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Introduction to Data Assimilation, Subgrid-Scale Parameterization and Predictability

Christian Franzke

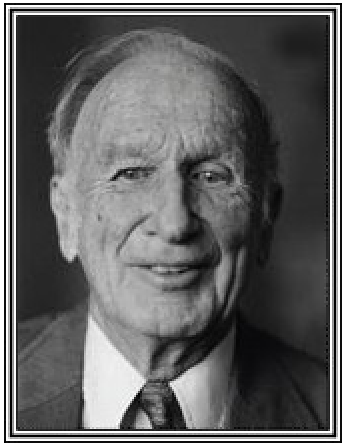
Meteorological Institute

Center for Earth System Research and Sustainability

University of Hamburg

Email: christian.franzke@uni-hamburg.de

Lorenz 1996 Model



Edward N. Lorenz
1917 – 2008

$$\frac{\mathrm{d}X_k}{\mathrm{d}t} = -X_{k-1}(X_{k-2} - X_{k+1}) - X_k + F - \frac{hc}{b} \sum_{j=J(k-1)+1}^{kJ} Y_j; \quad k = 1, \ldots, K \tag{1a}$$

$$\frac{\mathrm{d}Y_j}{\mathrm{d}t} = -cbY_{j+1}(Y_{j+2} - Y_{j-1}) - cY_j + \frac{hc}{b} X_{\mathrm{int}[(j-1)/J]+1}; \quad j = 1, \ldots, JK. \tag{1b}$$

Lorenz 1996

Time Discretization

$$\frac{\partial X}{\partial t} = F(X)$$

Simplest Scheme: Euler Forward (First Order)

$$\frac{\partial X}{\partial t} = \frac{X(t+\delta t) - X(t)}{\delta t}$$

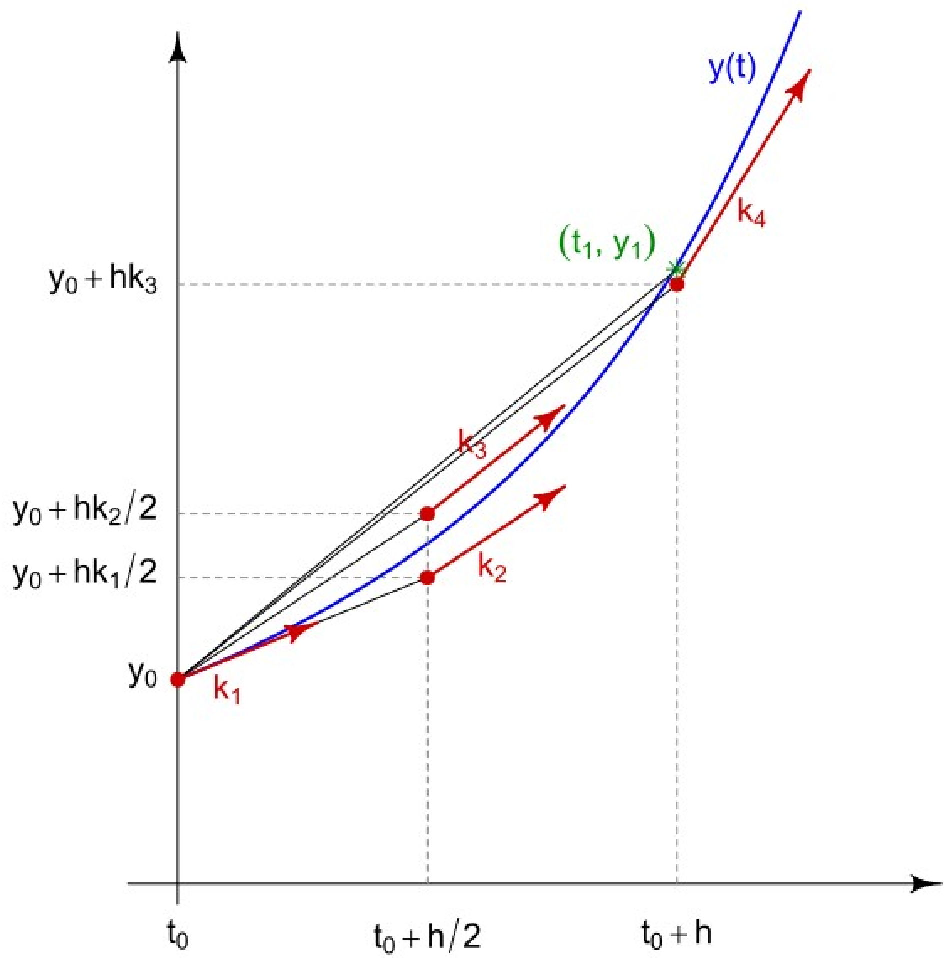
$$\Rightarrow \quad X(t+\delta t) = X(t) + \delta t \, F(X)$$

