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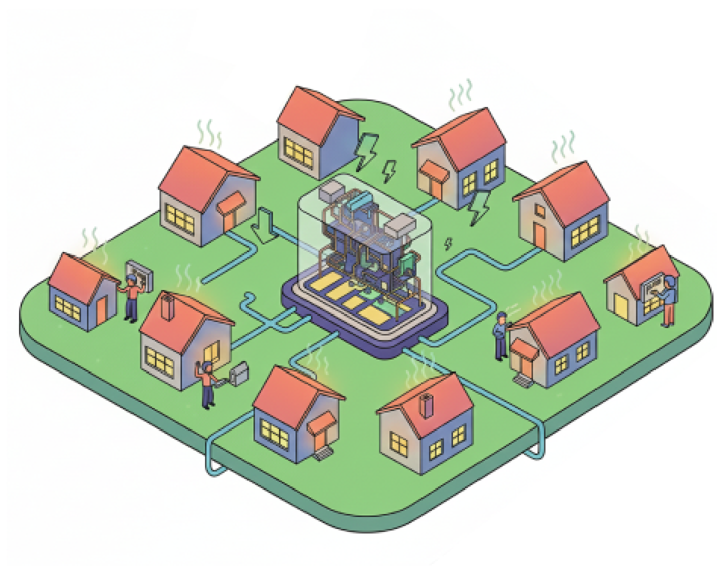

Master thesis

Buildings as Thermal Batteries

Dynamic Programming Control of Danish Household Heating for Cost and Emission Reduction

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Abstract

The transition to renewable energy requires flexible demand-side resources to balance wind and solar generation. Buildings can be utilised as thermal battery storage, as their mass absorbs and releases heat over periods of hours to days. Denmark has committed to 70% emission reductions by 2030 and net-zero by 2045, yet the building sector, which consumes 40% of national energy consumption, remains largely disconnected from grid flexibility services. This thesis show that cost-optimized dynamic programming control of electrically heated households achieves 18.89% electricity cost savings and 13.99% CO^2 emission reductions compared to a baseline hysteresis algorithm, while shifting 10–15% of peak electricity demand to off-peak periods for private consumption. A field experiment validated the approach, showing savings of 8.71% compared to the baseline, and a Monte Carlo simulation showed mean savings of 21.37%. A statistical analysis confirms a moderate positive correlation between spot prices and grid emissions (Spearman $\rho = 0.464$), indicating that optimising for cost implicitly results in emission reduction. Building insulation strongly determines a households flexibility potential, with new buildings achieving 25.7% savings compared to 6.8% for old buildings. Applied nationally to 216,791 electrically heated Danish households, this approach could deliver annual savings of 34.5 million EUR and avoid 12,421 tonnes of CO^2 emissions without requiring significant new infrastructure and shows that existing buildings can be used as distributed thermal batter storage, which can effectively support grid flexibility.

A GitHub repository with code is available at github.com/KaffeDiem/buildings-as-thermal-batteries

Contents

1 Introduction

This thesis investigates the potential of Danish households to function as thermal batteries, using cost-optimised control of heating systems to reduce electricity costs and potentially providing grid stability. By treating buildings' thermal mass as flexible energy storage, heating can be shifted away from peak demand periods without compromising occupant comfort.

The work addresses three questions: what cost savings can optimised heating, ventilation and air-conditioning (HVAC) control strategies deliver for individual households, what is the national impact if such control were widely adopted, and whether cost optimisation implicitly result in environmental benefits through the correlation between electricity prices and grid emissions.

The thesis proceeds as follows. section 2 establishes the context of Denmark's energy transition and the role buildings can play. section 4 provides technical background on the Danish electricity system and thermal storage principles. section 5 describes the thermal models, control algorithms, and simulation approach. section 7 presents a field experiment comparing control strategies. section 8 analyses the price-emission correlation in Danish electricity markets. section 9 scales the results nationally. section 10 and section 11 discuss the results and conclude.

2 Motivation

2.1 Denmark's Climate Ambitions

Denmark has committed to ambitious emission reduction targets under the Paris Agreement. The 2020 Climate Act mandates a 70% reduction in CO_2 emissions by 2030 relative to 1990 levels, with coal to be phased out entirely from electricity generation [Jensen et al., 2021]. At COP30 in 2025, the Danish government proposed an even more ambitious reduction target of 82 to 85% by 2035, net-zero by 2045, and 110% reduction by 2050 [Klima-, Energi- og Forsyningsministeriet, 2025

Achieving these targets requires not only expanding renewable generation but also developing flexibility mechanisms to meet the volatility of wind and solar power. The concept of Smart Energy Systems (SES) where renewable production, infrastructure, and consumption are integrated and coordinated [Jensen et al., 2021], requires new approaches to balancing supply and demand across different sectors.

2.2 The Building Sector as Battery Storage

The Danish building sector accounts for approximately 40% of national energy consumption [Jensen et al., 2021], yet it remains largely disconnected from grid flexibility services. This represents a missed opportunity, as the thermal mass of buildings stores heat naturally, allowing indoor temperatures to change slowly when heating is interrupted. This makes buildings able to act as thermal batteries.

Research suggests that the thermal storage capacity of Danish buildings is on the same level as the storage capacity of the entire electric vehicle fleet [Johra et al., 2024]. A 2020 study estimated that if all buildings not connected to district heating were heated by heat pumps, they would provide 545 GWh of thermal storage capacity [Jensen et al., 2020]. Unlike battery storage, this capacity requires almost no new hardware since many modern heat pumps already include smart controls which enables remote operation.

2.3 Virtual Power Plants and Grid Services

Individual households cannot participate directly in electricity markets to provide grid flexibility alone, as these markets typically require minimum bid sizes between 0.1 MW and 1 MW [Energinet, 2025]. Aggregating many households into virtual power plants (VPPs) addresses this limitation by combining their flexibility into a much larger resource. A VPP coordinates these distributed resources to operate as one entity to provide system services that maintains grid stability.

The aggregation of individual households benefit multiple market actors. The transmission service operator (TSO), Energinet, gains access to distributed flexibility without building new generation capacity and households receive economic compensation for maintaining grid flexibility [Johra et al., 2024]. The grid becomes more robust to fluctuations because it can now utilise distributed battery resources.

3 Problem Statement and Research Questions

Denmark's transition toward renewable energy sources (RES) has introduced volatility in electricity prices and grid load patterns. Buildings offer substantial thermal storage potential, but this resource remains largely untapped. The question is whether simple, implementable control strategies can unlock meaningful benefits for households and the grid.

This thesis addresses the following research questions:

1. What cost savings can a dynamic programming control strategy achieve for a heat pump or electric heater in a typical Danish household over an annual heating season?

2. What is the estimated national impact on peak electricity demand if cost-optimised HVAC control were adopted across the Danish building stock?

3. What is the correlation between electricity spot prices and CO_2 emissions intensity in the Danish grid, and does cost optimisation implicitly result in environmental benefits?

4 Background

4.1 The Danish Electricity System

The Danish transmission system is owned and operated by Energinet, an independent state-owned enterprise. The Danish electricity grid is split into two areas consisting of Western Denmark (DK1, covering Jutland and Funen) and Eastern Denmark (DK2, covering Zealand and surrounding islands). Interconnections with Sweden, Norway, Germany, the Netherlands, and Great Britain enable electricity trade and mutual support during supply shortfalls.

System Services

Maintaining grid stability requires balancing supply and demand in real time. Energinet depends on market actors to provide system services to achieve this balance in return for economic compensation. The system services differ in response speed, duration and technical requirements across the Danish DK1 and DK2 regions and a full overview can be seen in Table 0.1.

Table 0.1: System services in the Danish electricity grid [Energinet, 2025].

Service	Region	Direction	Response	Full Act.	Duration	Min. Bid
FCR	DK1	Symmetric	2 s	30 s	30 min	1 MW
FCR-N	DK2	Symmetric	2.5 s	60 s/3 min	1 h	0.1 MW
FCR-D	DK2	Asymmetric	2.5 s	7.5 s	20 min	0.1 MW
FFR	DK2	Up only	–	0.7–1.3 s	10 s	0.3 MW
aFRR	DK1/DK2	Asymmetric	30 s	5 min	2 h	1 MW
mFRR	DK1/DK2	Asymmetric	–	12.5 min	2 h	1 MW

Frequency Containment Reserves (FCR) respond within seconds to stabilise grid frequency. Automatic and Manual Frequency Restoration Reserves (aFRR, mFRR) restore frequency over longer time periods and Fast Frequency Reserve (FFR) addresses small fluctuations in the grid for short periods of time [Energinet, 2025].

Heat Pumps in the Danish Grid

The Danish Energy Agency estimates that heat pumps will supply approximately 16% of heating energy by 2030, with individual households consuming 4 TWh thermal energy annually by 2040 across 0.5 to 0.6 million homes [Zhang et al., 2022].

A 2022 study examined the potential for Danish heat pumps to provide system services [Zhang et al., 2022]. The study found that individual heat pumps, with typical capacities of 3 to 8kW, are too small for direct market participation but could contribute through VPP aggregation. However, the study found that the 2022 business case was weak due to electricity taxes making up approximately 64% of consumer prices, limiting the value of spot price optimisation. This is however subject to change from 2026 as state imposed taxes are subject to change from 72.7 to 0.8 øre/kWh which may benefit the business case [Skatteministeriet, 2025].

4.2 Buildings as Thermal Storage

The thermal mass of a building such as its walls, floors, furniture, and air acts as a thermal mass. When heating is applied, this mass absorbs energy and temperature rises gradually. When heating stops, the building temperature decreases to its surroundings at a rate relative to the temperature difference between indoors and outdoor ambient temperature. This behaviour can be characterised by a time constant τ which is elaborated upon in section 5.3. Large time constants indicate better insulation and greater flexibility for load-shifting, whereas small time constants indicate worse insulated houses.

A 2020 study characterised Danish household archetypes and found time constants ranging from 20 to 60 hours depending on construction period [Jensen et al., 2020] with new households with better insulation having larger time constants than older households. Buildings newer than 2006 are constructed to stricter energy standard and show the longest time constants and therefore offer the greatest flexibility for pre-heating strategies since they do not require constant heating to maintain their temperature.

4.3 Related Works

Optimal control of HVAC systems has been studied extensively. A 2011 study demonstrated that model predictive control (MPC) could reduce electricity costs for Danish supermarket refrigeration by 9 to 32% by optimising for price variations [Hovgaard et al., 2011]. The approach leveraged the thermal mass of refrigerated goods to shift cooling operation away from peak prices.

More recently, reinforcement learning (RL) algorithms have been applied to HVAC control in large buildings. A meta-study found that while RL shows promise, current algorithms struggle with the complexity of large commercial building environments, which include changing occupancy, weather, and prices. Furthermore, RL requires extensive training data [Al Sayed et al., 2024]. The slow training and limited adaptability of RL motivates the use of model-based approaches like dynamic programming or MPC algorithms for household use.

The correlation between electricity prices and emissions has been examined in several markets. A study of the German electricity grid found positive "medium-strong" correlation, concluding that cost optimisation results in implicit emission reductions [Gabrek and Seifermann, 2025]. The study found that while there is a correlation between price and emissions, emissions may be reduced further by not focusing entirely on price reduction. The study used Pearson and Spearman correlation coefficients which is also applied in this thesis.

5 Methodology

This section will describe the methodology used to reach the results in later sections. It covers thermal models, control strategies for HVAC systems, electricity price and emission correlation analysis and scaling results to a national level. Table 0.2 shows a complete list of notation with symbol and units used throughout the thesis.

