Multimedia Information Retrieval and Technology

Lecture 6. Weighting II

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- Ranked retrieval
  a. Term frequency, document freq, collection freq
  b. idf weighting
  c. tf-idf weighting
- The vector space model for scoring

RECAP:Term frequency tf

The term frequency $\mathrm{tf}_{t,d}$ of term *t* in document *d* is defined as the number of times that *t* occurs in *d*.

The log frequency weight of term t in d is

$$w_{t,d} = \begin{cases} 1 + \log_{10} \mathrm{tf}_{t,d}, & \text{if } \mathrm{tf}_{t,d} > 0 \\ 0, & \text{otherwise} \end{cases}$$

RECAP Document frequency

Document frequency df_t, defined to be the number of documents in the collection that contain a term *t*.

We define the idf (**inverse document frequency)** of term *t* by

$$\mathrm{idf}_t = \log_{10}(N/\mathrm{df}_t)$$

Exercise

The query "digital cameras"

The doc: "digital cameras and video cameras"

Assume the collection size is *N* = 10,000,000, Treat and as a stop word.

1) What is the raw tf for each term? The log frequency weight of each term?
2) Compute idf weight for each term.

	Doc1				Doc2	
	tf	wf	Df	idf	tf	wf
digital			10, 000			
video			100, 000			
cameras			50, 000			

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tf-idf weighting

We now combine the definitions of term frequency (tf) and inverse document frequency(idf), to produce a composite weight for each term in each document.

The tf-idf weight of a term is the product of its tf weight and its idf weight.

$$\text{tf-idf}_{t,d} = \text{tf}_{t,d} \times \text{idf}_t.$$

Best known weighting scheme in information retrieval

Note: the "-" in tf-idf is a hyphen, not a minus sign!

Alternative names: tf.idf, tf x idf

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tf-idf weighting

$$\text{tf-idf}_{t,d} = \text{tf}_{t,d} \times \text{idf}_t.$$

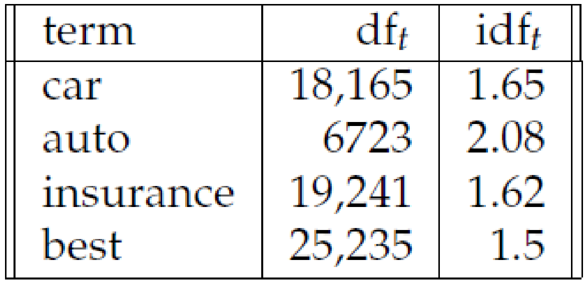
- Increases with the number of occurrences within a document **(term frequency component)**

- Increases with the rarity of document frequency of the term in the collection. **(idf component)**

Exercise

Consider the table of term frequencies for 3 documents denoted Doc1, Doc2, Doc3. Compute the tf-idf weights for the term **car** for each document, using log frequency weight and the idf values from table below.

term frequencies	Doc1	Doc2	Doc3
car	100	10	10
auto	1	0	0
insurance	0	10	100
best	10	10	10



term	df_t	idf_t
car	18,165	1.65
auto	6723	2.08
insurance	19,241	1.62
best	25,235	1.5

Score for a document with a given query

$$\text{Score}(q,d) = \sum_{t \in q \cap d} \text{tf.idf}_{t,d}$$

There are many variants

- How “tf” is computed (with/without logs)
- Whether the terms in the query are also weighted
- …

Exercise

Query: “best car insurance”

Consider the table of term frequencies for 3 documents denoted Doc1, Doc2, Doc3, Using log term frequency weight and the idf values from table below, calculate the score for Doc 1, Doc 2 and Doc 3 on this Query respectively.

