

Image Enhancement

Lecture 3

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Outline

- Introduction
 - AWGN, S&P noise
- Mapping function
- Histogram
 - Image histogram
 - Histogram Equalization
- Image Averaging
- Spatial filtering
 - LPF, HPF
- Practical filters
 - Averaging filter
 - edge detection filters
 - sharpening filters
 - Order Statistic Filters
- Convolution Neural Networks (CNN)

Introduction

- Images may **suffer** from the following degradations:
 - Poor contrast: due to poor illumination or finite sensitivity of the imaging device
 - Noisy image: due to electronic sensor noise, signal amplifier, atmospheric disturbances, or compression
 - Aliasing effects: due to sampling
 - Spatial blurring: due to aperture effects or motion
 - Others

Noisy image

- Additive white Gaussian noise (AWGN)

Original Image



Noisy Image



AWGN

1. Gaussian: probability density function

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(z-\bar{z})^2/2\sigma^2}$$

2. Code: AWGN

NewImage=

OldImage+K*Gaussian

```
void AddNoise(Mat img,double mu, double sigma,int k){  
    Mat outImage;  
    outImage.create(img.rows,img.cols,img.type());  
    for(int x=0;x        for(int y=0;y            double temp = img.at<uchar>(x, y)  
                +k*generateGaussianNoise(mu,sigma);  
            if(temp>PixelMax)  
                temp=PixelMax;  
            else if(temp<PixelMin)  
                temp=PixelMin;  
            outImage.at<uchar>(x, y) = temp;  
        }  
    }  
    Filter(outImage,Filter::NBJZ,3,3,1);  
    imshow("Output", outImage);  
    cvWaitKey(0);  
}
```

Noisy image

- Salt and pepper noise
 - also known as impulse noise. This noise can be caused by sharp and sudden disturbances in the image signal



Salt and pepper noise

1. Randomly select position
2. Set it 0 or 255

```
1 void AddNoise(Mat img,double SNR ){
2     Mat outImage;
3     outImage.create(img.rows,img.cols,img.type());
4     int SP = img.rows*img.cols;
5     int NP = SP*(1-SNR);
6     outImage = img.clone();
7     for(int i=0; i<NP; i++) {
8
9         int x = (int)(rand()*1.0/RAND_MAX* (double)img.rows);
10        int y = (int)(rand()*1.0/RAND_MAX* (double)img.cols);
11        int r = rand()%2;
12        if(r){
13            outImage.at<uchar>(x, y)=0;
14        }
15        else{
16            outImage.at<uchar>(x, y)=255;
17        }
18    }
19 }
20 Filter(outImage,Filter::NMBJZ,3,3,1);
21 imshow("Output", outImage);
22 cvWaitKey(0);
23 }
```

Image enhancement and restoration

- The **goals** of image enhancement are to:
 - Accentuate (filter) certain **features** for subsequent **analysis** or image display.
 - Discard unimportant image features.
 - Enhance otherwise **hidden** information
- On the other hand **image restoration** is a **process** that attempts to **reconstruct** or **recover** an image that has been **degraded**.
 - Remove noise
- **Enhancement** is the process of **manipulating** an image so that the result is more suitable than the original for a **specific** application.
 - Problem oriented
 - No general theory of measurement

Image enhancement and restoration

- Examples of image enhancement and restoration operations:
 - noise removal;
 - geometric distortion correction;
 - edge enhancement;
 - contrast enhancement;
 - image zooming;
- **Categories** of image enhancement
 - **Frequency** domain
 - **Spatial** domain (**pixel-domain**)

Pixel-domain image enhancement

- Mathematically pixel-domain image enhancement could be represented by the operator $T[.]$
- $T[.]$ operates on the input image im to produce an output image imo , there are two approaches for this :
 - Non-adaptive to the data content
 - Adaptive to the data content

Pixel-domain image enhancement

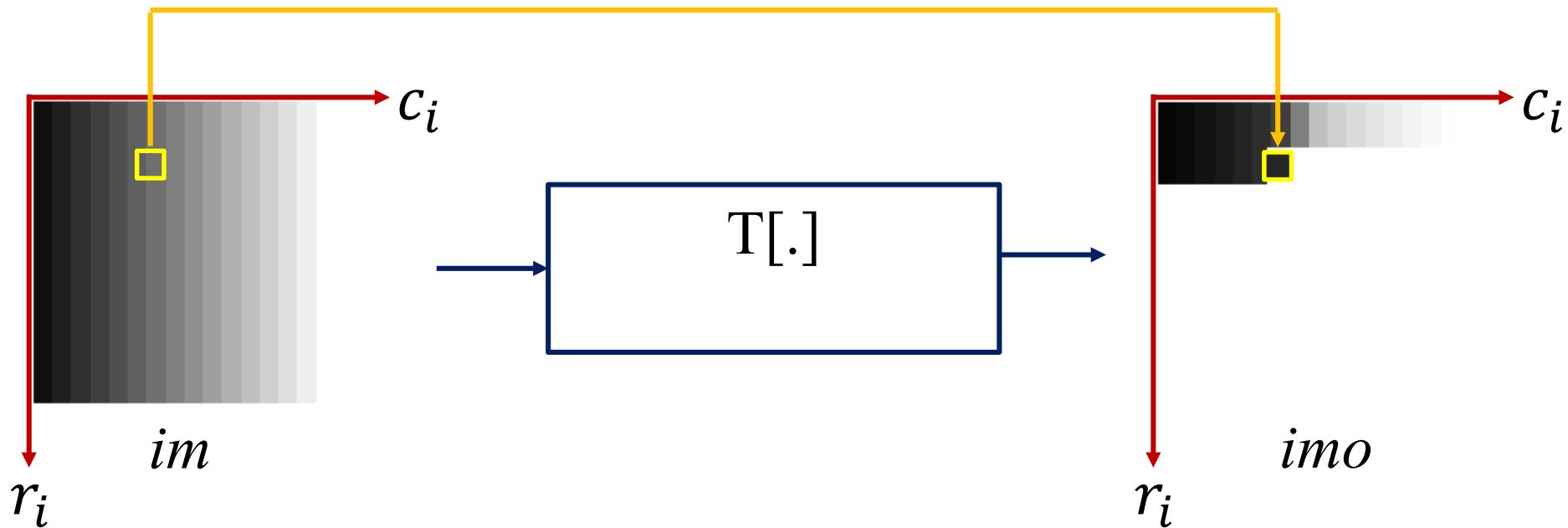
- Non-adaptive to image intensity
 - Usually used to compensate the image acquisition unit (e.g., gamma-compensation)
 - Usually used to overcome the limitation of the physical image acquisition unit (e.g., image negative)
- Adaptive to image intensity
 - Histogram based correction

Pixel-domain image enhancement

- $T[.]$ in term of principle of operation could be classified as :
 - Pixel-to-pixel mapping
 - Block-to-pixel mapping
 - Block-to-block mapping
 - Image-to-image mapping

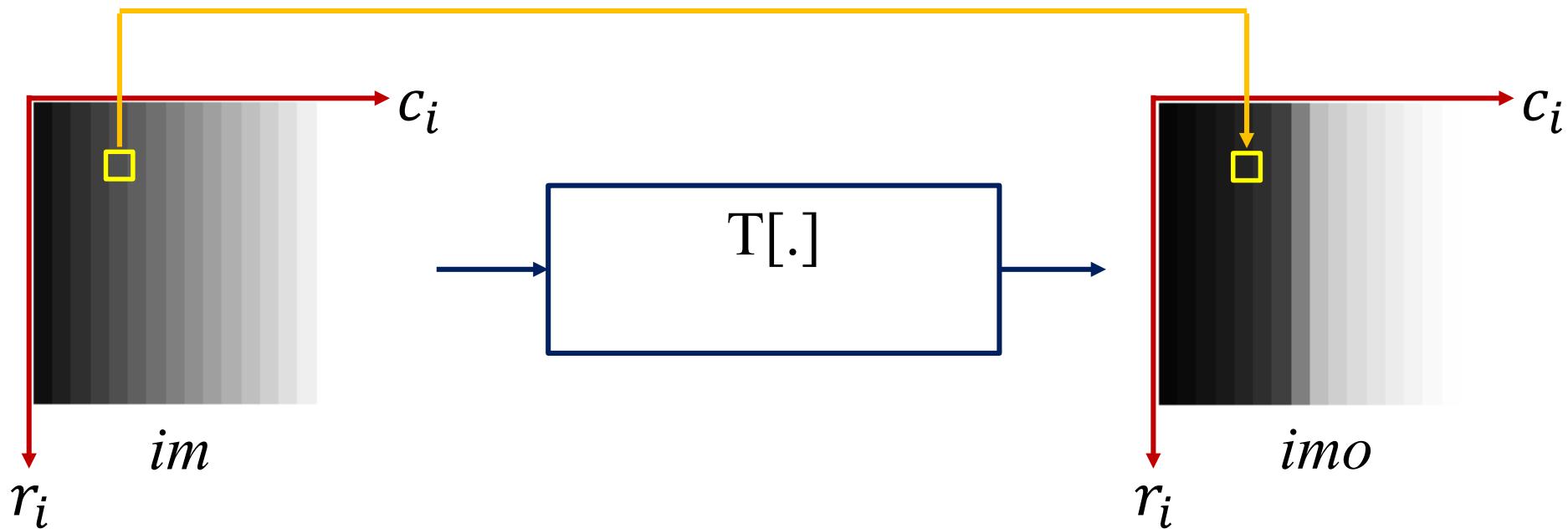
Pixel-domain image enhancement

- Pixel-to-pixel mapping
 - Operates on one pixel at a time
 - It is **easy to implement**



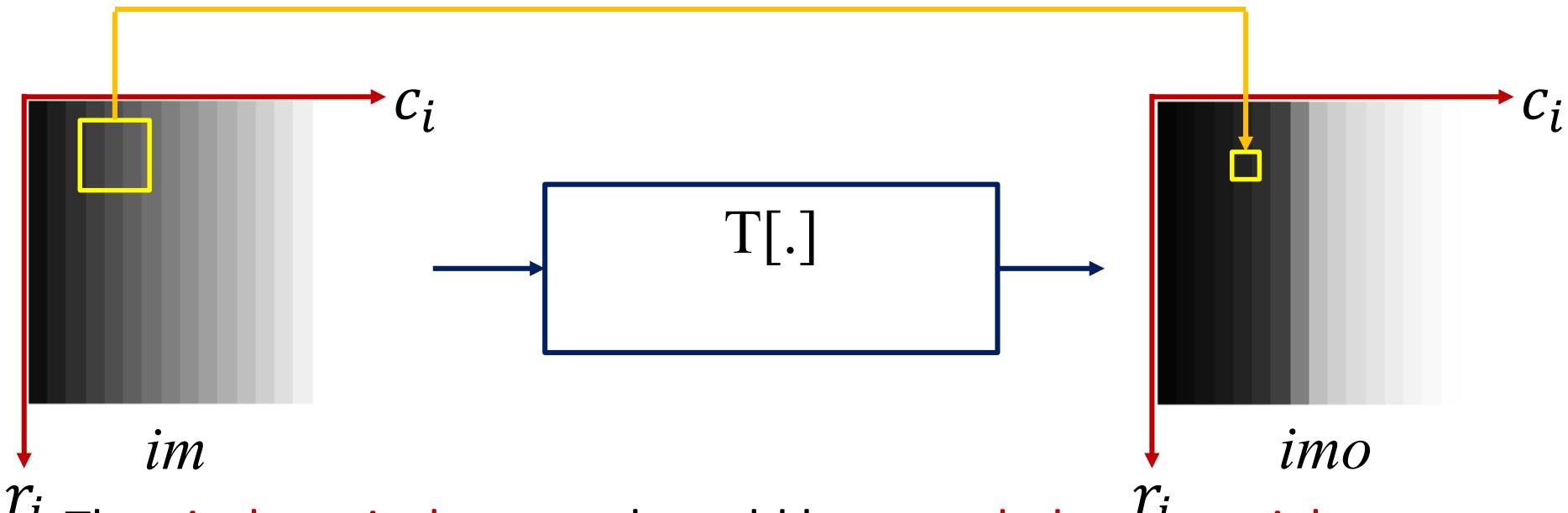
Pixel-domain image enhancement

- Pixel-to-pixel mapping
 - For gray images we could call it **intensity transform** (**mapping function**): $imo(r_i, c_i) = T[im(r_i, c_i)]$



Pixel-domain image enhancement

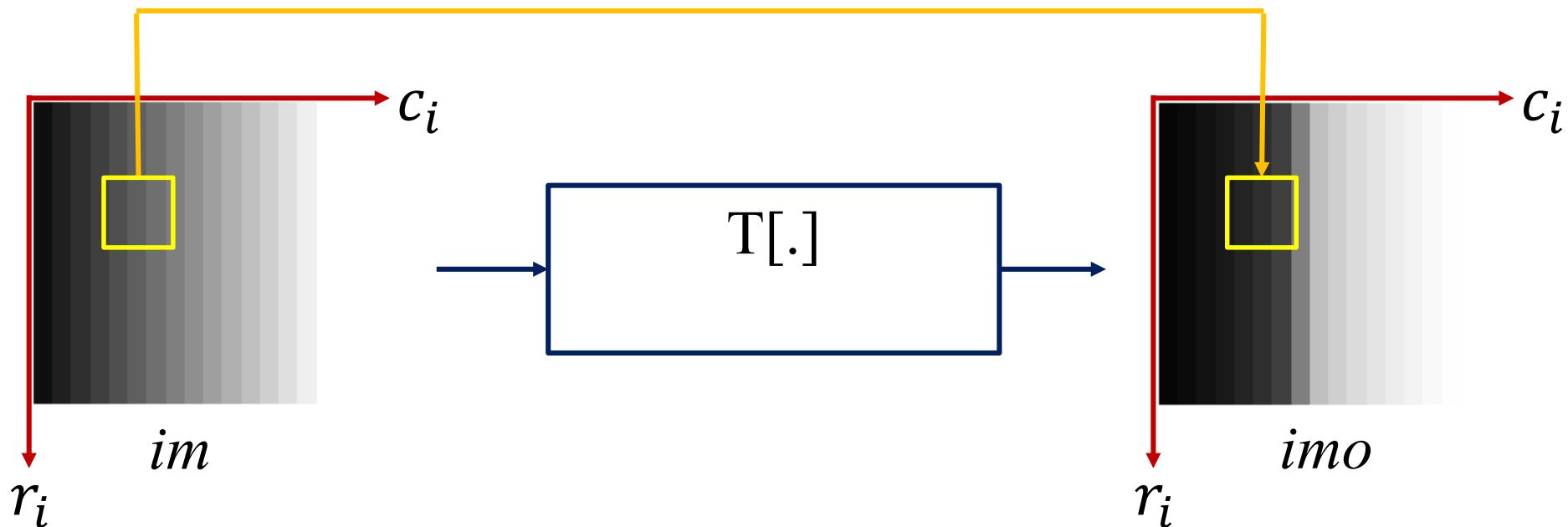
- Block-to-pixel mapping
 - Operates on a **set of ‘neighbourhoods’ N of each pixel**



The **pixel-to-pixel** approach could be regarded as a special case of the **block-to-pixel** approach, where the size of block is 1×1

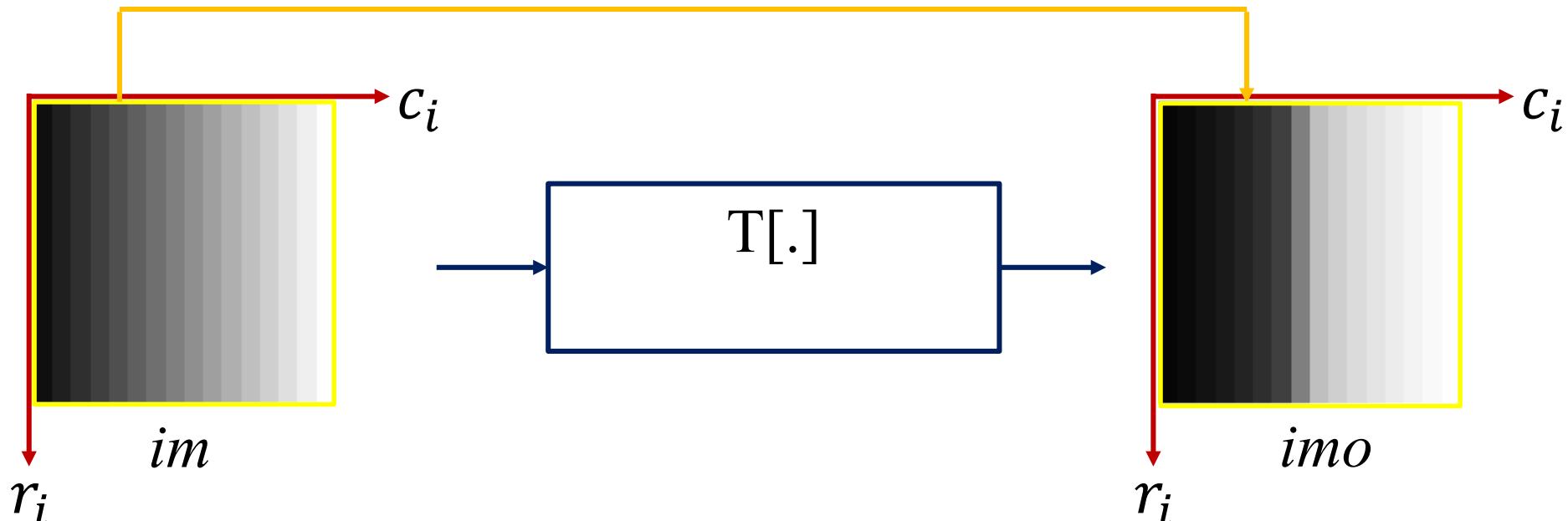
Pixel-domain image enhancement

- Block-to-block mapping
 - Operates on a **set of pixels** to produce a **set of pixels**



Pixel-domain image enhancement

- Image-to-image mapping
 - Operates on the **whole set of pixels** of the image



Is there any relationship between the block-to-block approach and the image-to-image approach ?

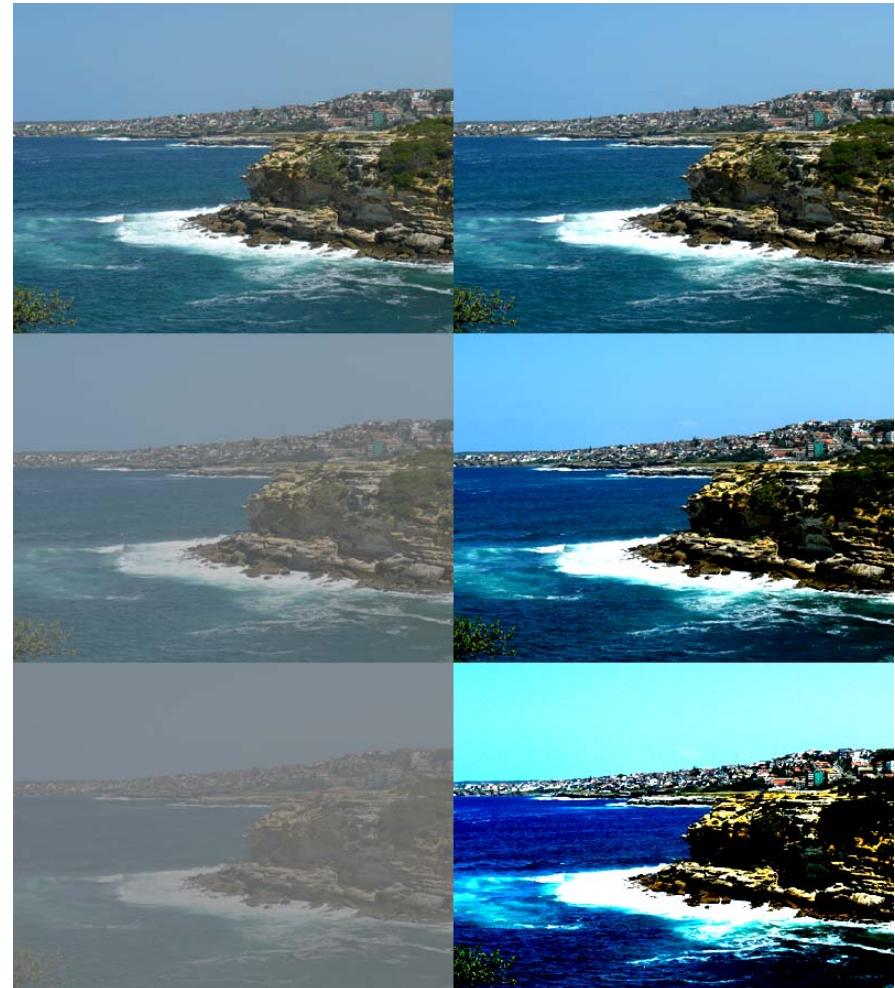
Pixel-domain image enhancement

- $T[.]$ in term of **principle of operation** could be classified as :
 - Pixel-to-pixel mapping
 - Block-to-pixel mapping
 - Block-to-block mapping
 - Image-to-image mapping
- The previous approaches have different **characteristics** in term:
 - Complexity
 - Flexibility
 - Simplicity of the design stage

Examples

Contrast enhancement

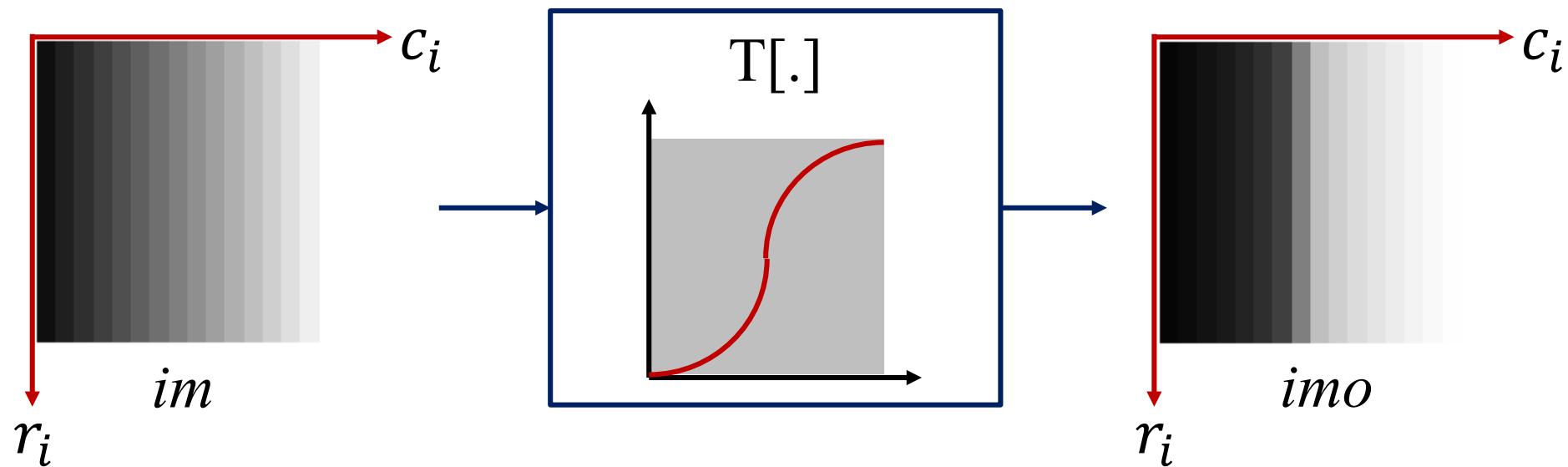
- To produce an image of **higher contrast** than the original image
 - Contrast is the **difference** in luminance or color that makes an object (or its representation in an image or display) distinguishable.
 - human visual system is more sensitive to **contrast** than absolute luminance



Changes in the amount of contrast in a photo

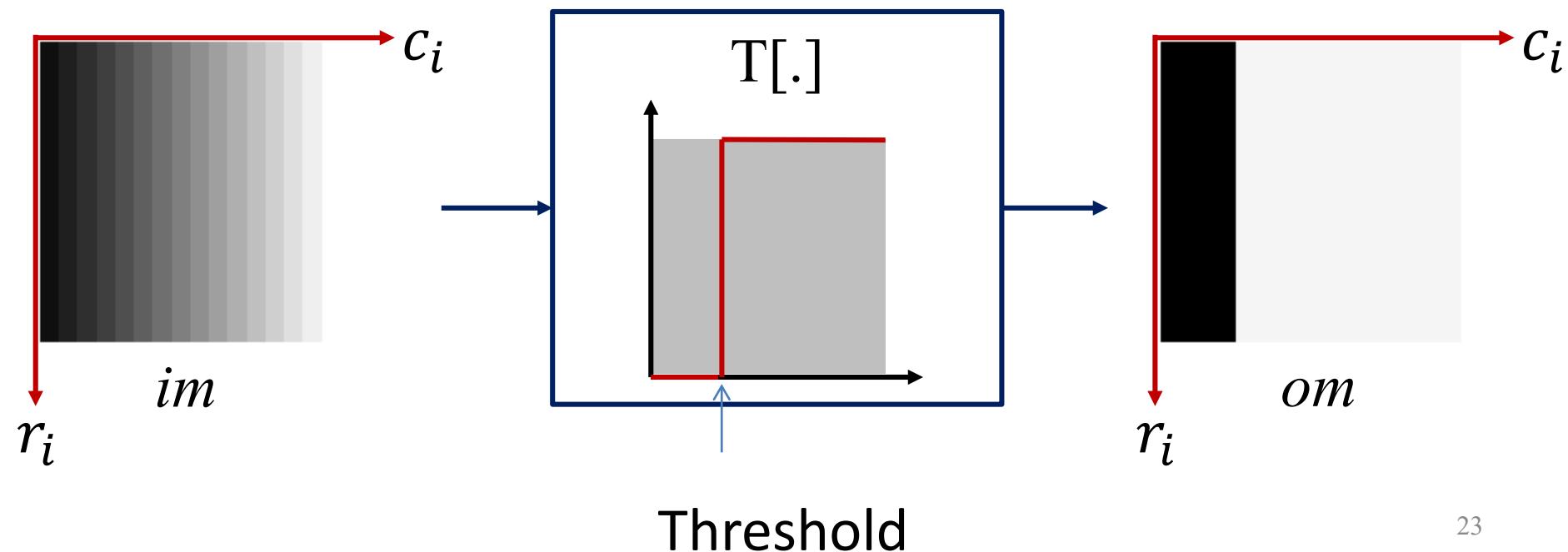
Contrast enhancement

- To produce an image of **higher contrast** than the original image



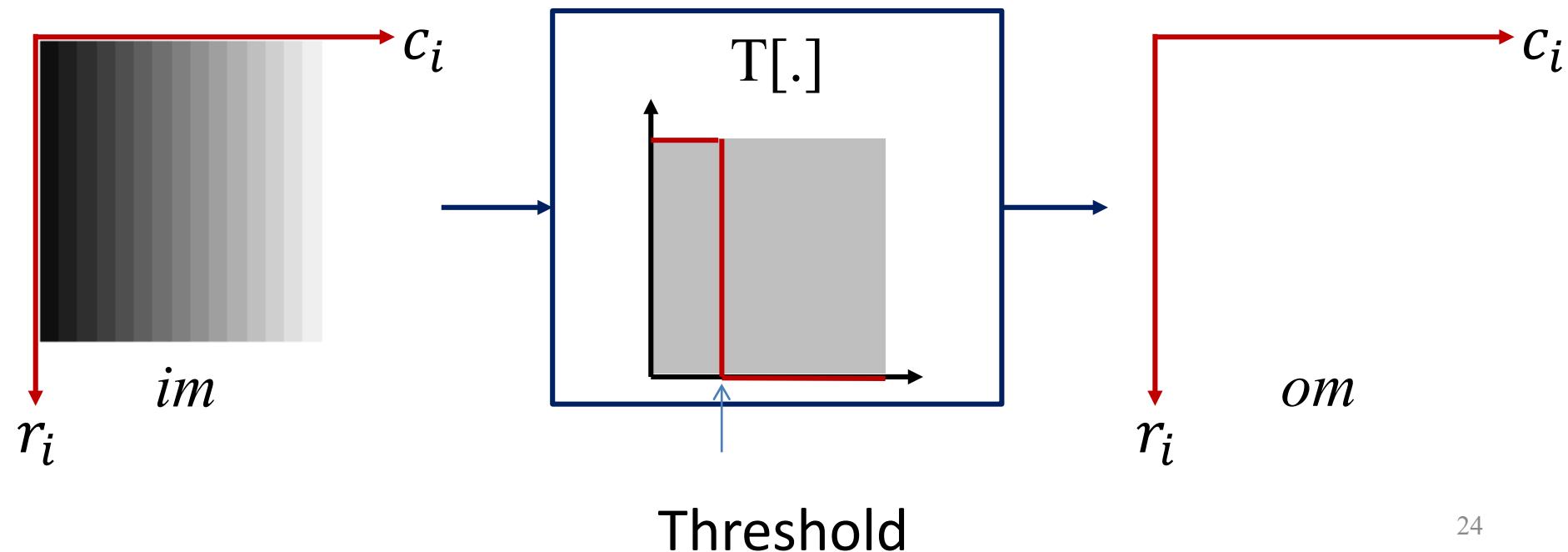
Thresholding

- Is used to **produce** a binary image



Thresholding

- What will happen with this mapping function ?