5.1 Thermal Model

Optimising HVAC systems requires a model that predicts how indoor temperature changes relative to heating and ambient conditions. A model that represents the building as a single thermal zone exchanging heat with its environment is used. While this simplifies the complex thermal dynamics of real buildings, it captures the essential behaviour relevant to control decisions at 15-minute intervals and is computationally efficient enough to run on embedded devices.

The model treats the building as a single thermal mass, where temperature changes according to the balance between heat input and heat loss:

$$\frac{dT_{\text{indoor}}}{dt} = \frac{P - h \cdot (T_{\text{indoor}} - T_{\text{ambient}})}{C} \tag{1}$$

The heater adds thermal energy at rate P (kW), while the building loses heat to its surroundings at a rate relative to the temperature difference $(T_{\text{indoor}} - T_{\text{ambient}})$ (°C) and the heat loss rate h (kW/°C). The thermal capacitance C (kWh/°C) determines how quickly the temperature responds to the heat flow. A larger thermal capacitance indicate greater thermal mass and slower temperature changes.

To use Equation 1 in a dynamic programming (DP) control strategy it must be discretised with time step Δt. For a given temperature state T_s and action $a \in \{\text{ON}, \text{OFF}\}$, the next temperature is:

$$T_s' = T_s + \frac{\Delta t}{C}\left(P_{\max} \cdot \mathbf{1}_{[a=\text{ON}]} - h(T_s - T_{\text{ambient}})\right) \tag{2}$$

where $\mathbf{1}_{[a=\text{ON}]}$ is an indicator function equal to 1 when heating is active and 0 otherwise. The resulting temperature is mapped to the nearest discrete state to maintain the finite state space required by the DP algorithm.

5.2 Control Strategies

Control strategies determine how HVAC systems maintain thermal comfort and indoor air quality. These strategies use sensors, such as thermometers and air quality monitors, to measure indoor conditions and determine control actions, such as switching a heat pump ON or OFF.

Table 0.2: Notation and symbols used throughout this thesis.

Symbol	**Unit**	**Description**
Thermal parameters		
T_{indoor}	°C	Indoor temperature
T_{ambient}	°C	Ambient (outdoor) temperature
C	kWh/°C	Thermal capacitance of building
C_{specific}	Wh/m^2°C	Specific heat capacity per unit floor area
h	kW/°C	Heat loss coefficient
τ	hours	Thermal time constant ($\tau = C/h$)
P_{loss}	kW	Heat loss rate
A	m^2	Floor area
Heating system parameters		
P	kW	Heater power input
P_{max}	kW	Maximum heater capacity
COP	–	Coefficient of performance
a	–	Control action $\in \{\text{ON}, \text{OFF}\}$
$\mathbf{1}_{[a=\text{ON}]}$	–	Indicator function (1 if heater on, 0 otherwise)
Control and optimisation		
Δt	min	Control time step
δ_T	°C	Temperature grid resolution for state space discretisation
T_{min}, T_{max}	°C	Comfort temperature bounds
δ	°C	Padding beyond comfort bounds
κ	–	Penalty scaling factor for soft constraints
T_s, s	–	Temperature state / state index
$g_t(T_s, a)$	EUR	Cost at time t for state T_s and action a
$V_t(T_s)$	EUR	Optimal value function at time t and state T_s
$\pi_t(T_s)$	–	Optimal policy at time t and state s
S	–	Number of discrete states
T	–	Planning horizon (number of time steps)
Economic and market parameters		
p_t	EUR/kWh	Electricity spot price at time t
E	kWh	Energy consumption per time step ($E = P_{\text{max}} \cdot \Delta t$)
Parameter estimation		
α	h/kWh	Regression coefficient ($\alpha = \Delta t/C$)
β	h^{-1}	Regression coefficient ($\beta = h \cdot \Delta t/C$)
Statistical analysis		
r	–	Pearson correlation coefficient
ρ	–	Spearman correlation coefficient
$\bar{x}$, $\bar{y}$	–	Sample means
R^{x_i}, R^{y_i}	–	Ranks of observations

Modern strategies can optimise for multiple objectives such as reducing energy costs while maintaining comfort under varying conditions.

Baseline: Hysteresis Control

The baseline control strategy used in this report is hysteresis control, sometimes referred to as bang-bang control. The hysteresis strategy maintains indoor temperature around a set point of 21 °Cby switching the system ON when the temperature falls below the set point and OFF when it rises above. Control strategies are then evaluated at fixed intervals such as every 5, 10 or 15 minutes.

Optimised: Dynamic Programming

The optimised control strategy is implemented with as a DP algorithm to determine the optimal sequence of ON and OFF switches during a time horizon. The time horizon is limited by data availability such as weather and price forecasts. The following section covers how the DP algorithm is implemented.

State space discretisation The dynamic state space requires discretisation of the continuos temperature state space into a finite grid. First, the comfort temperature range is defined by comfort bounds $[T_{min}, T_{max}]$, extended by padding δ which allows the algorithm to check temperatures slightly outside the comfort bounds if the savings are large.

The discrete temperature space is then:

$$\mathcal{T} = \{T_{min} - \delta, T_{min} - \delta + \delta_T, ..., T_{max} + \delta\} \tag{3}$$

Where δ_T is the grid resolution. A finer grid resolution allows the algorithm to make finer adjustments and be more precise at the cost of computation time.

Cost Function and Soft Constraints The algorithms goal is to minimize electricity cost while maintaining comfortable temperatures defined by the comfort bounds. Rather than enforcing hard constraints (such as forcing the algorithm to heat if the temperature drops below the comfort bounds), a soft penalty approach is used. This allows the algorithm to move slightly outside the comfort bounds if the potential cost savings are large.

The soft penalty function is defined as:

$$P(T) = \kappa(\max(0, T_{min} - T)^3 + (\max(0, T - T_{max})^3) \tag{4}$$

Where κ is the penalty scaling factor. Due to the cubic nature of the penalty function the algorithm will receive larger penalties further away from the comfort bounds.

The cost at time t for action a is defined as:

$$g_t(T_s, a) = p_t * E * 1_{[a=\text{ON}]} + P(T_s^{'}) \tag{5}$$

where p_t is the spot price for electricity (EUR/kWh) and $E = P_{\max} * \Delta t$ is the energy consumption per time step. $P_{\max}$ is always used as the DP strategy is binary and always heats with its maximum power or is turned off.

Bellman Equation and Backward Induction The optimal value function $V_t(T_s)$ represents the minimum cumulative cost from state T_s at time t to the end of the planning horizon. It satisfies the Bellman equation:

$$V_t(T_s) = \min_{a \in \{\text{ON,OFF}\}} \left[g_t(T_s, a) + V_{t+1}(T_s') \right] \tag{6}$$

where T_s' is the next state resulting from action a according to Equation 2. The terminal condition is $V_T(T_s) = 0$ for all states, since there are no costs at the end of the planning horizon.

The algorithm proceeds with backward induction over the planning horizon T (e.g., 24 hours at 15-minute time steps resulting in 96 steps). At each time step, both actions are evaluated for every state and the optimal action is stored in a policy table $\pi_t(T_s)$ for later reference.

Forward Simulation After computing the policy π the optimal action sequence is extracted with forward simulation starting from the current measured temperature T_0.

- Find the grid index s_0 closest to T_0.
- For $t = 0, 1, \ldots, T - 1$ apply action $a_t = \pi_t(s_t)$ and transition to the next state according to the thermal model.

The result is a complete optimal sequence of actions over the time horizon. The optimal sequence is then reevaluated for each time step (e.g., after 15 minutes) as the temperature and electricity price forecasts change.

Computational Efficiency The grid-based approach offers great computational advantages over continuous optimisation methods for a binary action space. With S states and T time steps the algorithmic complexity is $O(S \times T)$, making it a fast enough to run on embedded devices such as a Raspberry Pi.

5.3 Experiment Validation

To validate whether the field experiment was performed successfully and to compare actual performance against theoretical predictions, we must first establish key thermal parameters of the system. These parameters enable us to simulate expected behaviour under various weather conditions and assess how closely the experimental results match theoretical expectations.

Heat Loss Coefficient

The heat loss coefficient, denoted h, characterises the thermal insulation of a space. It determines the rate at which heat escapes from an indoor space per unit temperature difference between inside and outside. A lower value of h indicates better insulation, as less heat is lost for a given temperature difference.

The relationship between heat loss rate and temperature difference is given by:

$$P_{\text{loss}} = h \times (T_{\text{indoor}} - T_{\text{ambient}}) \tag{7}$$

where P_{loss} is the heat loss rate measured in kW.

Thermal Capacitance

The thermal capacitance C represents the amount of energy required to change the indoor temperature by 1 °C measured in kWh. It captures the combined thermal mass of the room, including the air, walls, furniture, and other materials that store heat. A higher C value indicates slow responses with regard to change in temperature. On the other hand, a low C value results in rapid temperature changes in response to heating or cooling.

Time Constant

The time constant τ provides a measurement of the systems thermal response speed and is defined as the ratio of thermal capacitance to heat loss coefficient:

$$\tau = \frac{C}{h} \tag{8}$$

The time constant τ represents the time required for the temperature difference between indoors and ambient outdoors temperature to decay to $1/e \approx 37\%$ of its initial value when no heating is applied. A larger time constant indicates that the room retains heat longer and may be better insulated. It also provides greater flexibility for pre-heating strategies.

5.4 Parameter Estimation

The Monte Carlo simulation in section 7 requires defining building characteristics, which are extracted from the field experiment in the same section. The thermal parameters h and C are estimated from the experiment data. To estimate the thermal parameters h and C the Equation 1 is discretised as with time step Δt:

$$\Delta T = \frac{\Delta t}{C} \cdot P - \frac{h \cdot \Delta t}{C} \cdot (T_{\text{indoor}} - T_{\text{ambient}}) \tag{9}$$

where $\Delta T = T_{\text{indoor}}(t+\Delta t) - T_{\text{indoor}}(t)$ is the observed temperature change over one time step. This equation is linear in the measured quantities P and $(T_{\text{indoor}} - T_{\text{ambient}})$, taking the form:

$$\Delta T = \alpha \cdot P - \beta \cdot (T_{\text{indoor}} - T_{\text{ambient}}) \tag{10}$$

where $\alpha = \Delta t / C$ and $\beta = h \cdot \Delta t / C$. The coefficients α and β are fitted using least squares regression on the experimental measurements. The estimated parameters are then defined as:

$$C = \frac{\Delta t}{\alpha}, \qquad h = \frac{\beta}{\alpha} \tag{11}$$

The time constant τ, representing the characteristic thermal response time of the building, is calculated directly from the estimated parameters using $\tau = C/h$.

5.5 Price-Emission Correlation Analysis

The following section covers the Pearson and Spearman correlation coefficient methodology. The coefficients are used to determine whether variables correlate. The methodology follows [Gabrek and Seifermann, 2025], which used the same approach to analyse the German electricity market.

Pearson and Spearman Coefficients

Table 0.3: Correlation coefficients as defined by [Backhaus et al., 2023].