	Doc1	Doc2	Doc3
car	100	10	10
auto	1	0	0
insurance	0	10	100
best	10	10	10

term	df_t	idf_t
car	18,165	1.65
auto	6723	2.08
insurance	19,241	1.62
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- Ranked retrieval
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RECAP:
Term-document incidence matrices

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	1	1	0	0	0	1
Brutus	1	1	0	1	0	0
Caesar	1	1	0	1	1	1
Calpurnia	0	1	0	0	0	0
Cleopatra	1	0	0	0	0	0
mercy	1	0	1	1	1	1
worser	1	0	1	1	1	0

1 if play contains word, 0 otherwise

RECAP: Term-document count matrices

Consider the number of occurrences of a term in a document:

Each document is a count vector : a column below

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	157	73	0	0	0	0
Brutus	4	157	0	1	0	0
Caesar	232	227	0	2	1	1
Calpurnia	0	10	0	0	0	0
Cleopatra	57	0	0	0	0	0
mercy	2	0	3	5	5	1
worser	2	0	1	1	1	0

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Binary → count → weight matrix

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	5.25	3.18	0	0	0	0.35
Brutus	1.21	6.1	0	1	0	0
Caesar	8.59	2.54	0	1.51	0.25	0
Calpurnia	0	1.54	0	0	0	0
Cleopatra	2.85	0	0	0	0	0
mercy	1.51	0	1.9	0.12	5.25	0.88
worser	1.37	0	0.11	4.15	0.25	1.95

Each document is now represented by a real-valued vector of tf-idf weights $\in \mathbb{R}^{|V|}$

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Documents as vectors

- So we have a |V|-dimensional real valued vector space!

Terms are axis of the space

- Documents are points or vectors in this space

Very high-dimensional: tens of millions of dimensions when you apply this to a web search engine.

- These are very sparse vectors - most entries are zero.

Vector space model

Vector space model: The representation of a set of documents as vectors in a common vector space.

Vector space model is fundamental to **a host of IR operations** including scoring documents on a query, document classification and document clustering.

Vector space model

We denote by $\vec{d}$ the vector derived from document d, with one component in the vector for each dictionary term.

The set of documents in a collection then may be viewed as a set of vectors in a vector space, in which there is one axis for each term.

Queries as vectors

A far more compelling reason to represent documents as vectors, we can also view a query as a vector.

Key idea 1: Do the same for queries: represent them as vectors in the space

Key idea 2: Rank documents according to their proximity to the query in this space

proximity = similarity of vectors

Queries as vectors

.

How:

rank more relevant documents higher than less relevant documents

Formalizing vector space proximity

First cut: distance between two points

(= distance between the end points of the two vectors)

