

Introduction to Data Assimilation, Subgrid-Scale Parameterization and Predictability

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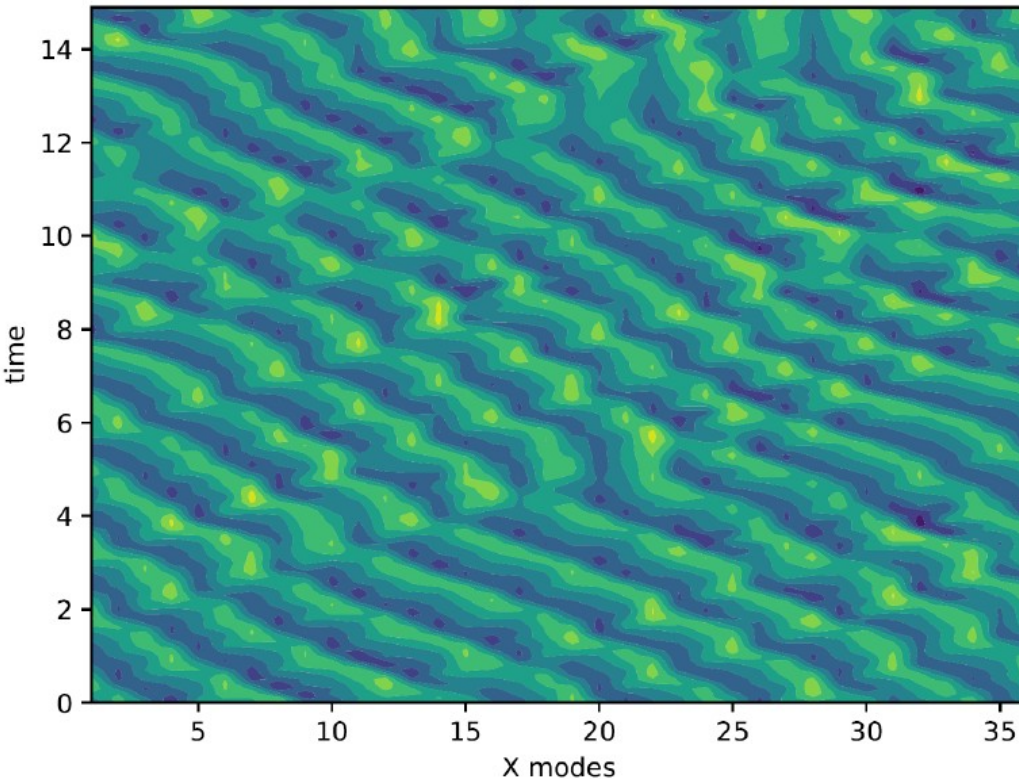
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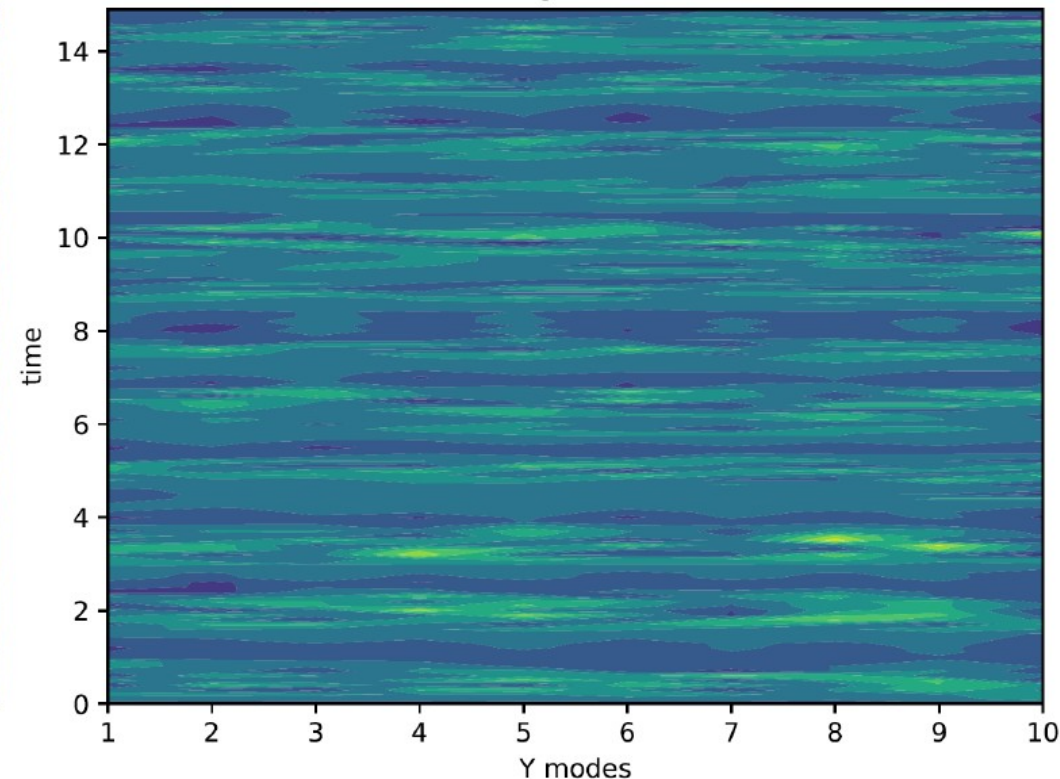
Lorenz 1996 Model