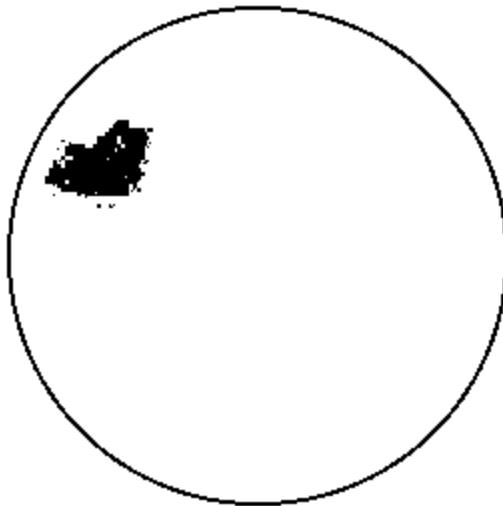
Thresholding

- Bleeding detection in the WCE (Wireless Capsule Endoscopy) images



Thresholding

- Bleeding detection in the WCE images

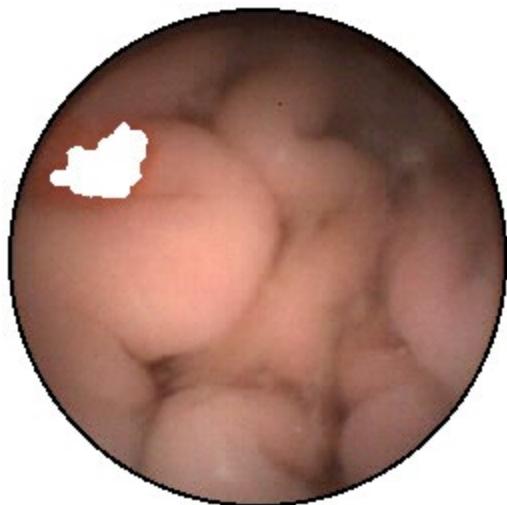


- RGB → HSI (color space) → blood detection → threshold and masking

Thresholding

- Bleeding detection in the WCE images

Masked bleeding area Original image



Scene text detection and recognition



观前街

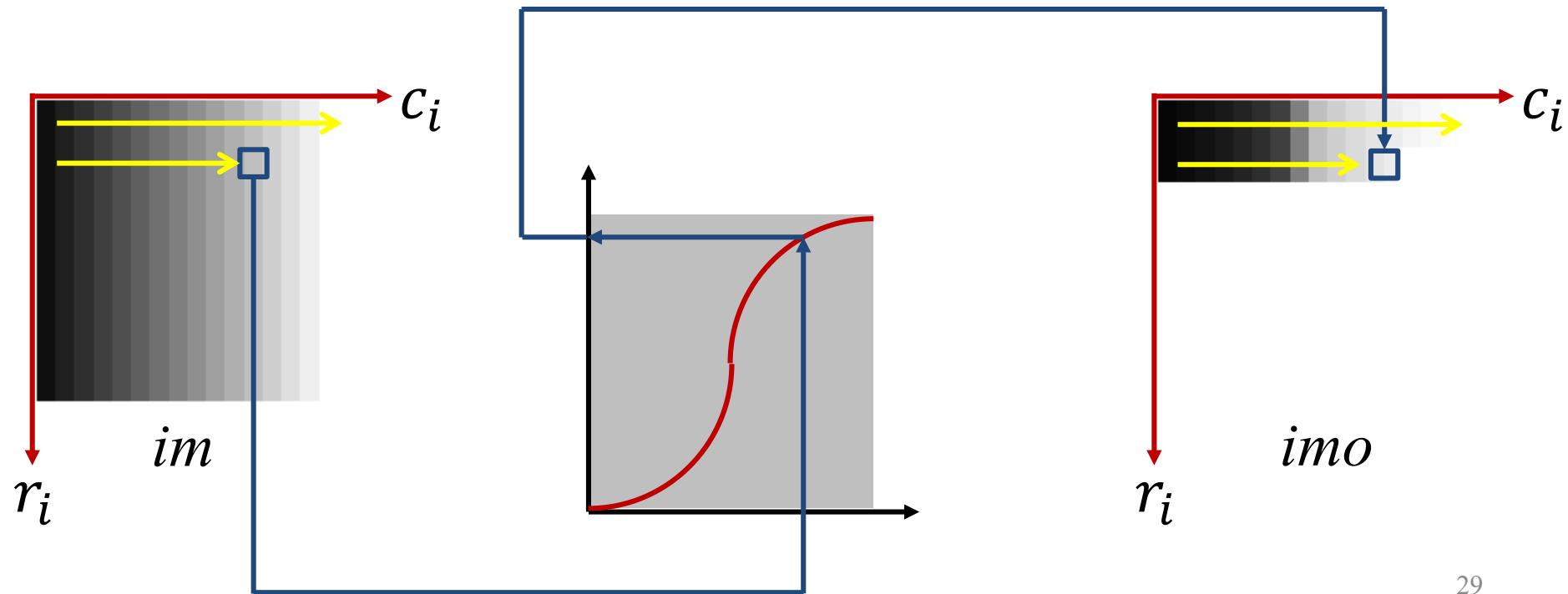


Guanqian
Street

Text-to-speech

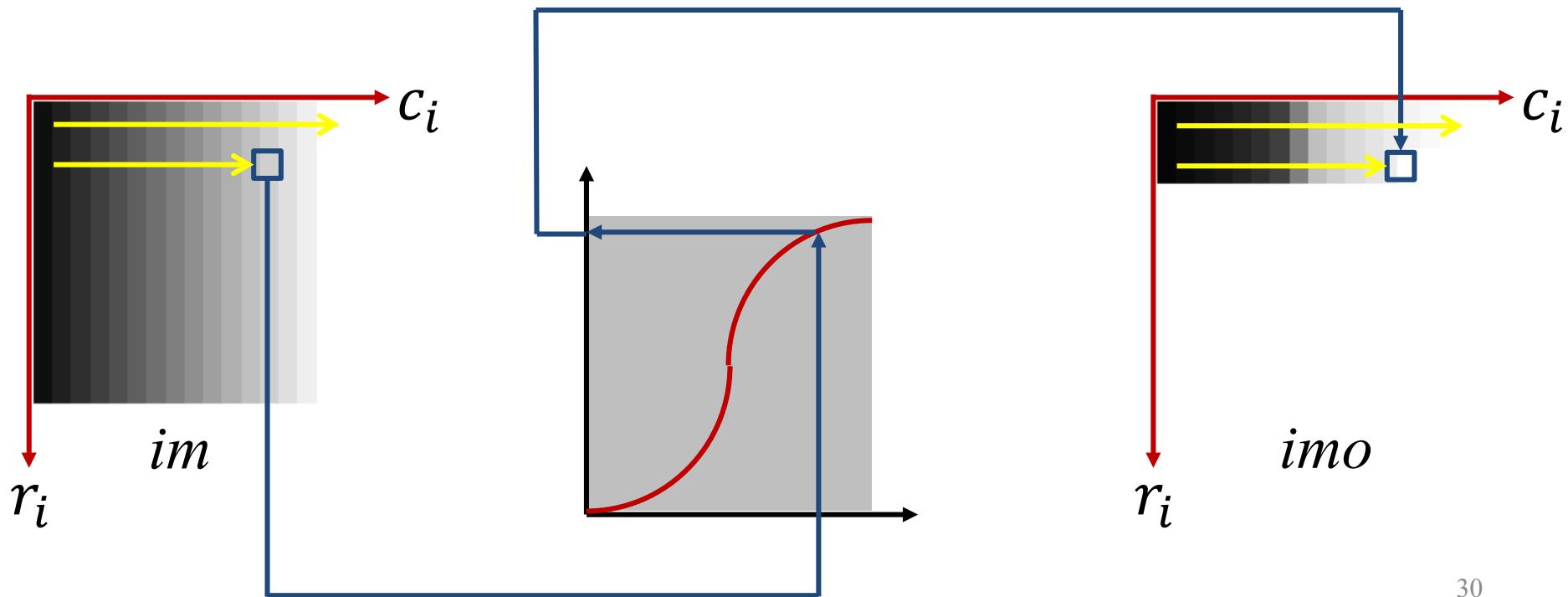
Graphical representation of Mapping function

- The operation $imo(r_i, c_i) = T[im(r_i, c_i)]$ could be described as a mapping function which is shown graphically :



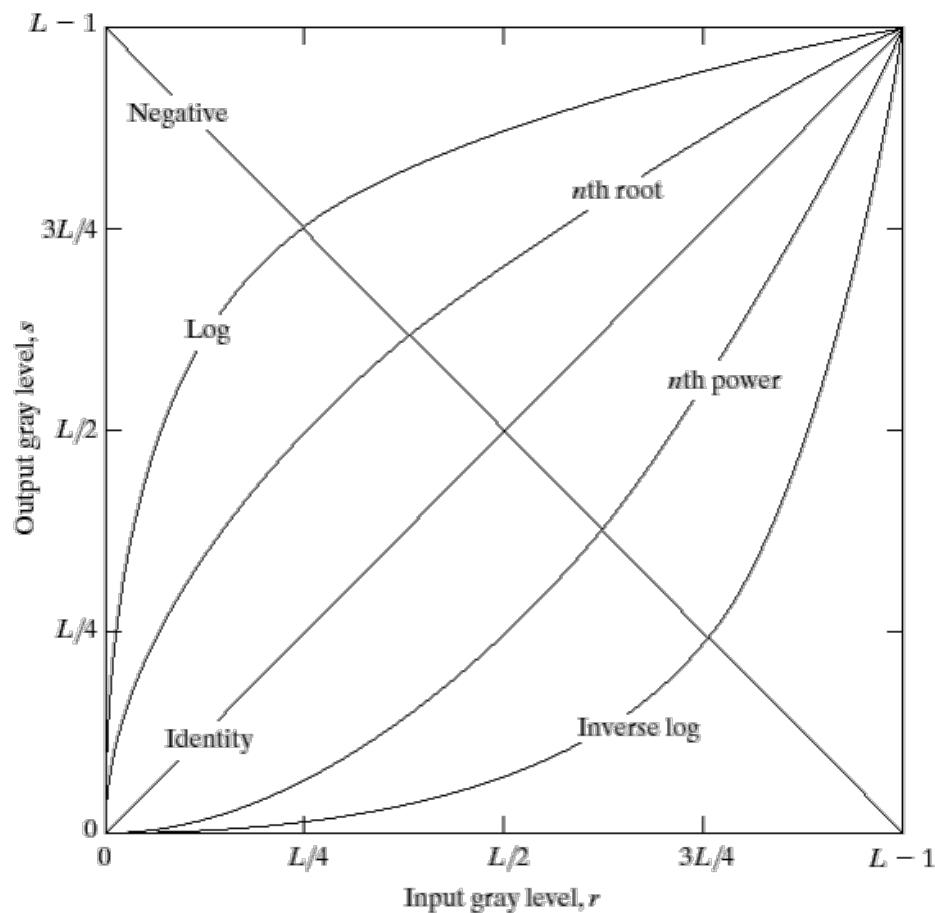
Graphical representation of Mapping function

- The operation $imo(r_i, c_i) = T[im(r_i, c_i)]$ could be described as a mapping function which is shown graphically :



Mapping function

Mapping function



- Some **mathematically described** mapping functions:
 - Negative and **identity** transform
 - **Power-law** transform
 - **Log-transformation**

Image negative

- Particularly suitable to enhance white/gray details embedded in dark regions

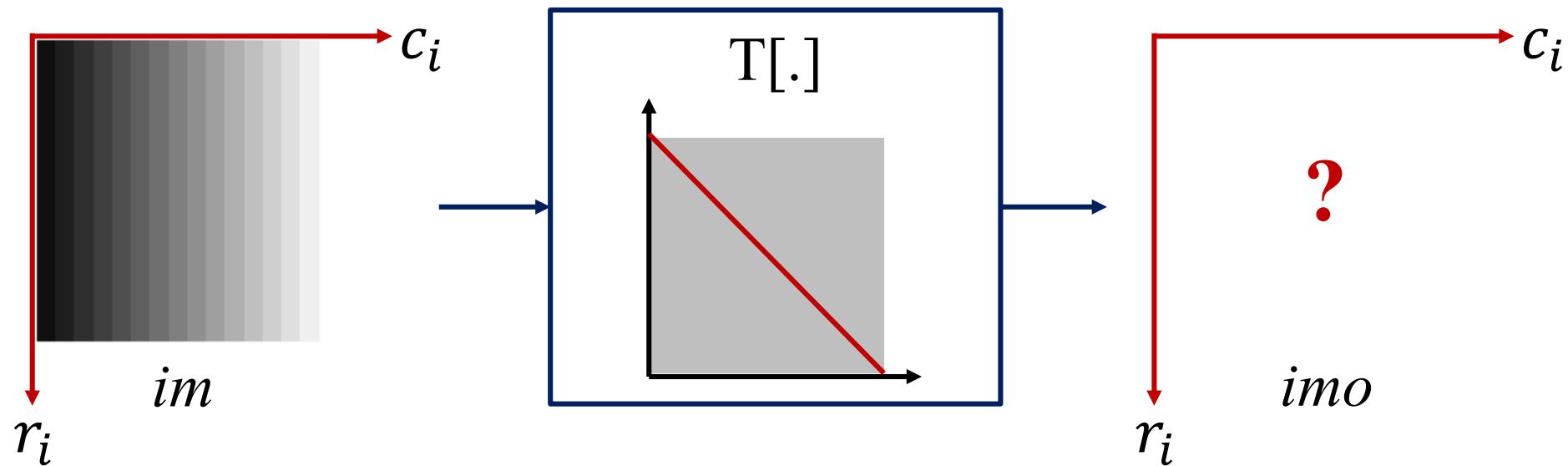


Image negative

- Particularly suitable to enhance white/gray details embedded in dark regions

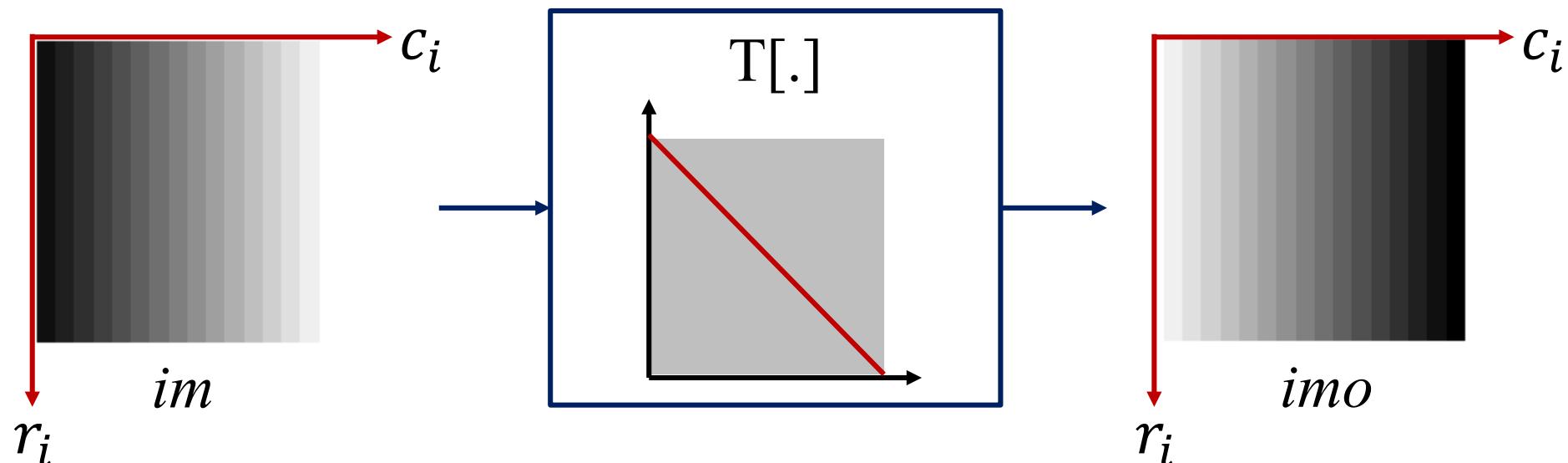
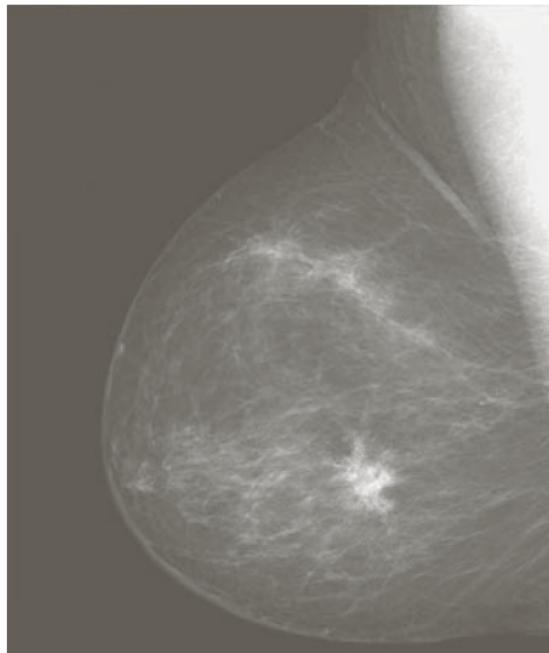


Image negative

- Digital **mammogram** showing a small **lesion**

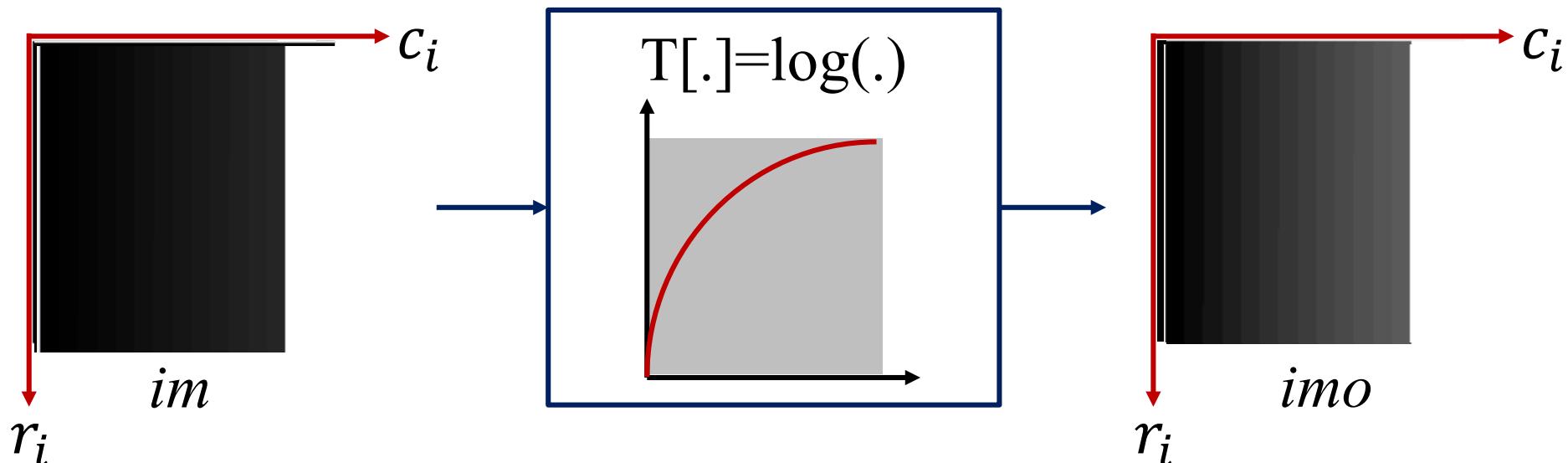
It is easier to see a dirty spot over white background



From: Gonzales & woods; Digital image processing 3rd

Log transforms

- This transform is used to **expand** the **range** of dark levels, while **compressing** the bright level range.
- It is **used to compresses** the **dynamic range** of images with large dynamic to **make** it suitable for display



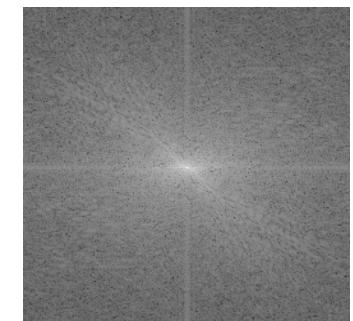
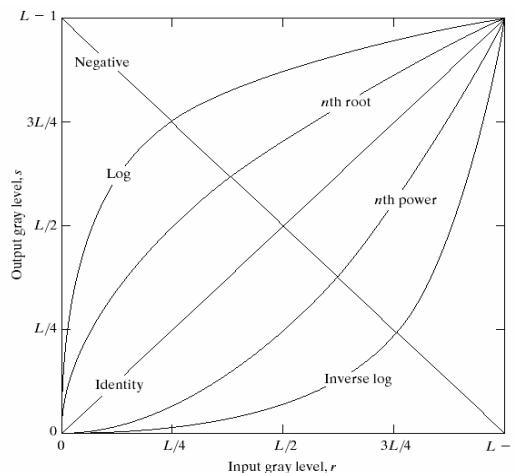
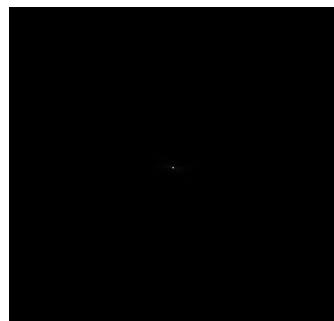
Log transforms

$$v = c \log(1 + u)$$

lenna



$\text{FFT}(\text{lenna})$



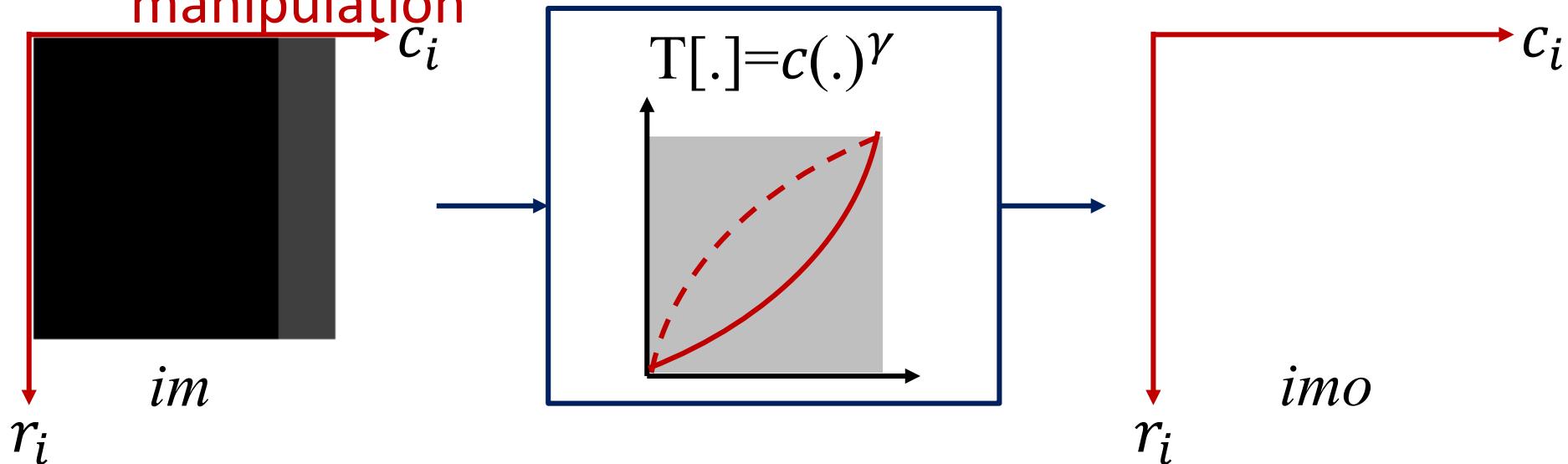
```
>> im = imread('lenna512.bmp')
>> a = abs(fftshift(fft2(double(im))));
```

>> max(max(a))

32505395

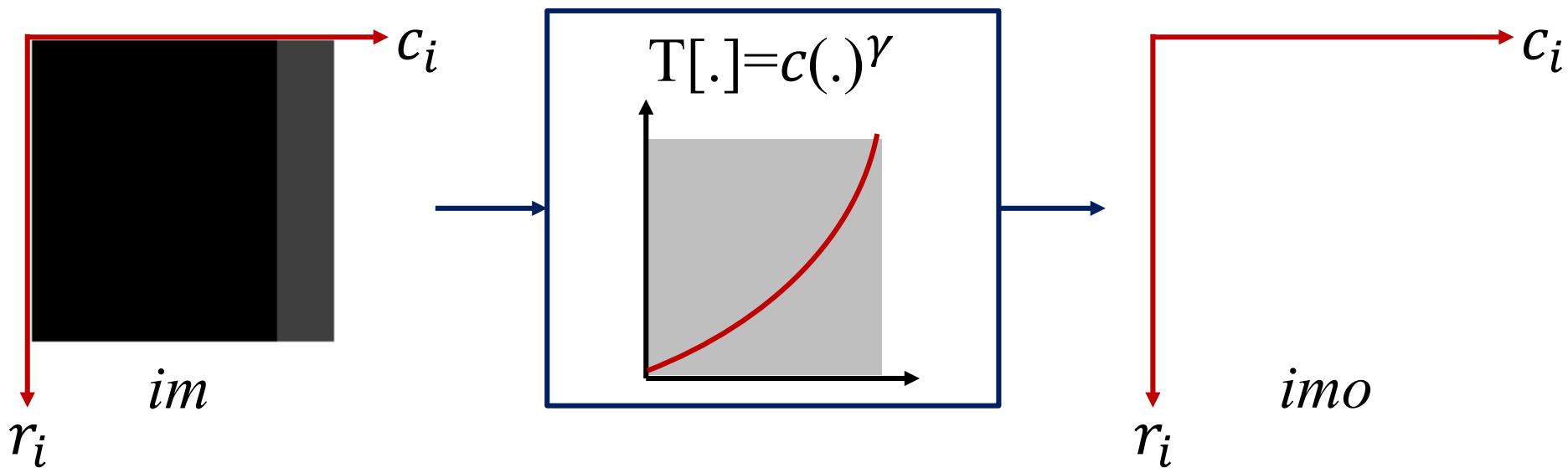
Power-law transforms

- This transform is good because we have a set of curves, rather than only one, which could be used to narrow certain range of gray levels and expand the other range → are useful for general purpose contrast manipulation



Power-law transforms

- $\gamma > 1$
 - Compresses dark range
 - Expands bright values



Power-law transforms

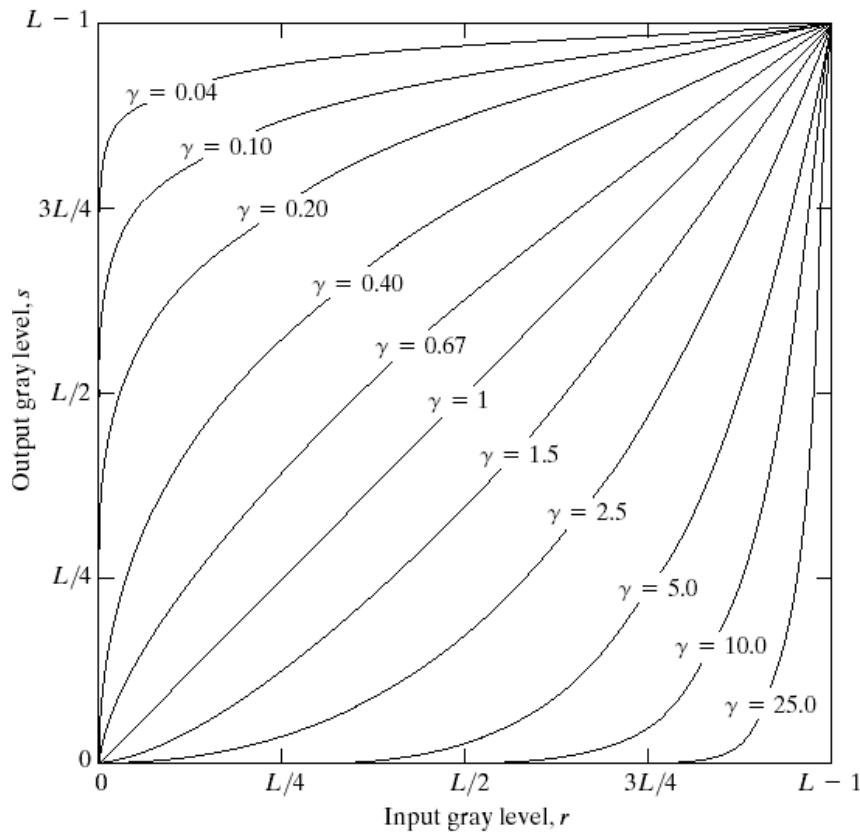
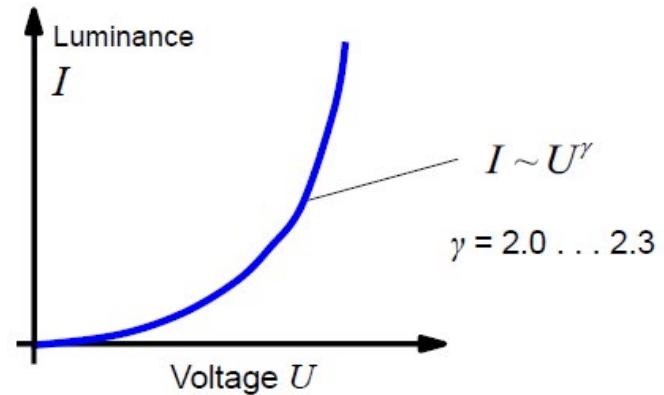


FIGURE 3.6 Plots of the equation $s = cr^\gamma$ for various values of γ ($c = 1$ in all cases).