Value	Interpretation
$\lvert r\rvert = 1$	Perfect correlation
$\lvert r\rvert \geq 0.7$	Strong correlation
$\lvert r\rvert \leq 0.3$	Weak correlation
$\lvert r\rvert = 0$	No correlation

Correlation coefficients measures the relationship of two variables. By performing a statistical test a correlation coefficient is obtained from -1 to 1, where 0 means no correlation and $|1|$ is perfect correlation. Positive values indicate that values increase together, whereas negative values indicate that as one value increase another will decrease. [Backhaus et al., 2023] defines the interpretation of correlation as seen in Table 0.3. The Pearson correlation coefficient is calculated according to Equation 12 by dividing the covariance of the two variables with their standard deviation.

$$r = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2 \sum_{i=1}^{n}(y_i - \bar{y})^2}} \tag{12}$$

While the Pearson correlation works with linear relationships it is not very robust to outliers. The Spearman correlation builds on the Spearman correlation and ranks the variables in order to disregard the rate of change for a variable. Is is defined as:

$$r = \frac{\sum_{i=1}^{n}(R^{x_i} - \bar{R^x})(R^{y_i} - \bar{R^y})}{\sqrt{\sum_{i=1}^{n}(R^{x_i} - \bar{R^x})^2 \sum_{i=1}^{n}(R^{y_i} - \bar{R^y})^2}} \tag{13}$$

Both equations are covered in-depth in [Gabrek and Seifermann, 2025].

Significance Testing

To determine whether data is correlated a null hypothesis test with H_0: "There is no correlation between electricity prices and CO_2 emissions" is tested with the standard significance level of $\alpha = 0.05$. Both Pearson and Spearman coefficients are used for the null hypothesis test.

5.6 National Scaling

To estimate the theoretical potential of the DP control strategy introduced in subsection 5.2 and its impact on a national scale, a simulation is performed across different building typologies with varying levels of insulation.

Building Archetype Selection

A previous study identified building archetypes and characteristics, which this thesis builds upon [Jensen et al., 2020]. Each archetype is defined by a time constant τ (see section 5.3), which captures the buildings ability to maintain temperatures and determines its flexibility for load-shifting. Typical floor areas for each construction period are obtained from [Danmarks Statistik, 2025] by dividing the total area (m^2) by the amount of households ("Parcelhus") for each period heated by heat pumps or electric ovens.

Table 0.4 presents the eight archetypes from different periods used in this study. Three archetypes (1930-1950, 1999-2006, and Post-2006) were not explicitly defined in [Jensen et al., 2020], and their time constants may not reflect reality. The general trend is however that newer buildings have higher time constants, as building regulation requires improved insulation.

Building Period Distribution

The building archetypes defined in Table 0.4 are mapped to the distribution of detached houses ("Parcelhus") from [Danmarks Statistik, 2025], which includes data on heating type and construction period. The raw data from Statistics Denmark is reported in 5-year intervals which is mapped to the building archetype periods.

Table 0.4: Building periods with average floor area and time constants. Time constants are obtained from[Jensen et al., 2020]. Floor areas are obtained from [Danmarks Statistik, 2025]. Entries marked with † are estimates.

Construction Period	Floor Area A (m^2)	Time Constant τ (hours)
Pre-1930	162	44.0
1930–1950†	144	32.4
1951–1960	134	38.0
1961–1972	148	50.0
1973–1978	152	40.0
1979–1998	148	46.0
1999–2006†	172	55.0
Post-2006†	181	70.0

As of 2025, Denmark has 65,100 detached houses heated by electric heaters and 151,691 heated by heat pumps [Danmarks Statistik, 2025]. Table 0.5 presents the distribution of these households across construction periods.

Table 0.5: Distribution of electrically heated Danish households by construction period and heating type.

Construction Period	Electric Heaters	Heat Pumps	Total
Pre-1930	18,330	44,105	62,435
1930–1950	3,789	14,678	18,467
1951–1960	2,440	9,301	11,741
1961–1972	11,917	30,796	42,713
1973–1978	11,127	8,896	20,167
1979–1998	16,055	9,773	25,828
1999–2006	571	3,549	4,120
Post-2006	727	30,593	31,320
Total	65,100	151,691	216,791

Heat Loss of Archetypes

For each archetype the thermal capacitance C and heat loss coefficient h is derived from the time constant τ and floor area A. The thermal capacitance is defined as:

$$C = C_{\text{specific}} \times A \tag{14}$$

where $C_{\text{specific}} = 100$ Wh/m^2 °C is the heat capacity per square meter following [Wittchen and Kragh, 2012]. The heat loss coefficient h is then derived from the time constant relationship in Equation 8 where $h = \frac{C}{\tau}$.

6 Data

This thesis combines data from multiple sources to enable the field experiment validation, price-emission correlation analysis, and national-scale simulations. All data covers the period 2021–2025 and is merged and combined as a best-effort.

6.1 Weather Data

Hourly ambient temperature measurements are obtained from the Danish Meteorological Institute [Danish Meteorological Institute, 2025]. For the national simulation, data from the Odense weather station is used as representative of Danish conditions, given its central location. The field experiment uses indoor temperature measurements from a thermometer described in section 7.

6.2 Electricity Market Data

Hourly day-ahead spot prices for the DK1 and DK2 price areas are obtained from Energinet [Energinet, a]. Prices are reported in EUR/MWh and converted to EUR/kWh for household calculations. The analysis uses DK2 prices.

Notably, the spot price represents only a portion of the total consumer electricity cost. Tariffs, taxes, and grid fees approximately double the consumer price relative to the spot price. The simulations optimise against spot prices only, as these represent the variable component that responds to grid conditions. The taxes are also subject to change from 2026 [Skatteministeriet, 2025].

6.3 Emissions Data

Hourly CO_2 emissions intensity (gCO_2/kWh) is obtained from Energinet's environmental declaration data [Energinet, b]. The dataset reports both preliminary and finalised values where only finalised values are used in the analysis to ensure accuracy.

6.4 Building Stock Data

Data on the Danish buildings are obtained from Statistics Denmark [Danmarks Statistik, 2025]. The dataset provides the number of detached houses ("Parcelhus") by construction year and heating type. As of 2025, Denmark has 65,100 detached houses heated by electric heaters and 151,691 heated by heat pumps. The raw data reports buildings in five-year intervals, which are aggregated to match the archetype periods defined in Table 0.4.

6.5 Building Thermal Characteristics

Thermal parameters for building archetypes are derived from two sources. Time constants τ are obtained from [Jensen et al., 2020], which characterised Danish household archetypes based on measurements from buildings in Middelfart. Typical floor areas for each construction period are obtained from the Danish TABULA building typology study [Wittchen and Kragh, 2012].

For archetypes not explicitly characterised in the source literature time constants are interpolated based on construction standards and insulation requirements of the respective periods (”Guesstimates”).

6.6 System Services Market Data

Historical capacity prices for frequency containment reserves (FCR-D) are obtained from Energi Data Service [Energi Data Service, 2025]. The 2024 data shows average capacity prices of 11.63 EUR/MWh for FCR-D up and 35.84 EUR/MWh for FCR-D down. System service specifications are obtained from Energinet [Energinet, 2025].

Figure 0.1: An annex representative of the electric heater experiment room. The experiment was not performed in this exact annex but in a smaller room connected with a holiday home.

7 Field Experiment

This section covers the field experiment where a baseline hysteresis control strategy is compared to a DP control strategy from subsection 5.2. The thermal parameters are then extracted from the experiment to perform a Monte Carlo simulation to validate the results, and determine the theoretical cost saving potential of the experiment.

7.1 Experiment Setup

The experiment is performed on an electric heater in a vacation home. Simple electric heaters serve as great experiment subjects as they are portable and lack lack built-in thermometers unlike more advanced HVAC systems which may regulate temperatures with internal thermometers. The lack of built-in regulation makes simple electric heaters easy to control with smart plugs.

Hardware

A thermometer measures indoor temperature at each time step $\Delta t = 15$ minutes and saves readings to a Raspberry Pi, which serves as the central controller. The controller then runs the DP algorithm to determine the optimal control sequence over the planning horizon.

The smart plug, which allows the current power consumption to be read and the electric heater to be switched between ON and OFF, receives the first control action in the optimal control sequence via Wi-Fi.

Once a new time step has elapsed the process is repeated. Figure 0.2 presents the flow of the control sequence repeated at every time step. Pictures of the hardware and actual heater can be seen in the appendix, section 3.

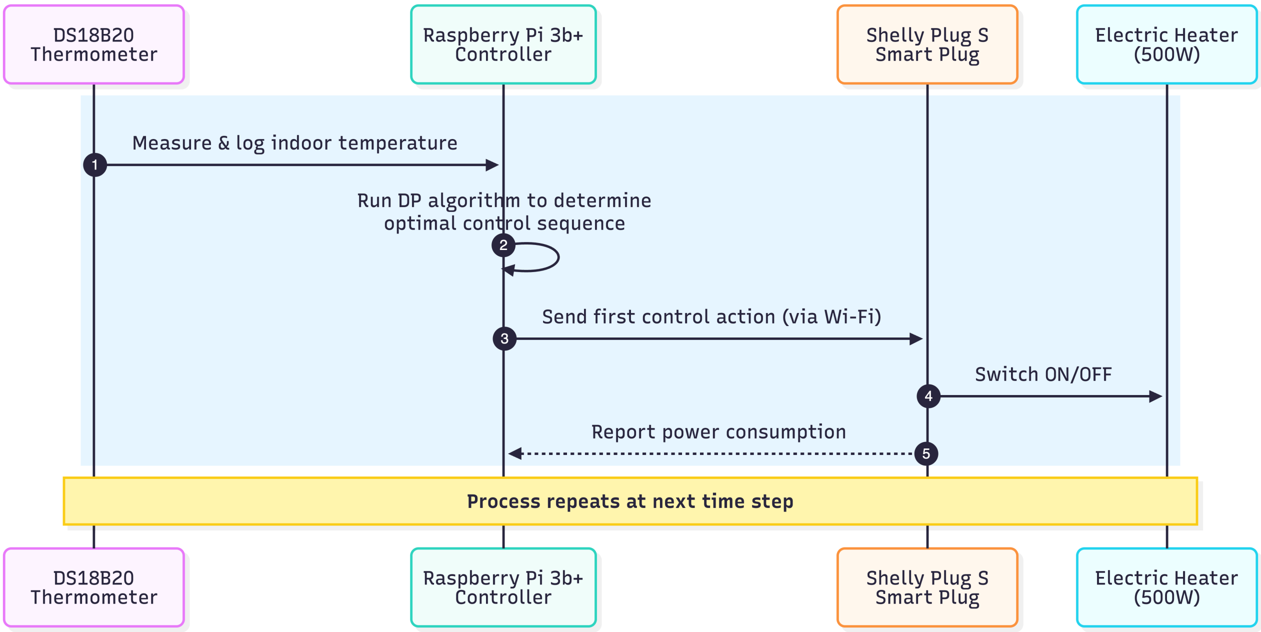

Figure 0.2: Sequence diagram showing the control sequence of the DP control strategy. The hysteresis control strategy uses the same approach with a much simpler algorithm to determine the control action.

7.2 Results

Figure 0.3 shows the performance of the hysteresis control strategy. During the experiment the heater accumulated an electricity cost of 0.339 EUR at an average spot price of 0.1075 EUR.

Figure 0.4 presents the performance of the DP control strategy with the parameters from Table 0.8. Note how the DP algorithm preheats from 06:00 to 10:00 to mitigate the morning electricity price peak.

Figure 0.5 compares the accumulated costs of both strategies. The DP controller achieved an accumulated cost of 0.309 EUR, representing a 8.71% reduction compared to the hysteresis controller. During the DP experiment over-all electricity prices were higher than the hysteresis control strategy as the experiments were performed on different days. During the DP experiment cumulative spot prices for 1 kWh reached 8.44 EUR compared to 8.03 EUR during the hysteresis experiment. Despite this disadvantage the DP control strategy still achieved a reduction in electricity price cost by pre-heating before price-peaks.