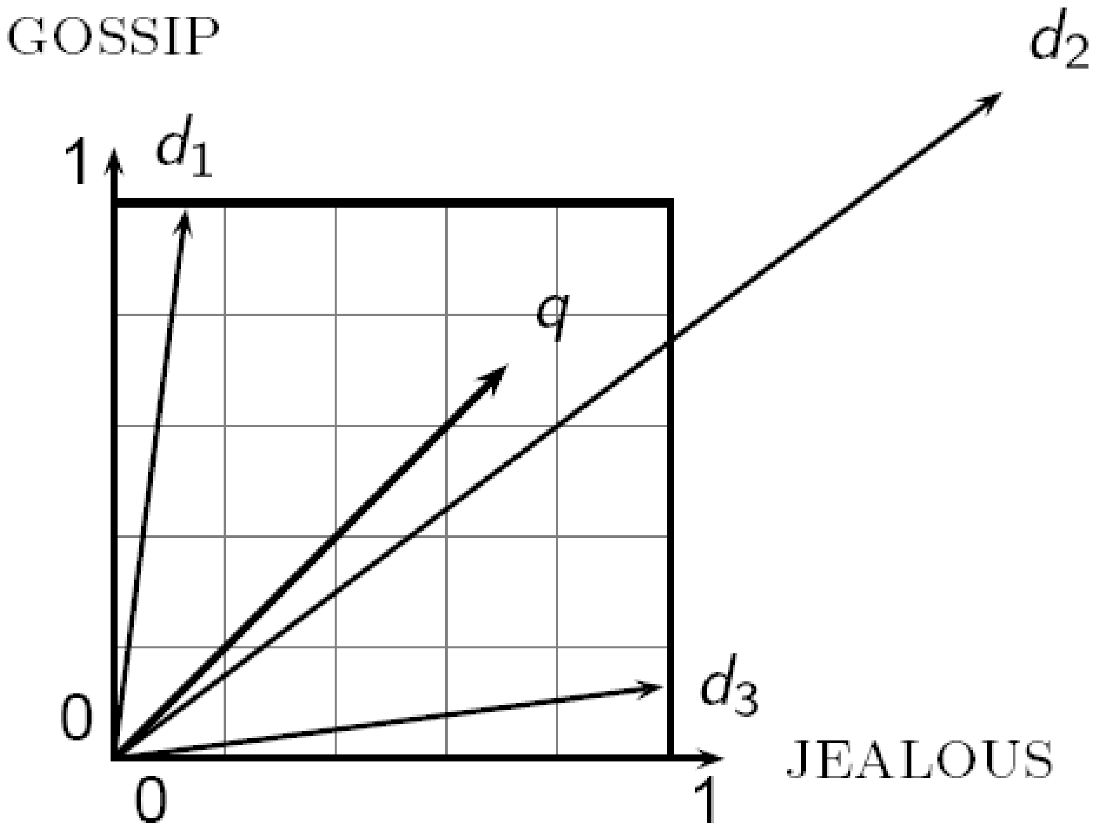
Euclidean distance?

Euclidean distance is a bad idea . . .

. . . because Euclidean distance is large for vectors of different lengths.

Why distance is a bad idea

The Euclidean distance between $\vec{q}$ and $\vec{d_2}$ is large even though the distribution of terms in the query $\vec{q}$ and the distribution of terms in the document $\vec{d_2}$ are very similar.



Use angle instead of distance

Experiment: take a document *d* and append it to itself. Call this document *d*′.

“Semantically” d and d′ have the same content

The Euclidean distance between the two documents can be quite large.

The angle between the two documents is 0, corresponding to maximal similarity.

Key idea: Rank documents according to angle with query.