Power-law transforms

- **Variety** of image capturing **devices**, printing, and display **respond** following a **power law**.
 - To **correct** the **power-law response** of this systems **gamma-correction** is used.
 - Example: CRT produce images darker than the intended input (the **intensity-to-voltage** response is power-law, $\gamma=1.8\text{--}2.5$)

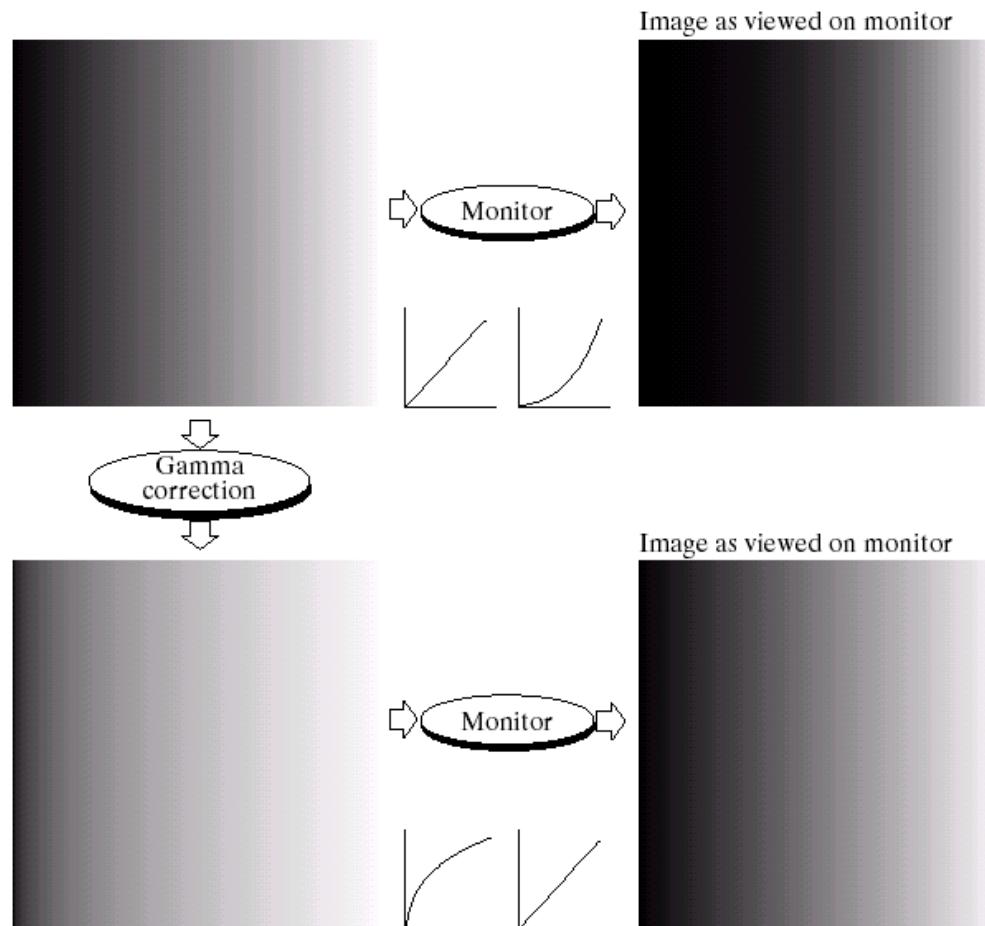


- Example: Cameras $\gamma = 1.0\text{--}1.7$

Gamma-correction

a b
c d

FIGURE 3.7
(a) Linear-wedge gray-scale image.
(b) Response of monitor to linear wedge.
(c) Gamma-corrected wedge.
(d) Output of monitor.



Gamma-correction

Original
Image



Gamma
Correction
 $c=1, \gamma=4$



Gamma
Correction
 $c=1, \gamma=3$



Gamma
Correction
 $c=1, \gamma=5$

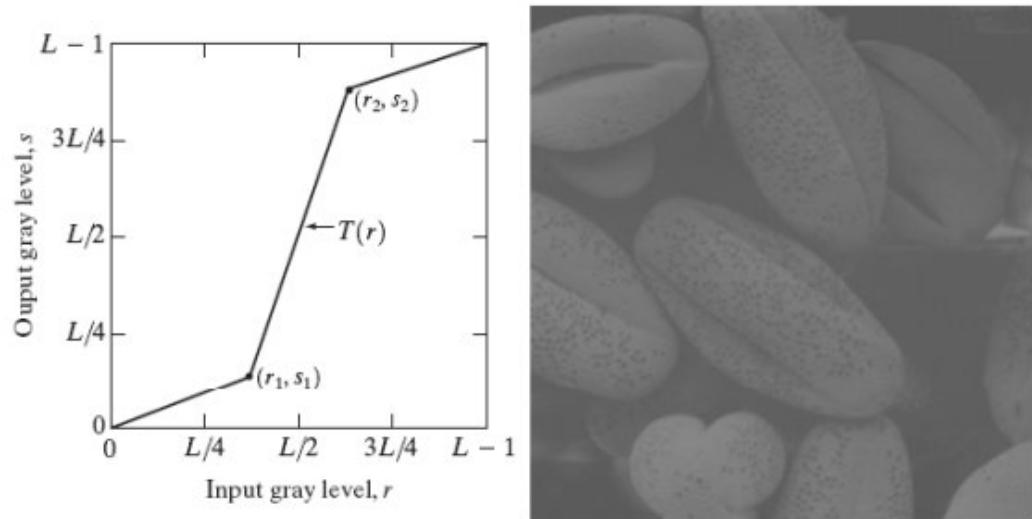


Piecewise-Linear Transformation Functions

Piecewise-Linear Transformation Functions

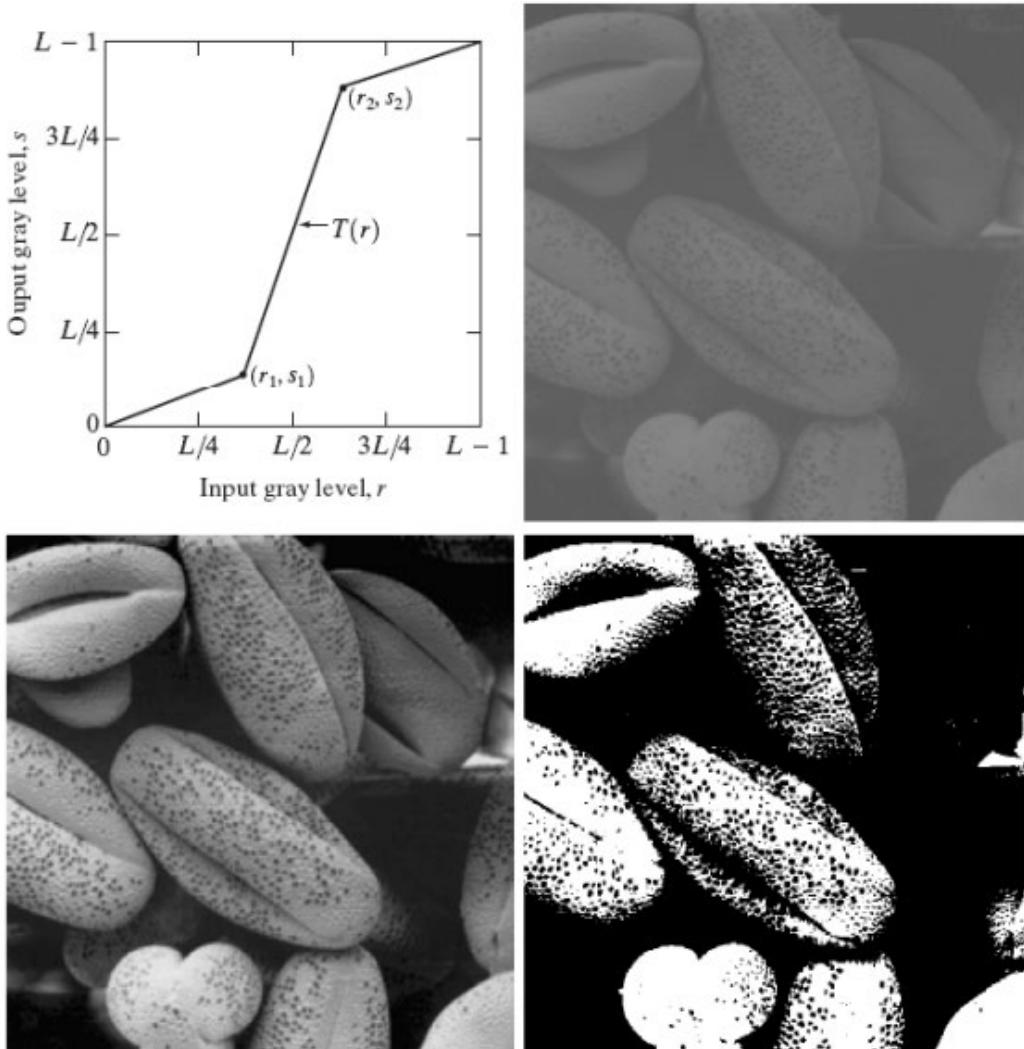
- Different from previous functions which could be described mathematically, **this category** of functions is **described graphically**, this makes them suitable for variety of applications
- **Allows** to make the **mapping function arbitrary complex**
 - Contrast enhancing
 - Gray level slicing

Contrast enhancing by stretching



- The function is assumed single valued and monotonically increasing function, aims to **increase the dynamic range** of the gray levels

Contrast enhancing by stretching



- The function is assumed single valued and monotonically increasing function, aims to **increase the dynamic range** of the gray levels
- **How** to decide the **shape** of the mapping function ?

Gray level slicing

- Highlighting a specific range of gray levels.

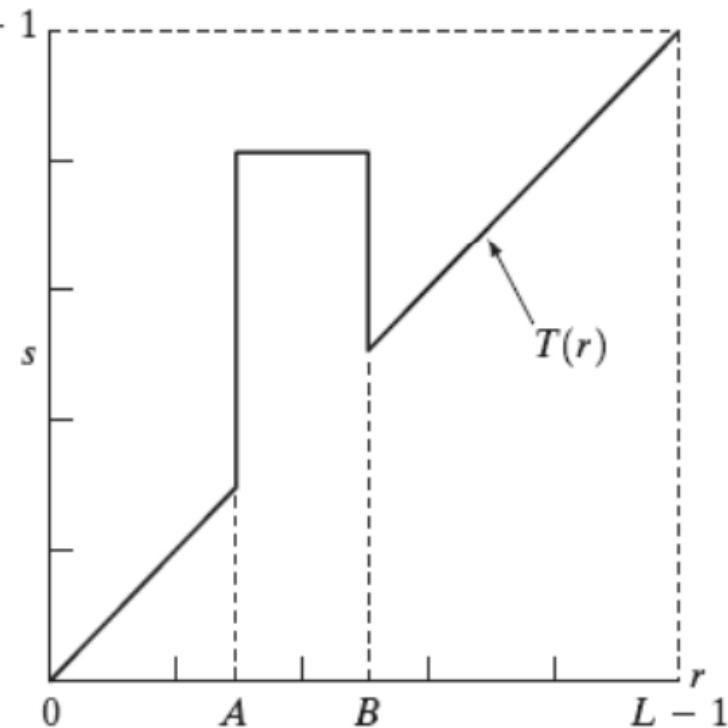
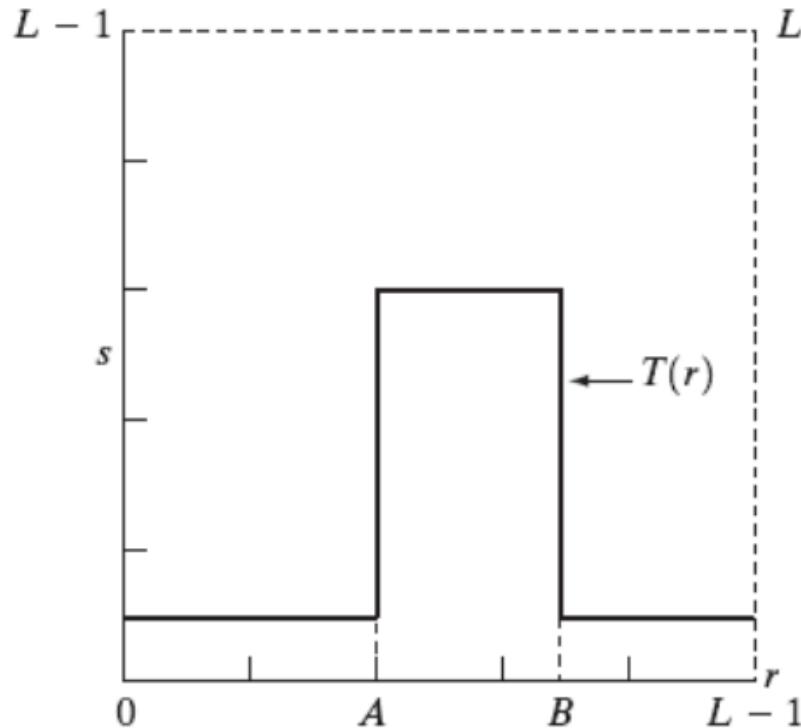
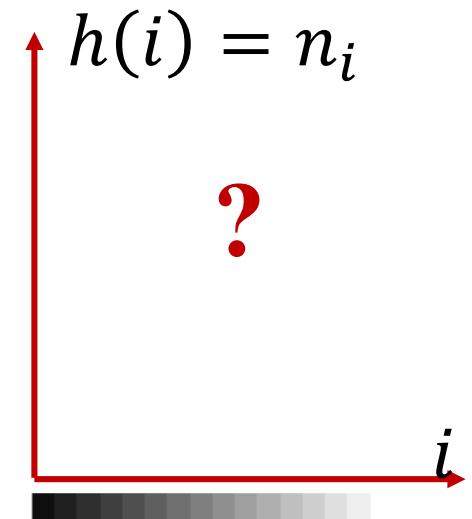
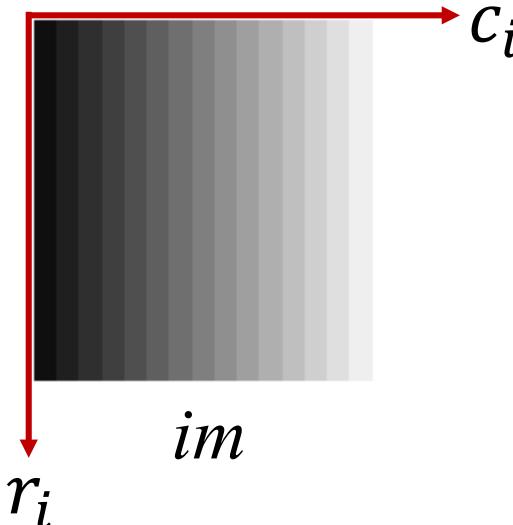


Image histogram

Image histogram

- Image histogram is a **graphical** representation of the **distribution** of gray levels in an image
- Is a **discrete function** $h(i) = n_i$ where **i is a gray level** and n_i is the **number of pixels** having that gray level

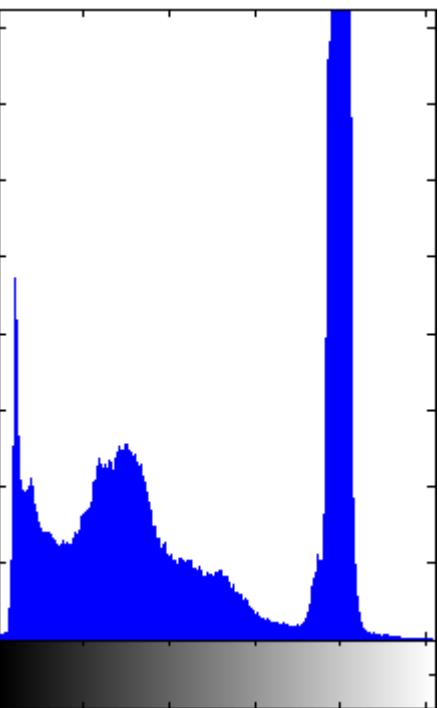


Normalized image histogram

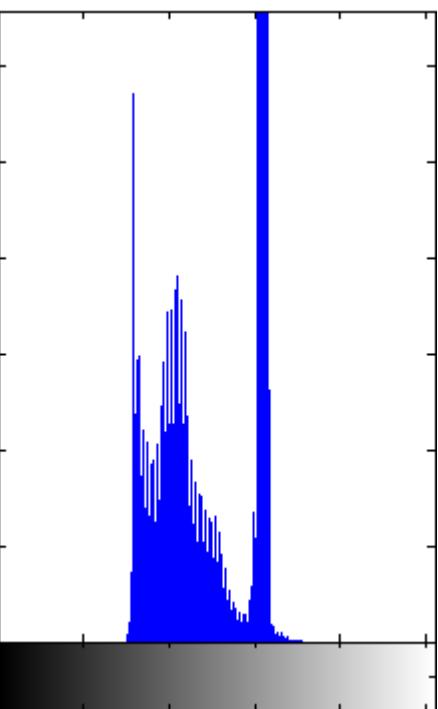
- In the **Normalized Histogram**, h_n , each value of the histogram is **divided by the total number** of pixels in the image $N = \sum_{\forall i} n_i$
$$h_n(i) = \frac{n_i}{N}$$
- When $N \rightarrow \infty$ then $h_n(i)$ represents the **estimate** of the **probability** of the **gray level** i .
 - In this case $h_n(i)$ becomes **equivalent** to the **Probability Density Function** (pdf) of a random *variable representing the gray level*.

Image histogram

- “unbalanced” histograms do not fully utilize the dynamic range
 - Low contrast image: narrow luminance range
 - Under-exposed image: concentrating on the dark side
 - Over-exposed image: concentrating on the bright side
- “balanced” histogram gives more pleasant look and reveals rich details

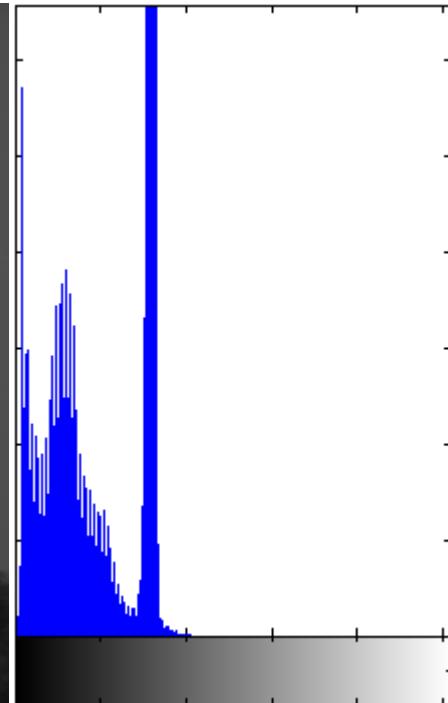
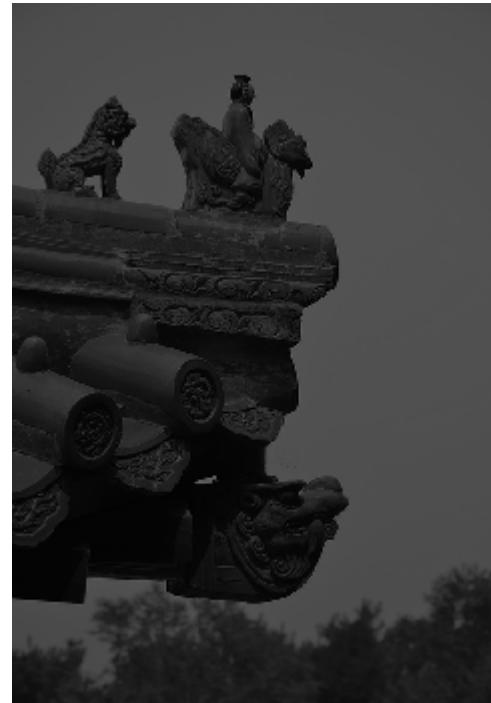


- High-contrast image
 - Covers the **entire** range of possible **gray levels**, tend to distribute them uniformly.
 - Exhibits a large variety of gray tones
 - We call it also **“balanced” histogram**

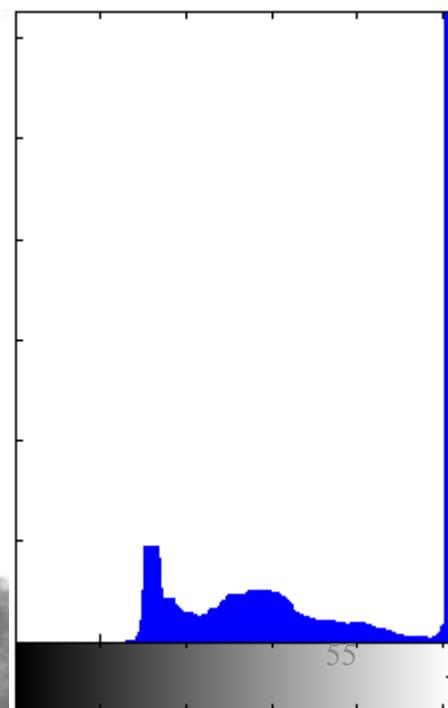


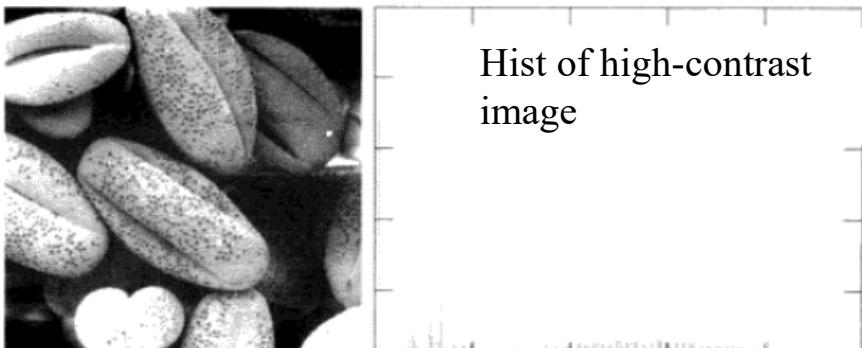
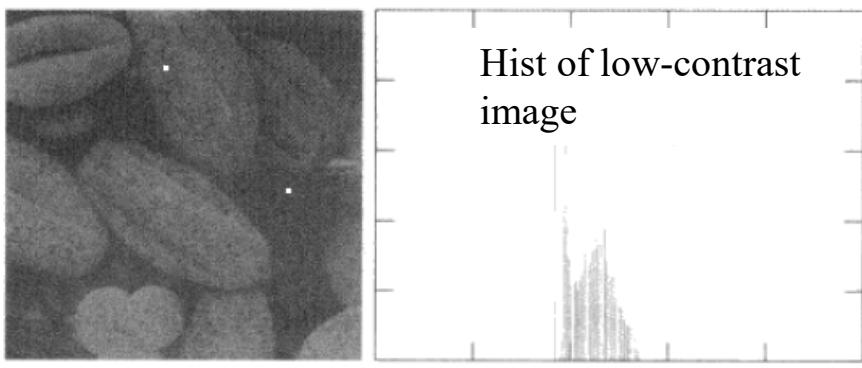
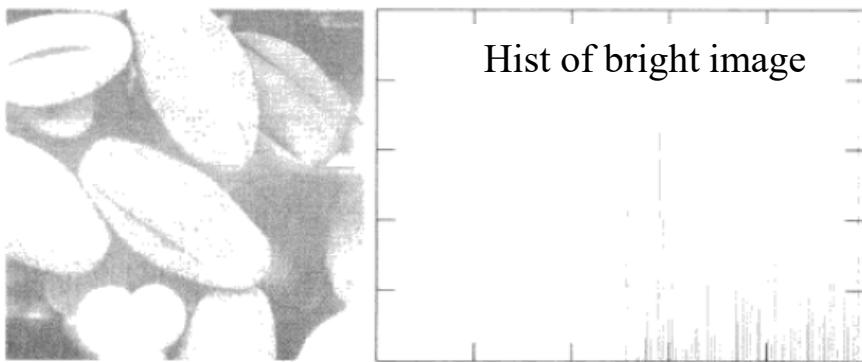
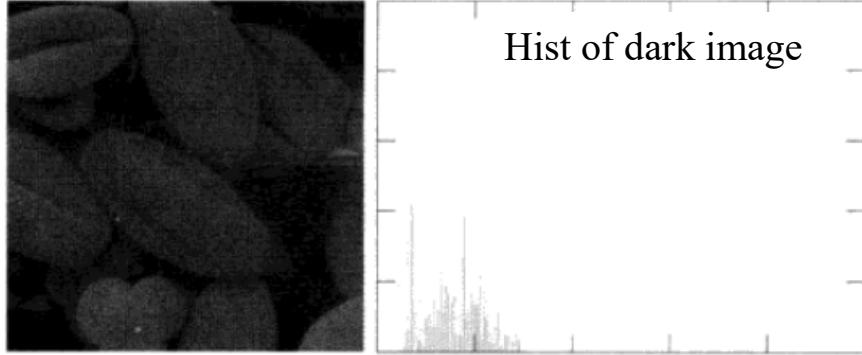
- Low-contrast image
 - The histogram is narrow and centered toward the middle of the gray scale.
 - So the image **looks dull** and **washed-out**.

- Dark image
 - The histogram is **biased** toward the **dark** side of the gray scale.
 - In **photography** we call it:
Under-exposed image



- Bright image
 - The histogram is **biased** toward the **bright** side of the gray scale.
 - In **photography** we call it:
over-exposed image



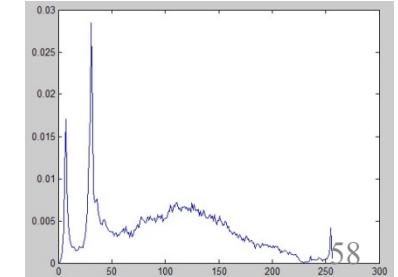
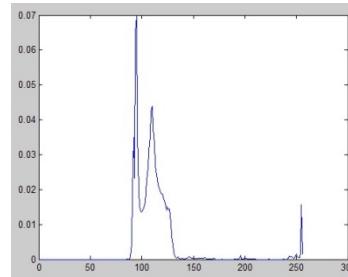
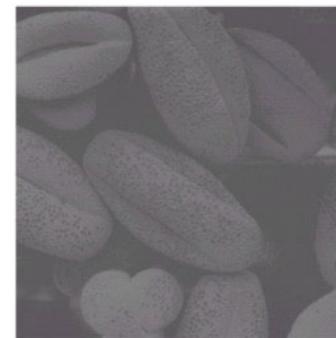
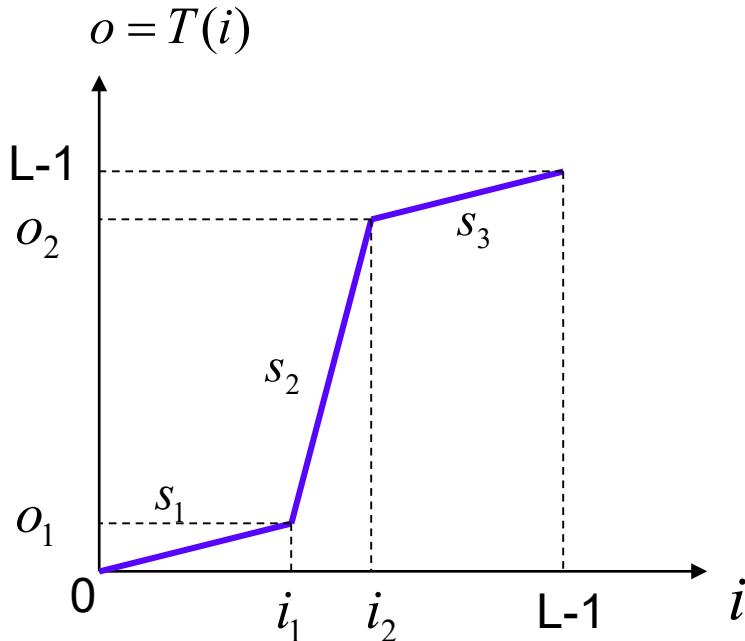


Low dynamic range

- A **low dynamic range** image could be **due to the following reasons:**
 - Poor illumination
 - Wrong setting of the lens **aperture** and/or **shutter speed**
- In a low contrast image the **full dynamic range** is not **exploited**
- Contrast enhancing by stretching
 - Is a **simple** technique to expand the histogram to fill entire gray-scale range

Contrast enhancing by stretching

- Piece-wise linear function could be designed based on the histogram of the original image,
 - The slope in the stretching region is greater than 1.
 - The slope in the compressing region is smaller than 1.



