Resampling the SV data stream

Resampling here will be presented in the context of substation automation according to IEC 61850 standard where data is in the form of SV stream of line voltages and currents which are periodic functions of time. In this context resampling might be necessary for at least two reasons.

1. The sampling of voltage and current signals is not coherent, which means that the sampling frequency $f_s$is not a whole multiple of power line frequencyf_{line}. In that case the number of samples per period, *SPP*, is not a whole number which leads to spectrum leakage when applying DFT to data stream. This is a common situation as $f_{line}$is usually not exactly equal to $f_{linenom}$ = 50 Hz (or 60 Hz).Coherent sampling can only be achieved in laboratory setup.

2. There is a need to lower data rate. This may be the case when voltage and current signals are sampled at 96 kHz but some IEDs determine the signal spectrum with the use of FFT so *SPP*should be a whole power of 2. As will be shown later, resampling is not a computationally intensive process so it might be rational to reduce sampling rate and then compute the spectrum by FFT.

Resampling technique, when applied to data stream at constant rate $f_s$such as SV,results in anew data stream at a rate$(N/M)\cdot f_s$ where N and M are whole numbers called respectively the interpolation and decimation coefficients.When the aim of resampling is to restore the SV data rate to nominal value, SPP_{nom}, which is 80 or 256 as defined by IEC 61850-9-2LE, M and N coefficients are close to each other.When the aim is to lower the data rate,M is greater than$N$.

The resampling procedure, Fig. 1, consists of three operations. In the upsampling operation N-1 zero samples are inserted between each pair of the original data. In the interpolation operation the resulting stream is passed through the interpolation low pass filter. The decimation operation consists of taking every M-th sample at the output of the interpolation filter. The role of the interpolation filter which will be further called the resample filter is twofold. First it produces interpolated values between each original data points and secondit restricts the bandwidth of the upsampled signal to prevent the spectrum aliasing [1,2]. How the signal spectra at various stages of the resampling process change and how the N and M coefficients determine the resample filter bandwidth can be found in reference works [1,2].

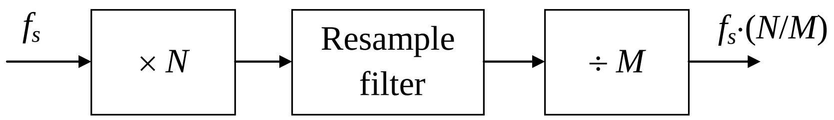

Fig. 1 Block diagram of the resampling process

The ideal resample filter should have a flatpass bandup to the highest frequency component of the signal and sufficient attenuation at $(N/(2M))\cdot f_s$(with the assumption$M \geq N$) to prevent spectrum aliasing [1,2].

Though in principle resampling can generatea data rate arbitrary close to coherent sampling, albeit at the cost of growing N and M, there are practical considerationslimiting the resampling resolution accuracylike uncertainty inf_{line} determination

Determination of *N* and *M* coefficients

As the reason to resample SV data stream is to reduce the spectrum leakagewhen using DFT, the question arises how the relative difference of SPP from the whole number of samples per period affects the accuracy of spectrum determination.The relationship betweenthe relative error in fundamental harmonic amplitude determination (for which we write $errA1$ from now on) and the relative deviation of the number of samples per period from the nominal value, $\delta SPP = (SPP - SPP_{nom})/SPP_{nom}$,for small values of δSPP, can be expressed by the following formula

$$errA1 = k \cdot \delta SPP\ . \tag{1}$$

The linear relationship between $errA1$ and δSPP is valid over $\delta SPP \in$ <-0.02, 0.02>.The k coefficientis equal to -0.5and it has been obtained empiricallyby calculating the DFT of a sinusoidal signal of varying frequency sampled at a constant rate.

Formula(1) is the basis, or at least a starting point, for the determination of the resampling process parameters$N$ and M.As$M \cong N$or$M > N$, it is better,considering the resolution of resampling procedure, to vary $M$with varying $f_{line}$and keep the N constant.The value of M, for which SPP best approximates SPP_{nom}, for givenf_{line} and N, is computed from the formula

$$M = round\left(\frac{N \cdot f_s}{f_{line} \cdot SPP_{nom}} \right). \tag{2}$$

The value of M computed for $f_{line} = f_{linenom}$ will be further denoted by M_{nom}.