Lorenz 1996 Model – Waves, hence Hovmoeller diagramm

Hovmüller diagramm of X modes



Hovmüller diagramm of Y modes



Lorenz 1996 Model

Definition:

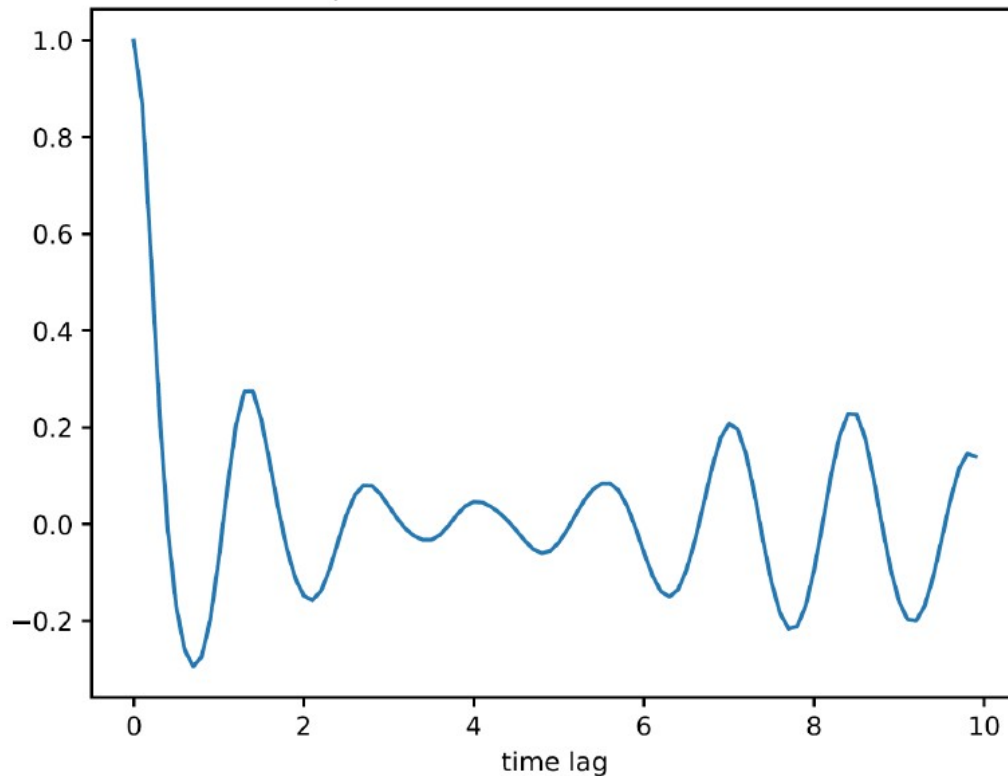
$$\text{Autocorr}(\tau) = \frac{E[(X_t - \mu)(X_{t+\tau} - \mu)]}{\sigma^2}$$