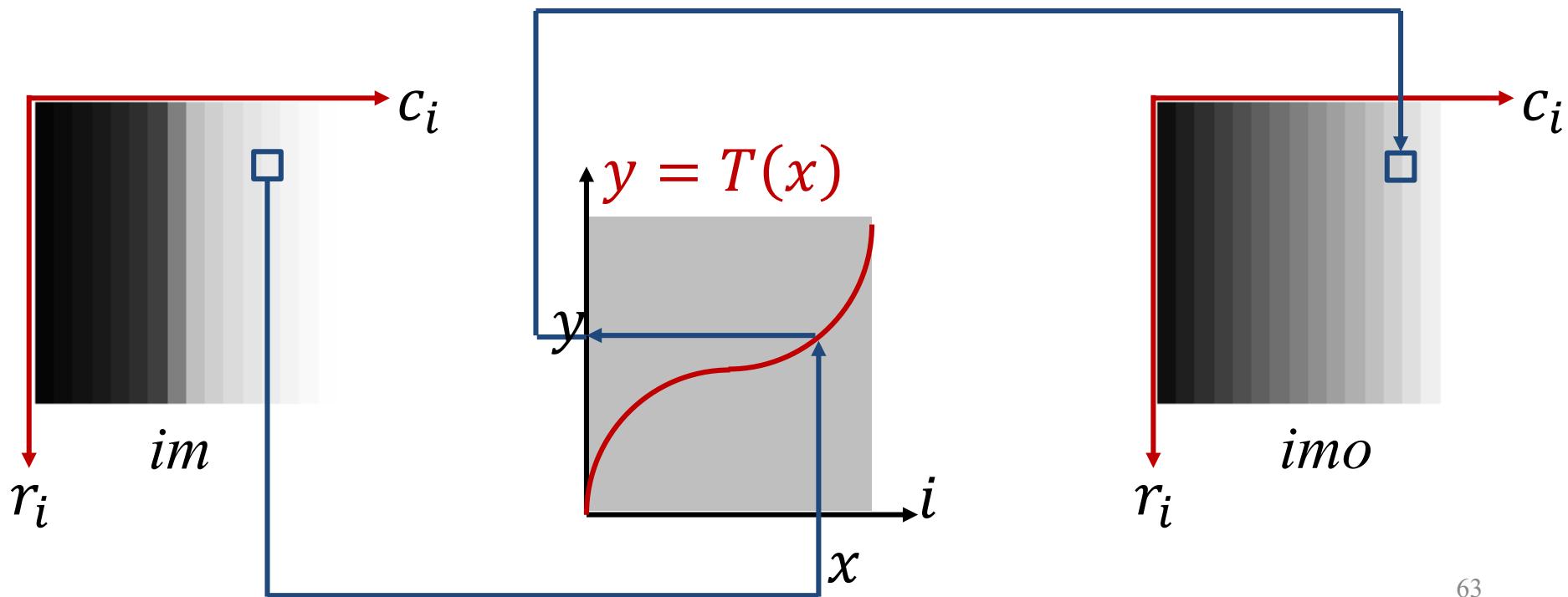
Histogram Equalization

Histogram Equalization

- It is a technique to enhance contrast in ‘natural’ images, and it is a kind of **nonlinear image-to-image mapping**
- The **target** is to **find** gray level **transformation** function T to **map** the image **intensity** i such that the **histogram** of $T(i)$ is ‘**equalized**’
 - An image with a perfectly **flat histogram** contains **largest** possible amount of **information**

Histogram Equalization

- The goal is to map each luminance level to a new value such that the output histogram has approximately uniform distribution.

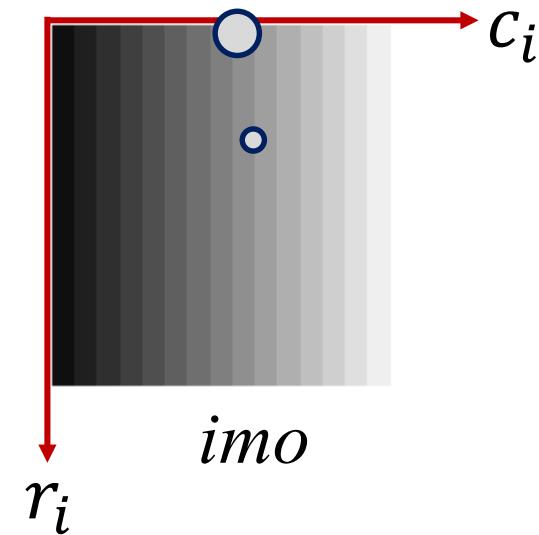
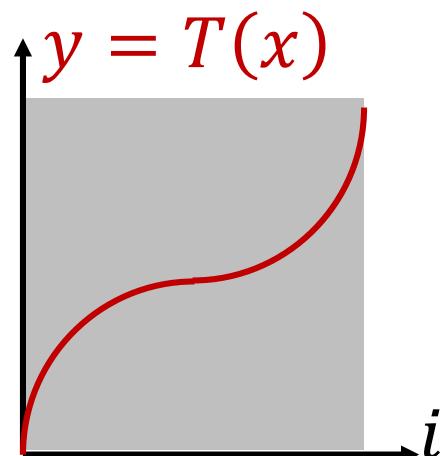
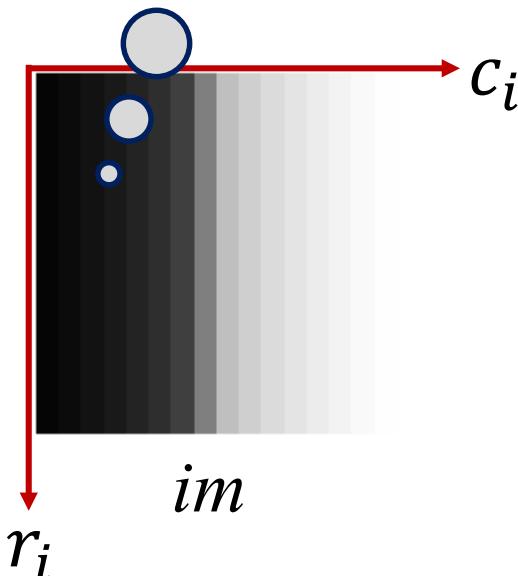


Histogram Equalization

Can you distinguish the various dark band?

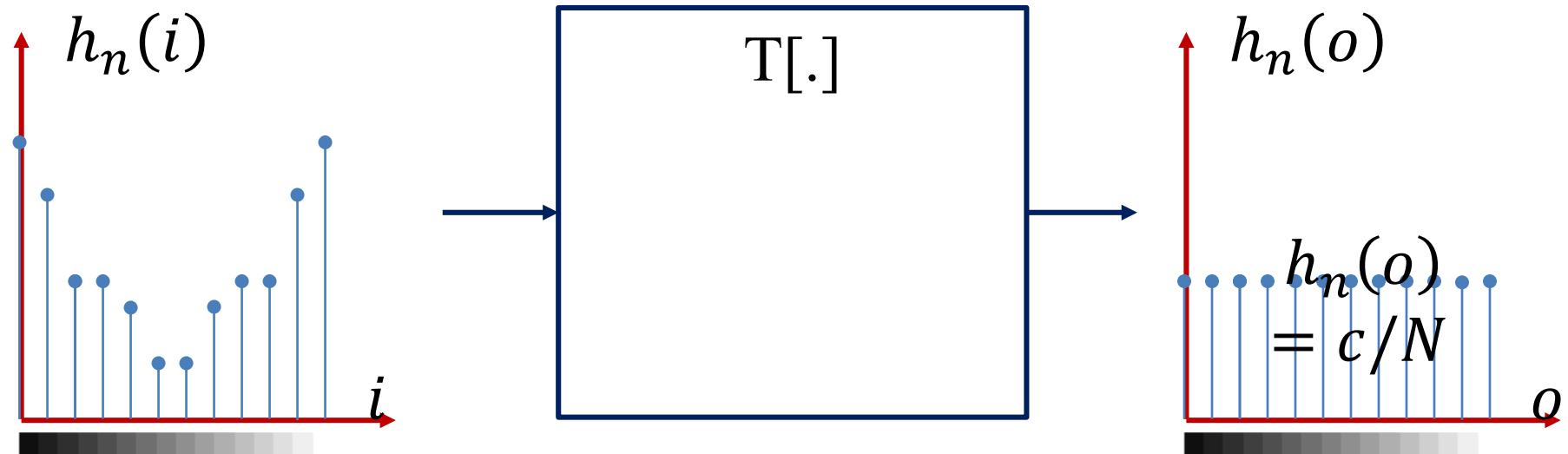
to map each luminance such that the output has a more uniform distribution.

Here you distinguish more gray bands → more information



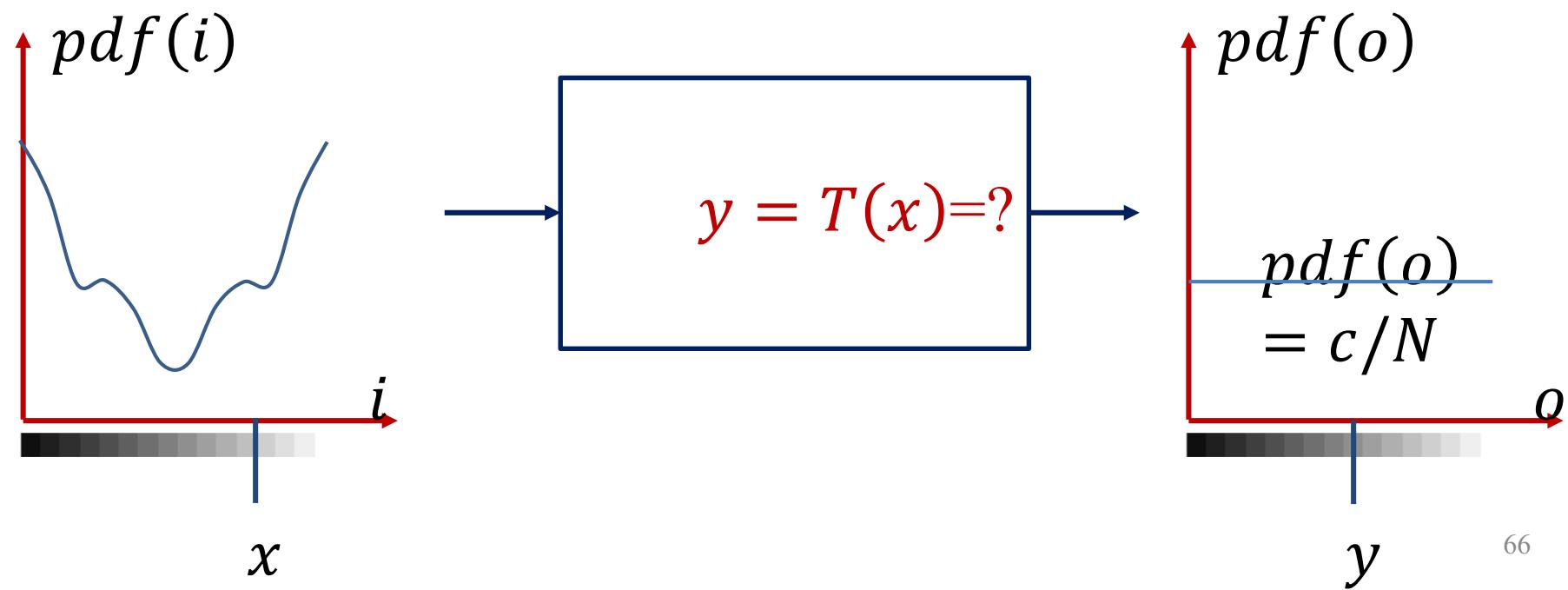
Histogram Equalization

- How to find $y = T(x)$



Histogram Equalization

- The first step to **simplify** this process is to assume that the functions are continuous.



Histogram Equalization

- To find $T(x)$ we have to notice that the number of remapped pixels is the same as the originals (preserving the quantities)

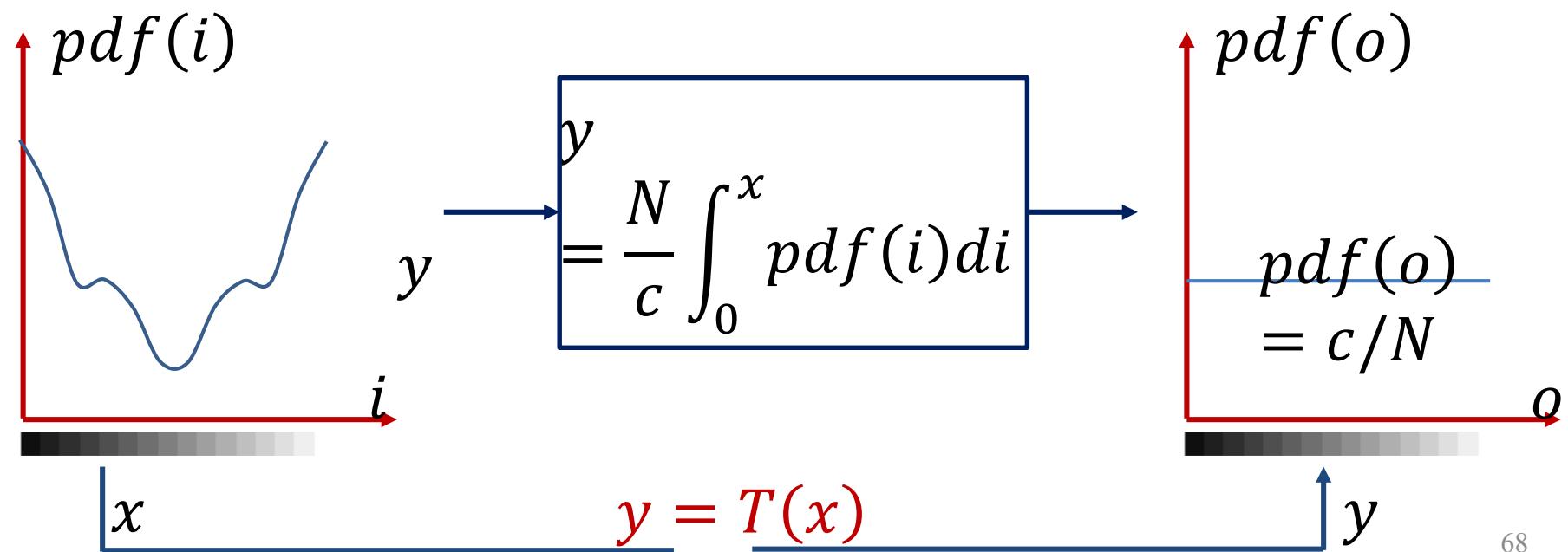
$$\begin{aligned}\int_0^x pdf(i)di &= \int_0^y pdf(o)do \\ &= \frac{c}{N} \int_0^y do \\ &= \frac{c}{N} y\end{aligned}$$

The *cdf* (Cumulative Distribution Function)

- Finally $\frac{c}{N} y = \int_0^x pdf(i)di$

Histogram Equalization

- Practically how to find $y = T(x)$



Histogram Equalization

- Discrete formulation

$$p_r(r_k) = \frac{n_k}{MN}, \quad k = 0, 1, 2, \dots, L-1$$

$$s_k = T(r_k) = (L-1) \sum_{j=0}^k p_r(r_j) = \frac{(L-1)}{MN} \sum_{j=0}^k n_j, \quad k = 0, 1, 2, \dots, L-1$$

Here: MN is the number of total pixels;

n_k is the number of pixels whose gray value equals k

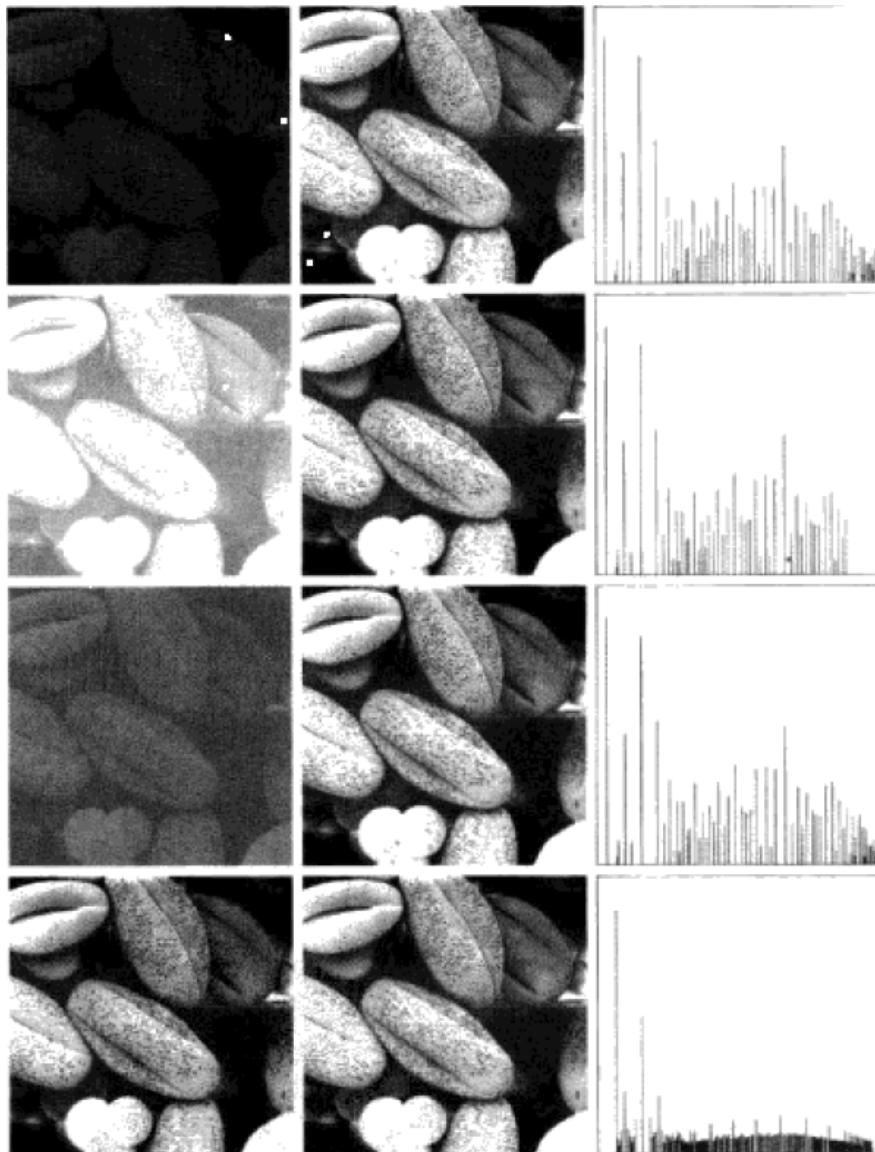
L : gray level

r_k is the gray value of the pixels in the original image

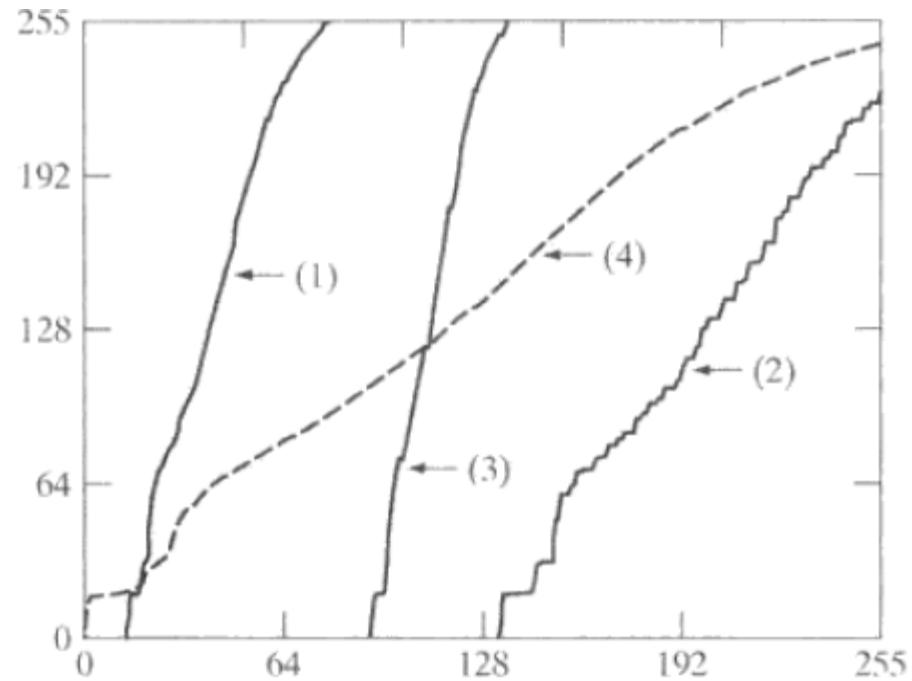
s_k is the gray value of the pixels in the new image

- Histogram equalization transformation function can be **automatically obtained based** on the information of the **histogram of the input** image.

Histogram Equalization



- **Left** column shows images with different contrast



Example

Image=0 2 5 6
1 2 4 5
1 3 3 0

Histogram:

Gray	Count	Prob.
0	2	0.167
1	2	0.167
2	2	0.167
3	2	0.167
4	1	0.083
5	2	0.167
6	1	0.083
7	0	0

Histogram Equalization

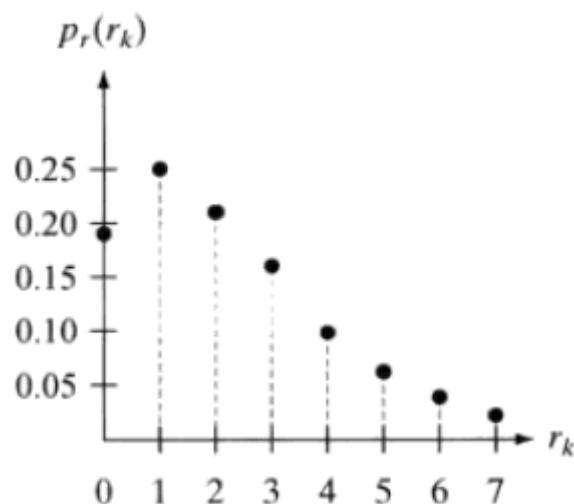
What is the variable?
What is the response?

$$s_k = T(r_k) = (L-1) \sum_{j=0}^k p_r(r_j) = \frac{(L-1)}{MN} \sum_{j=0}^k n_j, \quad k = 0, 1, 2, \dots, L-1$$

New Image=1 3 6 7
2 3 5 6
2 5 5 1

Example 2

r_k	n_k	$p_r(r_k) = n_k / MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02



Example 2

$$s_0 = T(r_0) = 7 \sum_{j=0}^0 p_r(r_j) = 7 p_r(r_0) = 1.33$$

$$s_1 = T(r_1) = 7 \sum_{j=0}^1 p_r(r_j) = 7 p_r(r_0) + 7 p_r(r_1) = 3.08$$

$$s_2 = 4.55, s_3 = 5.67, s_4 = 6.23, s_5 = 6.65, s_6 = 6.86, s_7 = 7.00$$

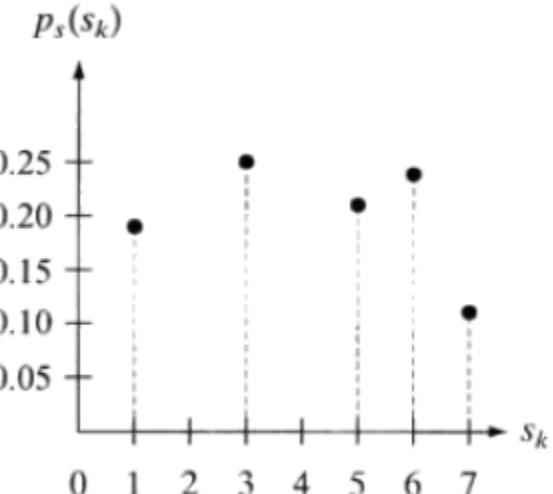
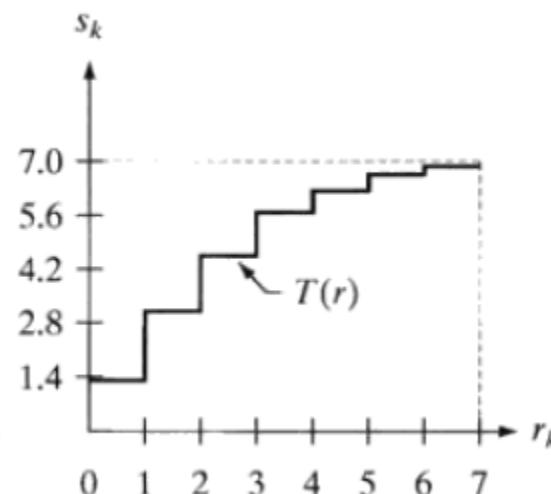
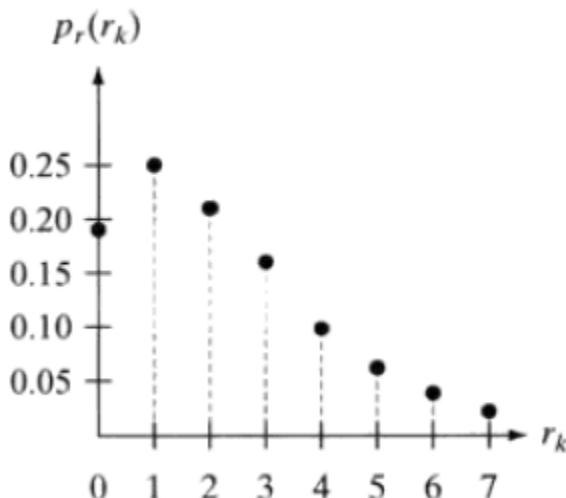


Image Averaging

- Consider a noisy image $g(x,y)$ formed by addition of noise $\eta(x,y)$ to an original image.
- $$g(x,y) = f(x,y) + \eta(x,y)$$
- Here we assume that at every pair of coordinates (x,y) the noise is uncorrelated and has zero average value.
- We try to reduce the noise content by adding a set of noisy images, $g_i(x,y)$.
- It can be shown that as the number of noisy images used in the averaging process increases, the noisy image resembles the original image $f(x,y)$.

Image Averaging

- Let $\check{g}(x,y)$ is obtained by averaging K different noisy images.
- $\check{g}(x,y) = (1/K) \sum g_i(x,y)$ where i varies from 1 to K.
- Then
 - $E\{\check{g}(x,y)\} = f(x,y)$ and
 - $\sigma^2 \check{g}(x,y) = (1/K) \sigma^2 \eta(x,y)$
- Here $E\{\check{g}(x,y)\}$ is the expected value of \check{g} and $\sigma^2 \check{g}(x,y)$ and $\sigma^2 \eta(x,y)$ are the variances of \check{g} and η , all at coordinates (x,y) .
- The standard deviation at any point in the average image is
 - $\sigma_{\check{g}(x,y)} = (1/\sqrt{K}) \sigma_{\eta(x,y)}$
- Now we realize that as K increases, the variability (noise) of the pixel values at each location (x,y) decreases.

Original Image



Noisy Image

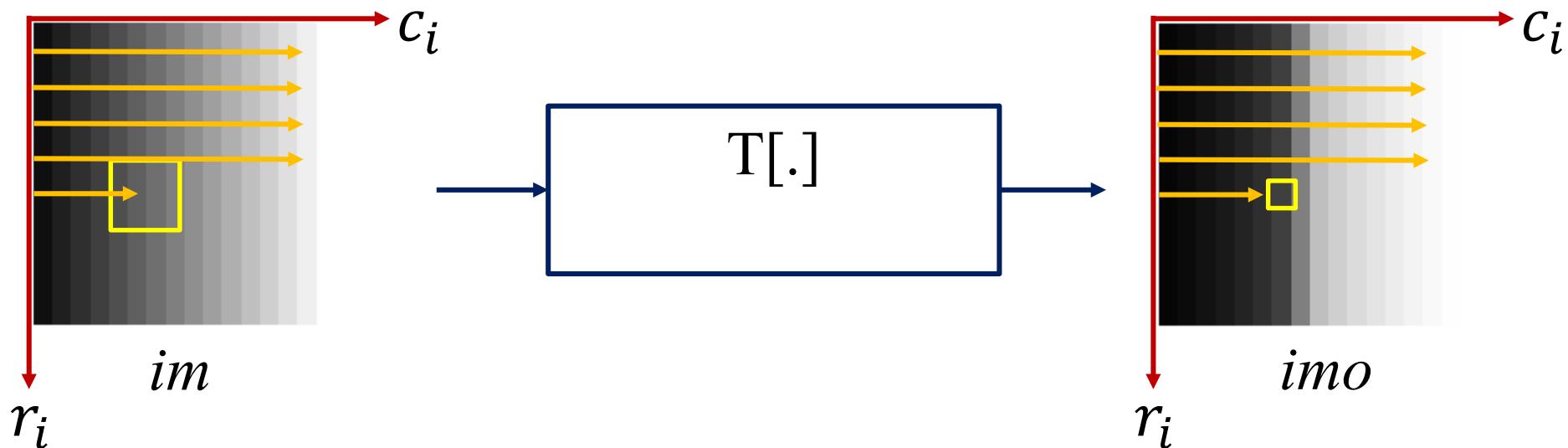


Averaged Noisy Image



Spatial Filtering

- Spatial filtering is a **block-to-pixel** mapping
 - Operates on a **set of ‘neighbourhoods’ N** of each pixel, and it is normally computed in the **spatial domain** and **not in frequency domain**



Spatial Filtering

a	b	c
d	e	f
g	h	i

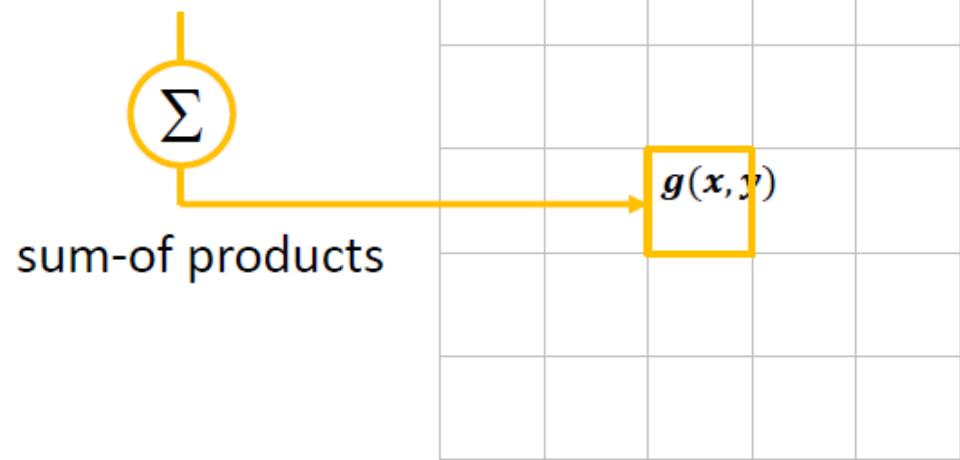
a	b	c
d	e	f
g	h	i

w1	w2	w3
w4	w5	w6
w7	w8	w9

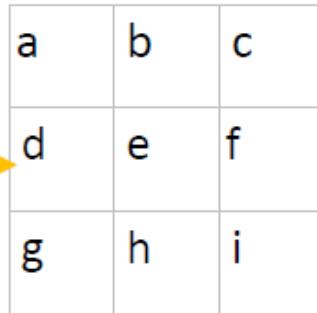
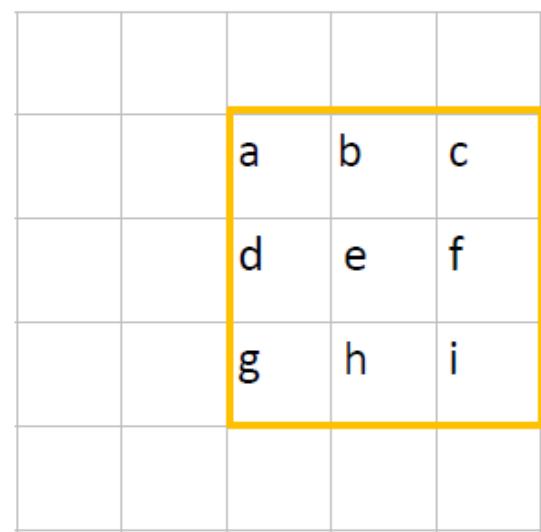
3×3 Filter
mask/window/kernel

a·w1	b·w2	c·w2
d·w4	e·w5	f·w6
g·w7	h·w8	i·w9

The **sum-of-products** is usually used as the value for the **position** of the **center** of the sub-image in the output image



Spatial Filtering

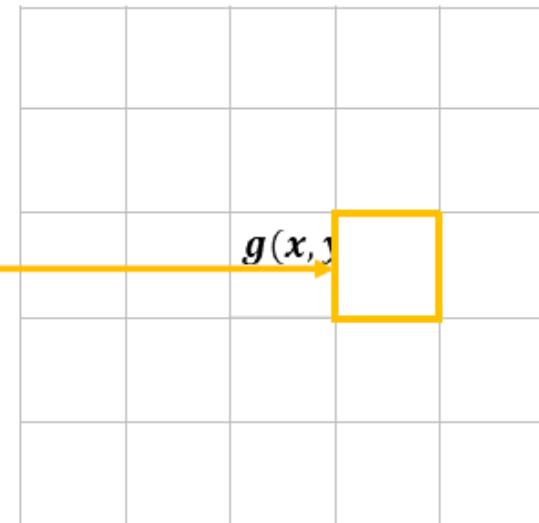


w1	w2	w3
w4	w5	w6
w7	w8	w9

a·w1	b·w2	c·w2
d·w4	e·w5	f·w6
g·w7	h·w8	i·w9

3×3 Filter
mask/window/kernel

The diagram shows a summation symbol (Σ) with three lines pointing to it from the products in the 3x3 filter table above. Below the symbol is the text "sum-of products".