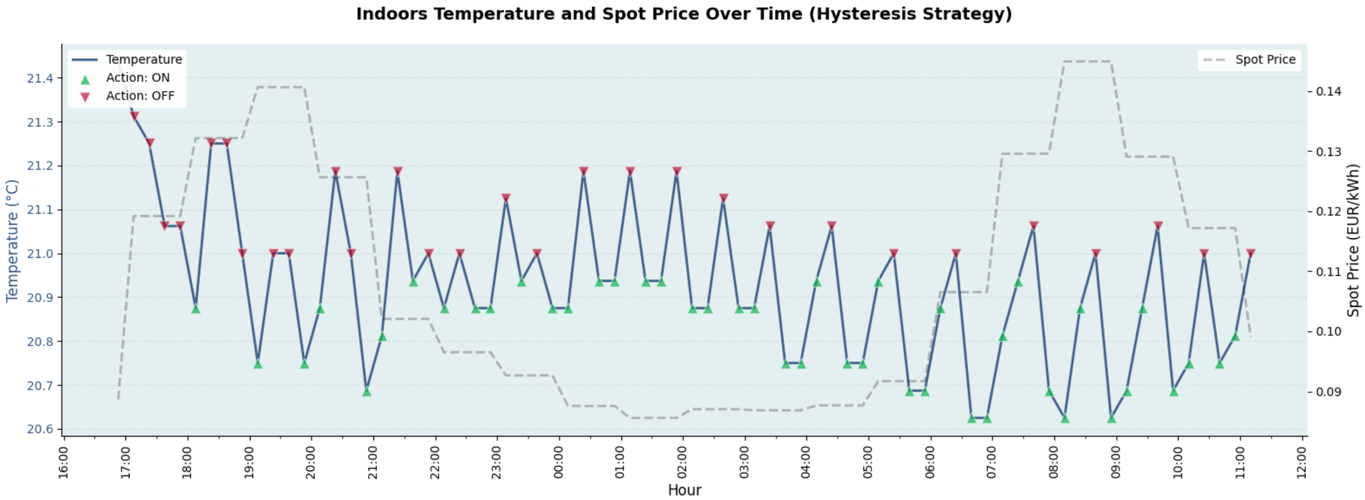

Figure 0.3: The result of the hysteresis control strategy. The control strategy switches to ON when the temperature drops below 21 °C, and switches to OFF during the next time step if the temperature is higher than 21°C.

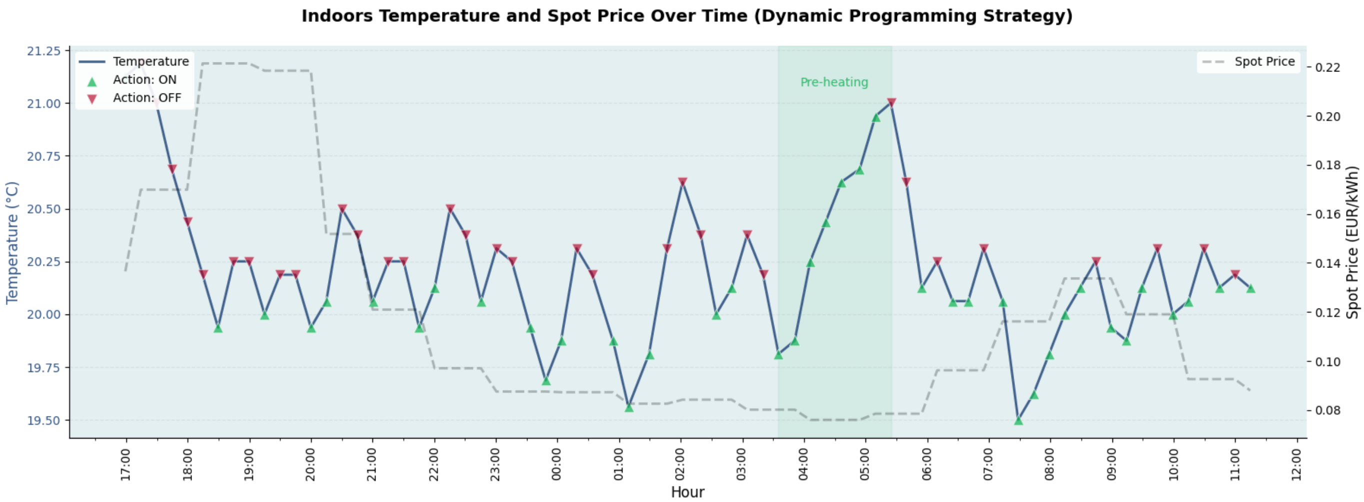

Figure 0.4: The result of the DP control strategy. The electric heater toggles between ON or OFF within the comfort band of 20 to 23 °C to minimise cost. A highlight shows the control strategy performing pre-heating before the electricity price peak in the morning to optimise for cost.

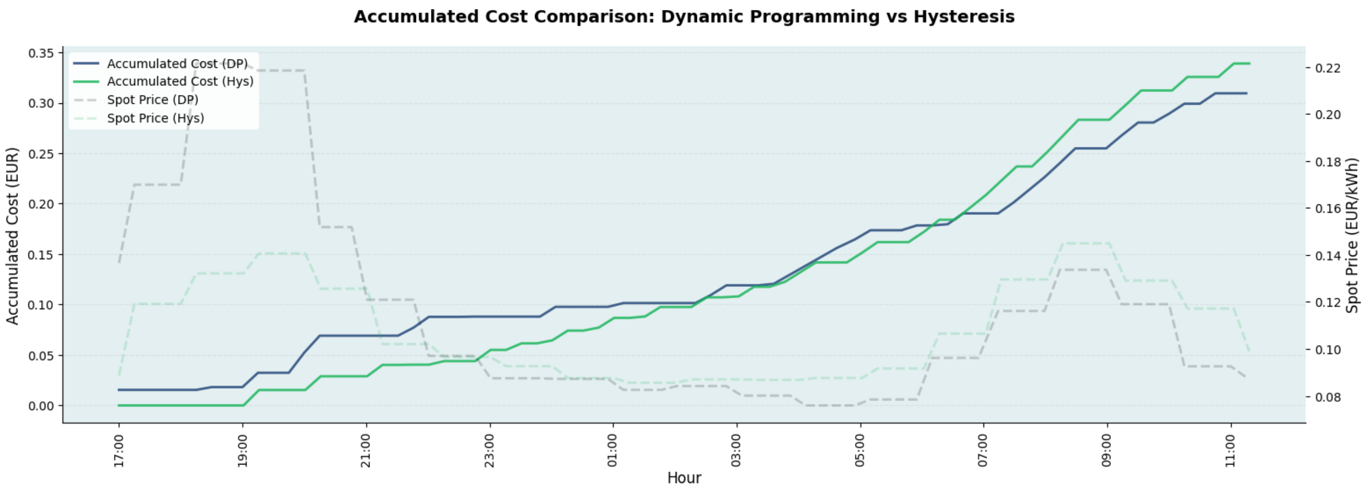

Figure 0.5: A comparison of the accumulated cost of the two tested control strategies. The DP control strategy resulted in a cost reduction of 8.71% compared to the hysteresis strategy.

7.3 Monte Carlo Simulation

A single field experiment, while valuable, captures only one specific set of weather conditions. A Monte Carlo simulation across a wider range of temperature and price conditions with the same characteristics as the field experiment estimates the theoretical savings potential.

Simulation Parameters

Table 0.6: Thermal parameters estimated from the field experiment used to perform a Monte Carlo simulation.

Parameter	Symbol	Value
Heat loss coefficient	h	0.022 kW/°C
Thermal capacitance	C	0.29 kWh/°C
Time constant	τ	13.1 hours

The simulation requires deriving the thermal parameters from the experiment data which can be seen in Table 0.6. The simulated room has identical characteristics to the experimental room shown in Figure 0.1.

Results Across Weather Conditions

Figure 0.6 presents the results of the Monte Carlo simulation, in which 1000 days were randomly sampled from a three-year period. Comparing the DP

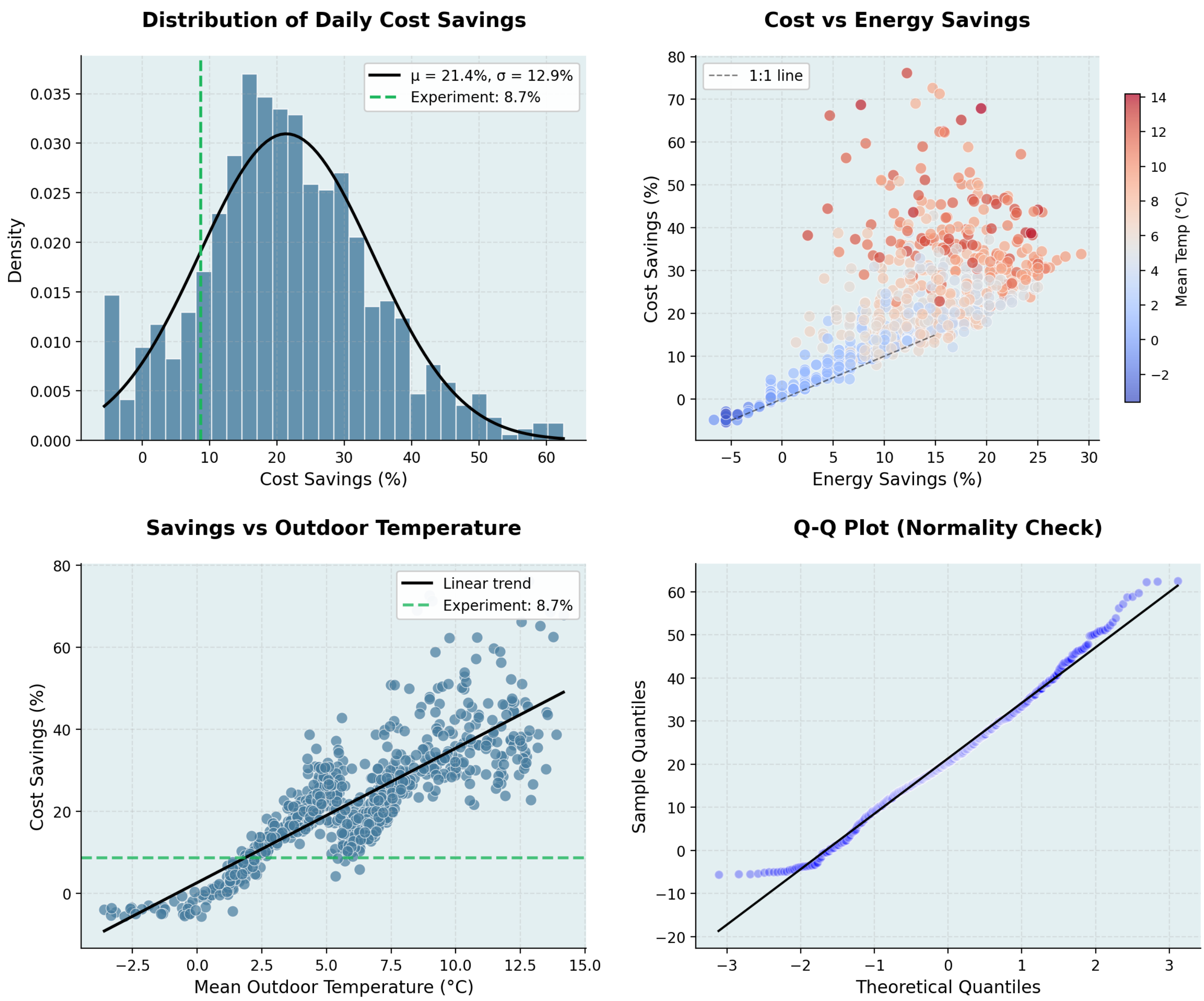

Figure 0.6: The result of running a Monte Carlo simulation showing the theoretical potential cost savings across randomly sampled days. The green line indicates the results of the field experiment and shows that it performed worse than expected.

control strategy against the baseline hysteresis strategy, the simulation results in a mean cost saving of $\mu = 21.37\%$ with a standard deviation of $\sigma = 12.89\%$.