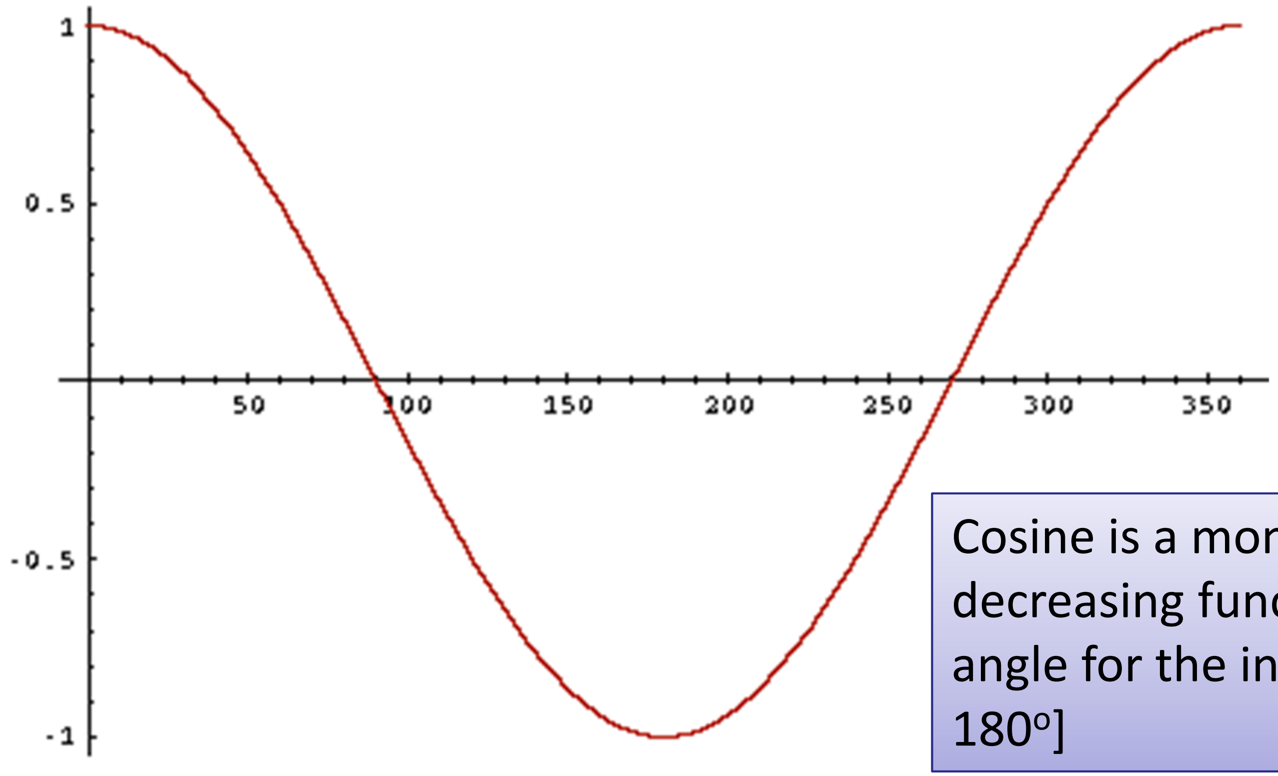
From angles to cosines

The following two notions are equivalent.

-- Rank documents in <u>decreasing</u> order of the angle between query and document

-- Rank documents in <u>increasing</u> order of cosine(query,document)

From angles to cosines



Cosine is a monotonically decreasing function of the angle for the interval [0^o, 180^o]

But how should we be computing cosines?

Cosines Similarity

- Compute the **cosine similarity** of their vector representations:

$$similarity(\overrightarrow{d_1}, \overrightarrow{d_2}) = \frac{\overrightarrow{d_1} \cdot \overrightarrow{d_2}}{|\overrightarrow{d_1}||\overrightarrow{d_2}|}$$

- Cosine similarity = dot product of length-normalized vectors.

Cosine(query,document) Similarity

The dot product (also known as inner product) $\vec{x} \cdot \vec{y}$ of two vectors is defined as $\sum_{i=1}^{M} x_i y_i$.

$$cos(\vec{q}, \vec{d}) = \frac{\overbrace{\vec{q} \cdot \vec{d}}^{\boxed{\text{Dot product}}}}{|\vec{q}||\vec{d}|}$$

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Length normalization

Let $\vec{d}$ denote the document vector for *d*, with *M* components $d_1 \ldots d_M$.

The Euclidean length of $\vec{d}$ is defined as

$$|\vec{d}| = \sqrt{\sum_{i=1}^{M} {d_i}^2}$$

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Length normalization

A vector can be (length-) normalized by dividing each of its components by its length:

$$\frac{\vec{d}}{|\vec{d}|}$$

Dividing a vector by its Euclidean length makes it **a unit (length) vector.**

Effect on the two documents *d* and *d′* (d appended to itself) from earlier slide: they have identical vectors after length-normalization.

Long and short documents now have comparable weights.

