Optimal artifact substats

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Kaeya generally needs ATK%, CR (crit rate) and CD (crit damage) for his artifact substates, but the optimal distribution of the three substat rolls depends on the artifact main stats and the weapon you chose. This document serves to provide a generalized formula to be used in the other calculations in the KaeyaBuilds repositories - a formula to where one can plug in different main stats and weapon stats.

In the derivations below we make several assumptions:

- Each of the five artifacts at level 0 has ATK%, CR, CD and a flex stat as substats. We don't take the flex slot into consideration, even though it could be a useful substat like flat ATK or ER (energy recharge).
- Substat rolls are not perfect. For example, ATK% substat rolls for a 5* artifact could be any of the these four numbers 4.1% / 4.7% / 5.3% / 5.8% according to https://genshin-impact.fandom.com/wiki/Artifacts. We assume we only roll the second best, that is 5.3%.

The only part of the damage formula that involves artifact substats is:

$$D = (B \times (1 + A + O) + F) \times (1 + (CR + OR) \times (CD + OD)) \quad (1)$$

Where D is damage; B is base attack; A is ATK% from artifact substats; O is all other ATK%; F is flat attack; CR and CD are the crit rate and damage from artifact substats; OR and OD are all other crit rate and crit damage. We want to optimize D given the constraint:

$$N = \frac{A}{N_A} + \frac{CR}{N_{CR}} + \frac{CD}{N_{CD}} \quad (2)$$

Where N is the total number of substat rolls that went into A, CR and CD. $N_A = 5.3\%$, $N_{CR} = 3.5\%$ and $N_{CD} = 7.0\%$, they are the second best rolls for artifact substats, see the assumptions above. We write down the Lagrangian of the above constrained system:

$$\mathcal{L} = D - \lambda \left(N - \frac{A}{N_A} - \frac{CR}{N_{CR}} - \frac{CD}{N_{CD}} \right) \tag{3}$$

We take derivatives of the Lagrangian with respect to A, CR and CD to obtain their values that optimizes D.

$$\frac{\partial \mathcal{L}}{\partial A} = B \times (1 + (CR + OR) \times (CD + OD)) + \frac{\lambda}{N_A} = 0 \tag{4}$$

$$\frac{\partial \mathcal{L}}{\partial CR} = (B \times (1 + A + O) + F) \times (CD + OD) + \frac{\lambda}{N_{CR}} = 0 \tag{5}$$

$$\frac{\partial \mathcal{L}}{\partial CD} = (B \times (1 + A + O) + F) \times (CR + OR) + \frac{\lambda}{N_{CD}} = 0 \tag{6}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = N - \frac{A}{N_A} - \frac{CR}{N_{CR}} - \frac{CD}{N_{CD}} = 0 \tag{7}$$

Now we solve the above four equations of four variables. From Eq. (5) and (6) we can obtain:

$$CD + OD = (CR + OR) \times \frac{N_{CD}}{N_{CR}} = 2(CR + OR) \tag{8}$$

$$CD = 2(CR + OR) - OD \tag{9}$$

Eq. (8) shows the well known 1:2 CR:CD ratio. However, the optimal CR:CD ratio for artifact substats is Eq. (9). With Eq. (4) and (9), we have:

$$\lambda = -N_A \times B \times (1 + 2(CR + OR)^2) \tag{10}$$

Plug Eq. (10) into Eq. (6), we get:

$$A = \frac{N_A}{N_{CR}} \frac{1 + 2(CR + OR)^2}{CR + OR} - \frac{F}{B} - O - 1 \tag{11}$$

Now we have expressed everything in terms of CR, plug them into Eq. (7)

to solve for CR:

$$4(CR+OR)^2 - N_{CR}(N+O+\frac{F}{B}+\frac{OR}{N_{CR}}+\frac{OD}{N_{CD}}+1)(CR+OR)+1=0 \tag{12}$$

$$CR = \frac{-b \pm \sqrt{b^2-4ac}}{2a} - OR \tag{13}$$

$$a = 4 \tag{14}$$

$$b = -N_{CR}(N+O+\frac{F}{B}+\frac{OR}{N_{CR}}+\frac{OD}{N_{CD}}+1) \tag{15}$$

$$c = 1 \tag{16}$$

Quadratic Eq. (12) has an exact solution Eq. (13).

In summary, the optimal substat roll distribution given weapon and artifact main stat N, O, OR and OD is:

$$CR = \frac{1}{N_{CR}}\left(\frac{-b \pm \sqrt{b^2-4ac}}{2a} - OR\right) \tag{17}$$

$$CD = \frac{1}{N_{CD}}\left(2(CR+OR) - OD\right) \tag{18}$$

$$A = \frac{1}{N_A}\left(\frac{N_A}{N_{CR}}\frac{1+2(CR+OR)^2}{CR+OR} - \frac{F}{B} - O - 1\right) \tag{19}$$