The figure reveals two clear trends. First, cost savings increase with higher mean outdoor temperatures. This is expected, as warmer conditions decrease the temperature difference ΔT and provides greater flexibility for the DP control strategy to pre-heat when electricity prices are cheap. When ΔT is large the electric heater must maintain comfort temperatures by consistently heating, leaving no room for flexibility. Second, there is a strong correlation between cost savings and energy savings, indicating that the DP strategy utilises the lower minimum comfort temperature of 20°C compared to the 21°C set-point of the hysteresis strategy. During a one-year period with 180 heating days this results in cost savings of 28.10 to 35.68 EUR within the 95% confidence interval.

8 Price-Emission Correlation in Denmark

This section examines the relationship between electricity spot prices and CO_2 emissions intensity in the Danish electricity grid. Understanding this correlation is important for evaluating whether cost-optimised HVAC control strategies implicitly result in emission reduction.

8.1 Correlation Analysis

Following the methodology of [Gabrek and Seifermann, 2025], which examined the German electricity market the Danish DK2 price area is analysed with Pearson and Spearman correlation coefficients. The analysis covers hourly observations from 2021 to 2024.

Monthly and Annual Patterns

Figure 0.7 presents the Pearson correlation between electricity prices and CO_2 emissions by month and year. The results reveal a consistent positive correlation across most months and years, with values ranging from -0.126 to 0.605.

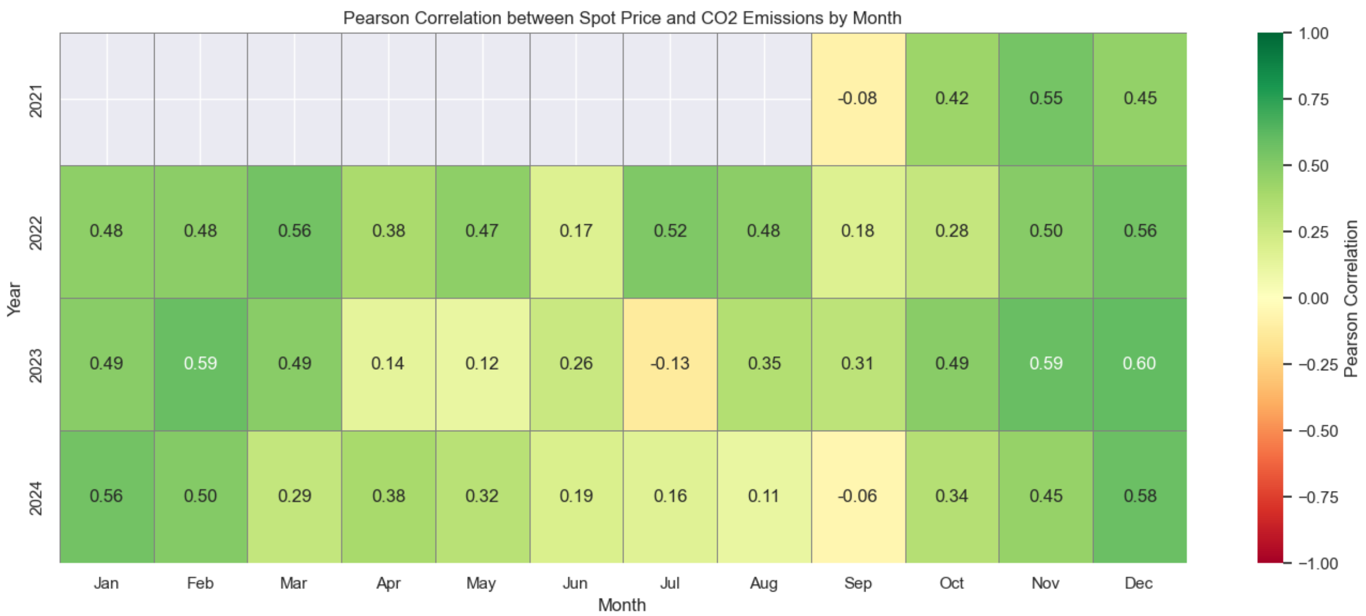

Figure 0.7: Pearson correlation of electricity prices and emissions by month and year. Positive values indicate that higher prices correlate with higher emissions.

The figure presents several patterns. Winter months consistently show stronger correlations, with values typically in the range of 0.45–0.60. This reflects the Danish generation mix during winter, when lower renewable output increases reliance on fossil fuel imports and thermal generation, simultaneously raising both prices and emissions. The strongest monthly correlations occur in December, reaching 0.605 in 2023 and 0.580 in 2024.

Summer months show weaker and sometimes negative correlations. July 2023 shows a negative correlation of -0.126, and September 2021 and 2024 also show weak negative values.

Hourly Patterns

Figure 0.8 presents the correlation by hour of day and month, aggregated across all years in the dataset. This reveals how the price-emission relationship varies throughout the daily load cycle.

The hourly analysis shows that correlations are generally positive across all hours and months, with mean and median values of 0.416 and 0.418 respectively. The strongest correlations occur during evening hours in March, reaching 0.713 at hour 22.

July is the only month with negative correlations, occurring during midday hours (11:00–14:00) with values between -0.081 and -0.037.

8.2 Significance Testing

To formally test whether the observed correlations are statistically significant, a null hypothesis is tested with:

- H_0: There is no correlation between electricity prices and CO_2 emissions ($\rho = 0$)
- H_1: There is a correlation between electricity prices and CO_2 emissions ($\rho \neq 0$)

We test at the standard significance level of $\alpha = 0.05$ using both Pearson and Spearman correlation tests on the complete dataset.

Test Results

Table 0.7 presents the results of the hypothesis tests.

Table 0.7: Hypothesis test results for the price-emission correlation.

Test	Correlation	p-value	Conclusion
Pearson	0.379	< 0.001	Reject H_0
Spearman	0.464	< 0.001	Reject H_0

Both tests show p-values far below the significance threshold ($p < 0.001$), providing strong statistical evidence to reject the null hypothesis. According to the correlation guidelines in Table 0.3 from [Gabrek and Seifermann, 2025], both coefficients fall within the moderate correlation range ($0.3 < |r| < 0.7$).

Figure 0.8: Pearson correlation of electricity prices and emissions by hour of day and month, aggregated across 2021–2024.

The higher Spearman coefficient ($\rho = 0.464$) compared to the Pearson coefficient ($r = 0.379$) suggests a stronger ranked relationship than linear relationship. This indicates that while prices and emissions consistently move in the same direction, the relationship is not strictly linear. Outliers and non-linear effects, such as price spikes during peak demand, may explain this difference. The figures and results for the Spearman correlation analysis is available in subsection 1.1

9 National-Scale Potential

This section presents the results of running a national simulation that compares the optimised DP control strategy to the hysteresis control strategy, and determines the cost and emission savings nationally. Simplifying assumptions and their implications are discussed in section 10.

Simulation Parameters

Table 0.8 summarises the parameters applied to all archetypes in the national simulation.

Table 0.8: Simulation parameters for the national scaling analysis.

Parameter	Description	Value
Heater capacity	Maximum electrical power for all heating systems	5 kW
COP (heat pump)	Coefficient of performance for heat pumps	3
COP (electric heater)	Coefficient of performance for electric heaters	1
Heating season	Months included in simulation	Oct–Apr
Control interval	Time step for control decisions (Δt)	15 min
Specific heat capacity	Specific heat capacity measured per m^2 (C_{specific})	100 Wh/m^2°C
Hysteresis control strategy		
Set-point	Target temperature for ON/OFF switching	21 °C
Dynamic programming control strategy		
Comfort bounds	Allowable temperature range	20 to 23 °C
Padding	Extension beyond comfort bounds (δ)	± 1 °C
Planning horizon	Look-ahead window for optimisation	24 h (96 steps)

Heat pump and electric heater COP values follow [Jensen et al., 2020]. A COP of 3 means that for every 1 kWh of electrical energy consumed, the heat pump delivers 3 kWh of thermal energy, whereas electric heaters convert electricity directly to heat with no amplification.

9.1 Results

The simulation covers 765 complete days from the heating seasons of 2021 to 2024, comparing the DP control strategy to the baseline hysteresis control

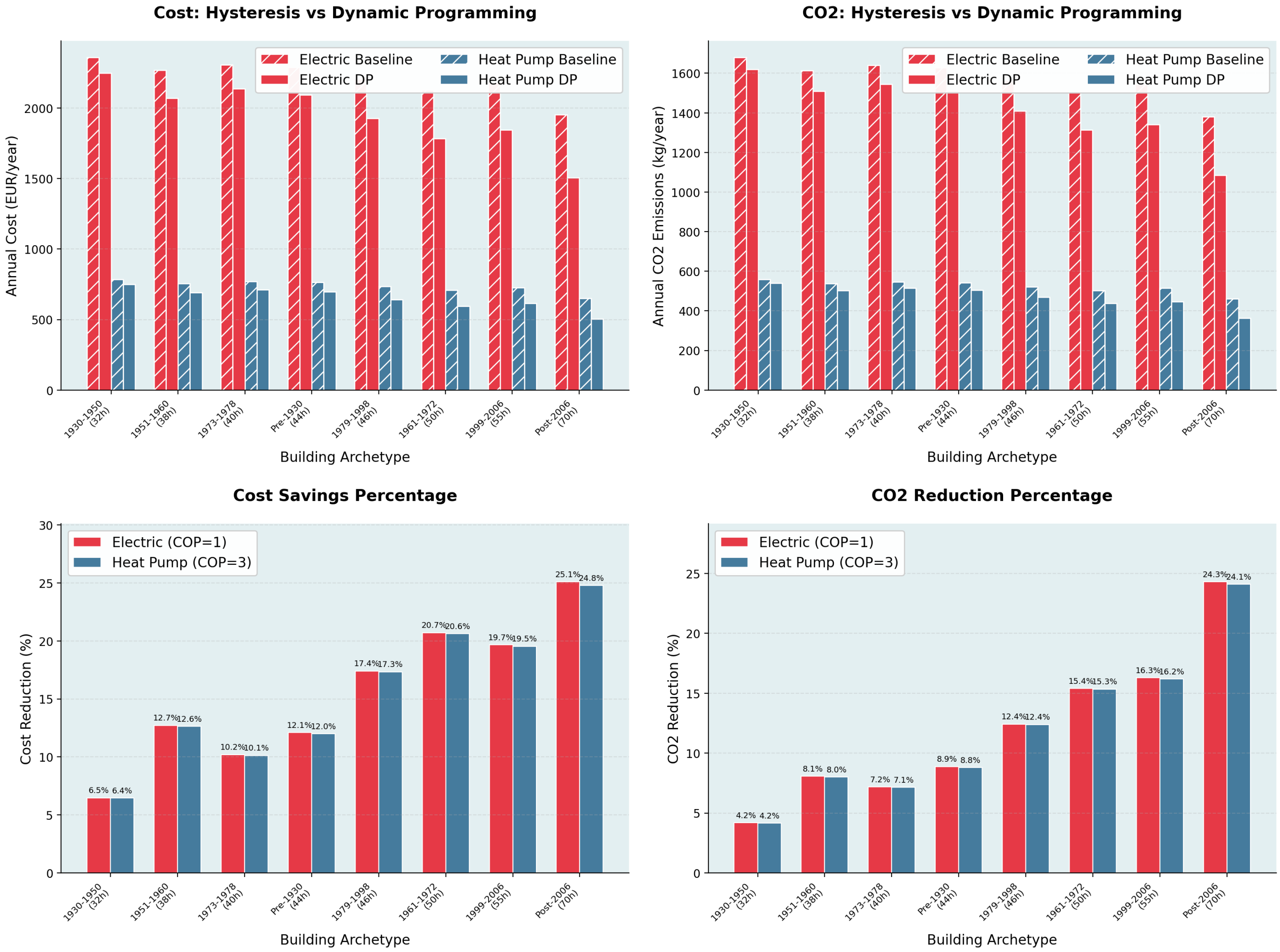

Figure 0.9: Savings in absolute values and percentage based on different building types and the type of heating. Electric heaters use $COP = 1$ while heat pumps use a $COP = 3$.

strategy across 216,791 electrically heated Danish households. The households are heated with electric heaters or heat pumps and kept separate for comparison. Figure 0.9 presents the simulation results across building archetypes from different construction periods. Cost savings vary substantially by construction period, ranging from 6 to 25%. The percentage savings are nearly identical for electric heaters and heat pumps for each archetype, as the COP affects absolute consumption but not the relative benefit of shifting the electricity consumption.