δSPPcan be expressed by the following relationship

$$\delta SPP = \frac{\frac{N}{M} \cdot \frac{f_s}{f_{line}} - SPP_{nom}}{SPP_{nom}}\ . \tag{3}$$

If we denote by M_{exact} the exact number (generally not a whole one) for which $(N/M_{exact}) \cdot (f_s/f_{line}) = SPP_{nom}$ and observe that $M = M_{exact} + x$for some x in the interval<-0.5, 0.5>, (3) can be transformed as follows

$$\delta SPP = \frac{\frac{N}{M_{exact} + x} \cdot \frac{M_{exact}}{N} \cdot SPP_{nom} - SPP_{nom}}{SPP_{nom}} = \frac{M_{exact}}{M_{exact} + x} - 1, \tag{4}$$

and taking into account that $M_{exact} >> 1$,

$$|\delta SPP| \approx \left| \frac{x}{M} \right| . \tag{5}$$

$|\delta SPP|$ attains the largest value when $x = \pm 0.5$, so

$$|\delta SPP| \leq \left| \frac{1}{2 \cdot M} \right| . \tag{6}$$

Equations (1) and (6) can be used to determine the resampling parameters N and Mneeded to keep the error $errA1$, due to leakage,within prescribed limit. As f_{line} is usually very close to $f_{linenom}$, M does not change much around M_{nom} so the M_{nom}can be obtained from (6).For example, if we want to limit to 50 ppm the contribution of δSPP to $errA1$, we get from (1) that $\delta SPP < 10^{-4}$ and substituting this into (6) we obtain that $M_{nom} \geq 5000$.If $SPP_{nom} = (f_s/f_{line})$ which is a typical case in IEC 61850 substation automation, N is then equal to M_{nom}.Generally if a new nominal number of samples per line period is required, SPP_{nomnew}, a formula below should be used

$$N = M_{nom} \cdot SPP_{nomnew} \cdot \frac{f_{line}}{f_s} . \tag{7}$$

Keeping Nconstant, the value of M for each value of f_{line}isnext computed by formula (2). Figure 2 shows the values of Mforf_{line}changing over the range <49 Hz, 51 Hz>, with$N = 5000$, $f_s = 12800$ and$SPP_{nom} = 256$.

