared:                  0.018
Method:                 Least Squares   F-statistic:                     43.31
Date:                Thu, 19 Aug 2021   Prob (F-statistic):           5.76e-11
Time:                        18:19:28   Log-Likelihood:                -2369.9
No. Observations:                2292   AIC:                             4744.
Df Residuals:                    2290   BIC:                             4755.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
NLR           -0.0190      0.003     -6.581      0.000      -0.025      -0.013
const          3.0272      0.116     26.067      0.000       2.799       3.255
==============================================================================
Omnibus:                    69028.020   Durbin-Watson:                   0.005
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              179.858
Skew:                          -0.108   Prob(JB):                     8.80e-40
Kurtosis:                       1.645   Cond. No.                         328.
==============================================================================
```

Appendix H: Error correction model – regressions

```
                                 OLS Regression Results
=======================================================================================
Dep. Variable:                    ΔNGA   R-squared (uncentered):                  0.027
Model:                            OLS   Adj. R-squared (uncentered):             0.026
Method:                 Least Squares   F-statistic:                             31.65
Date:                Thu, 19 Aug 2021   Prob (F-statistic):                   2.76e-14
Time:                        18:19:28   Log-Likelihood:                         1594.9
No. Observations:                2291   AIC:                                    -3186.
Df Residuals:                    2289   BIC:                                    -3174.
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
```

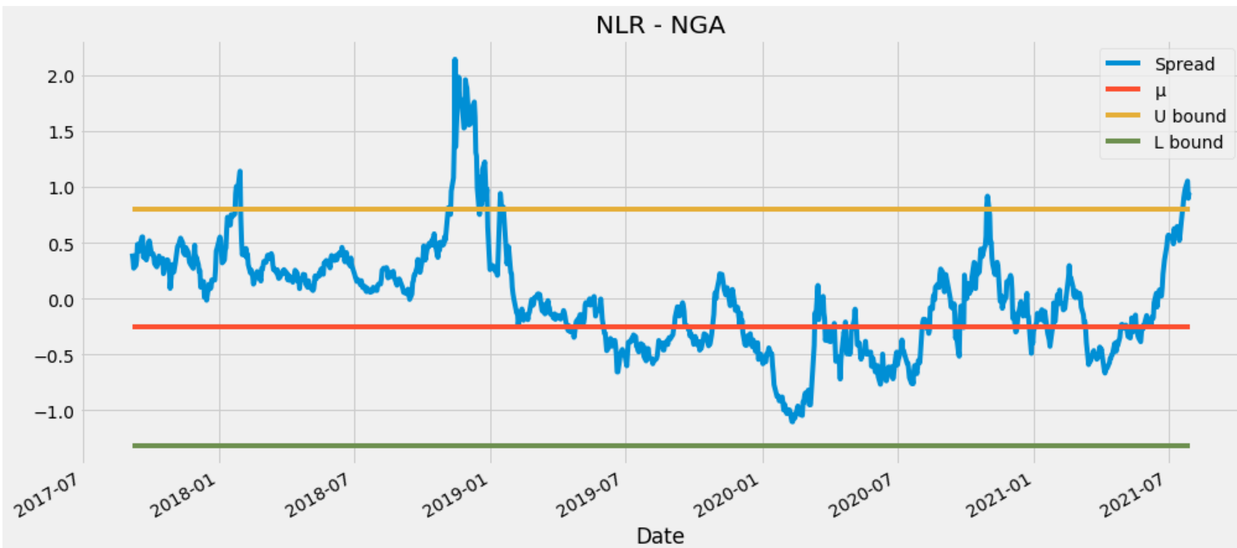
```
ΔNLR           0.0250      0.004      6.309      0.000       0.017       0.033
e_t-1         -0.0102      0.002     -4.526      0.000      -0.015      -0.006
==============================================================================
Omnibus:                      588.779   Durbin-Watson:                   2.173
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             6727.870
Skew:                           0.878   Prob(JB):                         0.00
Kurtosis:                      11.209   Cond. No.                         1.76
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specifie
d.
                                 OLS Regression Results
=======================================================================================
Dep. Variable:                   ΔNLR   R-squared (uncentered):                   0.024
Model:                            OLS   Adj. R-squared (uncentered):              0.023
Method:                 Least Squares   F-statistic:                              27.68
Date:                Thu, 19 Aug 2021   Prob (F-statistic):                    1.33e-12
Time:                        18:19:28   Log-Likelihood:                         -2188.4
No. Observations:                2291   AIC:                                      4381.
