Problem 1.

(a) The set of $\{f^i(x) \mid i \geq 0\}$

(b) The orbit *begins* to repeat itself after n iterations, that is $x_n = x_0$ and $x_n \neq x_i$ for all $0 \leq i \leq n-1$

(c) $|f'(x_0)| < 1$

(d) Every point in B is either dense in A or a limit of A.

Problem 2.

(a)

$$11111\ldots - 001111\ldots = 110000\ldots$$

(b) We took every string of length n, and appended it to our point in Σ.

$$0\ 1\ 00\ 01\ 10\ 11\ 000\ 001\ 010\ 011\ 100\ 101\ 110\ 111\ 0000\ldots$$

Problem 3.

(a) True

$$\begin{aligned} s &= 0.0222\ldots \\ 3s &= 0.2222\ldots \\ 2s &= 0.2 \\ s &= 0.1 \end{aligned}$$

(b) True, for example the rationals in $[0,1]$ have length 0 and are dense in the real interval $[0,1]$.

(c) False, $1/9 \leq 1/6 \leq 2/9$.

(d) False, p_+ and p_- are fixed.

Problem 4.

(a)
- Periodic points are dense in Λ
- All orbits are transitive
- The system is sensitive to initial conditions (implied by the first 2).

(b)
- There is only one periodic point at $x = 0$, which is not dense in λ.
- $x = 0$ is an attracting fixed point, so orbits aren't transitive. For example, the orbit of $x_0 = 1$ will not visit the neighborhood of anything greater than 1.

- Every point goes to 0, so the system is not sensitive under initial conditions.

Problem 5.

$$x = \frac{1}{2}(x^3 + x) \implies 2x = x^3 + x \implies x^3 - x = 0 \implies x(x^2 - 1) = 0$$

So $x = 0, 1, -1$ are our fixed points.

$$f'(x) = \frac{3}{2}x^2 + \frac{1}{2}$$

$|f'(0)| = 1/2$ so $x = 0$ is attracting, $|f'(1)| = 2$ so $x = 1$ is repelling, and $|f'(-1)| = 2$ so $x = -1$ is also repelling.

Problem 6.

$f(0) = 1,\ f(1) = 2,\ \ = f(2) = 0$, so the orbit of 0 is a period 3 orbit.

$$f'(x) = -3x + 5/2$$

$$|f^3(0)| = |f'(0) \cdot f'(1) \cdot f'(2)| = |(5/2)(-1/2)(-7/2)| = |35/2| > 1$$

so the period 3 orbit of $x = 0$ is repelling.

Problem 7.

$$h \circ f = g \circ h$$

Let $h(x) = cx$, then

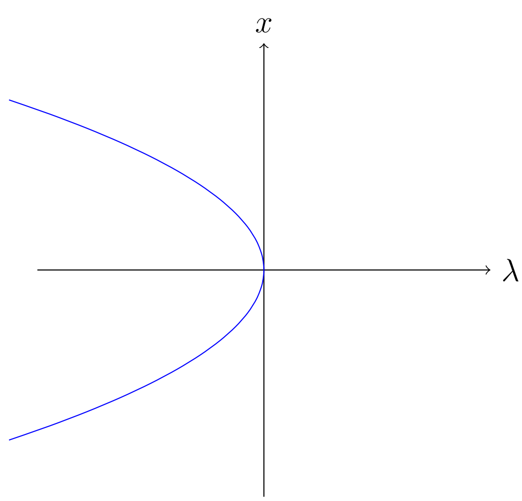
$$h(f(x)) = cx^3 \qquad\qquad g(h(x)) = \frac{c^3}{4}x^3$$

$$4c = c^3 \implies c^3 - 4c = 0 \implies c(c^2 - 4) = 0$$

so $c = 0, 2, -2$. Let $c = 2$. Then

$$h \circ f = 2x^3 = \frac{2^3}{4}x^3 = g \circ h$$

Problem 8. This is a saddle node bifurcation, since a fixed point pops into existence and then there are 2 fixed points from that point on. A graph of the diagram looks as follows.



Problem 9. There is a fixed point at $x = 0$, and 2 period 2 points at $x = \pm 1$. When $x_0 < -1$, the orbit diverges to $-\infty$. When $-1 < x_0 < 1$ and $x_0 \neq 0$, the orbit converges to 0. When $x > 1$, the orbit diverges to ∞.