The **sum-of-products** is usually used as the value for the **position** of the **center** of the sub-image in the output image

Spatial Filters

- Spatial convolution:
 - Convolution involves ‘overlap – multiply – add’ with ‘convolution mask’
- Mathematically the convolution operator of an image $h(x,y)$ with a filter $f(\theta, \vartheta)$ is :

$$g(x, y) = \sum_{\forall \theta} \sum_{\forall \vartheta} f(\theta, \vartheta) h(x - \theta, y - \vartheta)$$

Spatial Filters

- It is more natural to think about the **center** of the mask with **odd sizes**, whereas, **even** masks are **awkward** for **implementation**.
- Special consideration is given when the center of the filter **approaches** the **border** of the image; solutions: **cropping**, **padding**.

Matlab: Spatial-Filtering

- Example from MATLAB help

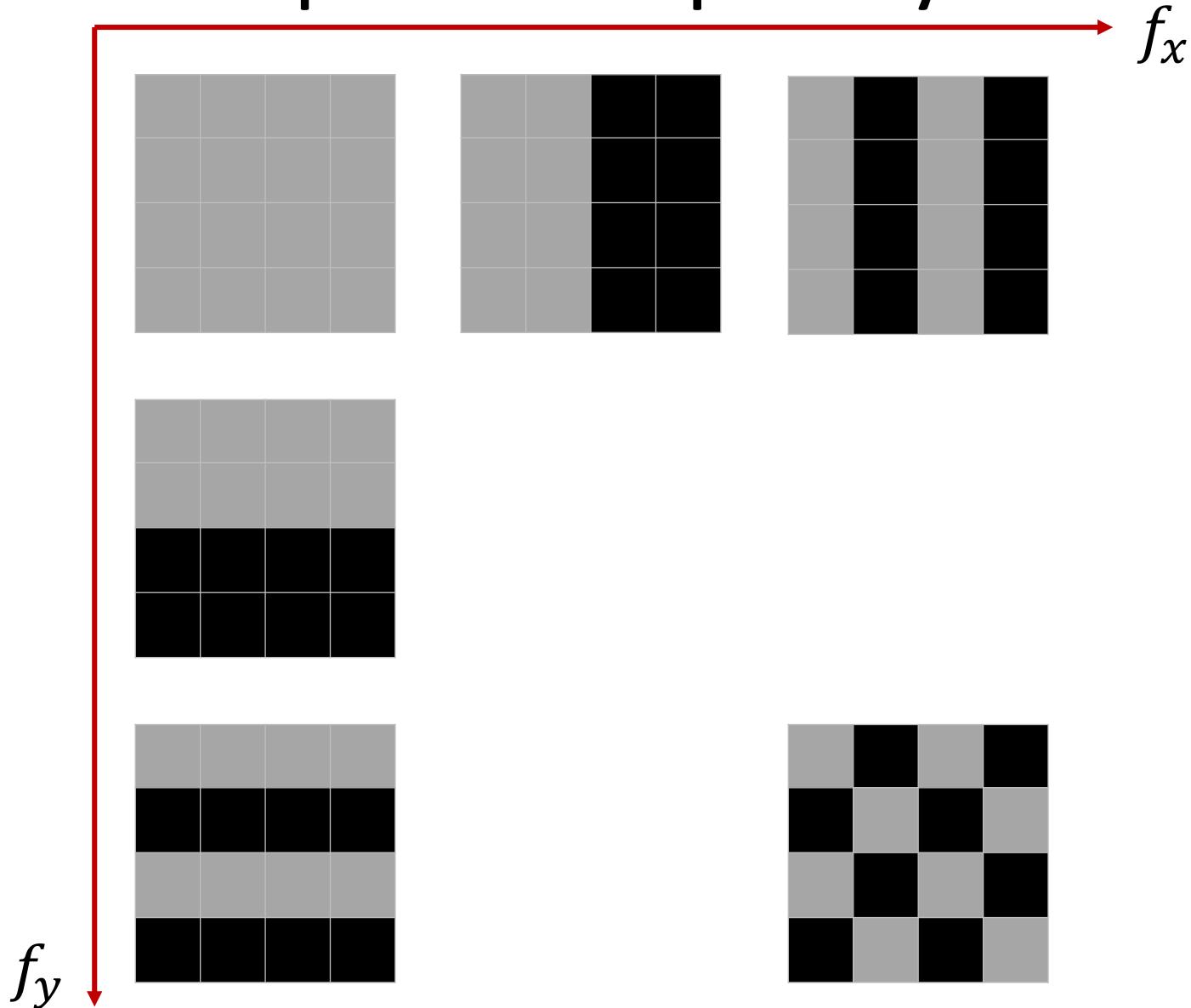
```
» im = imread('peppers.png');  
» h = fspecial('motion', 50, 45);  
» imo= imfilter(im , h);  
» figure, imshow(imo)
```

Spatial frequency

Spatial frequency

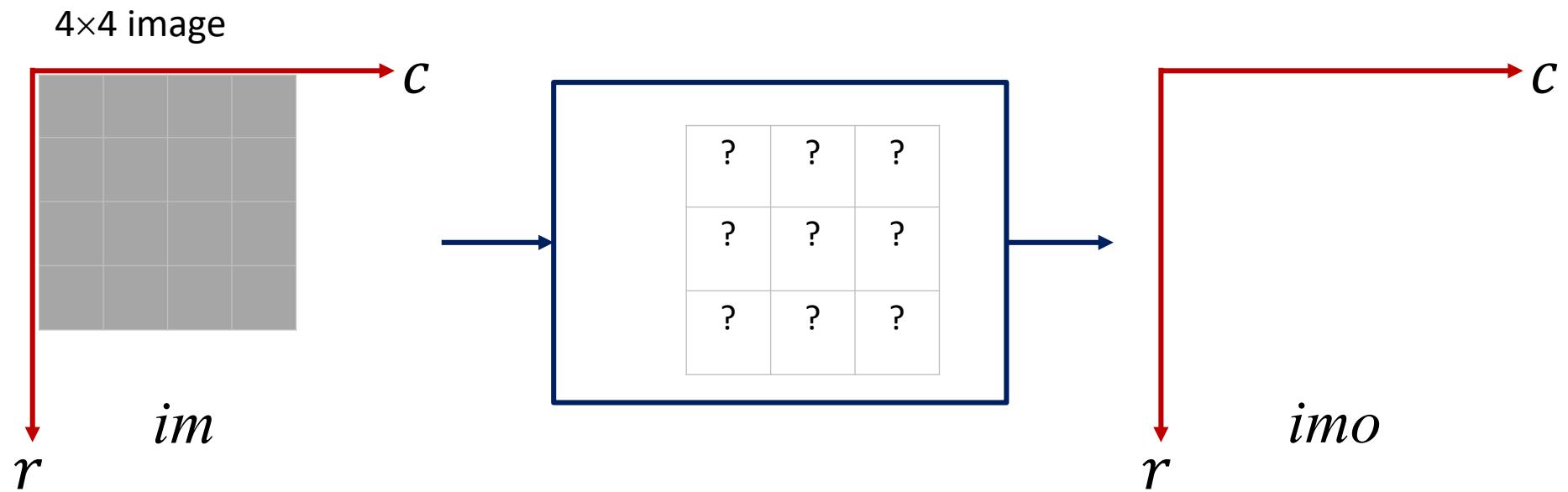
- Spatial Frequency refers to the rate of variations of image brightness with position in space.
 - High spatial frequencies correspond to rapidly varying brightness → fine detail.
 - Low spatial frequencies correspond to slowly changing brightness → flat areas or uniform texture areas.

Spatial frequency

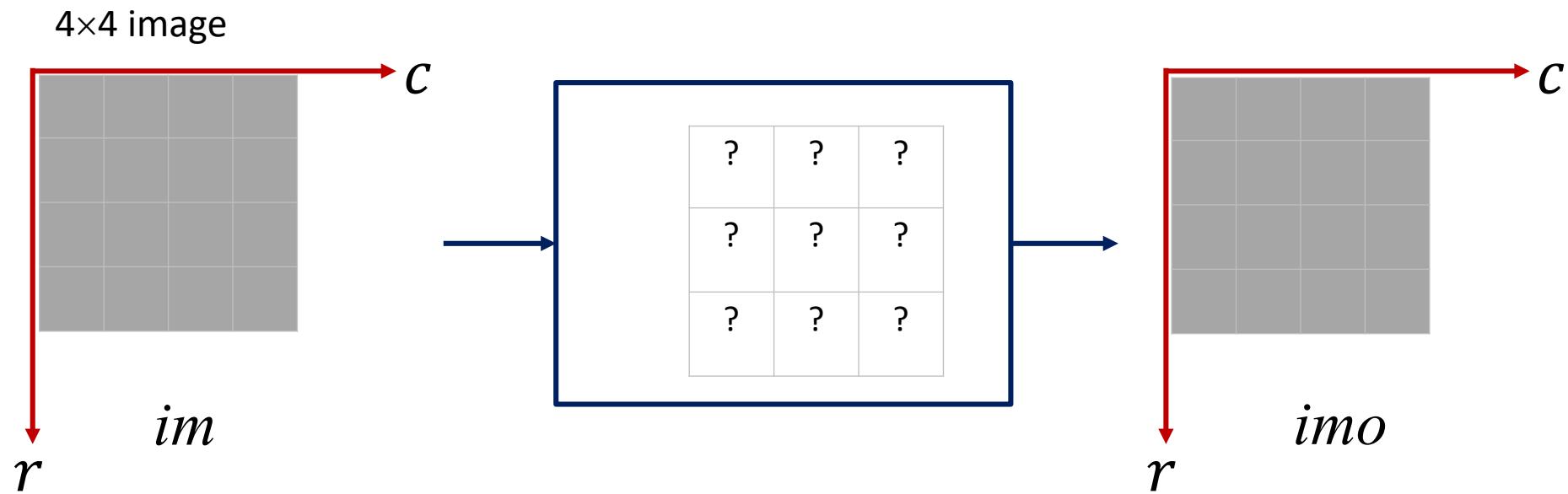


LPF and HPF filters

Low pass filters



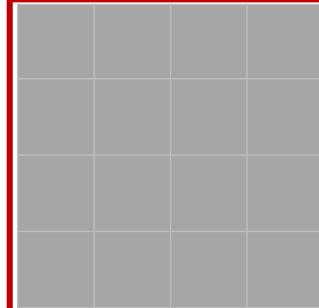
Low pass filters



Low pass filters

4×4 image

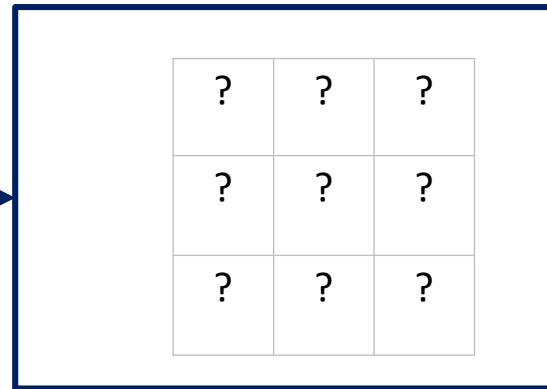
$\rightarrow c$



r

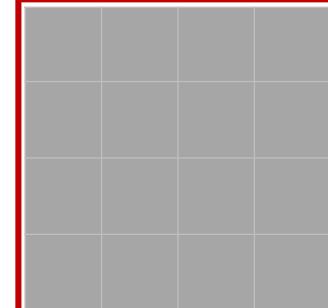
im

c



\rightarrow

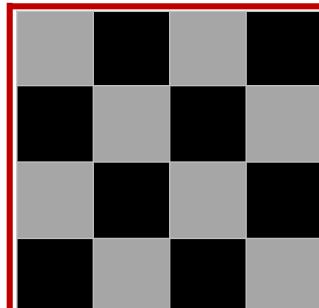
$\rightarrow c$



r

imo

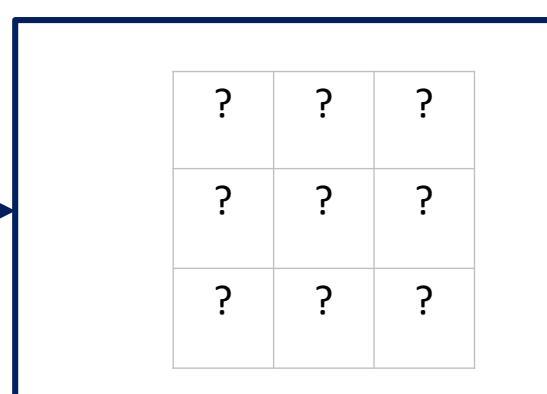
$\rightarrow c$



r

im

c



\rightarrow

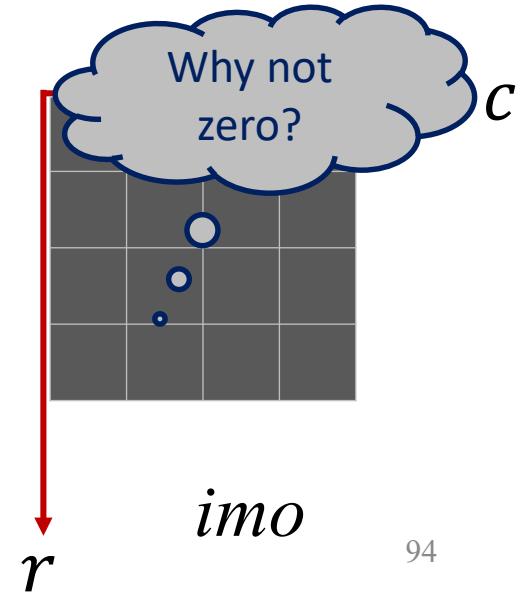
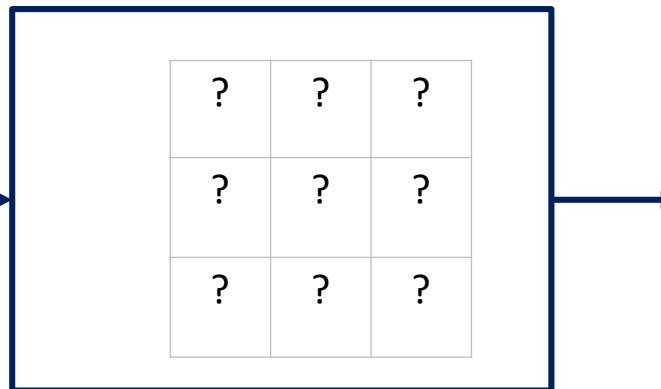
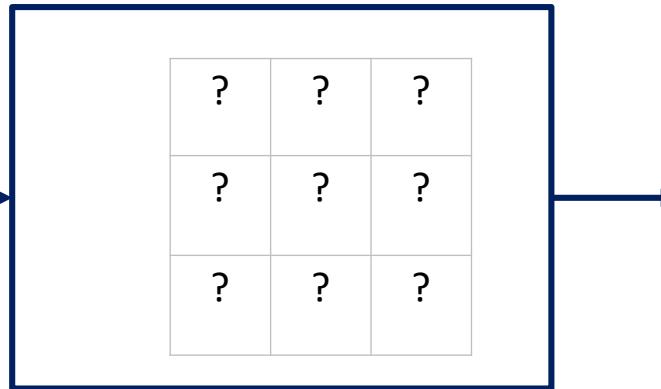
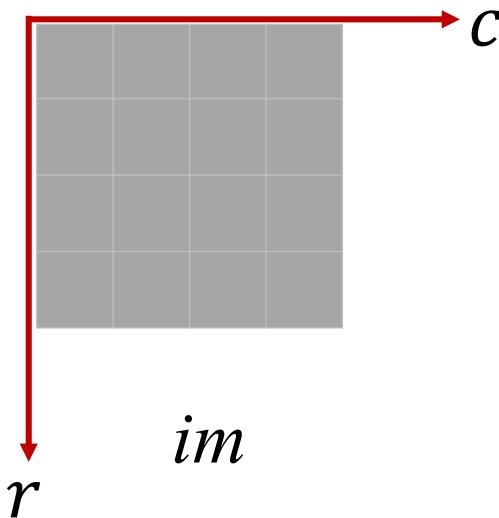
$\rightarrow c$

r

imo

Low pass filters

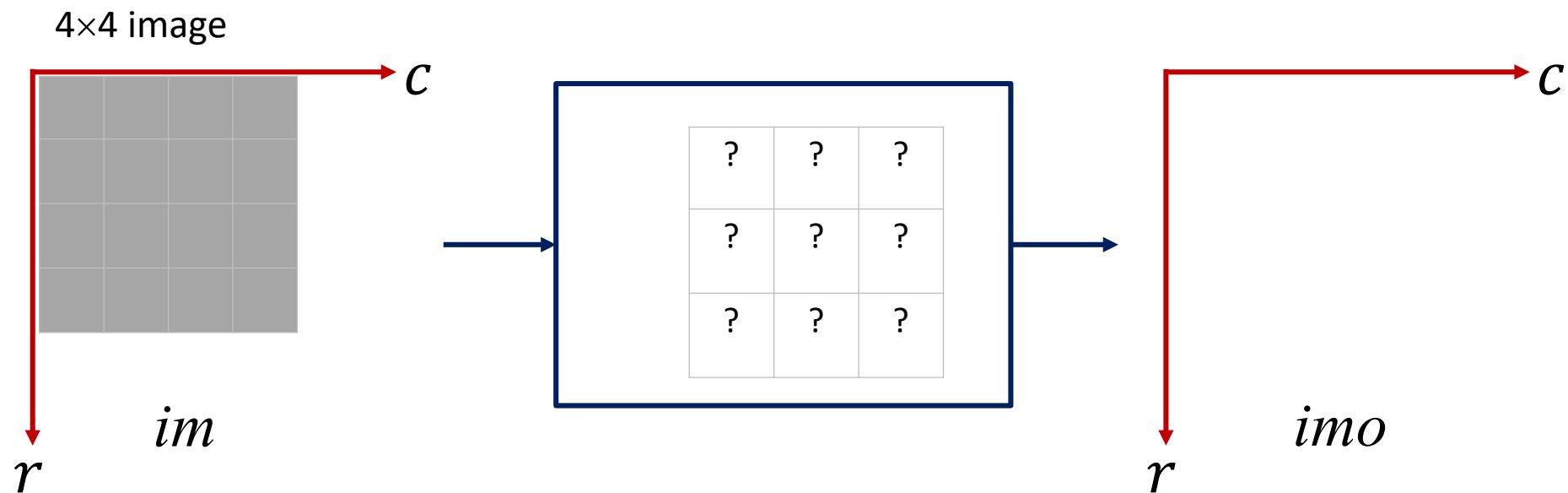
4×4 image



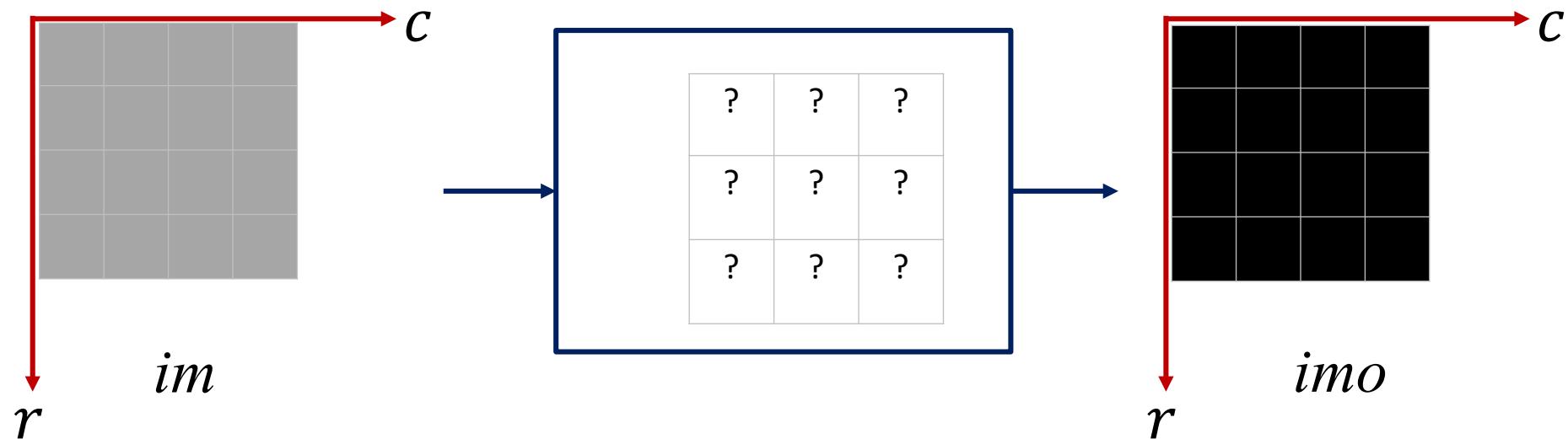
Low pass filters

- Low pass filters **attenuate** a **specific** range of **high frequency** content of the image.
- High frequency content correspond to boundaries of the objects and edges, so **LPF** filtered images **results** in:
 - Smooth images.
 - Blurred borders.
- Give an example of a 3x3 LPF filter!

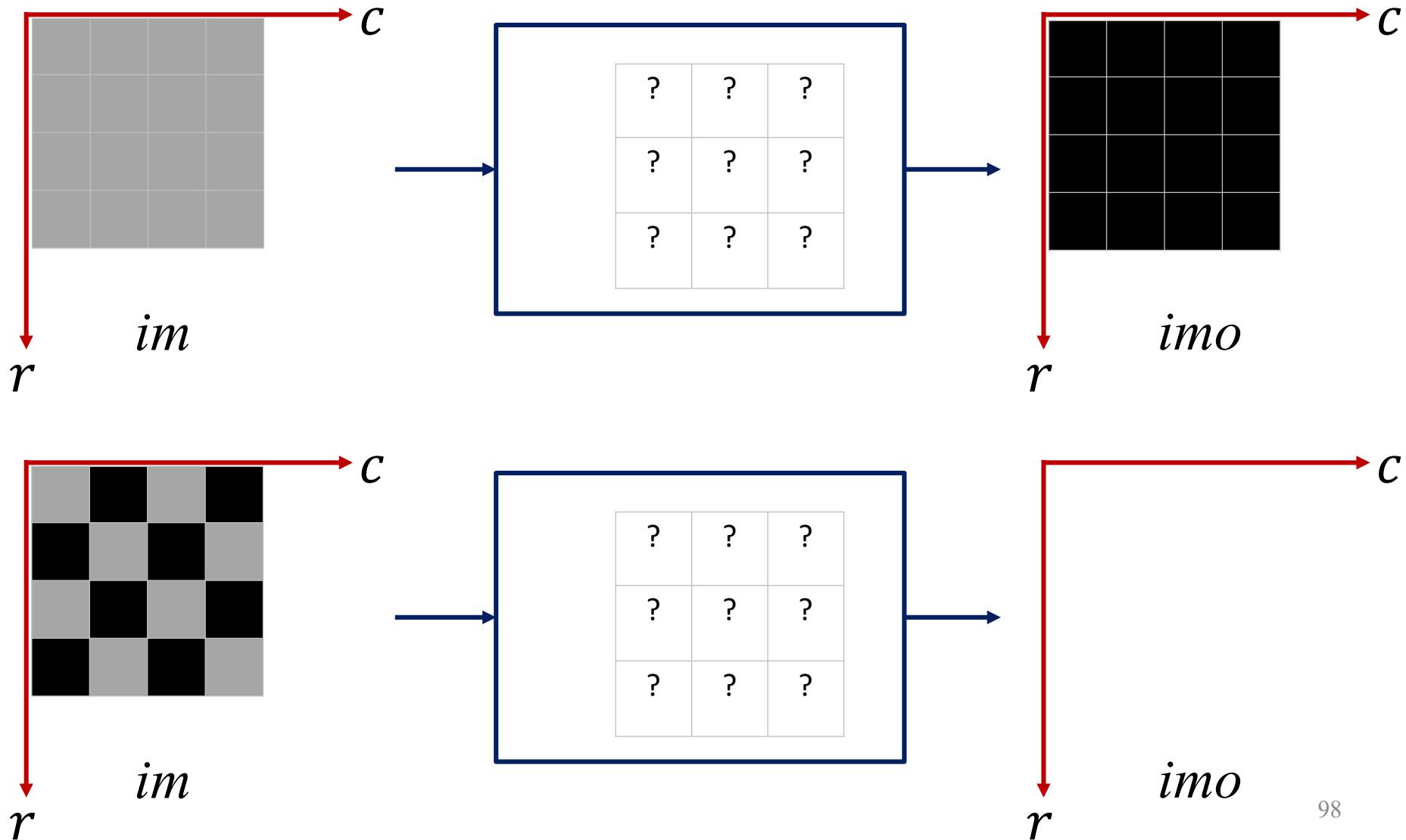
High pass filters



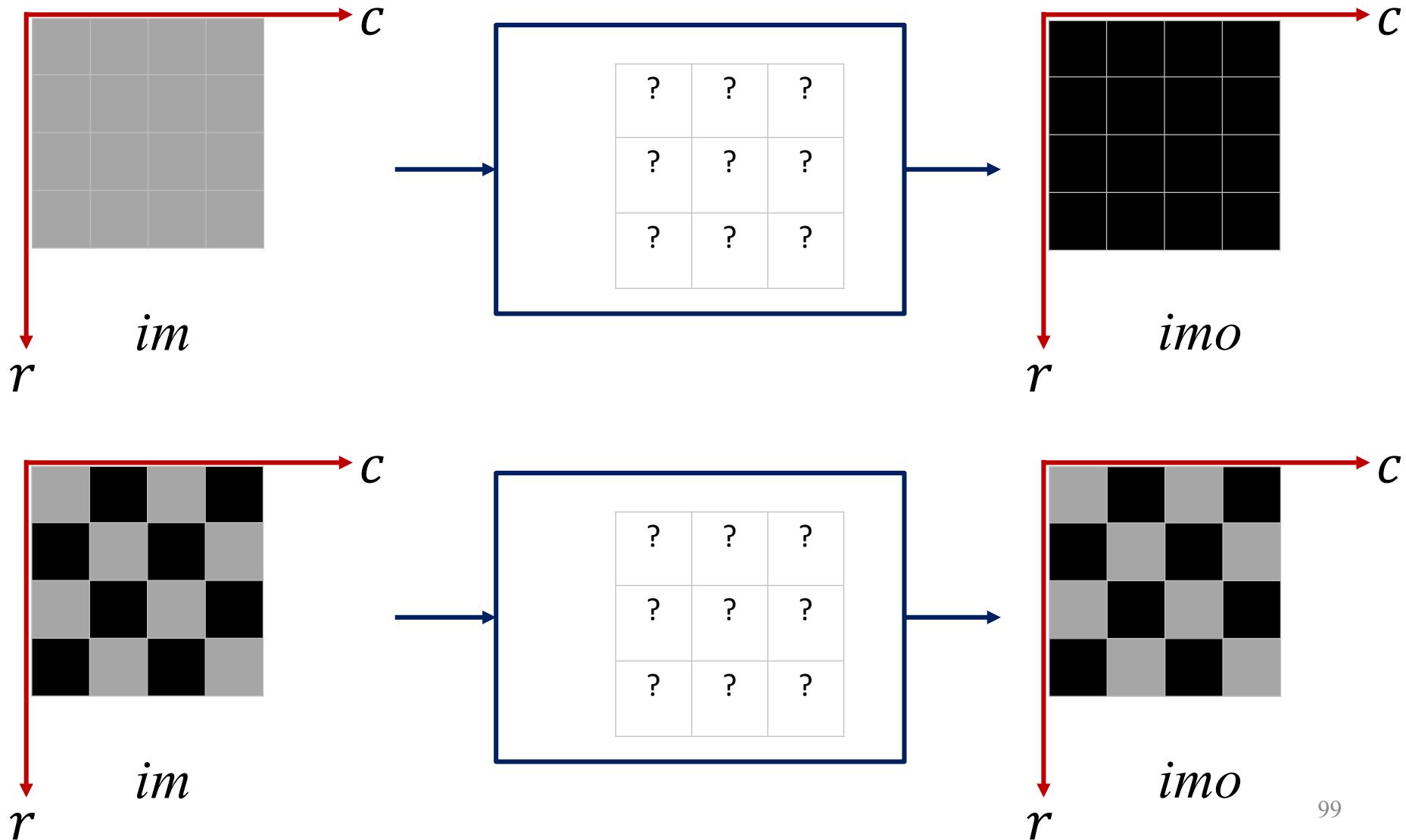
High pass filters



High pass filters



High pass filters



High pass filters

- High pass filters let the high frequency content of the image pass through the filter and block the low frequency content, so:
 - “Texture” information **get removed** so the results are **darker** images.
 - **Edges get emphasized.**

Practical filters: Averaging filter

Why 1/9 ?