There is a clear trend of archetypes with longer time constants τ also resulting in greater savings, indicating that well-insulated houses achieve higher savings. The buildings from the 1930-1950 period with $\tau = 32$ hours managed to save only 6.8% of costs whereas buildings with longer time constants $\tau = 70$ hours newer than 2006 managed to reduce costs by 25.7%.

Aggregate National Impact

Across the entire building stock, the DP strategy achieves weighted average cost savings of 18.89% and CO_2 emission reductions of 13.99%. This translates to annual savings of 34.5 million EUR and 12,421 tonnes of avoided CO_2 emissions.

9.2 Peak Shifting Potential

Beyond cost and emission savings, the DP strategy shifts electricity consumption away from peak demand periods. Since spot prices indicate a high grid load, cost optimisation naturally incentivise pre-heating during off-peak hours and reducing consumption during morning and evening peaks where prices are highest.

Aggregating the simulation results across all households, the DP strategy reduces peak-hour consumption by an estimated 10 to 15% compared to the baseline hysteresis control. Figure 0.10 presents the shift visually.

On average this translates to a demand reduction during peak periods of approximately 200 MWh across all private consumption, assuming average heater capacities of 5 kW.

9.3 VPP Qualification Analysis

This section evaluates whether aggregated Danish households with optimised HVAC control could qualify to provide system services to the TSO.

Comparison against Service Requirements

Electric heaters and heat pumps can be activated almost instantly by switching a smart plug or utilising heat pumps with pre-installed remote-control

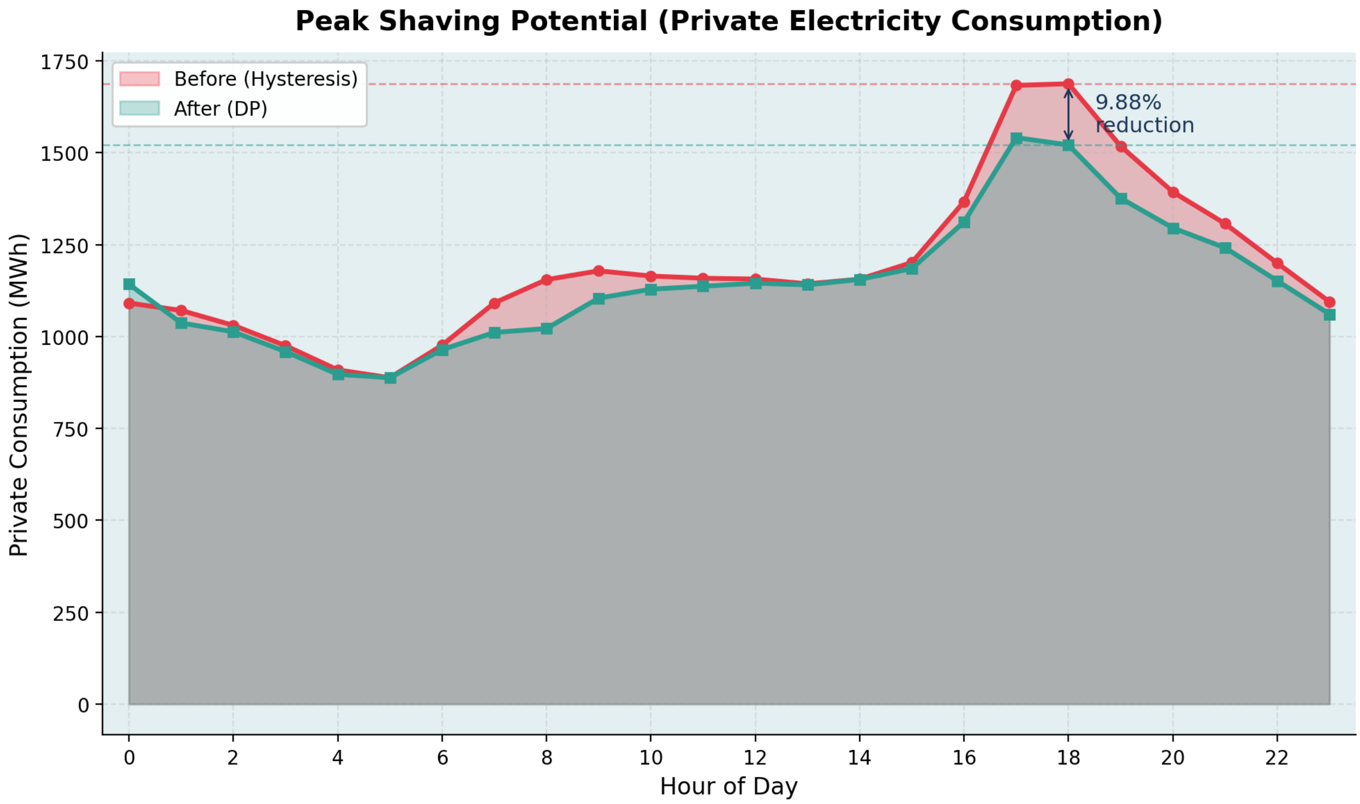

Figure 0.10: The peak shifting potential of the national simulation. The figure presents "the average day" during the simulation period where all days in the simulation data are averaged together.

capabilities, satisfying the response time requirements for all services in Table 0.1. The critical constraint is the duration. Buildings must sustain demand reduction without violating the comfort bounds for residents.

The shorter-duration services (FFR, FCR, FCR-D, FCR-N) requiring 10 seconds to 1 hour are feasible for all building archetypes. The 2-hour services (aFRR, mFRR) require buildings with large time constants τ. Archetypes with $\tau \geq 20$ hours can participate reliably, while older, poorly insulated buildings are limited to mild weather conditions or periods when starting from the upper comfort bound.

Heating systems are well-suited for upward regulation (load reduction by switching OFF) but can also provide downward regulation when temperatures permit additional heating. The symmetric services (FCR, FCR-N) require flexibility in both directions and therefore household must be in the middle of their comfort bounds to participate.

Aggregation Thresholds

Table 0.9 presents the minimum household aggregation required for each service, assuming 5 kW heater capacity.

Table 0.9: Minimum household aggregation required for system service qualification.

Service	Min. Bid	Households Required
FCR-N / FCR-D (DK2)	0.1 MW	20
FFR (DK2)	0.3 MW	60
FCR (DK1) / aFRR / mFRR	1 MW	200

The thresholds would be met by fewer than 0.1% of the 216,791 electrically heated Danish households. However, not all households can participate at any given moment, as some may be close to their comfort bounds with no remaining flexibility or already providing system services. Assuming an availability factor of 50% a VPP targeting a 1 MW bid would need to aggregate 400 households to meet the minimum bid size. Furthermore, households would only be able to bid during the heating season and would offer most of its flexibility during the shoulder months where the flexibility potential is largest, since winter months may require constant heating, as shown in section 7.

Economic Feasibility

Based on correspondence with Energinet (see section 2), FCR-D is the most suitable service for household heating systems due to its minimal energy delivery requirements compared to other reserve products. FCR-D operates as a pay-as-clear capacity market in DK2, where accepted bids receive the marginal clearing price.

Historical market data from 2024 shows average capacity prices of 11.63 EUR/MWh for FCR-D up and 35.84 EUR/MWh for FCR-D down [Energi Data Service, 2025]. Table 0.10 presents the estimated annual revenues for a 1 MW portfolio (200 households with 5 kW heaters), assuming a conservative 50% bid acceptance rate during the heating season (5,112 hours).

Table 0.10: Estimated annual FCR-D revenue potential per household.

Bidding Strategy	Portfolio Revenue	Per Household
FCR-D up only	29,725 EUR	149 EUR
FCR-D down only	91,604 EUR	458 EUR
Symmetric (up + down)	121,330 EUR	607 EUR

Several factors affect actual revenue. Households with heating systems are classified as Limited Energy Reservoir (LER) resources, which must demonstrate sufficient thermal flexibility to sustain service delivery [Energinet, 2024].

The well-insulated post-2006 buildings identified in Table 0.4 satisfy these requirements more easily than older archetypes. Additionally, energy delivered during activation is settled at imbalance prices, which typically rewards providers who help stabilise the grid.

Combining FCR-D revenues with the spot price savings from subsection 9.1 (28 to 160 EUR per household), total annual benefits could reach 180–770 EUR per household. The planned electricity tax reductions from 2026 will further improve the economic case by increasing the share of variable costs in consumer bills. However, since the DP already optimises for cost by shifting the load to cheaper hours that may affect the possibilities of bidding for system services.

10 Discussion

This section combines the results from the field experiment, price-emission correlation analysis, and national-scale simulation. It analyses the results in the context of Denmark's energy transition, compares them with related work, discusses practical implications for stakeholders and acknowledges the limitations of the study.

10.1 Summary of Key Findings

This thesis set out to answer three research questions regarding the potential of Danish households to function as thermal batteries through cost-optimised HVAC control.

First, the field experiment clearly showed that an optimised DP control strategy is able to reduce the costs of heating. The experiment managed to achieve cost savings of 8.71% even though the general price of electricity was higher than on the day of the baseline comparison experiment. The Monte Carlo simulation which was used to validate the results theoretically across 1,000 randomly sampled days showed an average theoretical saving of 21.37%. It also showed that weather conditions had a big role in the savings with high ambient temperatures resulting in more flexibility and larger savings. Furthermore, the national scaling simulation resulted in average savings of 18.89% which translates to annual savings of 28 to 160 EUR per household. However, the savings largely depended on the heating constants τ indicating that more well-insulated households performed better.

Second, the research question on national impact, the simulation across 216,791 electrically heated Danish households demonstrated aggregate annual savings of 34.5 million EUR and peak demand reductions of 10 to 15%. This corresponds to approximately 200 MWh of shifted load during peak periods. These figures suggest that widespread adoption of cost-optimised HVAC control could provide meaningful demand-side flexibility to the Danish grid without requiring significant new hardware or infrastructure investments.

Third, the research question on price-emission correlation was answered with a Pearson and Spearman analysis of four years of Danish electricity market data. Both Pearson ($r = 0.379$) and Spearman ($\rho = 0.464$) correlation coefficients were statistically significant ($p < 0.001$), confirming a "moderate" relationship between spot prices and grid emissions. This correlation shows that cost optimisation also lead to reduced emissions. The national simulation showed CO_2 emission reductions of 13.99%, corresponding to 12,421 tonnes of reduced emissions annually.