- Spatial Autocorrelation along X modes or Y modes averaged in time
→ Lag τ is here spatial quantity
- Temporal Autocorrelation along time for each X modes, Y mode and then averaged
→ Lag τ is a time

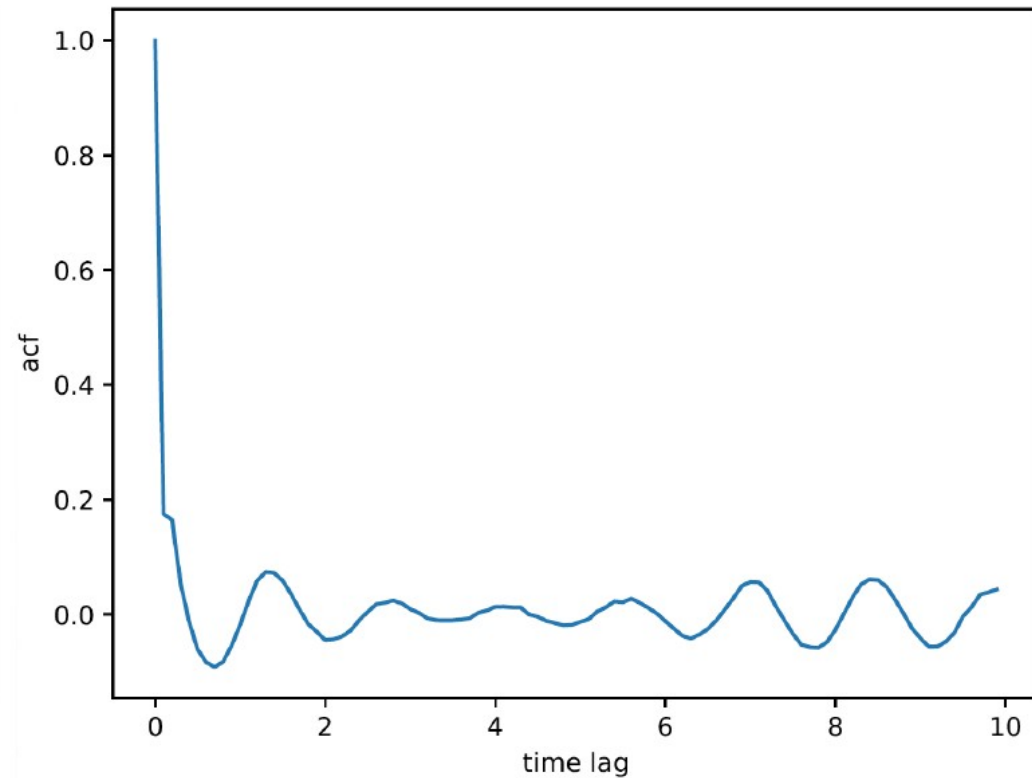
Lorenz 1996 Model

Temporal Autocorrelations

Temporal Autocorrelation of X modes



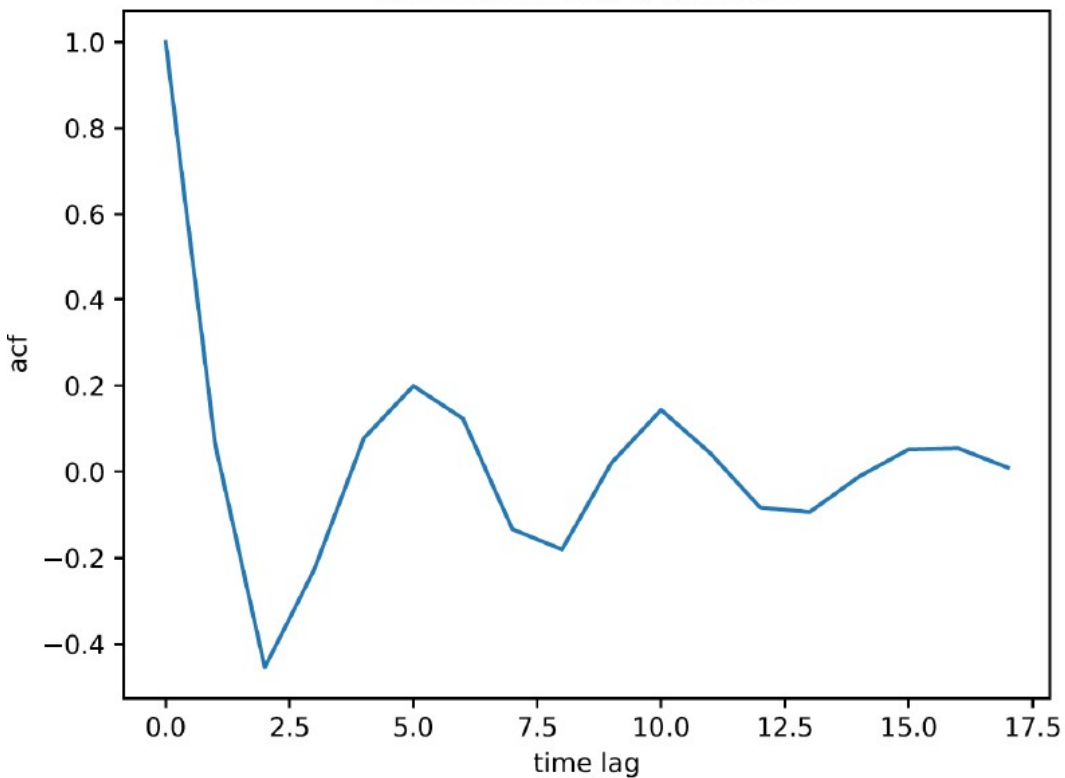
Autocorrelation of Y modes



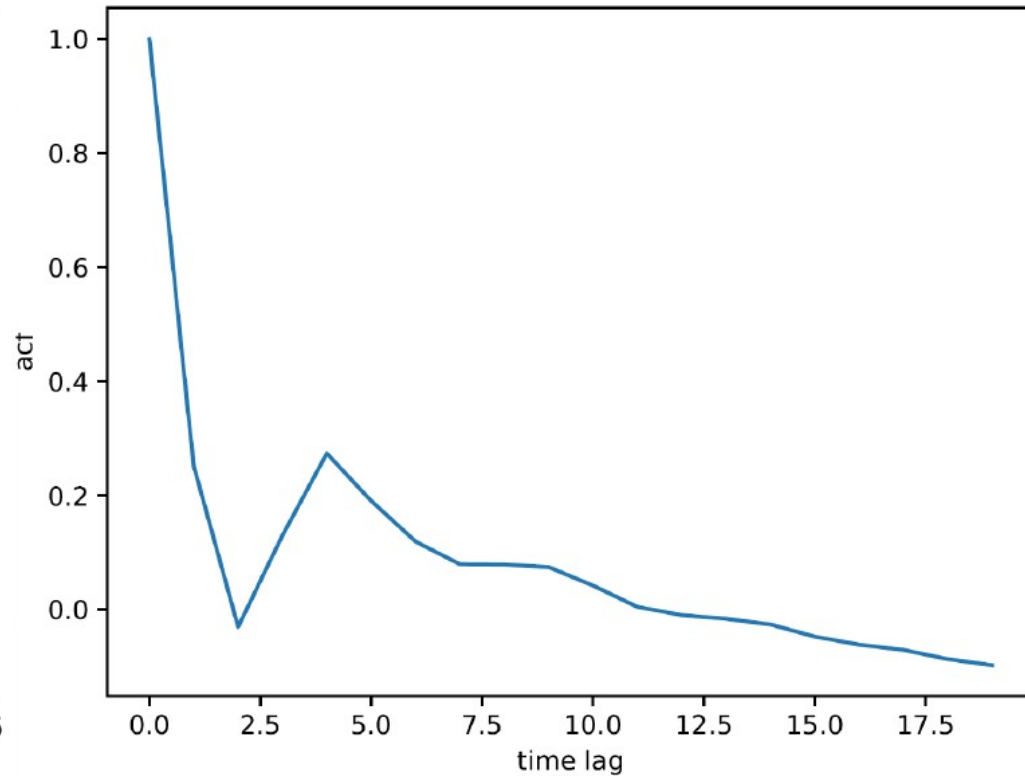
Lorenz 1996 Model