Averaging filter

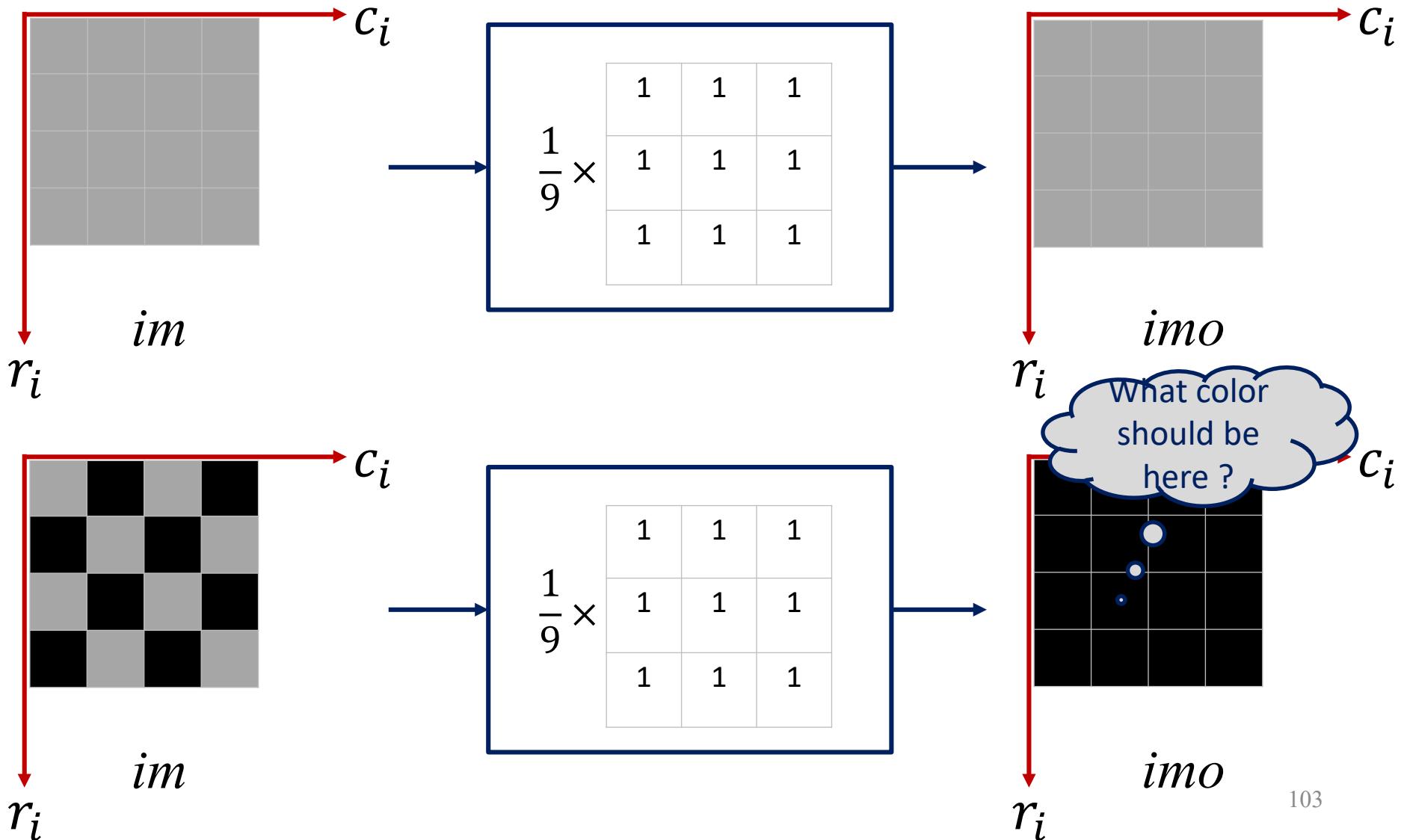
- The 3×3 averaging filter:

- $\frac{1}{9} \times$

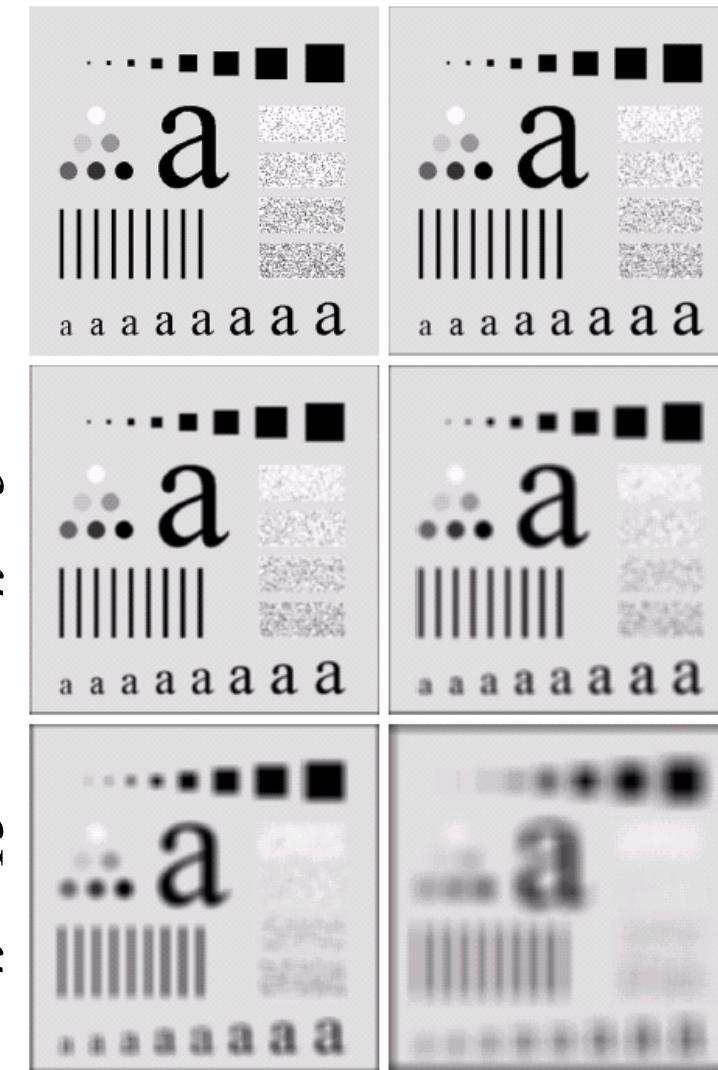
1	1	1
1	1	1
1	1	1

- This filter **is 45° rotation isotropic** (isotropic filter response is independent of the direction to which the filter is applied).
- It is a LPF

Averaging filter



Averaging filter



a
b
c
d
e
f

FIGURE 3.35 (a) Original image, of size 500×500 pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes $n = 3, 5, 9, 15, 25, 35$, and 55 , respectively. The black squares at the top are of sizes $3, 5, 9, 15, 25, 35, 45$, and 55 pixels, respectively; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their gray levels range from 0% to 100% black in increments of 20%. The background of the image is 10% black. The noisy rectangles are of size 50×120 pixels.

$n = 3$

$n = 9$

$n = 35$

Averaging filters
are a kind of
smoothing filters

- The **size of the mask** is related to the relative **size** of the **objects** that will be **blended with the background**.
- **Black border** is due to **padding** (expanding) the border of the original image with **zeroes**.

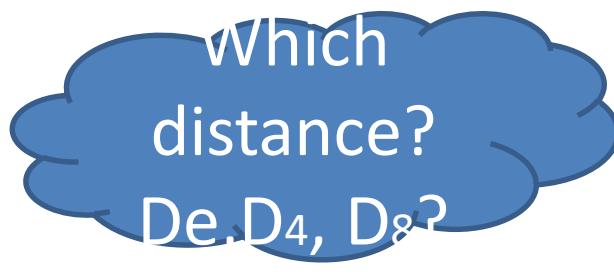
Smoothing filters

- Smoothing filters are **used for :**
 - Noise reduction
 - **Reduction of irrelevant detail** or false contours
 - Sometime for **image blurring**
- Undesirable side effect of smoothing filters
 - **Blur edges** (if blurring is not the main target)

Weighted average filters

- An example of 3×3 weighted average filter:

$$\frac{1}{16} \times \begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array}$$



- The **weights** aim to give more **importance** to some pixels at the expense of others.
 - In this example the **importance reduces** with the **distance** from the center of the filter.

Weighted average filters

- Due to the **uneven weights** the weighted average filter **reduces blurring** in the smoothing process.
- Normalization
 - To **avoid changing** the overall **brightness** due to the use of LPF the entries of the filter must sum to one. So in the previous example **1/16** is a normalization factor.

Matlab: Spatial-Filtering

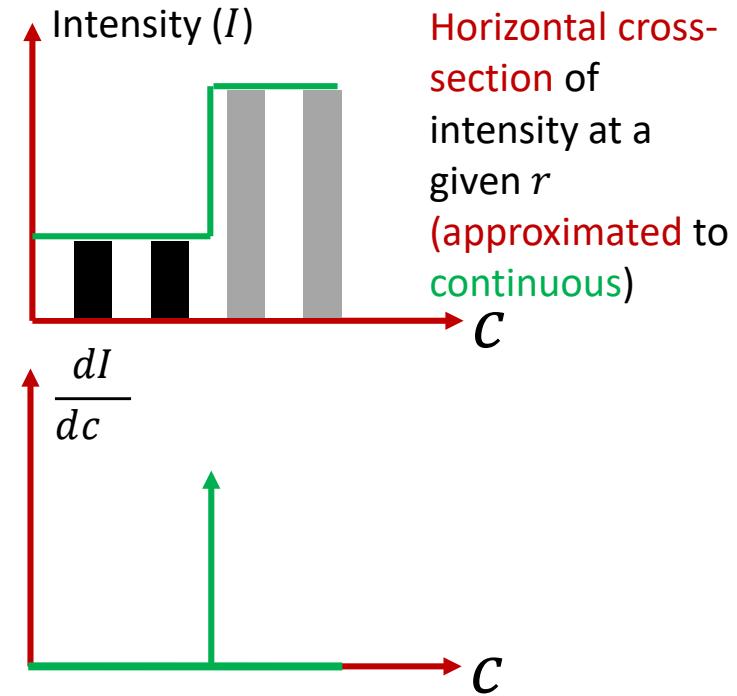
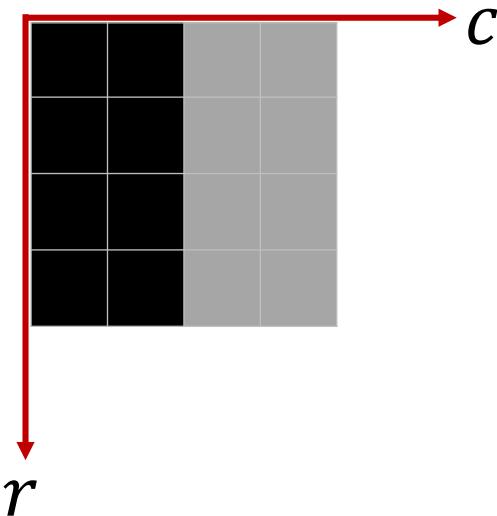
- Example from MATLAB



Practical filters: edge detection filters

Edge Detection Filters

- How to detect edges ?



- First derivative (**gradient**: $\frac{\partial f}{\partial x}$) allows to **detect** changes of continuous functions

Edge Detection Filters

- In the **discrete domain** the **derivative** could be approximated as :

$$\begin{aligned}\frac{\partial f}{\partial x} &\approx f(x) - f(x-1) \\ &\approx f(x+1) - f(x)\end{aligned}$$

- These are **fast to compute**, but the result can only be an **approximation** to the true gradient, and is usually an underestimate.

Edge Detection Filters

- Sobel operators :

Detect horizontal edges

-1	-2	-1
0	0	0
1	2	1

Detect vertical edges

-1	0	1
-2	0	2
-1	0	1

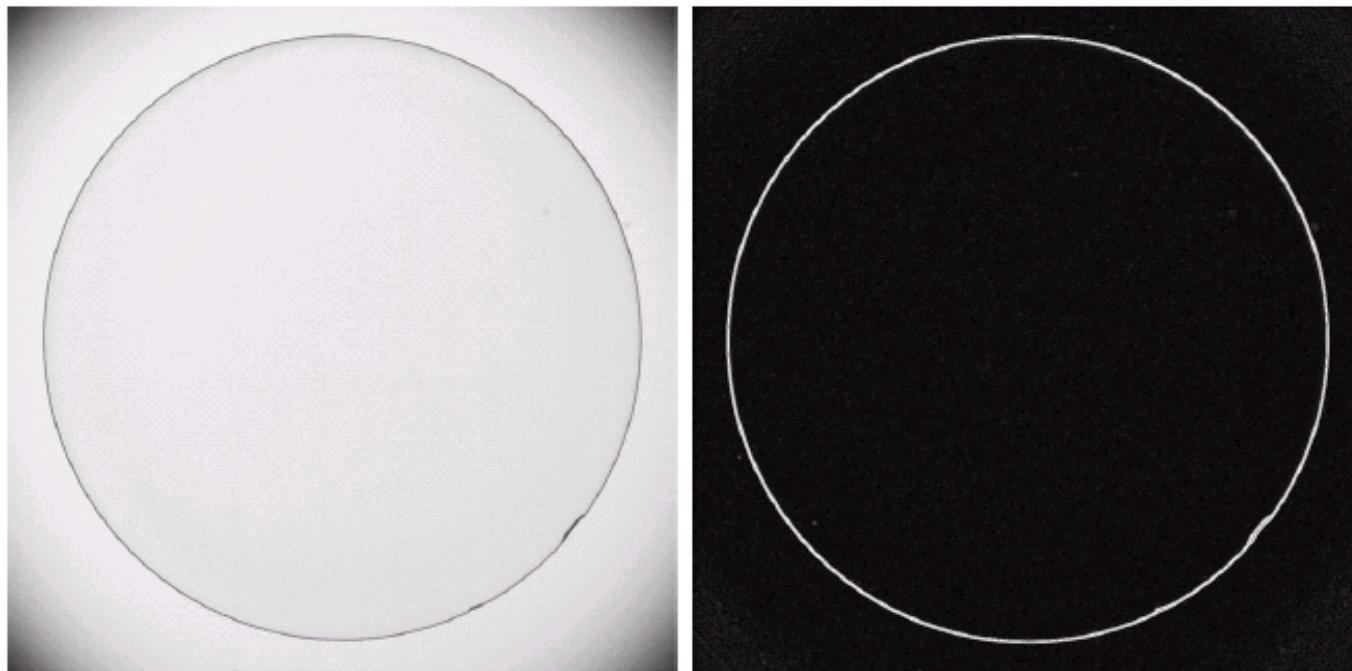
- The center point has more importance.
- Filter's coefficients sum to zero, indicating that a constant gray area would have zero response. Is it LPF or HPF filter ?

Edge Detection Filters

- Step 1: **Convolution** with gradient (Sobel) filter
 - Separately for horizontal and vertical **directions**
- Step 2: **Magnitude** of gradient
 - Norm of horizontal and vertical gradients
- Step 3: **Thresholding**
 - Threshold to detect edges

Edge Detection Filters

- Sobel operators :



a b

FIGURE 3.45
Optical image of contact lens (note defects on the boundary at 4 and 5 o'clock).
(b) Sobel gradient.
(Original image courtesy of Mr. Pete Sites, Perceptics Corporation.)

The horizontal and vertical edges are summed (ignoring sign) to give the result

Matlab: Edge Detection Filters

- Edge Detection in MATLAB

“All” edges

Horizontal edges



Vertical edges



Vertical edges

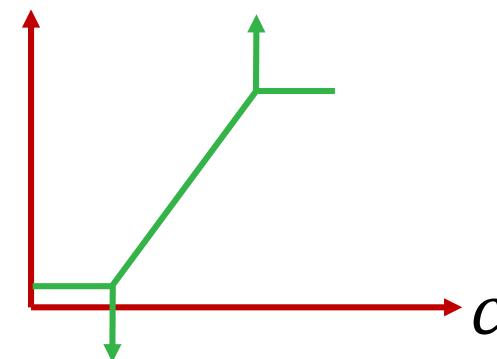
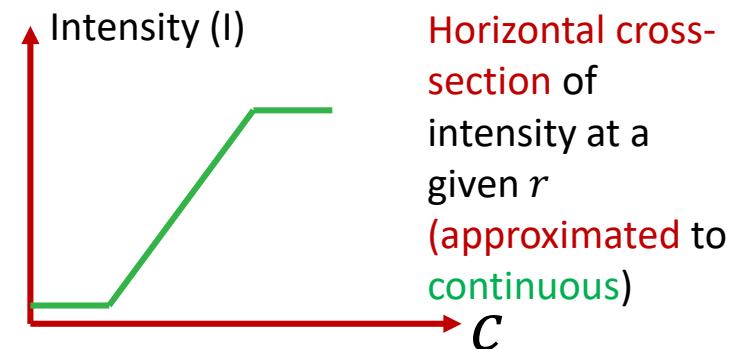
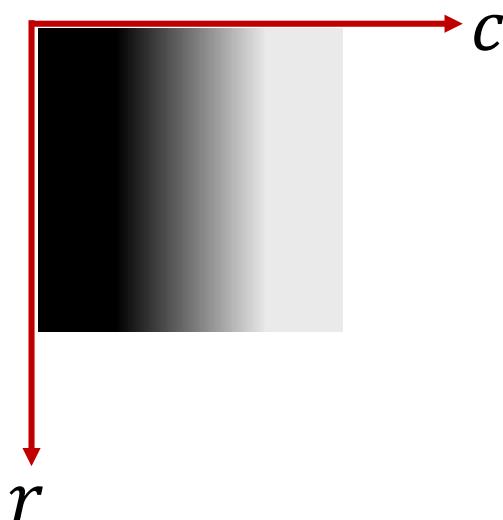


Practical filters: sharpening filters

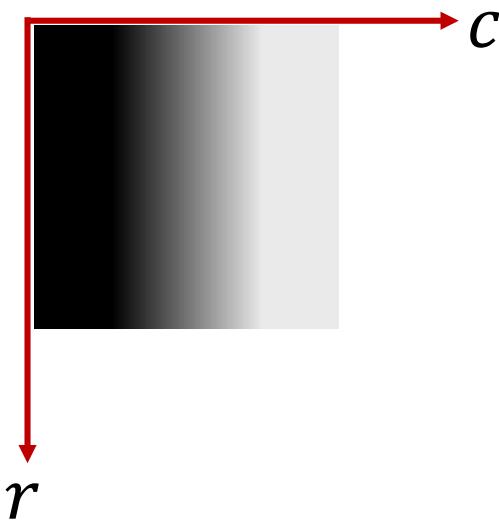
Sharpening Filters

- The principal **objective** of sharpening is to :
 - **highlight** fine **detail** in an image and/or to
 - enhance blurred details
- Unlike blur when **sharpening** an image we **want** to **accentuate** **high** frequencies
- How to sharpen an image ?

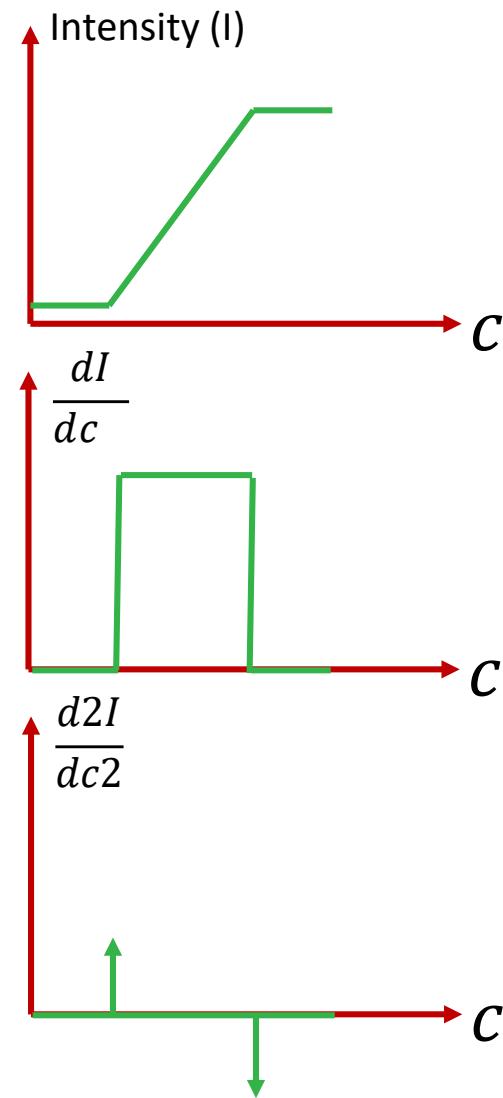
Sharpening Filters



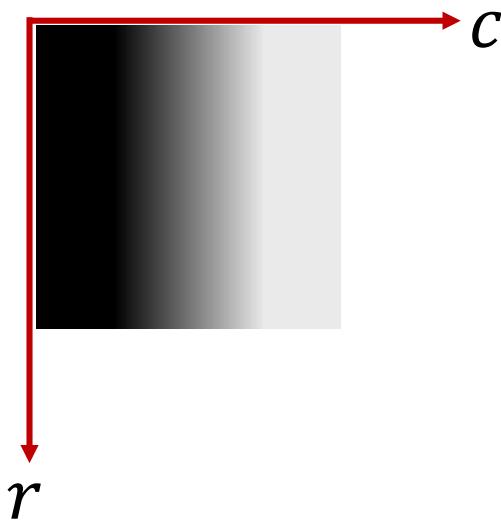
Sharpening Filters



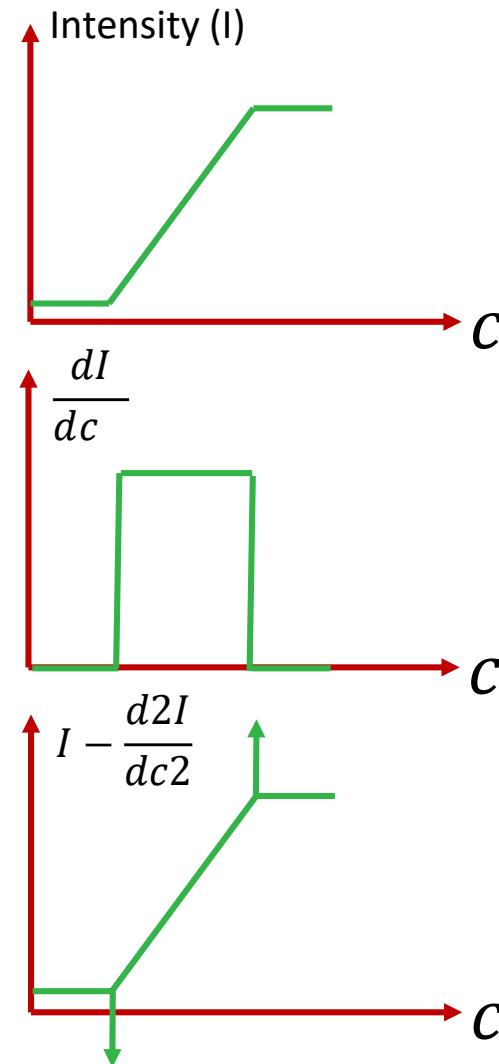
Can the second derivative enhance
sharpness ?



Sharpening Filters



The second derivative allows to enhance sharpness



Sharpening Filters

- First derivative

$$\begin{aligned}\frac{\partial f}{\partial x} &\approx f(x) - f(x-1) \\ &\approx f(x+1) - f(x)\end{aligned}$$

- Second derivative

$$\begin{aligned}\frac{\partial^2 f}{\partial x^2} &\approx \frac{\partial f}{\partial x} (f(x) - f(x-1)) \\ &\approx (f(x+1) - f(x)) - (f(x) - f(x-1)) \\ &\approx f(x+1) - 2f(x) + f(x-1)\end{aligned}$$

Sharpening Filter

Second derivative in discrete-domain:

$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

Two dimensional Laplacian:

$$\begin{aligned}\nabla^2 f = & f(x, y+1) + f(x, y-1) \\ & + f(x+1, y) + f(x-1, y) - 4f(x, y)\end{aligned}$$

Sharpening Filters

- 90° rotation isotropic second derivative filter

0	1	0
1	-4	1
0	1	0

- 45° rotation isotropic second derivative filter

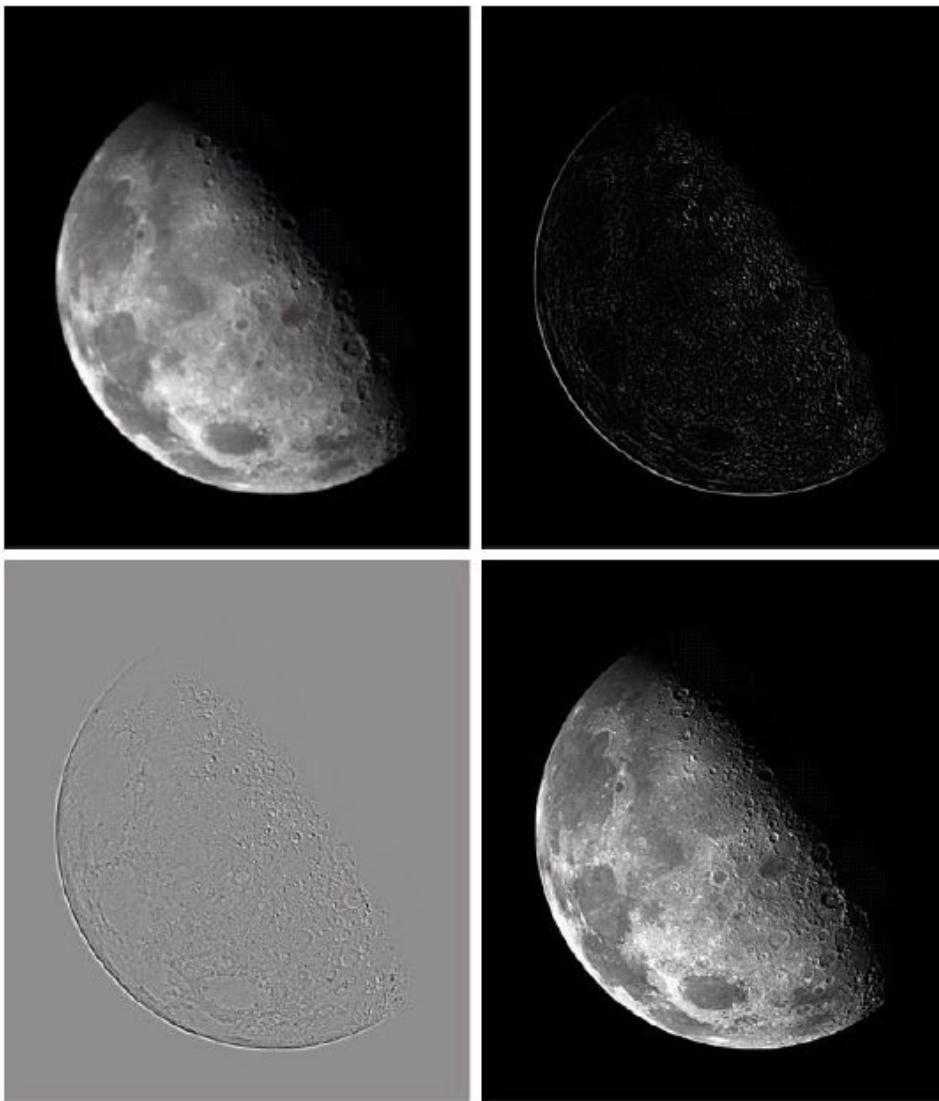
1	1	1
1	-8	1
1	1	1

Sharpening Filters

a
b
c
d

FIGURE 3.40

- (a) Image of the North Pole of the moon.
(b) Laplacian-filtered image.
(c) Laplacian image scaled for display purposes.
(d) Image enhanced by using Eq. (3.7-5).
(Original image courtesy of NASA.)