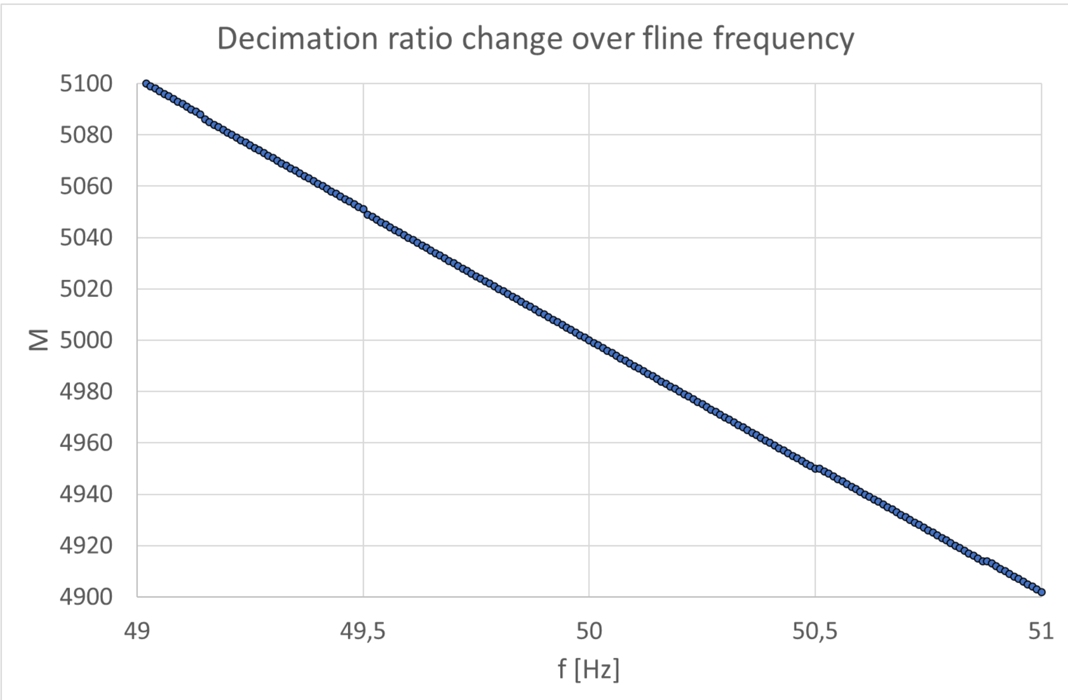

Fig. 2 Range of change of M for $f_{line} \in$<49 Hz, 51 Hz >

Resample filter design

Having obtained the values of N and Mcoefficients, the next step is to design the resample filter. This is a low pass FIR filter with the passband extending up to the highest frequency component of the signal and stopband starting at$(N/(2M))\cdot f_s$. If the value of N is held constant, a single resample filter is needed, designed with the maximum value of M, M_{max},computed at $f_{line} = f_{linemin}$. An example of such a filter computed with the use of **fir1** and **kaiseroid**GNU Octave functions has been shown in Figure 3.The parameters to the mentioned GNU Octave functions were$M = M_{max} = 5102$,$f_s = 12800$, $N = 5000$ and the passband width 2200 Hz. These functions need additionallyas parameters a passband ripple which was set to 0.0001% and a stopband attenuation which was set to 40 dB.The length$L$of the computed filter is 122664. Various tools besides GNU Octavecan be used to design such filter [3].