Time Discretization

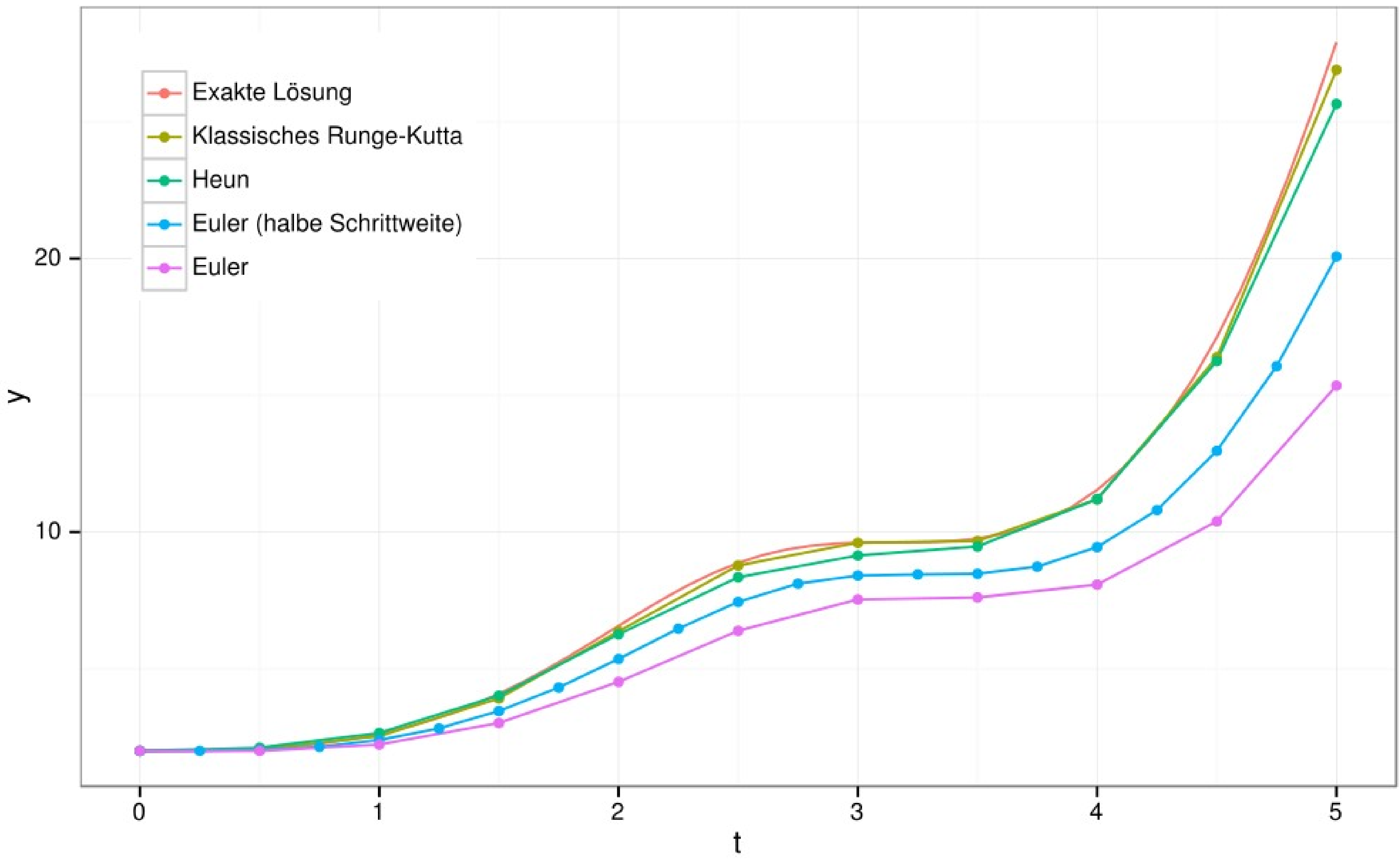
Runge-Kutta (4th Order)