- **Laplacian sharpening**

$$g(x, y) = f(x, y) - \nabla^2 f(x, y)$$

- **Simplification:**

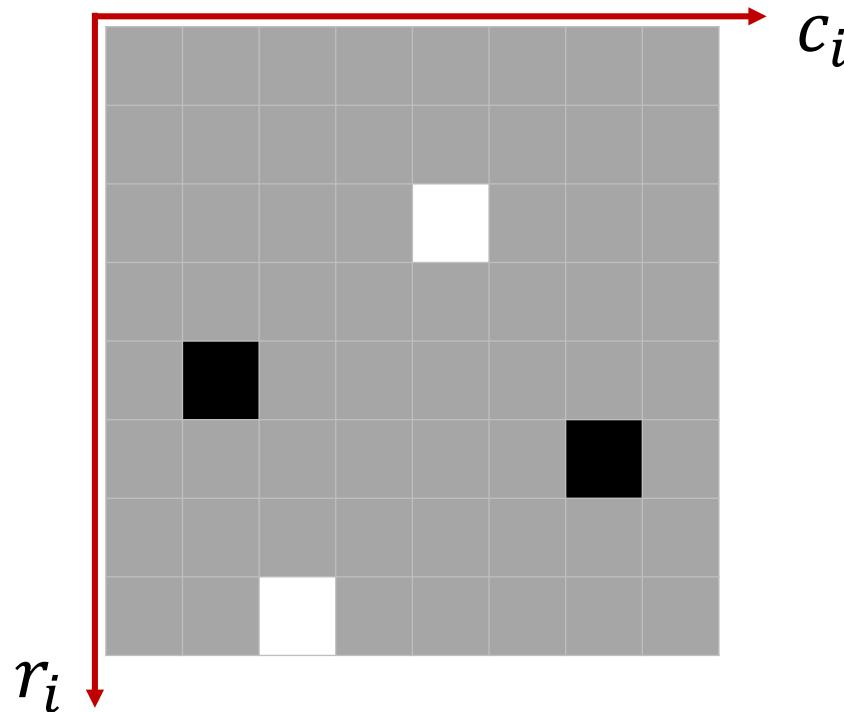
0	-1	0
-1	5	-1
0	-1	0

Sharpening Filters

- `originalRGB = imread('lenna512.bmp');`
- `h = fspecial('average', [3 3]);`
- `filteredRGB = imfilter(originalRGB, h, 'replicate');`
- `figure, imshow(originalRGB) ;`
- `figure, imshow(filteredRGB) ;`
- `w = [0 1 0`
- `1 -4 1`
- `0 1 0`
- `];`
- `lap_image = imfilter(filteredRGB, w, 'replicate');`
- `final_image = originalRGB - lap_image;`
- `figure, imshow(final_image) ;`

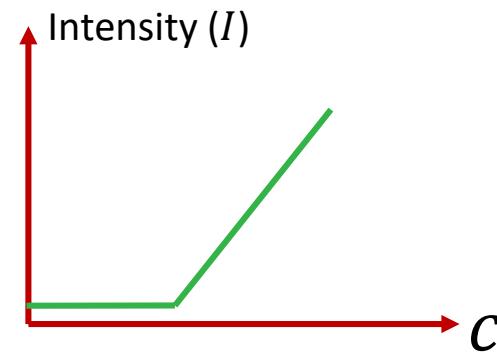
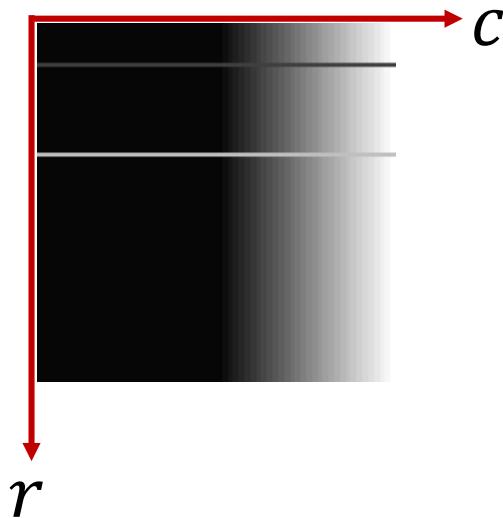
Problem: Salt & Paper noise

- How to remove S&P noise ?
- Is averaging (LPF) filter good for this kind of noise ?



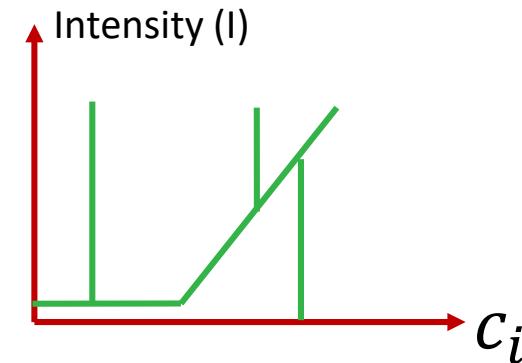
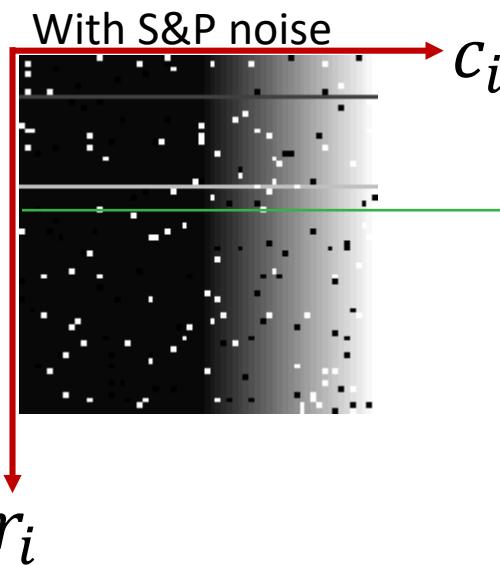
Case study: Salt & Paper noise

- Original image



Case study: Salt & Paper noise

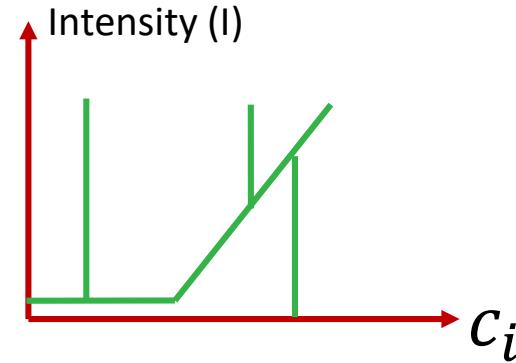
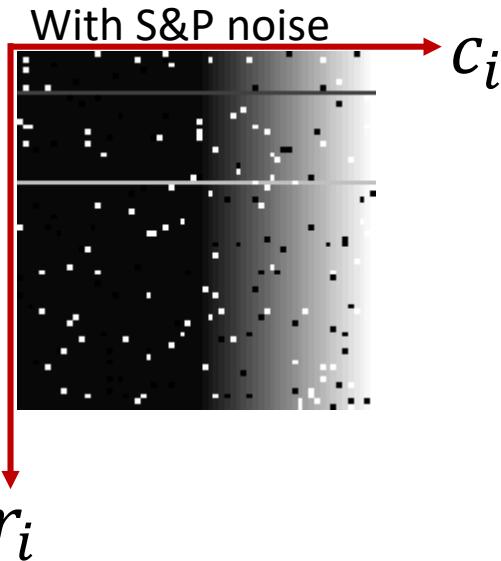
- Salt & Paper noisy image



- Sometime we call it **impulse/spike** noise

Case study: Salt & Paper noise

- Salt & Paper noisy image



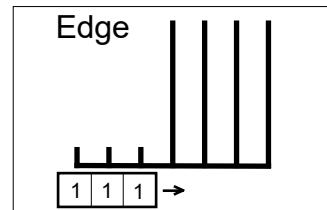
- How to remove S&P noise ?
- Is **averaging** (LPF) filter good to handle this kind of noise ?

Case study: Salt & Paper noise

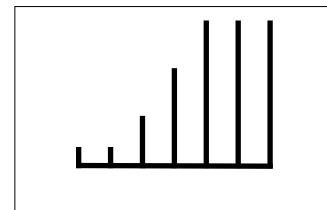
- Observations:
 - The **intensity** of natural images is normally **smooth**, without sudden and sharp changes → locally data is “**in order**”.
 - **S&P noise** (sometime related to “**defective pixel**”) appear as **sudden and sharp change** of intensity, as if it “**out-of-order**”.
 - To remove S&P noise we could use **order-statistics filters**.

Low Pass and Median Filters

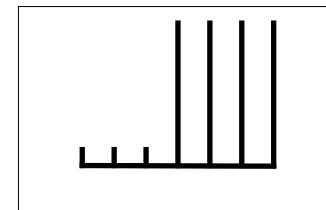
- The low-pass filter can provide image smoothing and noise reduction, but subdues and blurs sharp edges.
- Median filters can provide noise filtering without blurring.



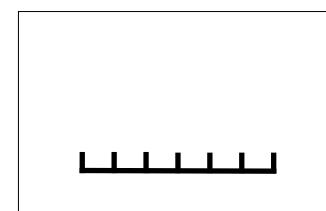
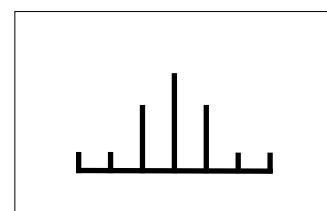
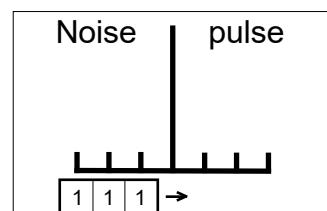
Original image



Averaging filter



Median filter

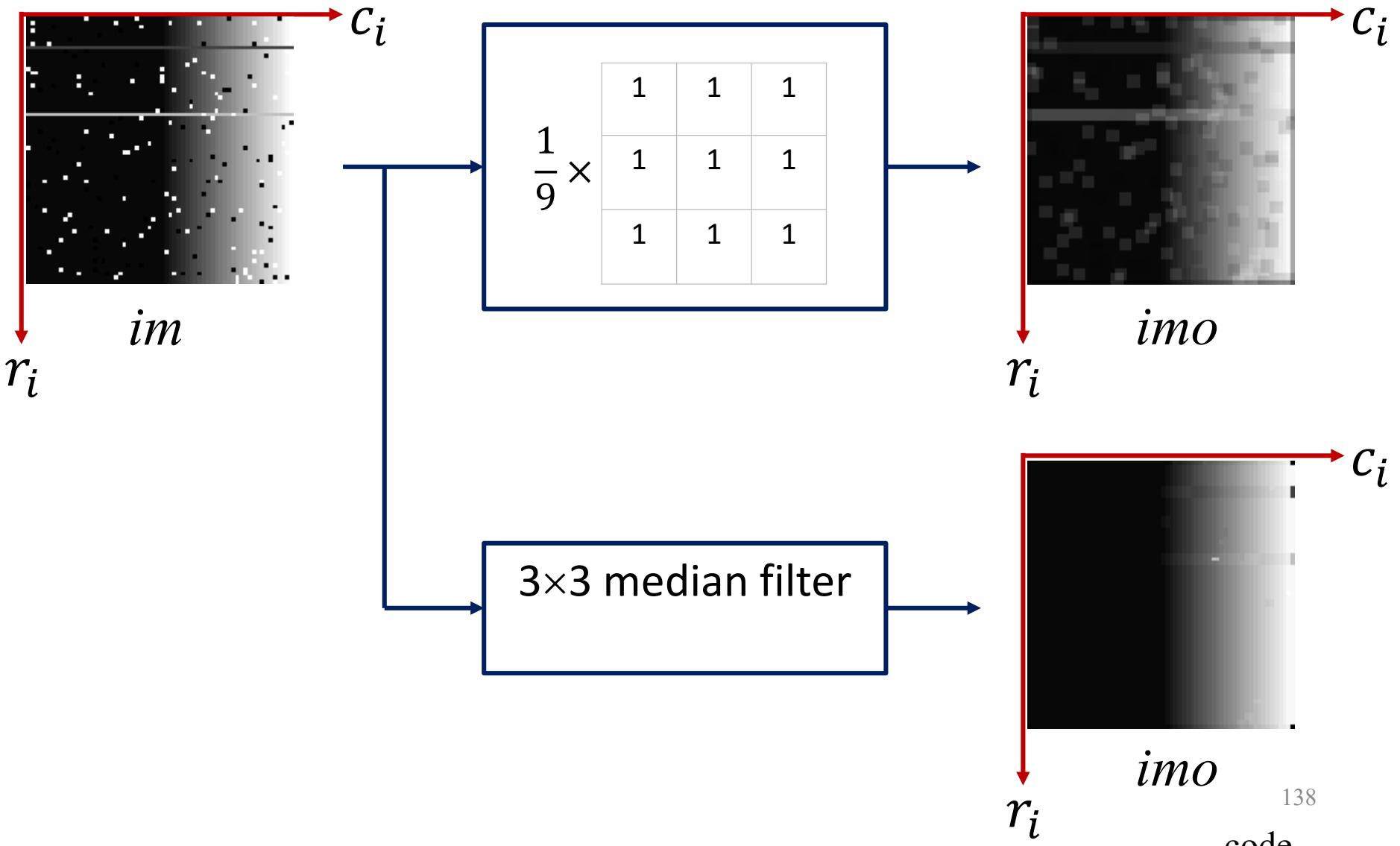


Order Statistic Filters

- Are **nonlinear** spatial filters.
- The median filter **replaces** the pixel at the **center** of the filter with the **median** value of the pixels falling beneath the mask, to do this it **orders** (ranks) the pixels of the sub-image and then **replaces** the value of the **center** pixel with the value determined by the **ranking result**.
- The median of a set of k pixel grey-levels is that grey level for which $k/2$ are smaller or equal, and $k/2$ are larger or equal.
- Note that the choice of neighborhood over which the median is computed has a significant influence over the filter's effect on edges, corners and other image discontinuities.

Order Statistic Filters

With 5% S&P noise



Order Statistic Filters

- Median filter does **not blur** the image but it **rounds the corners**.
- A median filter is **more effective** than convolution when the goal is to simultaneously **reduce noise** and **preserve edges**.

Order Statistic Filters

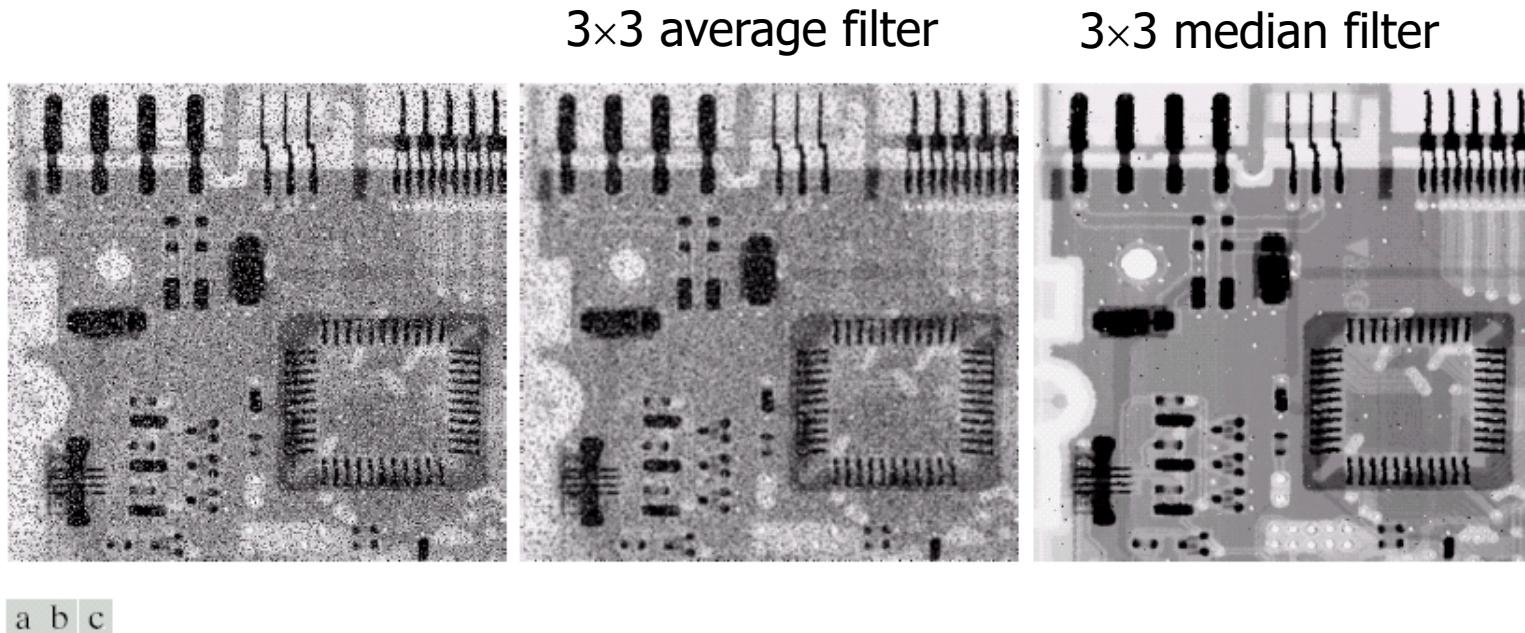


FIGURE 3.37 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3×3 averaging mask. (c) Noise reduction with a 3×3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

Order Statistic Filters

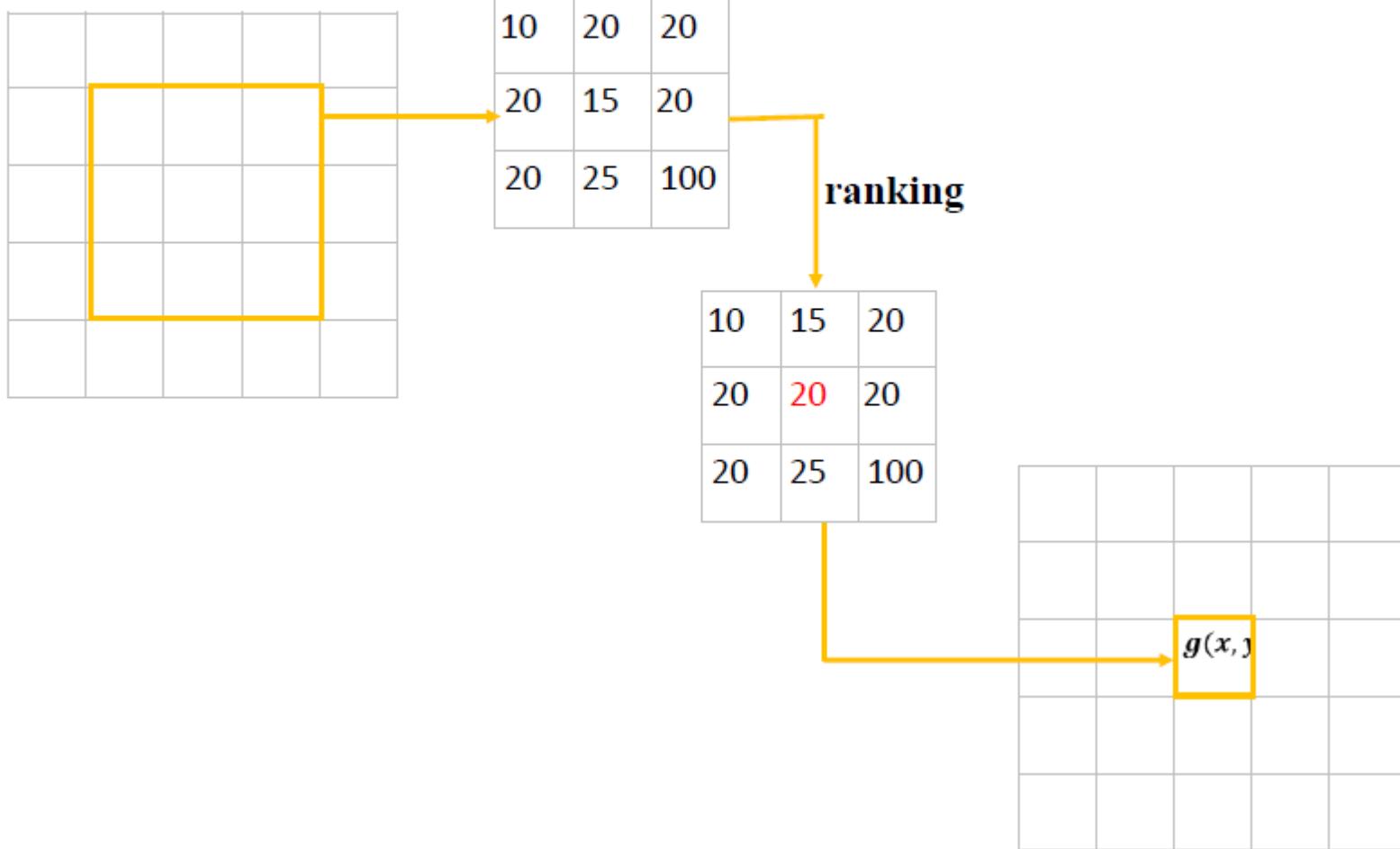
- Median filter eliminates isolated clusters of pixels that are light or dark with respect to their neighbors, and whose area is less than $n^2/2$.

3×3 Median filter [10 125 125 135 141 141 144 230
240] = 141

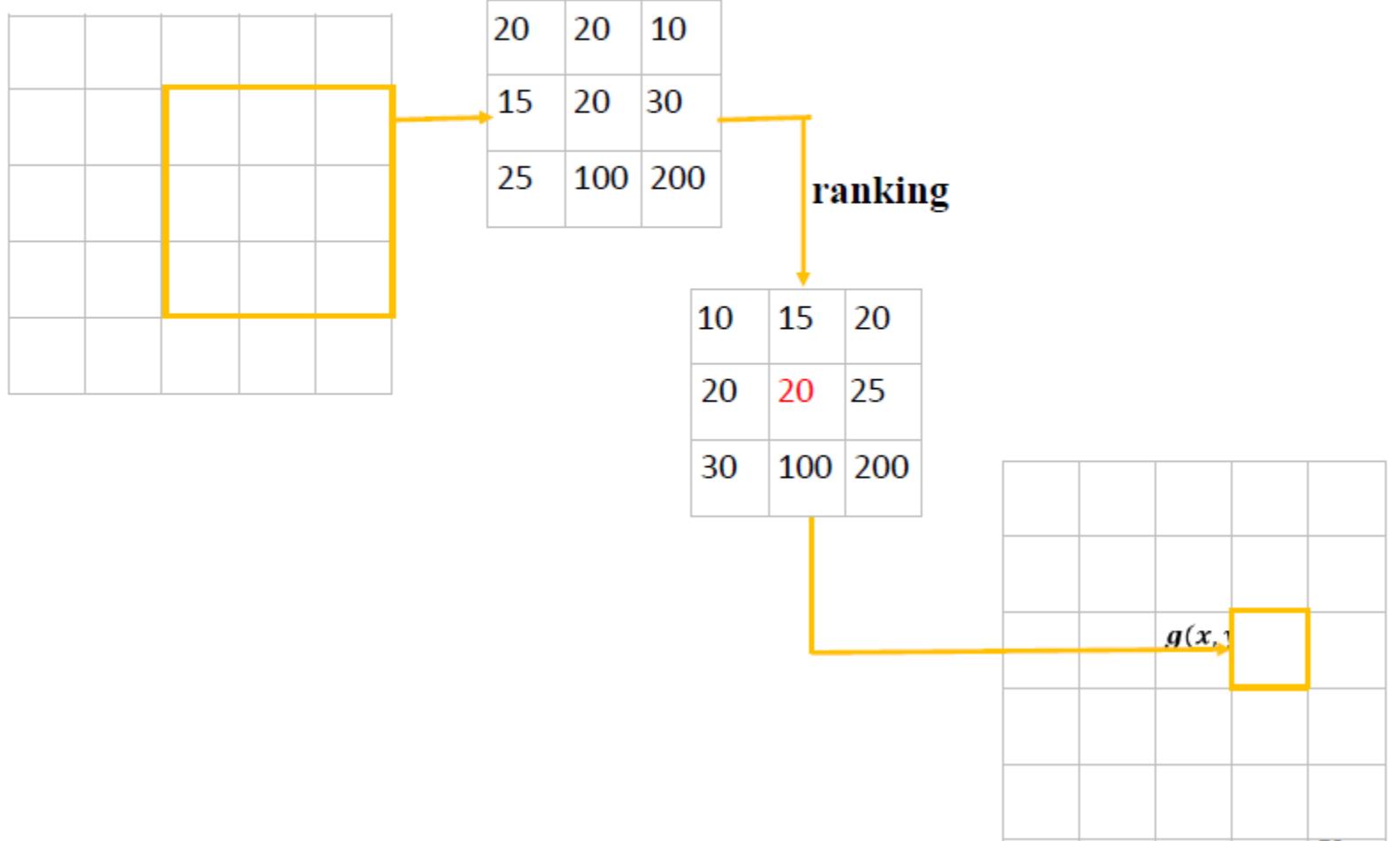
3×3 Max filter [10 125 125 135 141 141 144 230
240] = 240

3×3 Min filter [10 125 125 135 141 141 144 230
240] = 10

Order-Statistics filter (median filter)



Order-Statistics filter (median filter)



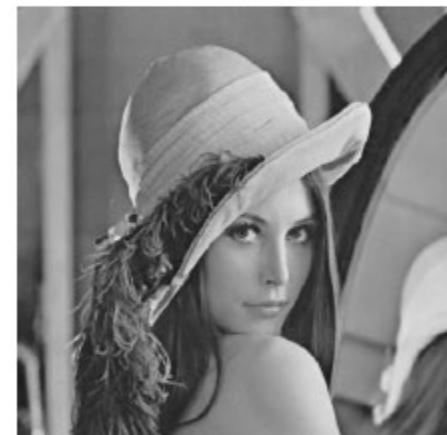
Order-Statistics filter (median filter)



original



salt & pepper



filtered image

In Summary

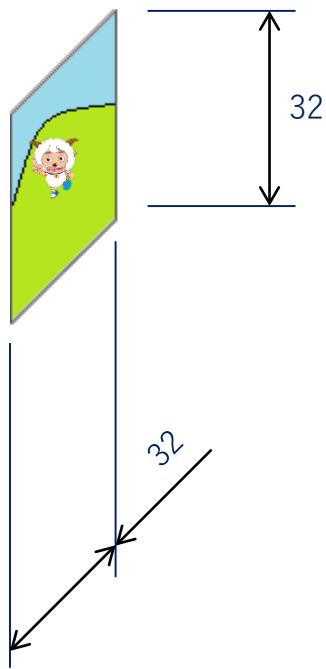
- Fixed parameters in the spatial filters
- Fixed rules in the order-statistics filters
- How to make it flexible? Adaptable?
 - Learning these parameters
 - CNN

Self-Reading

Convolutional Neural Networks (CNN)

