Homework Set 1.3

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Exercise 11

Using the complex product $(1+i)(5-i)^4$, derive

$$\pi/4 = 4\tan^{-1}(1/5) - \tan^{-1}(1/239)$$

First we shall solve for the product.

$$\begin{aligned}(1+i)(5-i)^4 &= (1+i)(24-10i)^2 \\ &= (1+i)(476-480i) \\ &= 956-4i\end{aligned}$$

Since both $1+i$, $5-i$, and $956-4i$ are in either quadraints I or IV, we can use $\arg(x+iy) = \tan^{-1}(y/x)$.

$$\begin{aligned}\arg((1+i)(5-i)^4) &= \arg(1+i) + 4\arg(5-i) = \arg(956-4i) \\ &= \tan^{-1}(1) + 4\tan^{1}(-1/5) = \tan^{-1}(-4/956) \\ &= \pi/4 - 4\tan(1/5) = -\tan^{-1}(1/239)\end{aligned}$$

Finally we can re-arange our eqaution to get

$$\pi/4 = 4\tan^{-1}(1/5) - \tan^{-1}(1/239)$$

Exercise 7d

Find the argument of the following complex number and write in polar form.

$$z = -2\sqrt{3} - 2i$$

$$\begin{aligned} r &= |z| = \sqrt{12+4} = 4 \\ \cos(\theta) &= -2\sqrt{3}/4 = -\sqrt{3}/2 \qquad \sin(\theta) = -2/4 = -1/2 \\ \theta &= 7\pi/6 \\ z &= 4(\cos(7\pi/3) + i\sin(7\pi/3)) \end{aligned}$$