$$y_{n+1} = y_n + \tfrac{h}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right),$$
$$t_{n+1} = t_n + h$$

$$k_1 = f(t_n, y_n),$$
$$k_2 = f\left(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_1\right),$$
$$k_3 = f\left(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_2\right),$$
$$k_4 = f\left(t_n + h, y_n + hk_3\right).$$



Time Discretization



Time Discretization

How to choose time step size?

Time Discretization

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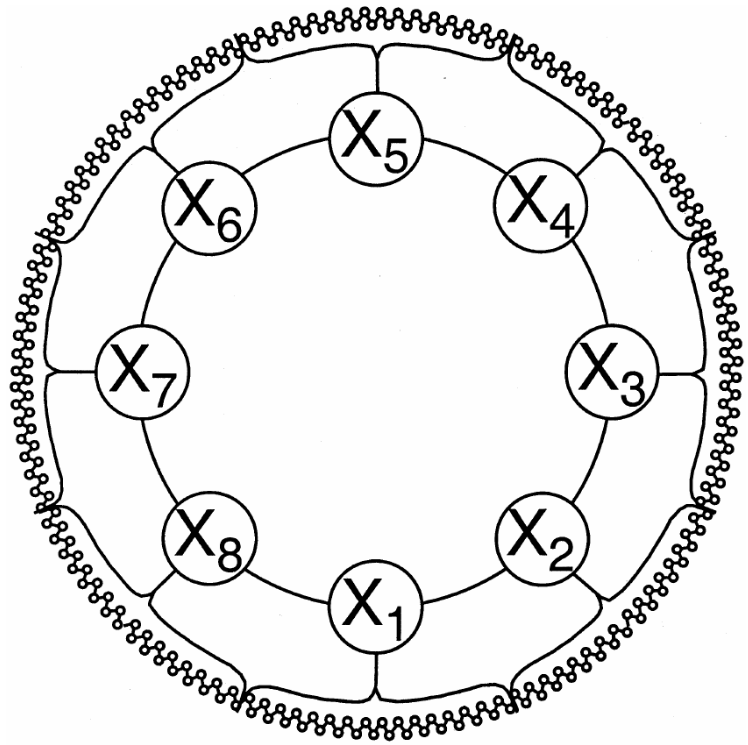
RK4: Error should decrease $\sim(\delta t)^4$

- Integrate model forward for 2 time units from same initial condition for different δt (factor 10)
- Then compare differences
- Repeat until error is at machine precision

(time unit $\neq$ δt; 1 time unit = N δt; δt= 1/N)

Lorenz 1996 Model



We are going to look at:

- F=10 - forcing
- h=1 - coupling strength
- c = b = 10 - scale separation (fluc 10x in Y's)
- K=36 - Number of X modes
- J=10 - Number of Y modes per X mode

How are we solving the system:

- Runge Kutta 4th order: time step 0.005
- Spinup: 100 time units
- Integration: 100 to 1000 time units

Wilks 2005