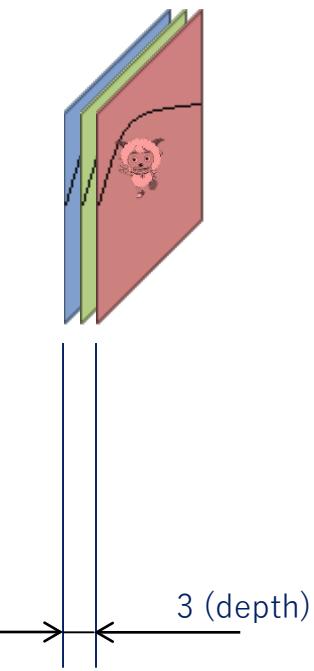
Convolutional Neural Networks

- Convolution layer



Convolutional Neural Networks

- Convolution layer



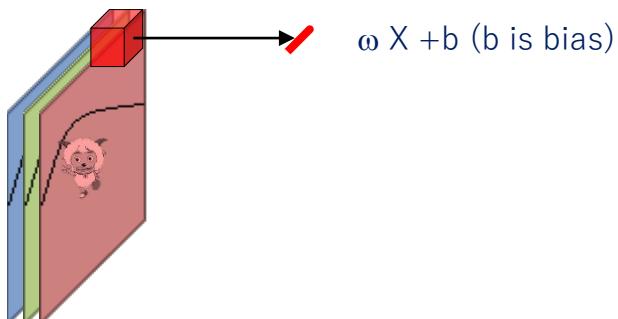
Convolutional Neural Networks

- Convolve the filter with the image



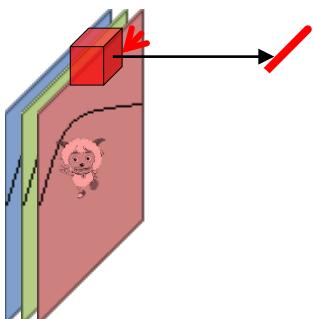
Convolutional Neural Networks

- Convolve the filter with the image



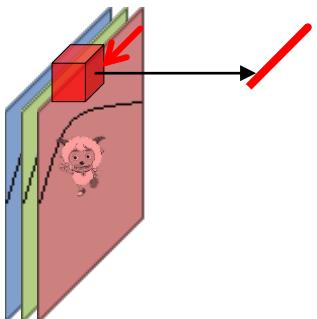
Convolutional Neural Networks

- Convolve the filter with the image



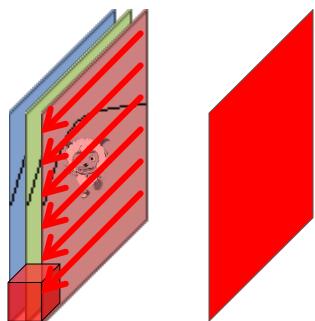
Convolutional Neural Networks

- Convolve the filter with the image



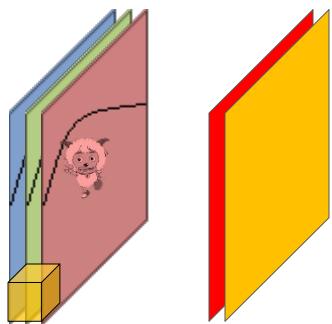
Convolutional Neural Networks

- Generate the first feature map in layer 1



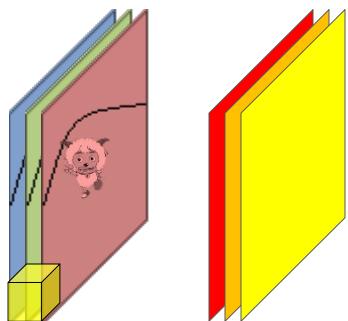
Convolutional Neural Networks

- Generate the second feature map in layer 1



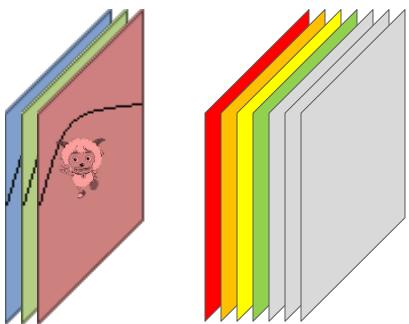
Convolutional Neural Networks

- Generate the third feature map in layer 1



Convolutional Neural Networks

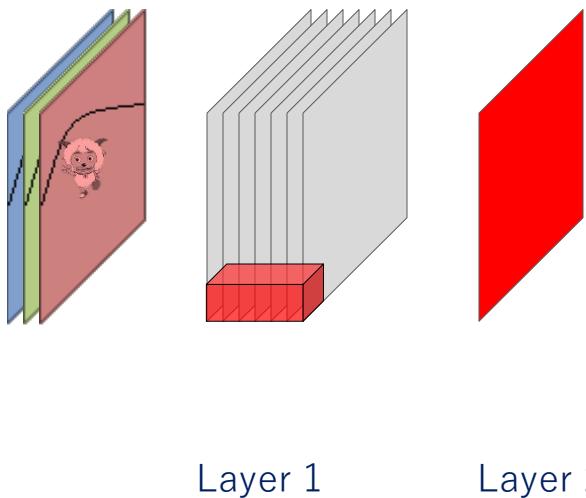
- Generating the first layer



Layer 1 : the features are stacked together to get a “new image” in this example it is an “image” with a depth of 6

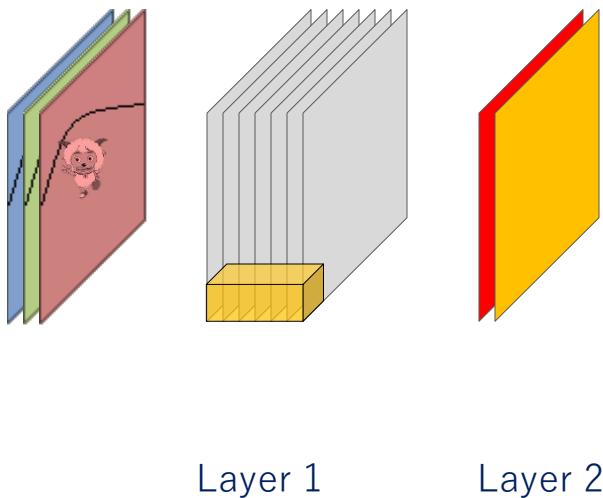
Convolutional Neural Networks

- Generate the first feature map in layer 2



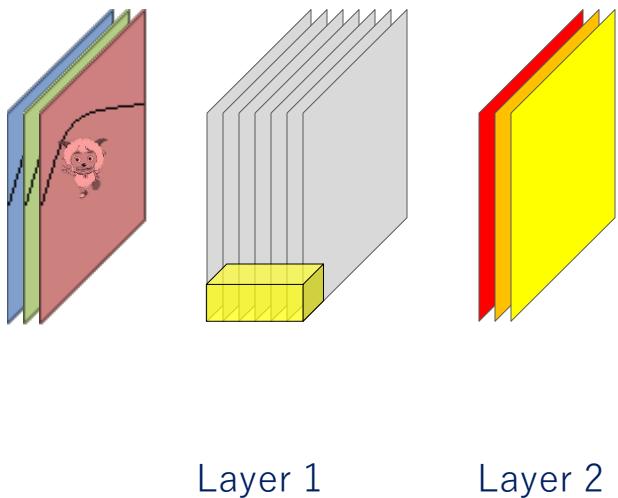
Convolutional Neural Networks

- Generate the second feature map in layer 2



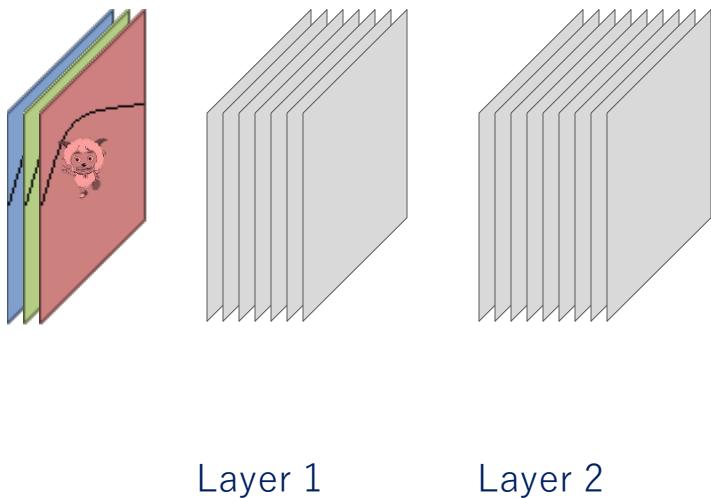
Convolutional Neural Networks

- Generate the third feature map in layer 2



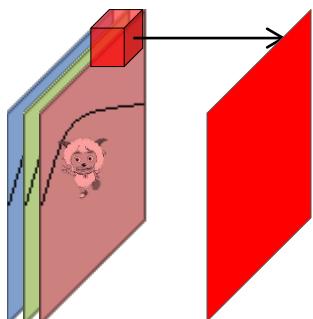
Convolutional Neural Networks

- Generating the second layer



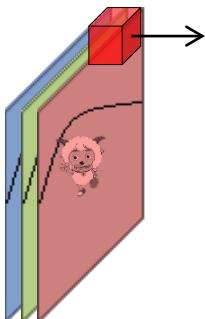
Convolutional Neural Networks

- Activation functions :



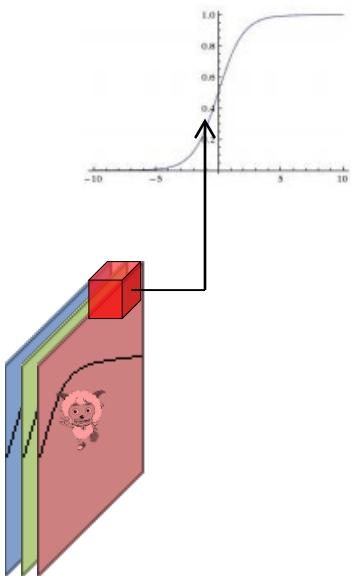
Convolutional Neural Networks

- Activation functions :
 - at the output of the convolution we might have a non-linear mapping function



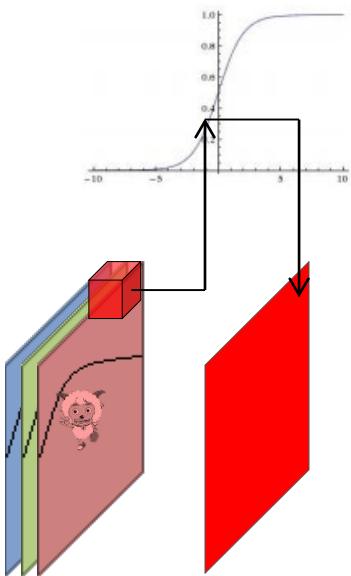
Convolutional Neural Networks

- Activation functions :



Convolutional Neural Networks

- Activation functions :

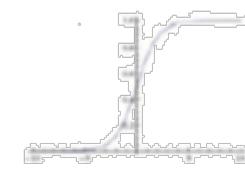


Convolutional Neural Networks

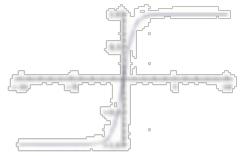
- Activation functions :

Sigmoid

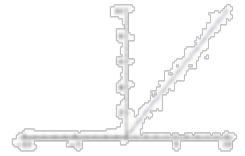
$$\sigma(x) = 1/(1 + e^{-x})$$



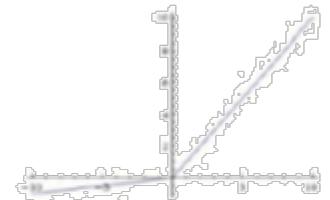
tanh $\tanh(x)$



ReLU $\max(0, x)$



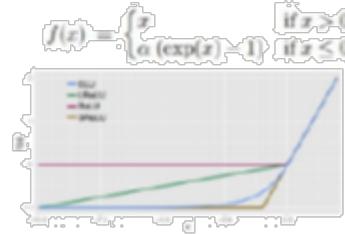
Leaky ReLU
 $\max(0.1x, x)$



Maxout

$$\max(w_1^T x + b_1, w_2^T x + b_2)$$

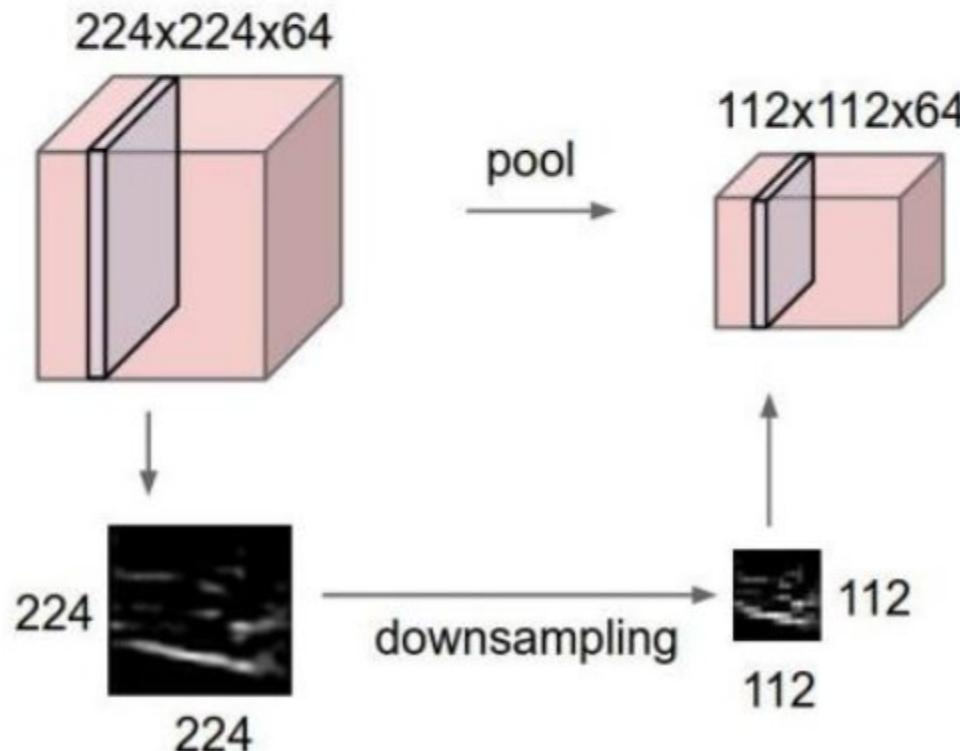
ELU



Convolutional Neural Networks

Pooling layer

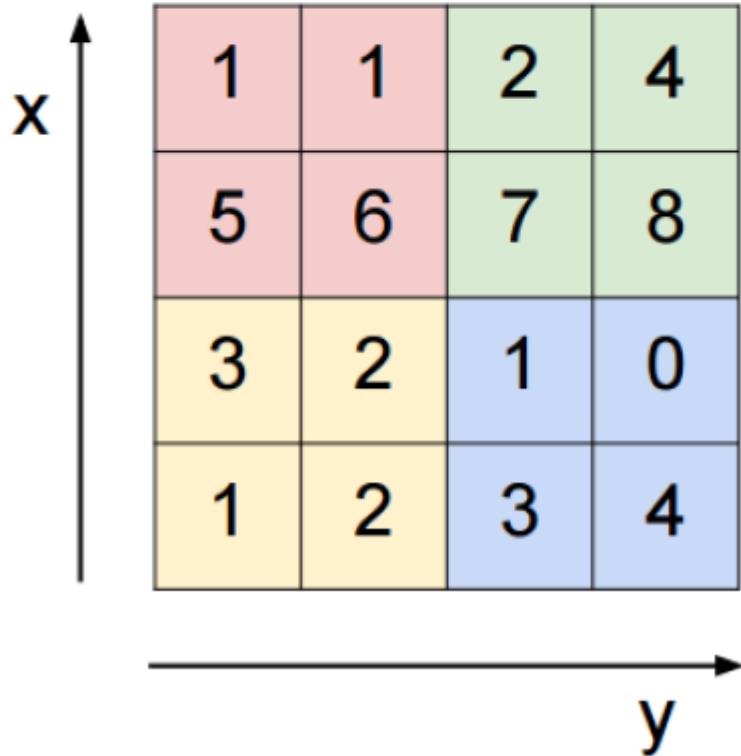
- makes the representations smaller and more manageable
- operates over each activation map independently:



Convolutional Neural Networks

MAX POOLING

Single depth slice



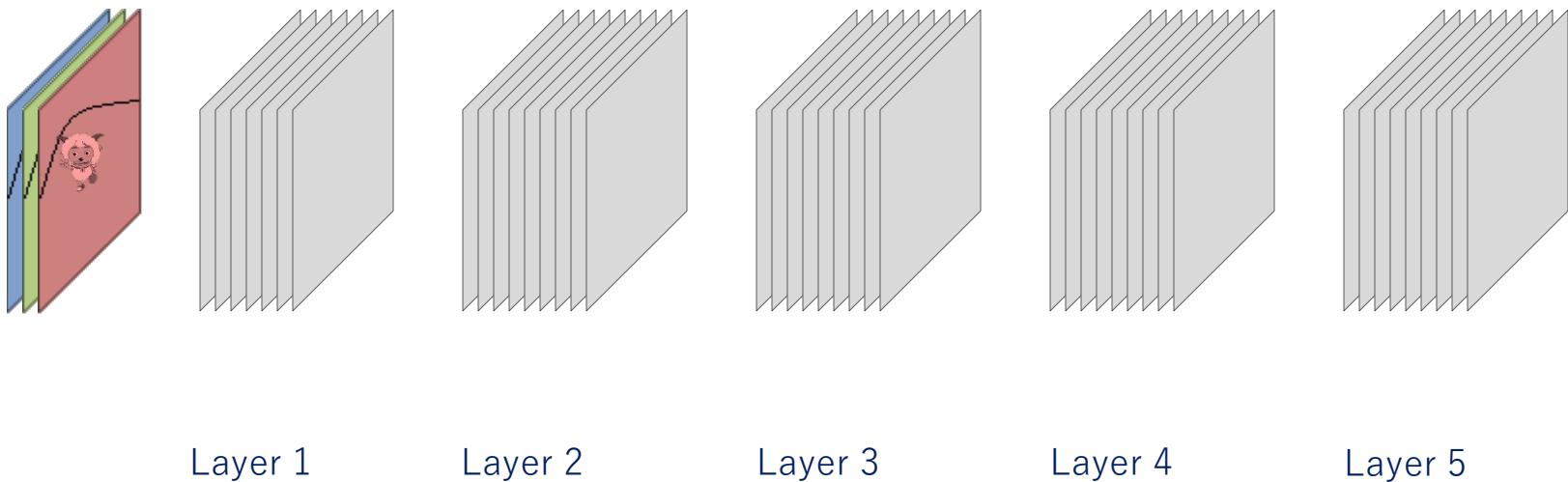
max pool with 2x2 filters
and stride 2



6	8
3	4

Convolutional Neural Networks

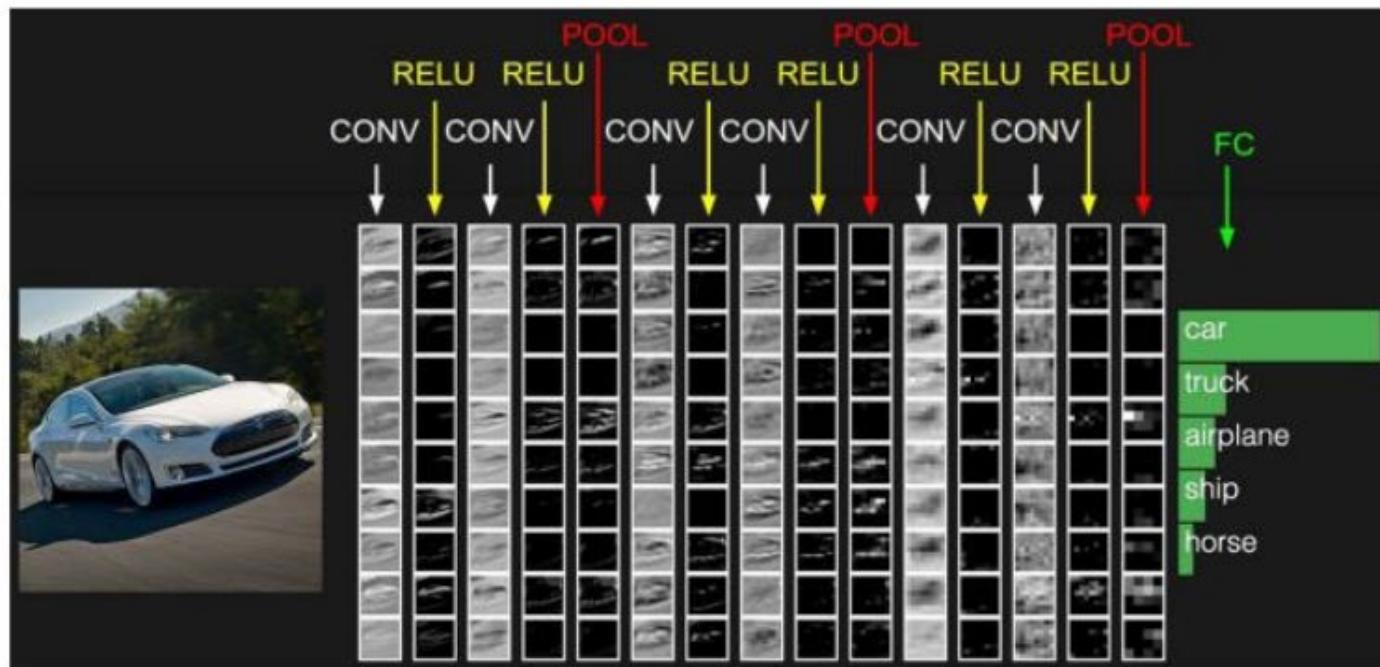
- ConvNet : is a sequence of convolution operations followed by activation functions



Convolutional Neural Networks

Fully Connected Layer (FC layer)

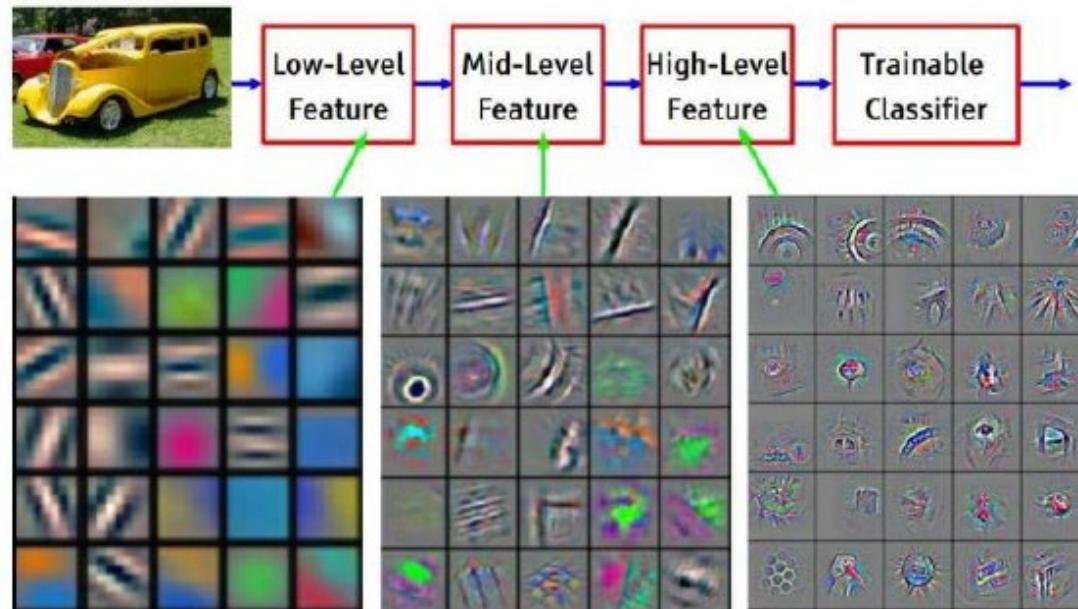
- Contains neurons that connect to the entire input volume, as in ordinary Neural Networks



Convolutional Neural Networks

Preview

[From recent Yann LeCun slides]

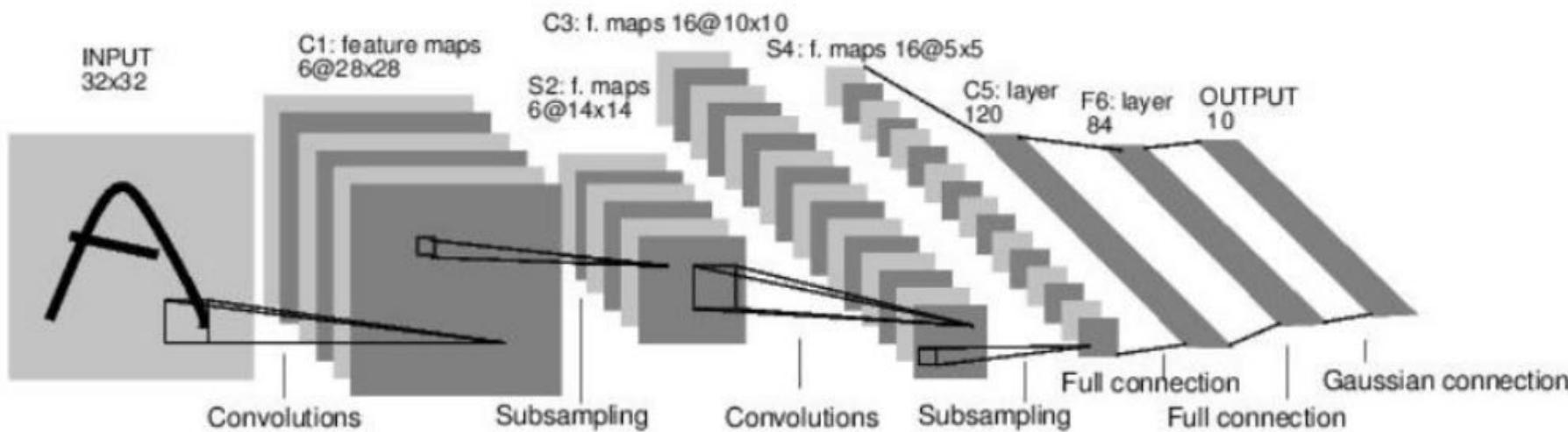


Feature visualization of convolutional net trained on ImageNet from [Zeiler & Fergus 2013]

Convolutional Neural Networks

Case Study: LeNet-5

[LeCun et al., 1998]

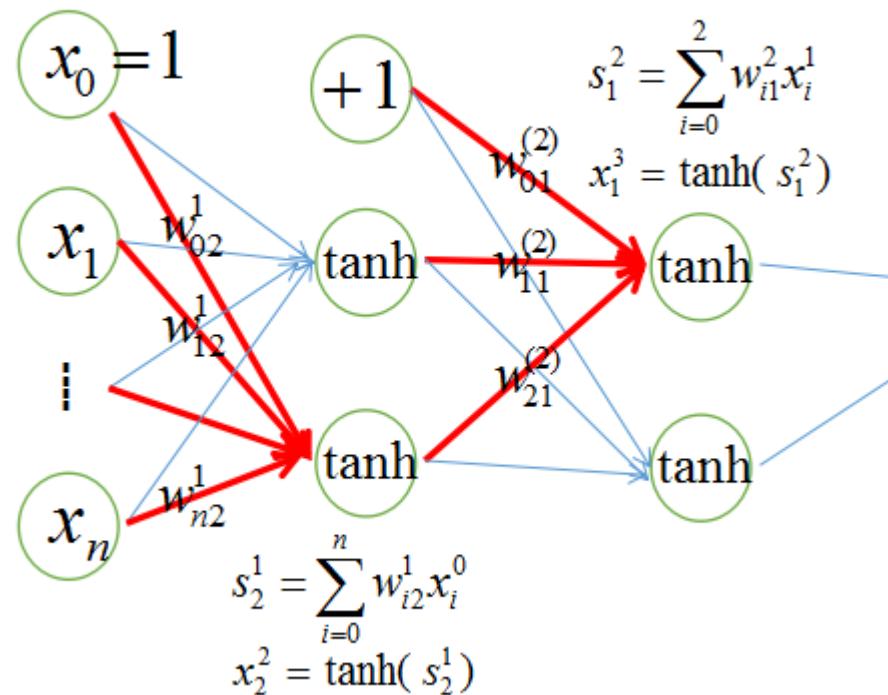


Conv filters were 5×5 , applied at stride 1

Subsampling (Pooling) layers were 2×2 applied at stride 2
i.e. architecture is [CONV-POOL-CONV-POOL-CONV-FC]

Neural Networks

- Multilayer Perceptron (MLP)



LeNet-5

- C1: Conv
 - Input : $32*32$
 - Kernel: $5*5$
 - Kernel number: 6
 - Stride: 1
 - NewSize=(OldSize-KernelSize)/Stride+1
 - Parameters: $(5*5+1)*6$
 - Connections: $(5*5+1)*6*28*28$

LeNet-5

- S2: Down-sampling
 - Input : $28*28$
 - Sampling size: $2*2$
 - Sampling method: $\text{sum} * a + b$
 - Stride: 2
 - NewSize= $(\text{OldSize}-\text{KernelSize})/\text{Stride}+1$
 - Parameters: $2*6$
 - Connections: $(2*2+1)*6*14*14$

LeNet-5

- C3: Conv
 - Input :last layer
 - Kernel size: 5*5
 - Kernel number: 16
 - Parameters: $6 * (3 * 25 + 1) + 6 * (4 * 25 + 1) + 3 * (4 * 25 + 1) + (25 * 6 + 1) = 1516$
 - Connections: $10 * 10 * 1516$

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	X			X	X	X			X	X	X	X		X	X	
1	X	X			X	X	X			X	X	X	X		X	
2	X	X	X			X	X	X			X		X	X	X	
3		X	X	X		X	X	X	X		X		X	X	X	
4			X	X	X	X	X	X	X		X	X	X		X	
5				X	X	X			X	X	X	X		X	X	

TABLE I

EACH COLUMN INDICATES WHICH FEATURE MAP IN S2 ARE COMBINED

LeNet-5

- S4: Down-sampling
 - Parameters: $2 * 16$
 - Connections: $16 * (2 * 2 + 1) * 5 * 5$
- C5: Full Connect
 - Parameters: $120 * (16 * 5 * 5 + 1)$
 - Connections: $120 * (16 * 5 * 5 + 1)$
- F6: Full Connect
 - Parameters : $84 * (120 + 1)$
- Output
 - Gaussian
 - softmax

Convolutional Neural Networks

Case Study: AlexNet

[Krizhevsky et al. 2012]

Full (simplified) AlexNet architecture:

[227x227x3] INPUT

[55x55x96] CONV1: 96 11x11 filters at stride 4, pad 0

[27x27x96] MAX POOL1: 3x3 filters at stride 2

[27x27x96] NORM1: Normalization layer

[27x27x256] CONV2: 256 5x5 filters at stride 1, pad 2

[13x13x256] MAX POOL2: 3x3 filters at stride 2

[13x13x256] NORM2: Normalization layer

[13x13x384] CONV3: 384 3x3 filters at stride 1, pad 1

[13x13x384] CONV4: 384 3x3 filters at stride 1, pad 1

[13x13x256] CONV5: 256 3x3 filters at stride 1, pad 1

[6x6x256] MAX POOL3: 3x3 filters at stride 2

[4096] FC6: 4096 neurons

[4096] FC7: 4096 neurons

[1000] FC8: 1000 neurons (class scores)