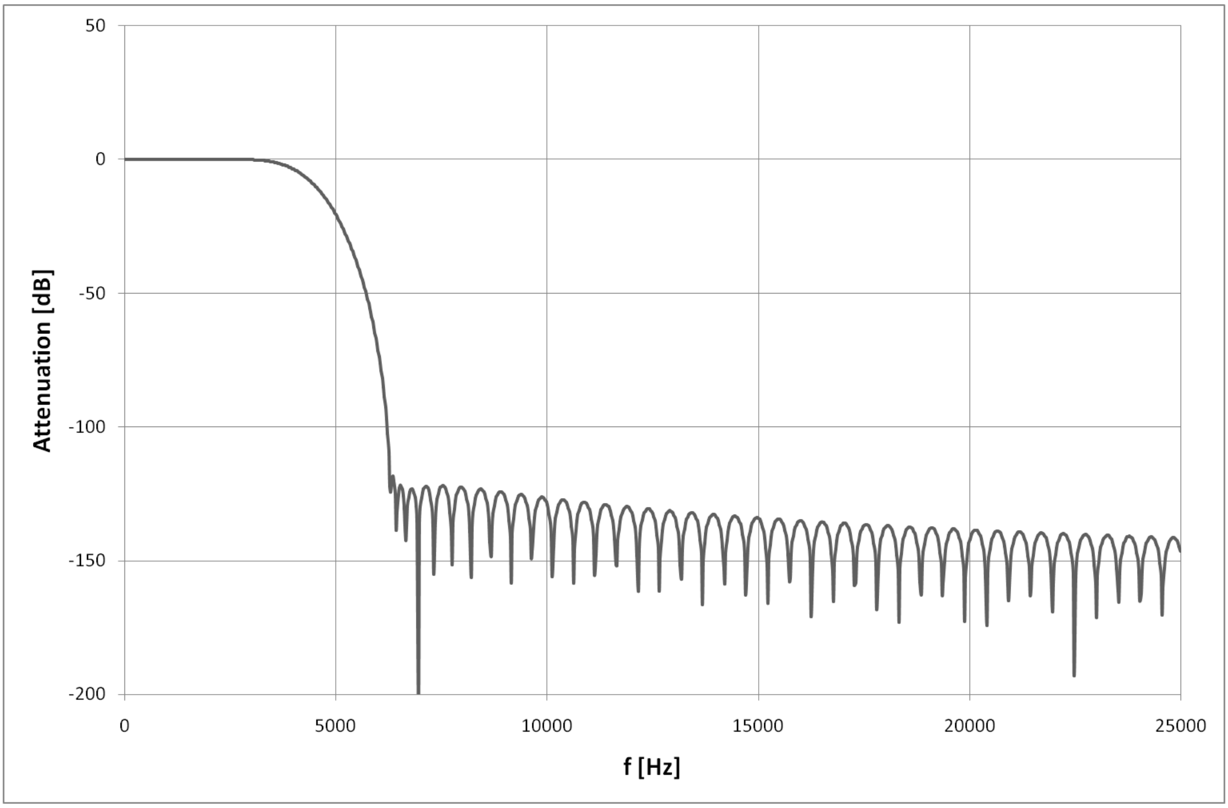

Fig. 3 Resample filter designed with fir1 and kaiseroid GNU Octave functions

The passband ripple of the filter determines how faithfully the original waveform of the signalis reproduced after the resampling procedure. With the accuracy target equal to 50 ppm the passband ripple should be no larger than 0. 0001%.

The length of the resample filter depends on N and M but also, quite dramatically, on the width of the transition band of the filter. The closer the passband edge is to the Nyquist frequency $(N/(2M))\cdot f_s$, the longer is the filter. Fig. 4presents the length of the resample filter computed with fir1 and kaiseroid as a function of the of the passbandwidth. The parameters to theGNU Octave functions were $M = 5001$, $f_s = 12800$, $N = 5000$,passband ripple equal to 0.0001% and a stopband attenuation equal to 40 dB.