10.2 Interpretation of Results

The Role of Insulation

The national simulation showed that houses with large time constants were able to reduce costs and emissions more efficiently than those with short time constants. Cost savings range from 6.8 to 25.7%. This variation is explained by the buildings ability to efficiently pre-heat when electricity prices are cheap. Furthermore, buildings with short time constants will perform worse during cold weather where the difference between indoor and ambient temperature is large, since they must constantly heat to maintain a comfortable temperature.

The field experiment underlined the results. The experimental room, with a time constant of only 13.1 hours, falls below the range of 20 to 60 hours reported for typical Danish households in [Jensen et al., 2020]. This poor insulation limited the room's flexibility and likely contributed to the experimental savings of only 8.71% falling below the Monte Carlo mean of 21.37%.

Understanding the Price-Emission Relationship

The moderate positive correlation between electricity prices and CO_2 emissions shows the underlying mechanics of the Danish electricity market. When RES are abundant, marginal generation costs are low and the grid is relatively clean. When renewable output falls short of demand, fossil fuel generation must be dispatched, raising prices and emissions.

For the DP control strategy this means that reducing cost will also implicitly lead to reduced emissions. However, since the correlation is not perfect with a correlation of $r = 0.379$ this means that the cost-optimal and emission-optimal solution is not always aligned. The 13.99% emission reduction from the national simulation, compared to 18.89% cost reduction, reflects this unaligned coupling.

Furthermore, the reverse correlation during July may be explained by very low consumption resulting in a negative correlation due to renewable energy sources not being activated. See section 2.

Field Experiment Performance

The field experiment achieved 8.71% cost savings compared to the baseline hysteresis strategy, which falls in the lower end of the Monte Carlo simulation where the mean was 21.37%. Several things explain this less-than-ideal performance.

First, the thermal parameters used in the DP algorithm were estimated from previous trial runs. The algorithm was initialised with a heating rate of 0.5 °C per 15-minute step, but the actual rate was closer to 0.25 °C per step. This caused the algorithm to start its pre-heating process too late and never reach its upper comfort bounds and perform worse than theoretically possible.

Second, the room had a time constant τ of 13.1 hours providing limited flexibility compared to typical Danish buildings. This underlines the fact that the experiment was performed in a vacation home and not a typical household.

Third, the spot prices during the DP experiment were higher than during the hysteresis experiment as the experiment was performed on two different days. Despite this disadvantage, the DP strategy still achieved cost savings.

These observations suggest that the experimental result represents a lower bound on achievable savings. With accurate parameter estimation and in a better insulated building, savings approaching the Monte Carlo simulation mean of 21.37% should be possible.

10.3 Comparison with Related Work

The findings of this thesis align well with prior research on thermal load optimisation and price-emission coupling.

The cost savings achieved in the national simulation (6.8 to 25.7% depending on archetype) are consistent with the 9 to 32% savings reported by [Hovgaard et al., 2011] for Danish supermarket refrigeration using MPC. Both studies uses thermal mass to shift electrical consumption away from expensive periods. The similarity in savings ranges, despite the different applications (refrigeration versus space heating), shows that these figures represent realistic expectations for thermal load optimisation in the Danish context. Furthermore, the savings reported in [Hovgaard et al., 2011] also included potential revenue from grid services, which the experiment and simulation in this thesis did not.

The price-emission correlation builds on the work of [Gabrek and Seifermann, 2025] on the German electricity market and puts it in a Danish context. That study found a "medium-strong" positive correlation and concluded that cost optimisation result in implicit emission reductions, while noting that emission-explicit optimisation could achieve greater reductions. The Danish results confirm this. The Spearman correlation of $\rho = 0.464$ falls within the moderate range, and the gap between cost savings (18.89%) and emission savings (13.99%) in the national simulation supports the conclusion that cost-only optimisation does not result in optimal emission reduction.

The building thermal characteristics used in the national simulation come from [Jensen et al., 2020], which characterised Danish building archetypes based on measurements in Middelfart. The time constants of 20 to 60 hours reported in that study provided the base for the archetype definitions in Table 0.4. This thesis extends that characterisation by simulating the economic and environmental of using this thermal storage capacity through optimised control.

10.4 Practical Implications

For Individual Households

The results suggest that Danish households with electric heating systems can achieve meaningful cost savings through optimised control with minimal hardware investment. A smart plug, thermometer, and simple controller (such as a Raspberry Pi) are enough to implement the DP control strategy from the field experiment.

Expected annual savings from spot price optimisation range from 28 to 160 EUR depending on building characteristics, heating system type, and weather conditions. Buildings newer than 2006, or those with large time constants and good insulation, are the ones that can expect the largest savings since they are the most flexible. These savings does not include the potential revenue from VPP participation, which could add 149 to 607 EUR annually per household based on 2024 FCR-D market prices.

The implementation requirements are accessible. Modern heat pumps increasingly include smart controls that enable remote operation, reducing the need for additional hardware. For simpler electric heaters, a smart plug provides sufficient control capability. The DP algorithm itself is computationally lightweight, running comfortably on embedded devices with $O(S \times T)$ complexity.

For Aggregators and Virtual Power Plants

The analysis identifies a clear opportunity for VPP aggregators targeting household heating systems. The minimum aggregation thresholds are achievable: 20 households for FCR-N/FCR-D services in DK2, or 200 households for the 1 MW services. With 216,791 electrically heated households nationally, even a small portion of the market would be able to support multiple VPPs .

The recommended strategy for VPPs is to prioritise buildings newer than 2006, which offer the greatest flexibility. These 31,320 households could support over 150 separate 1 MW bids if all of them were enrolled, though realistic participation rates would be lower.

FCR-D emerges as the most suitable service for household heating systems based on correspondence with Energinet. FCR-D's minimal energy delivery requirements align with the limited thermal mass of buildings. The asymmetric nature of FCR-D also matches the capabilities of heating systems, which can reduce load (FCR-D up) by switching OFF and increase load (FCR-D down) by switching ON when the comfort bounds allow.

However, aggregators must account for the interaction between spot price optimisation and system services. A DP strategy that has already shifted load to off-peak hours may have reduced flexibility available for ancillary services and it must therefore also coordinate potential revenue from system services such as shown in the Economic MPC presented in [Hovgaard et al., 2011].

For Policymakers

Several policy implications emerge from this analysis. The planned reduction in electricity taxes from 2026, which will lower the rate from 72.7 øre/kWh to 0.8 øre/kWh [Skatteministeriet, 2025], will substantially strengthen the business case for optimised consumption. Currently, the relatively high fixed costs of taxes and tariffs comprising approximately 64% of prices, limits the value of spot price optimisation (and as a result, emission reduction). After the tax reduction, a larger share of consumer costs will respond to market conditions, increasing the relative reward for flexible consumption.

The moderate price-emission correlation suggests an opportunity for policy intervention. Carbon pricing mechanisms that more directly link electricity costs to emissions would strengthen the alignment between economic and environmental incentives. If the price and emissions were more strongly coupled, cost-optimised HVAC control would result in greater emission reductions without requiring households to explicitly consider emissions.

Support for aggregator business models, as first shown in Market Model 3.0 [Energistyrelsen, 2021], would accelerate the development of demand-side flexibility. Clear regulatory frameworks for aggregator operations, standardised methods of communication for distributed resources, and streamlined pre-qualification processes for system services would reduce barriers to entry and encourage market development and new VPP developments.

Finally, building energy standard matter. As the previous results have clearly shown, the well-insulated buildings are much more flexible with regards to cost-optimising measures and as Figure 0.9 shows, their overall energy demands are much lower.

10.5 Limitations

Modelling Simplifications

The thermal model used in this thesis represents buildings as single thermal zones with uniform temperature. Real buildings have multiple rooms with different thermal characteristics. The single-zone approximation captures the essential dynamics relevant to 15-minute control decisions but may get all the complex interactions wrong.

The binary control strategy (heater ON or OFF) represents a simplification of actual HVAC capabilities. Modern heat pumps can change their heating output continuously, and even simple electric heaters may have multiple power settings. Continuous control could achieve smoother temperature profiles and potentially greater savings by avoiding the hard changes shown in hysteresis control. The Economic MPC approach demonstrated by [Hovgaard et al., 2011] represents one way of doing so.

The assumption of perfect price forecasting in the DP algorithm is optimistic. While day-ahead prices are published every day, the algorithm optimises over a 24-hour horizon that extends into periods where prices are not yet published. In practice, price forecasts would introduce shorter time horizons which could lead to lower savings.

The constant COP assumption for heat pumps (COP = 3) ignores the temperature dependence of actual heat pump performance. Heat pumps become less efficient as outdoor temperatures decrease, which is when heating demand is highest [Gøtske et al., 2024]. This simplification likely overstates the efficiency advantage of heat pumps in cold weather and may affect the accuracy of national energy consumption estimates.

Field Experiment Constraints

The field experiment compared two control strategies on different days, introducing weather variation and price differences as a factor. A more robust experimental design would test both strategies simultaneously in identical rooms. The single-room experiment setup was chosen for practical reasons but affects the results of the field experiment.

Parameter estimation from limited experimental data introduced uncertainty in the thermal model. The mismatch between assumed and actual heating rates demonstrates the challenge of accurate parameter identification. Online estimation methods that automatically update parameters based on observed behaviour could improve performance in deployed systems.

The experiment was done in a single location (a vacation home) with characteristics that may not be representative of detached households. The time constant of 13.1 hours falls below the typical range for Danish households, suggesting that the experimental room was worse insulated than an average household. Results from better-insulated buildings may show larger savings.

National Scaling Assumptions

The national simulation applied weather data from a single location (Odense) to all households across Denmark. Regional climate variations could affect both heating demand and the timing of optimal control actions. A more detailed analysis would use regional weather data.

Several building archetypes required estimated time constants because they were not explicitly mentioned in the source literature. Especially for buildings newer than 2006 their large time constant of $\tau = 70$ hours increase uncertainty.

The simulation assumes no interaction effects between households. In reality, if many households simultaneously pre-heat before a price peak, this coordinated behaviour could itself affect prices or cause local grid issues. The market impact of widespread adoption would depend on the price elasticity of supply and the capacity of distribution networks. These system-level effects are beyond the scope of the present analysis.

10.6 Future Work

Since the field experiment was performed on two different days, a more robust experimental design would strengthen the confidence of the DP algorithms ability to outperform different baseline algorithms. Furthermore, the experiment could be performed in buildings with larger time constants to confirm the key role of insulation with regards to performance. Longer experiments during an entire heating season would capture the full range of weather conditions and price patterns.

The Economic MPC presented in [Hovgaard et al., 2011] would be a natural next step of the binary DP strategy. The computational requirements are higher but remain possible for modern embedded systems and should be tested with modern heat pumps, as heat pumps are the preferred electrical heating method in modern buildings.

Integration with actual VPP operations would test the practical viability of household heating for system services. A pilot project aggregating several hundred households and participating in FCR-D markets would reveal operational challenges not apparent in simulation, including communication reliability and meeting the service requirements from Energinet.

Since this thesis focused on cost-savings with emissions as benefit, a natural development would be to focus on reducing emissions by performing the save simulations with an emissions-optimal objective. The difference between cost savings (18.89%) and emission savings (13.99%) in the national simulation suggests room for improvement.

Finally, the potential of commercial buildings deserves investigation. Office buildings with regular occupancy schedules offer opportunities for aggressive pre-heating during the night. A building that is empty until 09:00 could preheat to 25°Cduring cheap nighttime hours and disable heating through the morning price peak, achieving larger temperature variations than would be acceptable in residential buildings.