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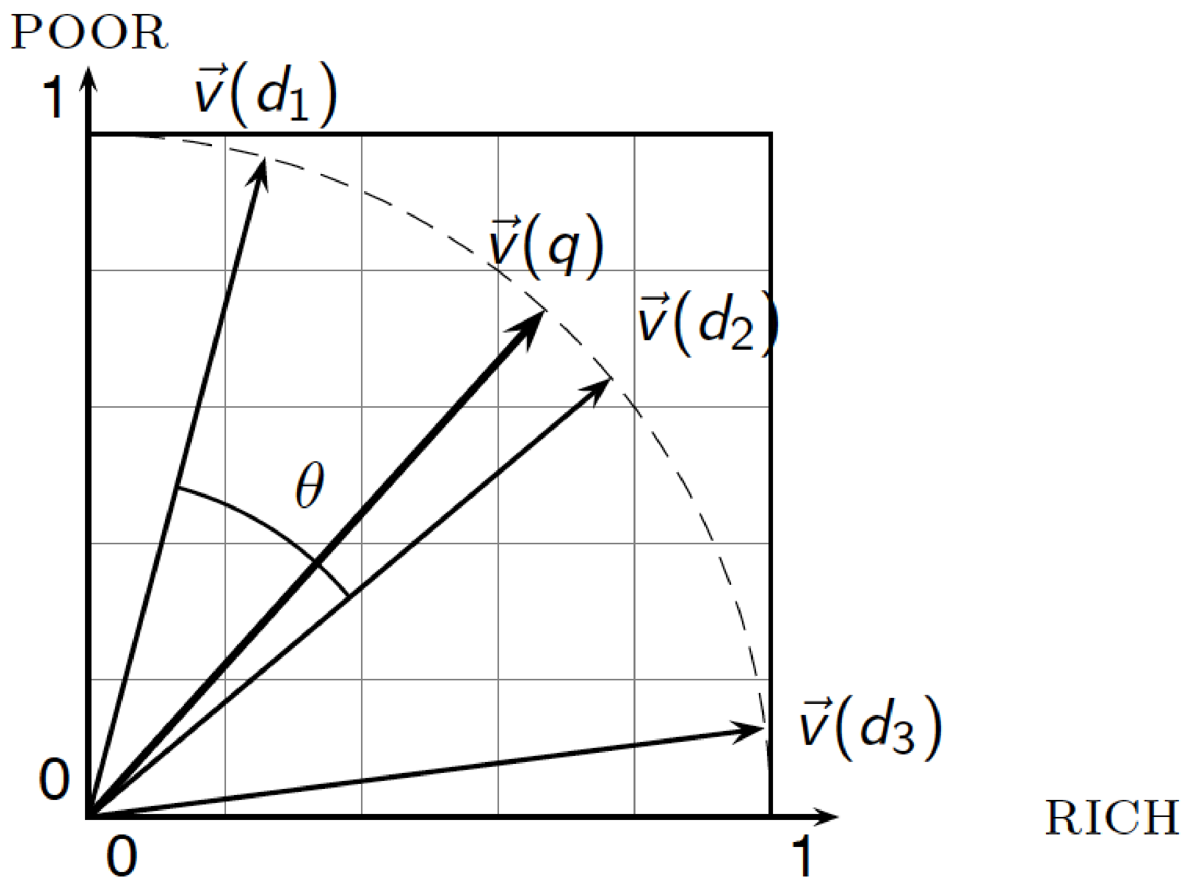
Cosine(query,document) Similarity

$$\cos(\vec{q},\vec{d}) = \frac{\vec{q}\bullet\vec{d}}{|\vec{q}||\vec{d}|} = \overbrace{\frac{\vec{q}}{|\vec{q}|}\bullet\frac{\vec{d}}{|\vec{d}|}}^{\text{Unit vectors}} = \frac{\sum_{i=1}^{|V|} q_i d_i}{\sqrt{\sum_{i=1}^{|V|} q_i^2}\sqrt{\sum_{i=1}^{|V|} d_i^2}}$$

q_i is the tf-idf weight of term i in the query;
d_i is the tf-idf weight of term i in the document.

Cosine similarity of $\vec{q}$ and $\vec{d}$ … or, equivalently, the cosine of the angle between $\vec{q}$ and $\vec{d}$.

Cosine similarity illustrated



POOR
1
$\vec{v}(d_1)$
$\vec{v}(q)$
$\vec{v}(d_2)$
θ
$\vec{v}(d_3)$
0
0
1
RICH