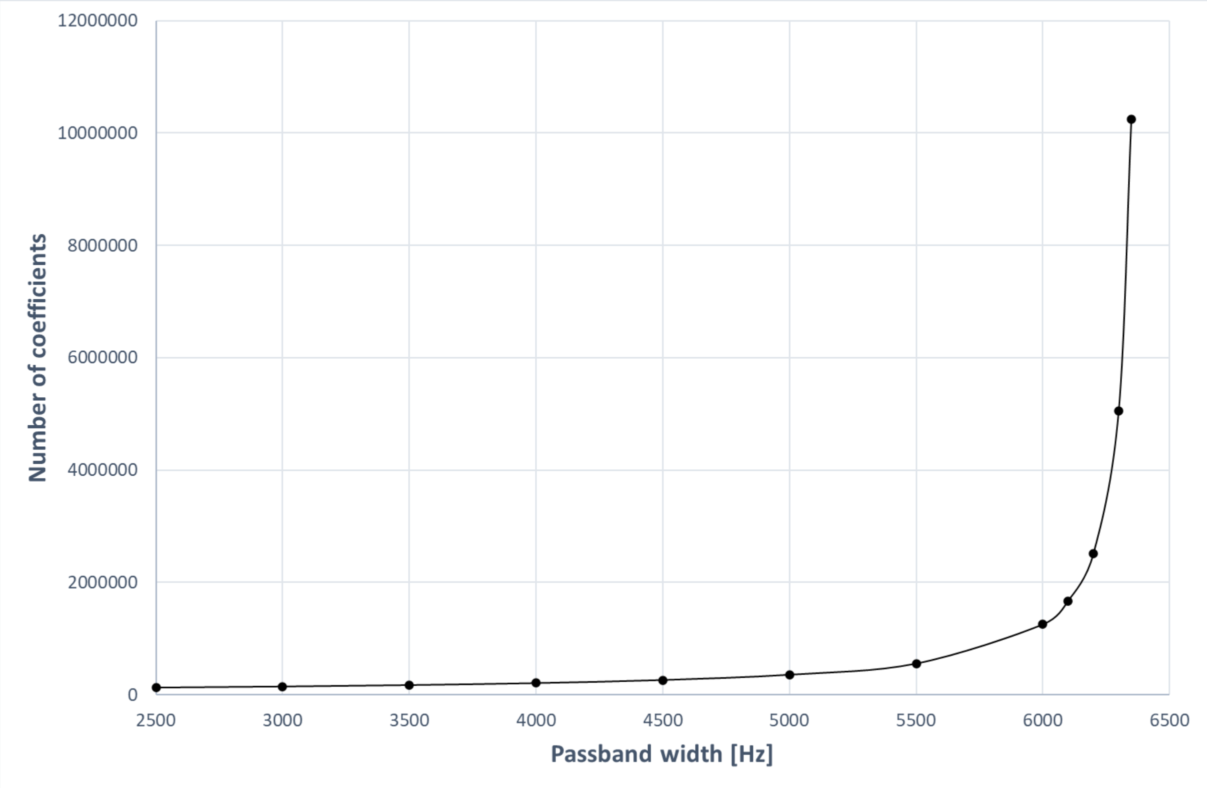

Fig. 4 Number of resample filter coefficients as a function of the passband width; N=5000, M=5001, f_s=12800, passband ripple = 0.0001%,stopband attenuation = 40 dB; Nyquist frequency is equal to 6400 Hz

Though the length *L*of the resampling filter may be quite large, the number of multiplications and summations to compute each sample of the resampled stream is the length of the filter divided by *N*. The total number of multiplications and summations to compute SPP_{nom} samples is thus $(L/N)\cdot Spp_{nom}$, which for the example filter from Fig. 3 is equal to 6400.

Noise

As follows from the above, the resampling procedure is low pass filtering. The resample filter is of FIR type which is quite insensitive to roundoff errors. If there is a need to filter out the unwanted noise, the bandwidth of the resample filter can be reduced below the highest frequency signal component. In this respect resampling is very robust when it comes to noise immunity.

Resampling procedure design algorithm

The steps that have to be taken in the design of resampling algorithm have been presented in the following flowchart, Fig. 5.

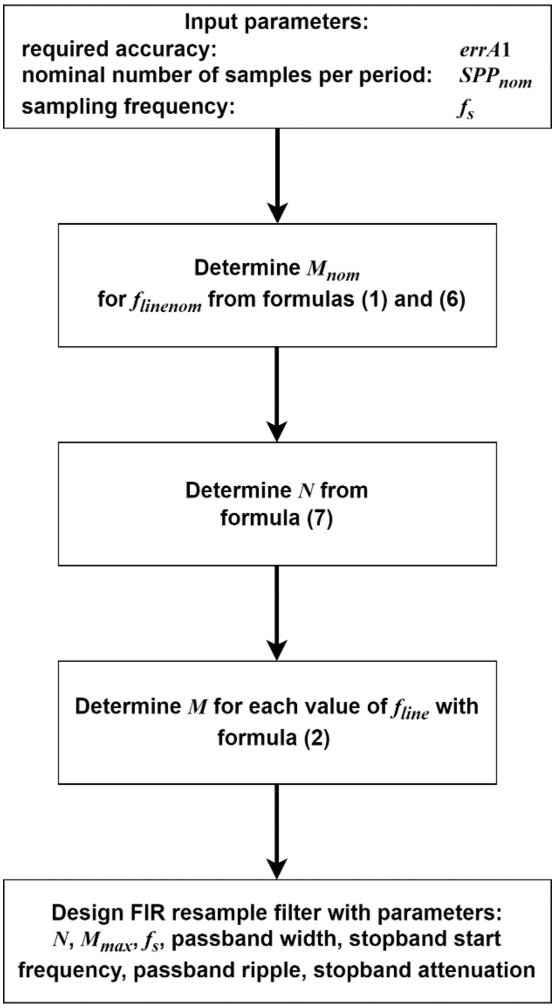

Fig. 5 Resample procedure design flowchart

Parameterization of the DC setting function

During some faults in power system, like for instance a short circuit across generator terminals [4], a decaying DC component may be present in a measured current signal. The amplitude and rate of DC component decay can give information about the nature of the fault. The DC component, together with sinusoidal component is modeled by the equation (8) and the aim is to parameterize the model, i.e., find the values of I_{sc}, *tau*, I_0, f and φ_0, given the signal samples.