Spatial Autocorrelations

Autocorrelation of X modes

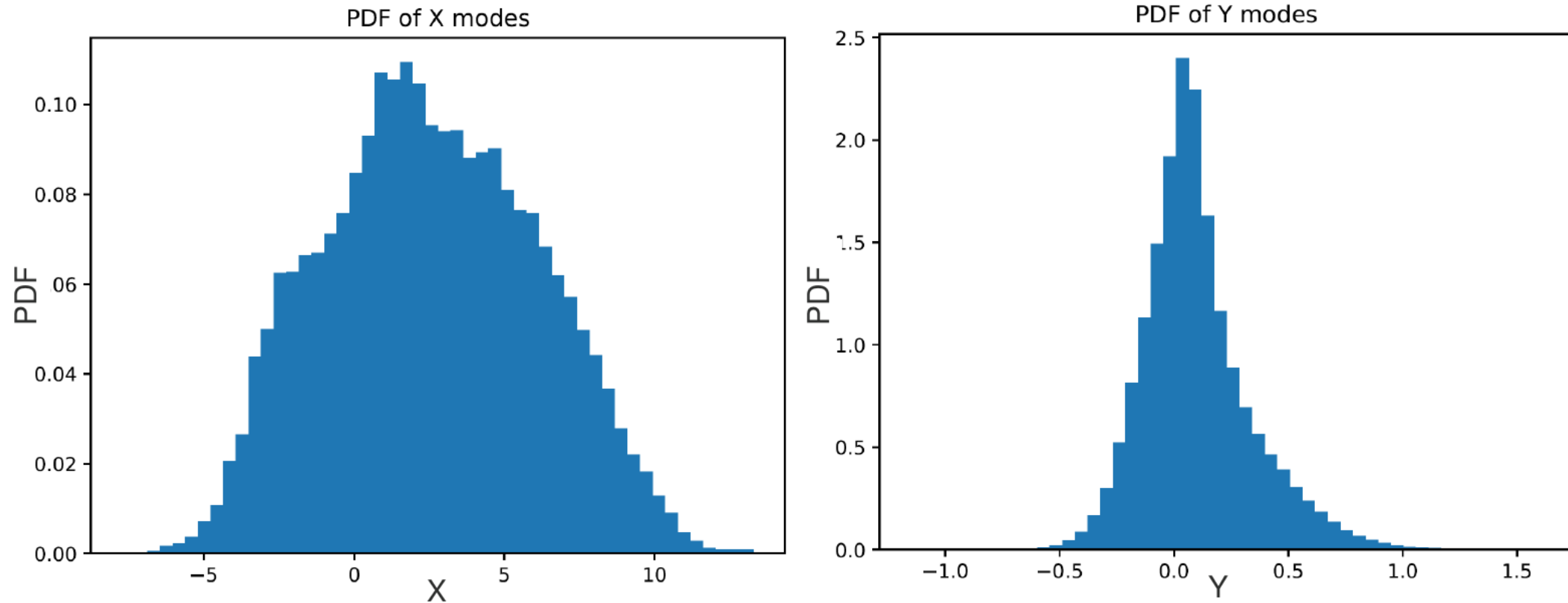


Autocorrelation of Y modes



Lorenz 1996 Model

Lorenz 1996 Model – Probability Density of X and Y Modes



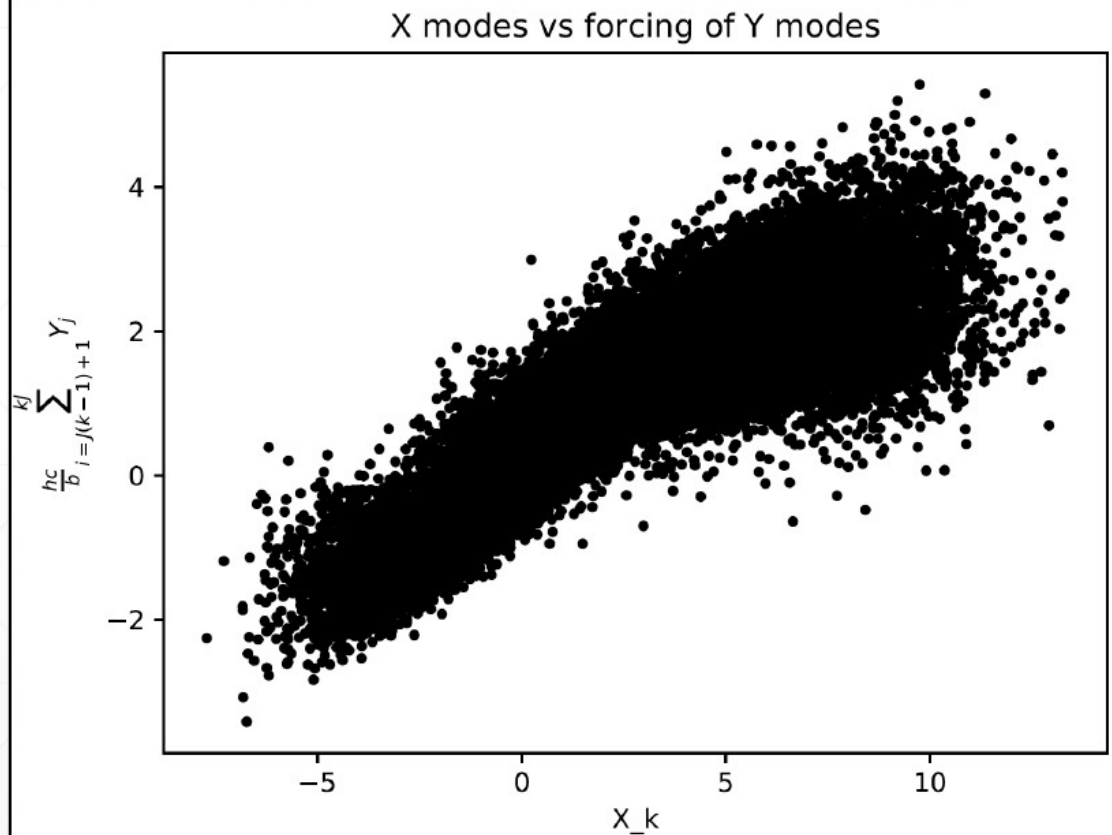
Lorenz 1996 Model

Lorenz 1996 Model – Forcing of Y modes onto X modes

- preparation for the Wilks 2005 “”
- Question: “Can the forcing by the Y modes on the X modes be simplified?”

x axis X_k

