Frequency response measurement

WAC consortium

April 28, 2025

Equations

Definitions:

- Digitizer: measures amplitude $D(f)$, have got frequency dependent gain $G(f)$. Time dependence of the gain is neglected.
- Thermal transfer standard: U is output DC voltage measured by the multimeter, and the overall gain is $\Delta(f)$. Thermal transfer standard and the multimeter are supposed to be stable in time.
- AC source: generates signal of amplitude with non-negligible time and frequency dependence $Z(f,t)$.

One measurement of the AC source by digitizer and by the thermal transfer standard is expressed as:

$$D(f,t) = Z(f,t) \cdot G(f) \tag{1}$$
$$U(f,t) = Z(f,t) \cdot \Delta(f) \tag{2}$$

If the measurement by digitizer and by the themral transfer standard is done simultaneously, or almost simultaneously, then:

$$D(f) = Z(f) \cdot G(f) \tag{3}$$
$$U(f) = Z(f) \cdot \Delta(f) \tag{4}$$

Two measurements are done, for measured and reference frequencies marked as f_M and f_R. If these measurements are done right after another, then the time dependence of the AC source can be neglected and only frequency dependence is taken into account:

$$t_\mathrm{R} \approx t_\mathrm{M} \implies Z\big|_{t_\mathrm{R}} = Z\big|_{t_\mathrm{M}} \tag{5}$$

Then we can write both measurements as:

$$D(f_\mathrm{M}) = Z(f_\mathrm{M}) \cdot G(f_\mathrm{M}) \tag{6}$$
$$U(f_\mathrm{M}) = Z(f_\mathrm{M}) \cdot \Delta(f_\mathrm{M}) \tag{7}$$
$$D(f_\mathrm{R}) = Z(f_\mathrm{R}) \cdot G(f_\mathrm{R}) \tag{8}$$
$$U(f_\mathrm{R}) = Z(f_\mathrm{R}) \cdot \Delta(f_\mathrm{R}) \tag{9}$$

Combining the equations:

$$\frac{G(f_{\mathrm{M}})}{G(f_{\mathrm{R}})} = \frac{D(f_{\mathrm{M}})}{D(f_{\mathrm{R}})} \cdot \frac{U(f_{\mathrm{R}})}{U(f_{\mathrm{M}})} \cdot \frac{\Delta(f_{\mathrm{M}})}{\Delta(f_{\mathrm{R}})} \tag{10}$$

For f_{M} varied through whole input bandwidth of the digitizer, the ratio returns the frequency response of the digitizer related to the gain at reference frequency f_{R}. If the reference frequency is small enough, the gain $G_{f_{\mathrm{R}}}$ can be measured using DC method.

DC offset is not result of measurement and processing!

AC-DC measurements:

$$\Delta_L = 1 + \delta(f) \tag{11}$$

$$U_{\mathrm{AC}_{\mathrm{in}}} = U_{\mathrm{DC}_{\mathrm{in}}} \cdot (1 + \delta \cdot 10^{-1}) \tag{12}$$

With:

$$\delta_{792} = \frac{U_{\mathrm{out}_{\mathrm{AC}}} - U_{\mathrm{out}_{\mathrm{DC}}}}{n \cdot U_{\mathrm{out}_{\mathrm{DC}}}} \tag{13}$$