

Homework Set 1.2

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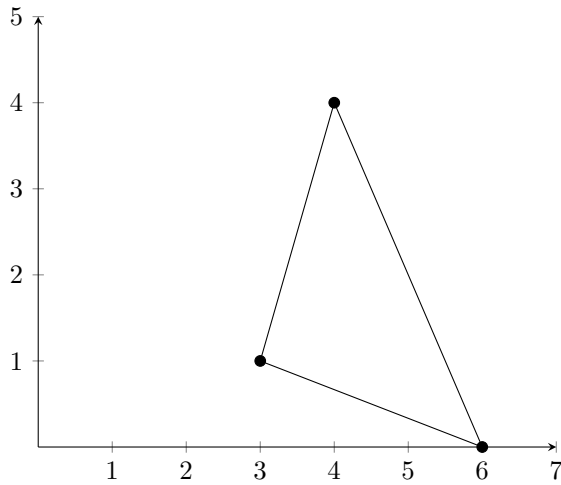
Exercise 6

Show that the points $3 + i$, 6 , and $4 + 4i$ are the vertices of a right triangle.

Proof. If the edges of the triangle formed by these vertices satisfy

$$A^2 + B^2 = C^2$$

Then they form a right a right triangle.



Let edge A be the distance from $3 + i$ to $4 + 4i$, B be the distance from $3 + i$ to 6 , and C be the distance from $4 + 4i$ to 6 . Then

$$A = \sqrt{(3 - 4)^2 + (1 - 4)^2} = \sqrt{10}$$

$$B = \sqrt{(3 - 6)^2 + (1 - 0)^2} = \sqrt{10}$$

$$C = \sqrt{(4 - 6)^2 + (4 - 0)^2} = \sqrt{20}$$

and

$$\sqrt{10}^2 + \sqrt{10}^2 = \sqrt{20}^2$$

□

Exercise 16

Prove that if $|z| = 1$ ($z \neq 1$), then $\operatorname{Re}[1/(1 - z)] = \frac{1}{2}$

Proof. Recall that $\operatorname{Re}[z] = (z + \bar{z})/2$ and $|z| = z\bar{z}$

$$\begin{aligned} \operatorname{Re}[1/(1 - z)] &= \frac{1}{2} \left(\frac{1}{1 - z} + \overline{\left(\frac{1}{1 - z} \right)} \right) = \frac{1}{2} \left(\frac{1}{1 - z} + \frac{1}{1 - \bar{z}} \right) \\ &= \frac{1}{2} \left(\frac{1 - \bar{z} + 1 - z}{1 - \bar{z} - z + z\bar{z}} \right) \\ &= \frac{1}{2} \left(\frac{1 - \bar{z} + 1 - z}{1 - \bar{z} - z + |z|^2} \right) \\ &= \frac{1}{2} \left(\frac{1 - \bar{z} - z + 1}{1 - \bar{z} - z + 1} \right) \\ &= \frac{1}{2} \end{aligned}$$

□

Exercise 17

Let a_1, a_2, \dots, a_n be real constants. Show that if z_0 is a root of the polynomial equation $z^n + a_1 z^{n-1} + a_2 z^{n-2} + \dots + a_n = 0$, then so is $\overline{z_0}$.

Proof. Let $f(z) = a_0 z^n + a_1 z^{n-1} + a_2 z^{n-2} + \dots + a_{n-1} z + a_n$. Since $a_1, a_2, \dots, a_n \in \mathbb{R}$, $a_i = \overline{a_i}$. Then

$$\begin{aligned} f(\overline{z_0}) &= a_0 \overline{z_0}^n + a_1 \overline{z_0}^{n-1} + \dots + a_n \\ &= (\overline{a_0})(\overline{z_0})^n + (\overline{a_1})(\overline{z_0})^{n-1} + \dots + \overline{a_n} \\ &= \overline{a_0 z_0^n + a_1 z_0^{n-1} + \dots + a_n} \\ &= \overline{0} \\ &= 0 \end{aligned}$$

□