y axis $\frac{hc}{b} \sum_{i=J(k-1)+1}^{kJ} Y_j$



Lorenz 1996 Model

Energy definition

$$E_X = \frac{1}{2} \sum_{k=1}^K X_k^2$$

$$E_Y = \frac{1}{2} \sum_{j=1}^{JK} Y_j^2$$

“Momentum” definition

$$M_X = \sum_{k=1}^K X_k$$

$$M_Y = \sum_{j=1}^{JK} Y_j$$

Energy tendency equations

$$\partial_t E_X = -E_X + F M_X - \frac{hc}{b} \sum_{k,j=1}^{K,J} X_k Y_j$$

$$\partial_t E_Y = -c E_Y + \frac{hc}{b} \sum_{k,j=1}^{K,J} X_k Y_j$$

Total Energy tendency equation

$$\begin{aligned} \partial_t E &= \partial_t E_X + \partial_t E_Y \\ &= -E_X - c E_Y + F M_X \end{aligned}$$



Lorenz 1996 Model

1) Write routines for plotting:

- PDF
- Autocorrelation
- Hovmoeller diagramm
- Plot Y Forcing on X modes
- energy cycle terms

2) Error Growth

- Do many integrations over short time with different time steps but same initial state
- Compare with a log-log plot
 - X axis: time
 - Y axis: averaged error wrt integration with the smallest timestep