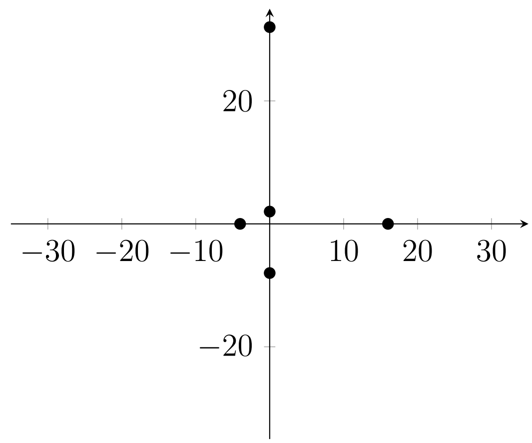
Exercise 2. Sketch the first five terms of the sequence $(2i)^n, n = 1,\ 2,\ 3, \ldots$, and describe the divergence of this sequence



This sequence doubles in magnitude and rotates 90° on each iteration, approaching $\pm\infty$ on both the real and imaginary axis.

Exercise 8. Use the Definition 2 to prove that $\lim_{z\to 1+i}(6z-4) = 2+6i$

Proof. Let $\delta = \varepsilon/6$. We shall show that for all $\varepsilon > 0$

$$|(6z-4)-(2+6i)| < \varepsilon$$

whenever $0 < |z-(1+i)| < \delta$.

$$\begin{aligned} |(6z-4)-(2+6i)| &= |6z-6-6i| \\ &= 6|z-(1+i)| \\ &< 6\delta \\ \implies |z-(1+i)| &< \delta \end{aligned}$$

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Exercise 11c. Find the following limit.

$$\lim_{z\to 3i} \frac{z^2+9}{z-3i}$$

The function $\dfrac{z^2+9}{z-3i}$ is not continuous at $z = 3i$ as it is not defined there, however for $z \neq 3i$ we get

$$\frac{z^2+9}{z-3i} = \frac{(z+3i)(z-3i)}{z-3i} = z+3i,$$

which is continuous, so

$$\lim_{z\to 3i} \frac{z^2+9}{z-3i} = \lim_{z\to 3i} z+3i = 6i$$