Df Residuals:                    2289   BIC:                                      4392.
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
ΔNGA           0.7054      0.108      6.561      0.000       0.495       0.916
e_t-1         -0.0097      0.003     -3.562      0.000      -0.015      -0.004
==============================================================================
Omnibus:                      741.997   Durbin-Watson:                   1.994
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            14834.175
Skew:                          -1.024   Prob(JB):                         0.00
Kurtosis:                      15.296   Cond. No.                         39.3
==============================================================================

      OLS Regression Results
=======================================================================================
Dep. Variable:                   ΔHOI   R-squared (uncentered):                   0.622
Model:                            OLS   Adj. R-squared (uncentered):              0.621
Method:                 Least Squares   F-statistic:                              1882.
Date:                Thu, 19 Aug 2021   Prob (F-statistic):                        0.00
Time:                        18:19:28   Log-Likelihood:                          5132.9
No. Observations:                2291   AIC:                                 -1.026e+04
Df Residuals:                    2289   BIC:                                 -1.025e+04
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
ΔOIL           0.0200      0.000     61.273      0.000       0.019       0.021
e_t-1         -0.0103      0.003     -3.698      0.000      -0.016      -0.005
==============================================================================
Omnibus:                     1338.666   Durbin-Watson:                   2.081
Prob(Omnibus):                  0.000   Jarque-Bera (JB):           127164.489
Skew:                          -1.859   Prob(JB):                         0.00
Kurtosis:                      39.309   Cond. No.                         8.53
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specifie
d.
                                 OLS Regression Results
=======================================================================================
Dep. Variable:                   ΔOIL   R-squared (uncentered):                   0.622
Model:                            OLS   Adj. R-squared (uncentered):              0.621
Method:                 Least Squares   F-statistic:                              1880.
Date:                Thu, 19 Aug 2021   Prob (F-statistic):                        0.00
Time:                        18:19:28   Log-Likelihood:                         -3280.9
No. Observations:                2291   AIC:                                      6566.
Df Residuals:                    2289   BIC:                                      6577.
Df Model:                           2
```

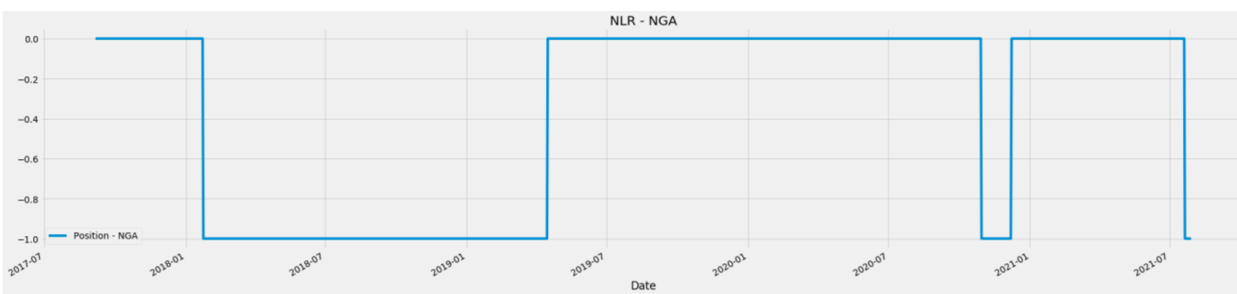
Covariance Type: nonrobust

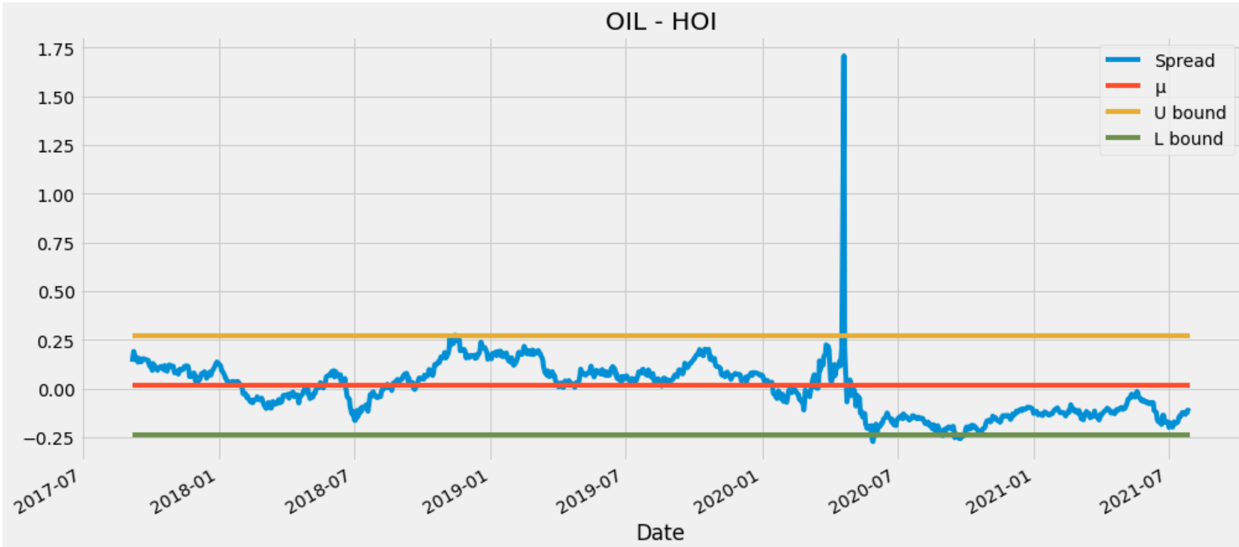
	coef	std err	t	P>|t|	[0.025	0.975]
ΔHOI	31.0198	0.506	61.298	0.000	30.027	32.012
e_t-1	-0.0115	0.003	-3.570	0.000	-0.018	-0.005

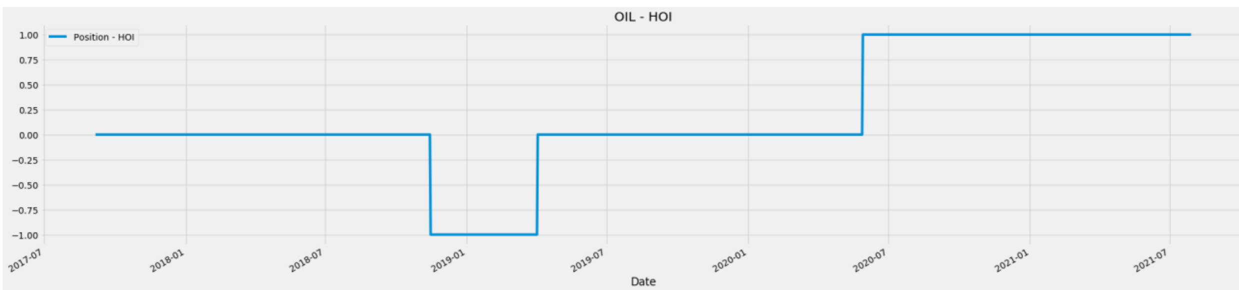
Omnibus:	972.518	Durbin-Watson:	2.117
Prob(Omnibus):	0.000	Jarque-Bera (JB):	97707.954
Skew:	1.040	Prob(JB):	0.00
Kurtosis:	34.926	Cond. No.	157.