$$i(t) = I_{sc} \cdot e^{-\frac{t}{tau}} + I_0 \cdot \sin(2 \cdot \pi \cdot f \cdot t + \varphi_0) \tag{8}$$

One method that works well is the nonlinear regression. Many software mathematical packages offer functions for fitting models to data. MATLAB has a **fit** function, Mathematica has a **FindFit** and the equivalent function in Python scipymodule is **curve_fit**. Here a **nonlin_curvefit** function from GNU Octave**optim** package has been used to fit data corrupted by Gaussian noise,(of zero mean and variance equal to 5% of the signal amplitude), to the model described by (8), Fig. 6.

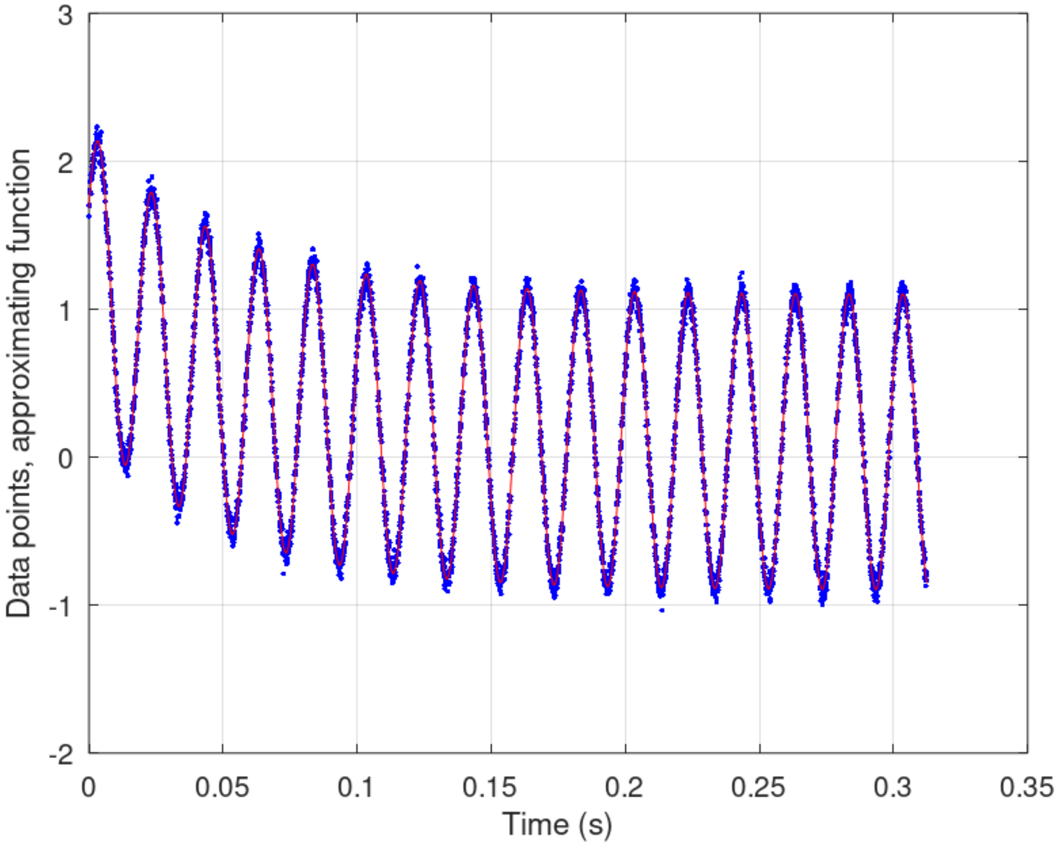

Fig. 6 Fitting a sinusoid signal plus dc decaying component data with GNU Octavenonlin_curvefit function

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