Tutorial 8

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1 Problem 1

(a) The sequence $y[n]$ is upsampled from $x[n]$ with upsampling rate L. Use the DFT spectrum of $x[n]$ to express $Y[k]$:

$$\begin{aligned} Y[k] &= \sum_{n=0}^{LN-1} y[n] W_{LN}^{nk} \\ &= \sum_{n=0}^{N-1} x[n] W_{LN}^{nkL} \end{aligned} \tag{1}$$

Use the unitary characteristics of the twiddling factor,

$$(W_N^n)^{QN+R} = W_N^{Rn} \exp(-2\pi j Q n) = W_N^{Rn} \tag{2}$$

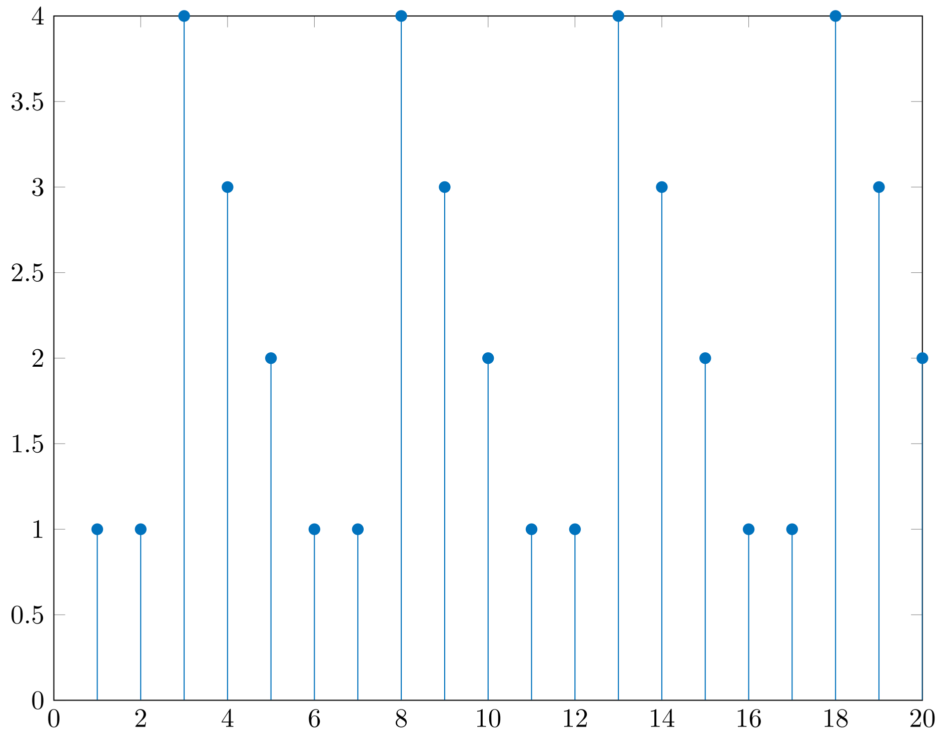
Where

$$Q = k//N \in \mathbb{N}, R = k \mod N \in \mathbb{N} \tag{3}$$

Substitute (3) into (1), thus

$$Y[k] = \mathrm{repmat}(X[k], L) \tag{4}$$

(b) Use MATLAB to plot the result:



2 Problem 2

(a)

```
x=[2,-5,6,-3,4,-4,0,-7,8];
y=circshift(x,-12);
y(-3-(-5)+1)
```

The answer is -4.

(b)

```
z=circshift(x,15);
z(2-(-5)+1)
```

The answer is -5.

3 Problem 3

Use the convolution characteristics of DFT to evaluate the discrete circular convolution.

```
ifft(fft([-3,2,-1,4]).*fft([1,3,2,-2]))
```

Therefore

$$y[n] = [3, 3, -9, 11] \tag{5}$$

4 Problem 4

(a) The circular convolution for two sequences with the same length can be expressed as the convolution between two extended sequence:

$$\begin{aligned} y_C[n] &= (g[n] * (\delta[n] + \delta[n-L]) * h[n])[L, 2L-1] \\ &= (y_L[n] * (\delta[n] + \delta[n-L]))[L, 2L-1] \\ &= (y_L[n])[L, 2L-1] + (y_L[n])[0, L-1] \end{aligned} \tag{6}$$

Where the L is the length of the operand and there should be zero-padding in the lase summation:

```
yl=conv(g,h);
l=length(yl);
yc=[yl(l+1:2*l),0]+yl(1:l+1)
```

(b) Use the formula derived above, the result of the circular equation is:

$$\begin{aligned} y_L &= [-6, 22, -3, -54, 77, 9, -28, 63, -6, 13, 12] \\ \Rightarrow y_C &= [-34, 85, -9, -41, 89, 9] \end{aligned} \tag{7}$$

This result corresponds to that calculated by IFFT.

5 Problem 5

Firstly, use the convolution, TD shifting and decimation property of DFT to simplify the $G[k], H[k]$:

$$\begin{aligned} G[k] &= \frac{1}{2}\left(X[2k] + W_N^k X[2k]\right), k \in [0, \frac{N}{2} - 1] \\ H[k] &= \frac{1}{2}\left(X[2k] - W_N^k X[2k]\right), k \in [0, \frac{N}{2} - 1] \\ \Rightarrow X[k] &= \frac{1}{2}(G[k]^{-1} + H[k]^{-1}) \\ &= (1 + W_N^m)G[m] + (1 - W_N^m)H[m], m = k \mod \frac{N}{2} \end{aligned} \tag{8}$$

6 Problem 6

(a) For arbitrary length N, the asymmetric sequence $x[n]$ can be written as

$$x[n] = x[0 : \left\lfloor \frac{N}{2} \right\rfloor] + x[0 : \left\lfloor \frac{N}{2} \right\rfloor + 1] + x_c \tag{9}$$

where the x_c can be zero for even N and non-zero for odd N.

7 Problem 7

Use DFT to evaluate the linear convolution. Pad the input vectors with zeros that the length of both padded vectors exceed $L_1 + L_2 - 1$:

```
x=[2,1,2];
y=[-4,0,-3,2];
x1=[x,zeros([1,length(y)-1])];
y1=[y,zeros([1,length(x)-1])];
c=ifft(fft(x1).*fft(y1));
```

The result is

$$w[n] = [-8, -4, -14, 1, -4, 4] \tag{10}$$