Appendix I – Spread and trading position – Z in (1.2, 1.3) – test set

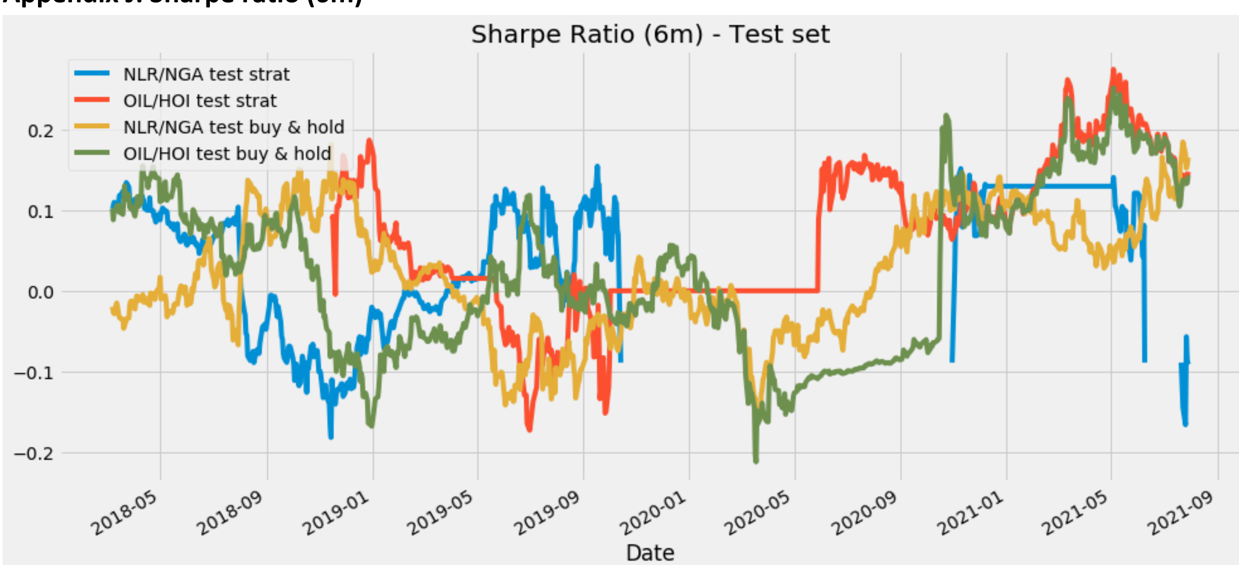


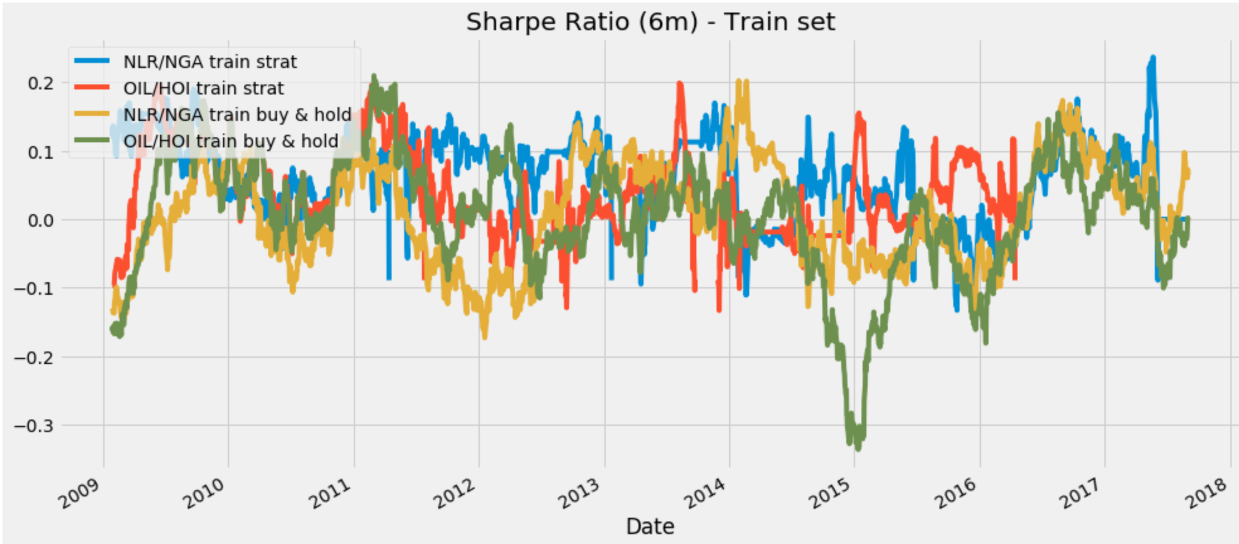




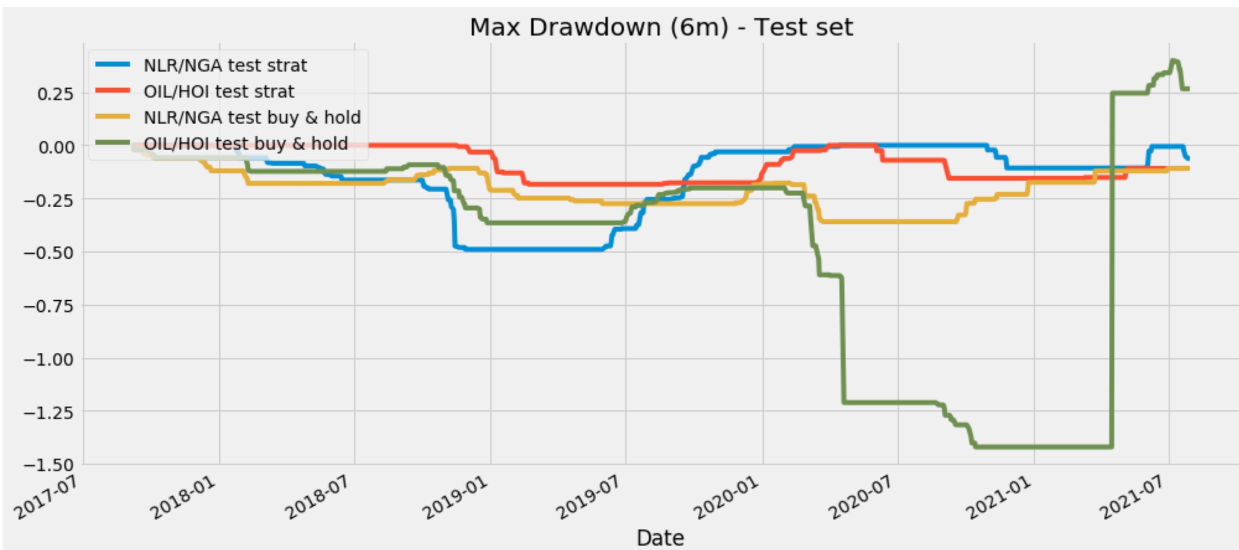


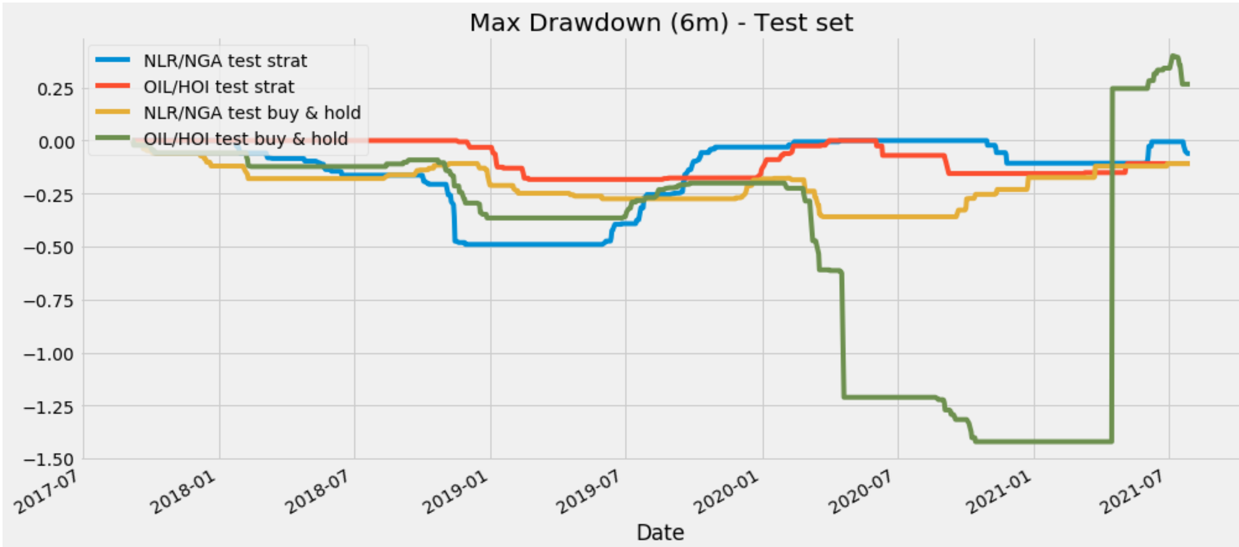
Appendix J: Sharpe ratio (6m)





Appendix K: Maximum drawdown (6m)




Max Drawdown (6m) - Test set
NLR/NGA test strat
OIL/HOI test strat
NLR/NGA test buy & hold
OIL/HOI test buy & hold
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−1.25
−1.50
2017-07
2018-01
2018-07
2019-01
2019-07
2020-01
2020-07
2021-01
2021-07
Date