11 Conclusion

This thesis explored whether Danish households can function as thermal batteries through cost-optimised control of heating systems. The field experiment shows that a DP control strategy achieves 8.71% electricity cost reduction compared to baseline hysteresis control, and a Monte Carlo simulation across 1,000 randomly sampled days showed theoretical savings of 21.37%. By scaling up the approach the national simulation confirmed average savings of 18.89%, translating to annual household savings of 28 to 160 EUR depending on the buildings ability to store heat.

For the national impact, the simulation across 216,791 electrically heated Danish households showed an aggregate annual savings of 34.5 million EUR and peak demand reductions of 10 to 15%, corresponding to approximately 200 MWh of peak load reduction in private consumption. These results show that national adoption of cost-optimised HVAC control could provide flexibility without requiring significant infrastructure investment.

The price-emission correlation confirmed a moderate positive relationship between spot prices and grid emissions (Spearman $\rho = 0.464$, $p < 0.001$). This correlation means that cost optimisation implicitly result in emission reduction, and the national simulation showed a potential reduction 13.99% of emitted CO_2 (12,421 tonnes annually) beside the cost savings. However, the difference between cost and emission reductions shows that emission-optimised reduction could result in greater environmental impact.

The thesis shows that even simply binary DP control strategies, which can be implemented and controlled with minimal hardware and infrastructure investments, can result in great savings. The fast run-time of the algorithm ($O(S \times T)$ makes it suitable for embedded devices such as heat pumps or directly in smart plugs. Furthermore, aggregating households into VPPs could provide system services such as FCR-D, with the minimum bid thresholds requiring only 20 to 200 households to participate.

In summary, this thesis demonstrates that existing buildings can serve as distributed thermal battery storage, supporting grid flexibility while reducing both costs and emissions for Danish households. The planned electricity tax reductions from 2026 will further strengthen this business case, making demand-side flexibility an increasingly attractive component of Denmark's energy transition.

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Appendix A

Appendix

1 Price-Emission Correlation

1.1 Spearman Correlation

Figure A.1 presents the Spearman correlation of electricity prices and emissions by years and months. It is observed that all months expect one shows positive values indicating a correlation between electricity prices and emissions while Figure A.2 shows the correlation based on month and hour of day. This is even more true during the winter where the range is 0.42 to 0.70 for

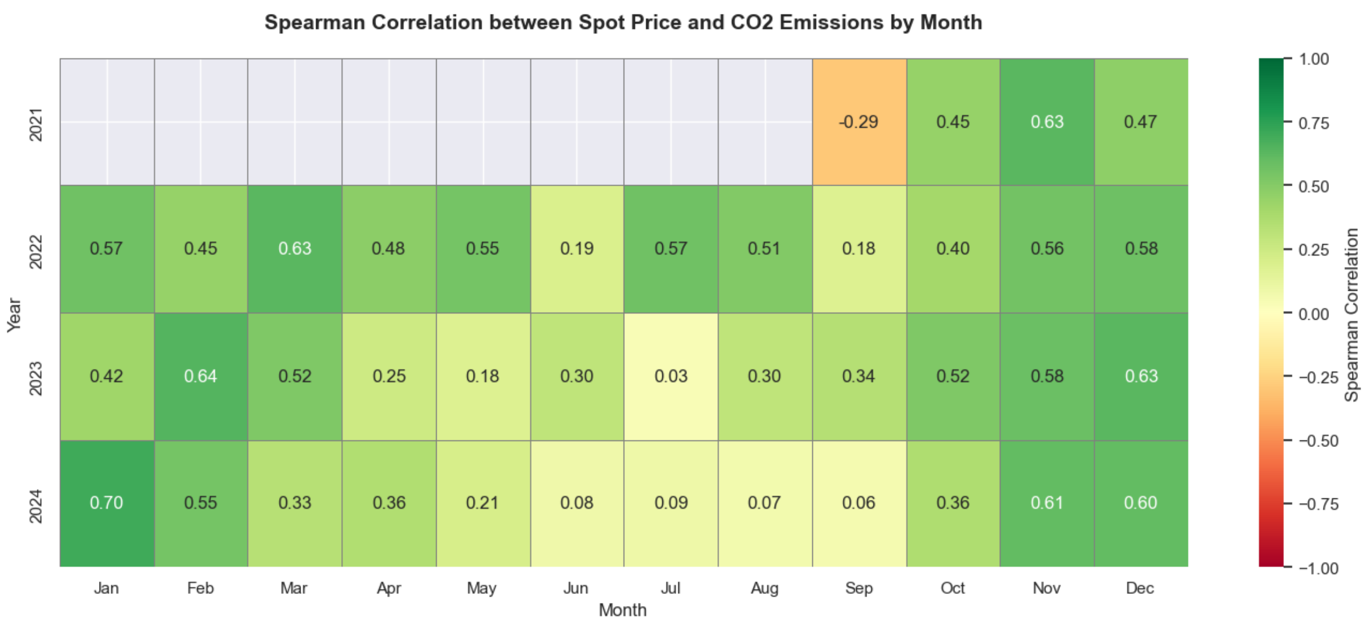

Figure A.1: Spearman correlation of electricity prices and emissions by month and year.

Figure A.2: Spearman correlation of electricity prices and emissions by hour of day and month.

Hej Kasper,

Selvtak 😊

Jeg synes, at deltagelse på FCR-D er det rette valg.
De andre reservemarkeder har lidt større krav til din energileverance, mens FCR-D stiller de mindste krav her.
Vær obs på, at der er krav om at rampe til 86% af det leverede bud inden for 7.5s og at buddet er 0.1 effekt, ikke energi (MW vs. MWh).

Det er også vigtigt, at være opmærksom på hvilke krav der er til energileverance for mindre enheder med begrænset energikapacitet.
I tilfældet med kølere eller varmvandstanke handler det om hvor længe man skal kunne slukke for sin enhed, før den SKAL tændes igen, for at dine temperaturer ligger sikkert.
Det hedder LER ressourcer og det kan du få lidt indledende info om her (energilager-og-systemydelser-22-februar-2024-samlet-praesentation.pdf), men det kræver nok lidt nærmere læsning.

Figure A.3: Confirmation that FCR-D is a good first target to provide system services for HVAC systems.

all of the observed years.

2 Energinet Mails

The following section covers correspondence with Energinet. Figure A.3 confirms FCR-D as the starting point for system services for HVAC systems and Figure A.4 contains an explanation of the reverse correlation seen during July in section 8.

3 Experiment Hardware

This section presents images of the hardware used in the experiment. Figure A.5 shows the electric heater controlled with a baseline hysteresis and DP control strategy while Figure A.6 shows the actual hardware used.

Hej Kasper,

Det gik lige lovlig stærkt i tirsdags med at svare 😊
CO2 intensiteten er jo selvfølgelig den du allerede har fat i.

Først vil jeg mene, at grunden til at du ser den korrelation som du gør er det med residualforbruget forårsaget af, at der er mere forbrug end VE i systemet.
Det er selvfølgelig påvirket af størrelsen af forbruget og mængden af VE.
Men mekanismen, der tillader det at ske er "merit order dispatch".
Du mødte den formentlig til intro til el-kursus, men den er koblet til det her fordi alle bud, der indgives på Day ahead sorteres fra billigst til højest ud fra deres marginal omkostninger, altså omkostninger til at producere energi, så ikke en levelized cost, der tager højde for etableringsomkostninger.

Det betyder, at VE som sol og vind stort set altid er dem, der ligger først i køen.
De eneste teknologier, der ligger i nærheden er hydro og atomkraft fra vores naboer.

Derfor, hvis forbruget er mindre end VE er der ikke noget residualforbrug, mens, hvis der er et residualforbrug, så skal det dækkes af fx kraftvarmeværker, der kan brænde biomasse, kul, naturgas osv. og så er det CO2 kommer i spil.

Jeg har lige tilføjet et lidt fortegnet eksempel på Merit Order Dispatch herunder.
Jeg håber det giver et lidt mere komplet billede af hvad der sker.

Figure A.4: Explanation of the reverse correlation seen during summer months in section 8.

Figure A.5: The electric heater used in the field experiment.

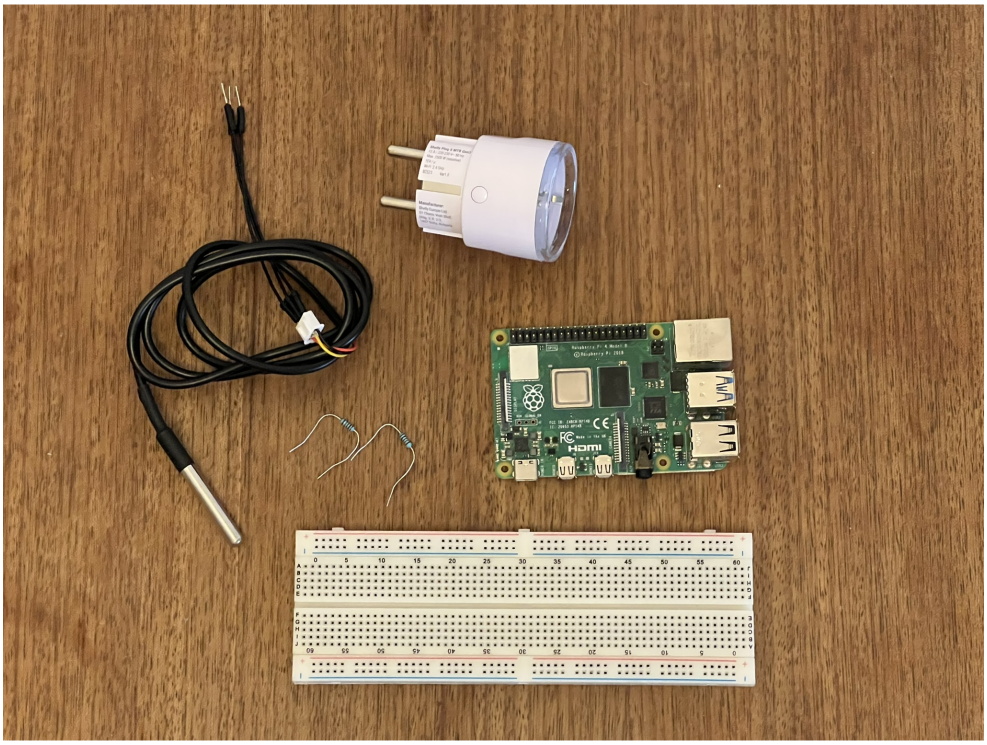

Figure A.6: The hardware used in the field experiment.

UCPH's AI declaration

Declaration of using generative AI tools
✚ **I/we have used generative AI as an aid/tool** *(please tick)* ☐ **I/we have <u>NOT</u> used generative AI as an aid/tool** *(please tick)* *If generative AI is permitted in the exam, but you haven't used it in your exam paper, you just need to tick the box stating that you have not used GAI. You don't have to fill in the rest.*
List which GAI tools you have used and include the link to the platform (if possible): *[Copilot (code suggestions), https://github.com/features/copilot]* *[Anthropic Opus 4.5 (commercial license), https://claude.ai/]* *[Google NotebookLM (commercial license), notebooklm.google.com]*

Describe how generative AI has been used in the exam paper:

1) *Purpose (what did you use the tool for?)*
2) *Work phase (when in the process did you use GAI?)*
3) *What did you do with the output? (including any editing of or continued work on the output)*

GenAI was has been used during the work phase for code generation and editing and ideation on project structure combined with searching for relevant source papers. Furthermore, GenAI has generated the front page illustration and table styling. GenAI has not been used as a source itself.

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